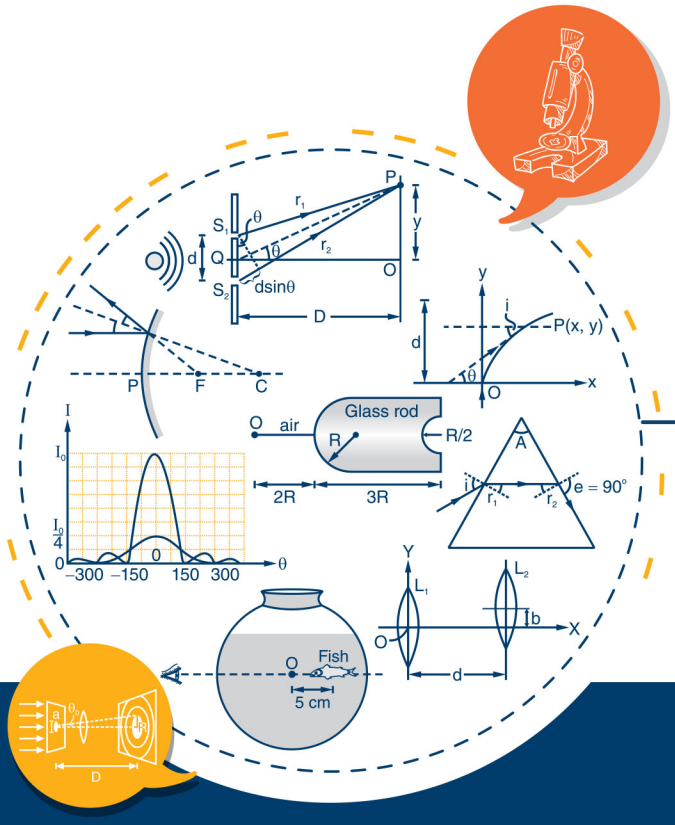
3 THIRD EDITION

Pearson SUCCESS COMPANION



JEE ADVANCED PHYSICS

Optics

Rahul Sardana

Pearson

About Pearson

Pearson is the world's learning company, with presence across 70 countries worldwide. Our unique insights and world-class expertise comes from a long history of working closely with renowned teachers, authors and thought leaders, as a result of which, we have emerged as the preferred choice for millions of teachers and learners across the world.

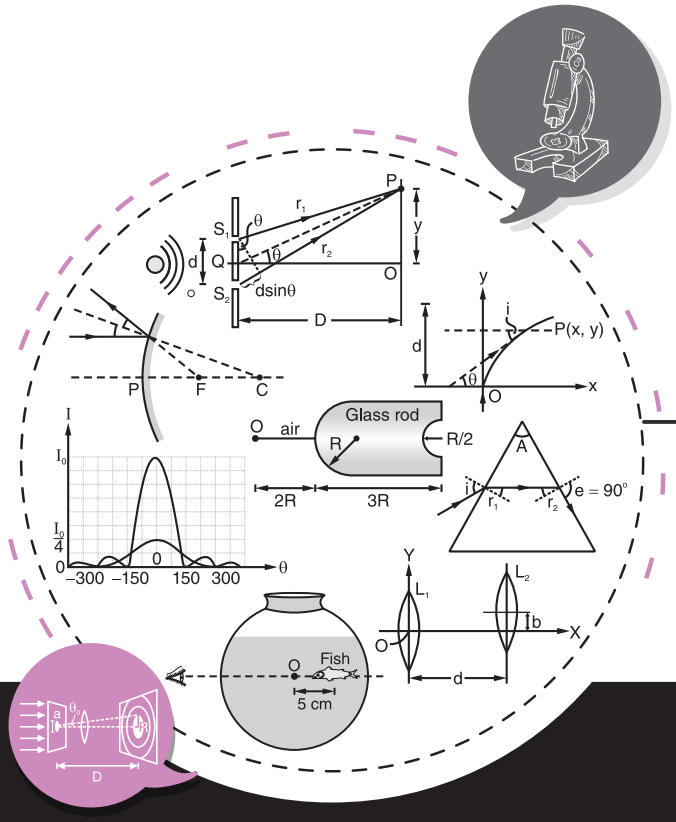
We believe learning opens up opportunities, creates fulfilling careers and hence better lives. We hence collaborate with the best of minds to deliver you class-leading products, spread across the Higher Education and K12 spectrum.

Superior learning experience and improved outcomes are at the heart of everything we do. This product is the result of one such effort.

Your feedback plays a critical role in the evolution of our products and you can contact us - reachus@pearson.com. *We look forward to it.*

This page is intentionally left blank

3 THIRD EDITION

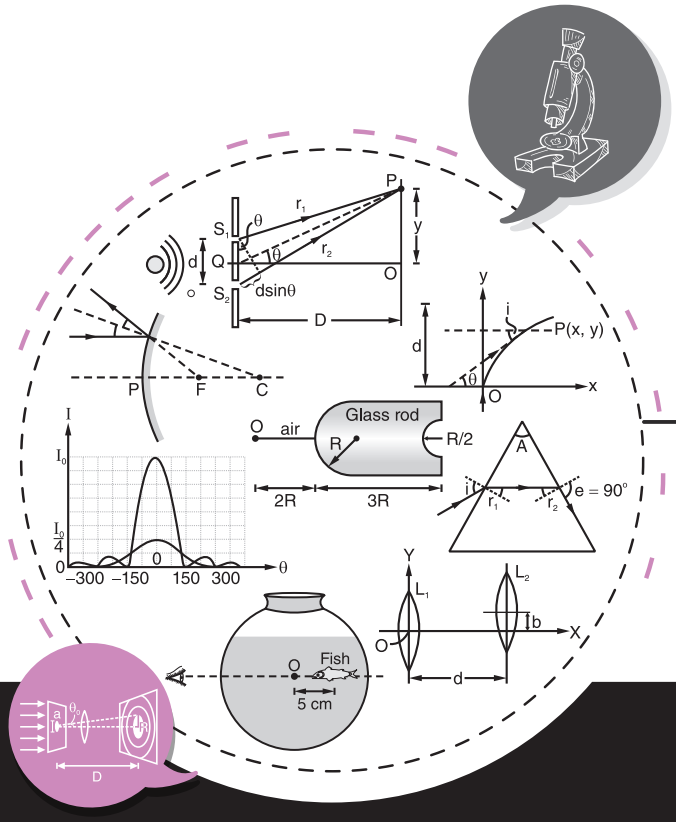


JEE ADVANCED PHYSICS

Optics

This page is intentionally left blank

3 THIRD EDITION



JEE ADVANCED PHYSICS

Optics

Pearson

Rahul Sardana

Copyright © 2020 Pearson India Education Services Pvt. Ltd

Published by Pearson India Education Services Pvt. Ltd, CIN: U72200TN2005PTC057128.

No part of this eBook may be used or reproduced in any manner whatsoever without the publisher's prior written consent.

This eBook may or may not include all assets that were part of the print version.
The publisher reserves the right to remove any material in this eBook at any time.

ISBN 978-93-539-4032-4
eISBN: 978-93-539-4247-2

Head Office: 15th Floor, Tower-B, World Trade Tower, Plot No. 1, Block-C, Sector 16,
Noida 201 301, Uttar Pradesh, India.
Registered Office: The HIVE, 3rd Floor, Pillaiyar Koil Street, Jawaharlal Nehru Road,
Anna Nagar, Chennai 600 040, Tamil Nadu, India.
Phone: 044-66540100
website: in.pearson.com, Email: companysecretary.india@pearson.com

CONTENTS

Chapter Insight xiv

Preface xix

About the Author xx

CHAPTER 1 RAY OPTICS 1.1

Reflection at Plane and Curved Surfaces 1.1

Nature of Light: An Introduction 1.1

Optics: An Introduction 1.2

Domains of Optics 1.2

Fundamental Laws of Geometrical Optics 1.2

Basic Terms and Definitions 1.3

Ray 1.3

Medium 1.3

Beam 1.3

Object(s) 1.4

Image(s) 1.4

Reflection of Light 1.5

Laws of Reflection 1.5

Fermat's Principle of Least Time 1.5

Laws of Reflection Using Fermat's Theorem 1.6

Vector Form of Laws of Reflection 1.6

Angle of Deviation (δ) 1.7

Two Identical Perpendicular Plane Mirrors 1.7

Reflection from a Plane Surface or Plane Mirror 1.8

Lateral Inversion 1.9

Field of View of an Object 1.10

Minimum Size of a Plane Mirror to See a Complete Image 1.11

Required Minimum Width of a Plane Mirror for a Person to See the Complete Width of his Face 1.13

Number of Images in Inclined Mirrors 1.13

Locating all the Images Formed by Two Plane Mirrors 1.14

Images Formed by Two Plane Mirrors 1.16

Rotation of a Plane Mirror 1.16
Velocity of Image in a Plane Mirror 1.18
Reflection from Curved Surfaces 1.22
Paraxial Rays 1.23
Focus, Focal Length and Power of a Mirror 1.23
Sign Conventions for Mirrors 1.23
Rules for Obtaining Image by Ray Tracing 1.24
Image Formation by Concave Mirror 1.25
Image Formation by Convex Mirror. 1.26
Relation Between Focal Length (f) and Radius of Curvature (R) 1.26
Mirror Formula 1.27
Newton's Formula. 1.28
Linear Magnification or Lateral Magnification or Transverse Magnification . . 1.29
Longitudinal Magnification or Axial Magnification 1.31
Superficial or Areal Magnification by a Spherical Mirror 1.32
Relation Between Object and Image Velocity for Curved Mirrors. 1.33
Finding Coordinates of Image of a Point 1.35
Graph of $\frac{1}{v}$ Versus $\frac{1}{u}$ 1.36
Graph of v Versus u 1.37
Effect of Shifting the Principal Axis of a Spherical Mirror 1.40
Splitting of a Mirror 1.41
Velocity of Image in Spherical Mirror 1.43

Refraction at Plane Surfaces 1.46

Refraction of Light at Plane Surfaces 1.46
Laws of Refraction. 1.46
Refractive Index (RI) 1.46
Absolute Refractive Index 1.46
Relative Refractive Index 1.46
Bending of a Light Ray 1.47
Refraction: Important Points 1.47
Light Incident on a Medium Having Variable Refractive Index 1.50
Concept of Optical Path Length (OPL) and Reduced Thickness 1.54
Laws of Refraction Using Fermat's Principle. 1.56
Vector Form of Snell's Law 1.57
Refraction Through a Composite Slab 1.57
Lateral Shift on Passing Through a Glass Slab 1.58
Apparent Depth 1.59

Shift of Point of Convergence or Divergence . . . 1.61
Multislabs . . . 1.62
Total Internal Reflection (TIR) . . . 1.68
Critical Angle . . . 1.69
Examples of Total Internal Reflection . . . 1.69
Optical Fibre . . . 1.70
Angle of Acceptance . . . 1.70
Field of Vision of a Fish . . . 1.71
Prism . . . 1.77
Refraction Through a Prism . . . 1.77
Condition of No Emergence . . . 1.78
Condition for Grazing Emergence . . . 1.79
Maximum Deviation . . . 1.79
Minimum Deviation . . . 1.82
White Light . . . 1.83
Variation of Refractive Index with Colour (Cauchy's Formula) . . . 1.84
Dispersion . . . 1.84
Dispersive Power of a Prism . . . 1.84
Combination of Two Prisms . . . 1.84
Colours of Objects and Colour Triangle . . . 1.87
Colour Triangle . . . 1.88
Rayleigh Law . . . 1.88
Colour of the Sky . . . 1.88
Colour of Clouds . . . 1.88

Refraction at Curved Surfaces and Lens . . . 1.90

Single Refracting Surface . . . 1.90
Sign Conventions . . . 1.90
Refraction of Light at Curved Surfaces . . . 1.90
Refraction at Convex Surface . . . 1.91
Refraction at Concave Surface . . . 1.93
Lateral or Transverse Magnification . . . 1.101
Longitudinal or Axial Magnification of Image . . . 1.101
Effect of Motion of Object or Refracting Surface on Image . . . 1.102
Thin Spherical Lenses . . . 1.105
Naming Convention for Lenses . . . 1.105
Convex or Converging Lenses . . . 1.106
Concave or Diverging Lenses . . . 1.106

Optical Centre of Lens 1.106
Principal Axis of a Lens 1.106
Principal Focus 1.106
Rules for Obtaining Images in Lenses 1.107
Thin Lens Formula for a Convex Lens 1.107
Thin Lens Formula for a Concave lens 1.108
Image Formation by Convex Lens 1.109
Image Formation by a Concave Lens 1.110
Variation Curves of Image Distance vs Object Distance for a Thin Lens . . . 1.110
Newton's Formula. 1.111
Linear or Lateral or Transverse Magnification (m) 1.115
Longitudinal or Axial Magnification by a Thin Lens 1.115
Effect of Motion of Object and Lens on Image 1.116
Lens Maker's Formula for Thin Lens 1.118
Lens Immersed in a Liquid 1.121
Displacement Method 1.122
Power of a Lens. 1.125
Lenses in Contact 1.127
Two Thin Lenses Separated by a Distance. 1.127
Lenses with One Silvered Surface 1.129
Defects of Images: Aberrations 1.135
Monochromatic Aberrations 1.135
Chromatic Aberration 1.137
Achromatism and Achromatic Doublet 1.138
Human Eye 1.139
Defects of Eye 1.140
Optical Instruments 1.141
Visual Angle 1.142
Magnifying Power or Angular Magnification (M) 1.142
Simple Microscope (Magnifying Glass) 1.142
Uses. 1.144
Compound Microscope 1.145
Magnifying Power (M) 1.145
Astronomical Telescope (Refracting Type) 1.148
Terrestrial Telescope 1.150
Galileo's Telescope 1.151
Limit of Resolution and Resolving Power. 1.151
Photometry 1.151

Solved Problems 1.154
Practice Exercises 1.173
Single Correct Choice Type Questions 1.173
Multiple Correct Choice Type Questions 1.206
Reasoning Based Questions 1.211
Linked Comprehension Type Questions 1.213
Matrix Match/Column Match Type Questions 1.222
Integer/Numerical Answer Type Questions 1.225
Archive: JEE Main 1.228
Archive: JEE Advanced 1.236
Answer Keys–Test Your Concepts and Practice Exercises 1.250

CHAPTER 2 **WAVE OPTICS** 2.1

Introduction 2.1
Newton's Corpuscular Theory 2.1
Wave Optics 2.1
Wavefronts and Rays 2.1
Huygen's Principle 2.2
Laws of Reflection on the Basis of Huygen's Theory 2.3
Law of Refraction on the Basis of Huygen's Theory 2.3
Interference 2.3
Sustained Interference 2.4
Coherent Sources 2.4
Methods of Producing Coherent Sources 2.4
Interference: Mathematical Treatment 2.4
Condition for Maxima: Constructive Interference 2.5
Condition for Minima: Destructive Interference 2.5
Phase Difference and Path Difference 2.6
Theory of Interference: Maxima and Minima 2.7
YDSE (Quantitative Treatment): Method 1 . . . . . 2.7
YDSE (Quantitative Treatment): Method 2 . . . . . 2.8
Fringe Width and Angular Fringe Width 2.11
Interference Experiment in Water 2.13
Fringe Visibility (V) 2.14
Intensity Distribution 2.14
Use of White Light in YDSE 2.15
Shape of Interference Fringes due to Different Types of Sources (in YDSE Setup) 2.16
When Two Point Sources in a Line are Placed Parallel to Screen 2.16

Two Point Sources in a Line Placed Normal to the Screen . . . 2.16
Two Rectangular Slit Sources in a Plane Parallel to Screen . . . 2.16
Missing Wavelength (s) in Front of any One Slit in YDSE . . . 2.17
Order of Fringes . . . 2.18
Optical Path . . . 2.20
Path Difference Between Two Parallel Waves Due to a Denser Medium in Path of One Beam . . . 2.20
Displacement or Shifting of Fringe Pattern in YDSE . . . 2.21
YDSE for Source not Placed at the Central Line . . . 2.24
YDSE When Incident Rays are not Parallel to Central Line . . . 2.25
Multiple Slit Interference Pattern . . . 2.30
Resultant Wave Amplitude (Using Phasors) . . . 2.30
Resultant Wave Equation . . . 2.31
Location of Secondary Minima(s) . . . 2.32
Location of Secondary Maxima(s) . . . 2.32
Coherent Sources by Division of Wavefront . . . 2.35
Fresnel's Biprism . . . 2.36
Determination of λ . . . 2.36
Lloyd's Single Mirror. . . . 2.36
Theory of Division of Amplitude . . . 2.38
Diffraction: Introduction and Classification . . . 2.42
Types of Diffraction . . . 2.43
Fraunhofer Diffraction at a Single Slit . . . 2.43
Explanation and Mathematical Treatment . . . 2.44
Diffraction Maxima Due to Single Slit . . . 2.46
Illumination Pattern Due to Diffraction by a Single Slit . . . 2.46
Fraunhofer Diffraction at a Circular Aperture . . . 2.48
Resolving Power and Rayleigh's Criterion . . . 2.50
Rayleigh's Criterion . . . 2.50
Resolving Power of a Microscope . . . 2.51
Resolving Power of a Telescope . . . 2.51
Human Eye . . . 2.52
Validity of Geometrical Optics and Fresnel's Distance (Z_F) . . . 2.52
Fresnel's Zone . . . 2.52
Interference and Diffraction: A Comparison . . . 2.52
Polarization of Light . . . 2.53
Plane of Vibration . . . 2.54
Plane of Polarisation . . . 2.54

Plane Polarised Light. 2.54
Polarization by Reflection 2.54
Linearly, Circularly and Elliptically Polarised Light 2.55
Polarization by Selective Absorption 2.56
Law of Malus 2.57
Explanation of the Law 2.57
Intensity Curve. 2.57
Polarisation by Scattering 2.58
Polarization by Double Refraction 2.59
Quarter Wave Plate 2.60
Half Wave Plate 2.60
Optical Activity and Specific Rotation (α). 2.61
Solved Problems *2.63*
Practice Exercises *2.79*
Single Correct Choice Type Questions *2.79*
Multiple Correct Choice Type Questions. *2.96*
Reasoning Based Questions. *2.98*
Linked Comprehension Type Questions *2.99*
Matrix Match/Column Match Type Questions. *2.106*
Integer/Numerical Answer Type Questions. *2.109*
Archive: JEE Main *2.110*
Archive: JEE Advanced *2.114*
Answer Keys–Test Your Concepts and Practice Exercises *2.120*

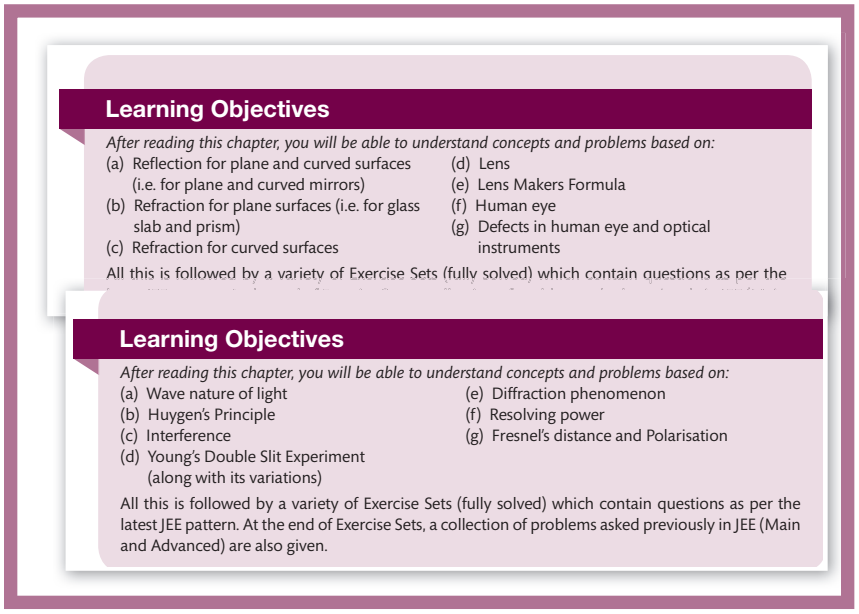
HINTS AND EXPLANATIONS

Chapter 1: Ray Optics. **H.3**
Chapter 2: Wave Optics **H.121**

CHAPTER INSIGHT

Learning Objectives

Help the students set an aim to achieve the major take-aways from a particular chapter.



Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Reflection for plane and curved surfaces (i.e. for plane and curved mirrors)
(b) Refraction for plane surfaces (i.e. for glass slab and prism)
(c) Refraction for curved surfaces
(d) Lens
(e) Lens Makers Formula
(f) Human eye
(g) Defects in human eye and optical instruments

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Wave nature of light
(b) Huygen's Principle
(c) Interference
(d) Young's Double Slit Experiment (along with its variations)
(e) Diffraction phenomenon
(f) Resolving power
(g) Fresnel's distance and Polarisation

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

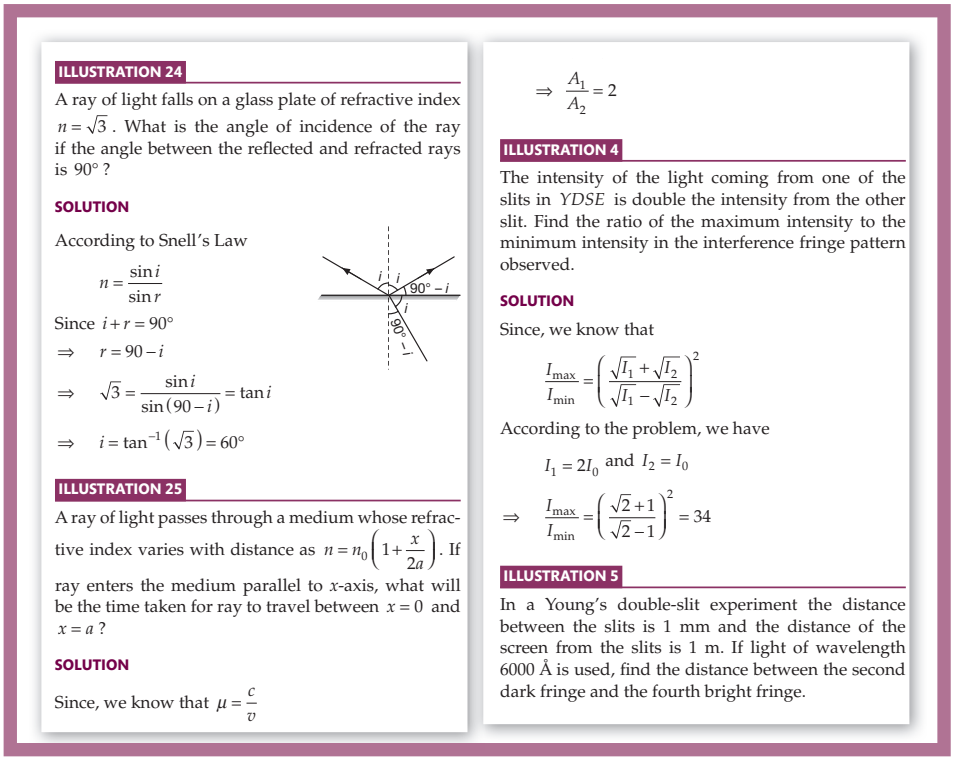


ILLUSTRATION 24

A ray of light falls on a glass plate of refractive index $n=\sqrt{3}$. What is the angle of incidence of the ray if the angle between the reflected and refracted rays is 90° ?

SOLUTION

According to Snell's Law

$$n=\frac{\sin i}{\sin r}$$

Since $i+r=90^\circ$

$\Rightarrow \quad r=90-i$

$\Rightarrow \quad \sqrt{3}=\dfrac{\sin i}{\sin(90-i)}=\tan i$

$\Rightarrow \quad i=\tan^{-1}(\sqrt{3})=60^\circ$

ILLUSTRATION 25

A ray of light passes through a medium whose refractive index varies with distance as $n=n_0\left(1+\dfrac{x}{2a}\right)$. If ray enters the medium parallel to x-axis, what will be the time taken for ray to travel between $x=0$ and $x=a$?

SOLUTION

Since, we know that $\mu=\dfrac{c}{v}$

$\Rightarrow \quad \dfrac{A_1}{A_2}=2$

ILLUSTRATION 4

The intensity of the light coming from one of the slits in $YDSE$ is double the intensity from the other slit. Find the ratio of the maximum intensity to the minimum intensity in the interference fringe pattern observed.

SOLUTION

Since, we know that

$$\frac{I_{max}}{I_{min}}=\left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$$

According to the problem, we have

$$I_1=2I_0 \text{ and } I_2=I_0$$

$\Rightarrow \quad \dfrac{I_{max}}{I_{min}}=\left(\dfrac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2=34$

ILLUSTRATION 5

In a Young's double-slit experiment the distance between the slits is 1 mm and the distance of the screen from the slits is 1 m. If light of wavelength 6000 Å is used, find the distance between the second dark fringe and the fourth bright fringe.

Theory with Illustrations

Elaborative and simple theory helps the students to understand the illustrations supporting the theory. Please note that theory and problem solving techniques are based on simple learning program IF → THEN → ELSE. I would suggest you not to attempt the illustrations without going through the theory of that section.

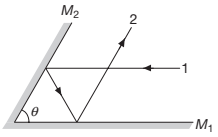
Test Your Concepts-I

Based on Reflection at Plane Surfaces

(Solutions on page H.3)

1. A ray of light travelling in the direction $\frac{1}{2}(\hat{i}+\sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i}-\sqrt{3}\hat{j})$. Find the angle of incidence.

2. A ray of light travels from point *A* to a point *B* after



Test Your Concepts-I

Based c

1. In a Young's Double Slit Experiment carried out a liquid of refractive index $\mu = 1.3$, a thin film of a is formed in front of the lower slit as shown in th figure. If a maxima of third order is formed at th origin *O*, find the

(a) thickness of the air film.

(b) positions of the fourth maxima.

The wavelength of light in air is $\lambda_0 = 0.78\ \mu$m ar $\frac{D}{d} = 1000$.

Test Your Concepts-II

Based c

1. A slit of width 0.025 m is placed in front of a len of focal length 50 cm. The slit is illuminated with light of wavelength 5900 Å. Calculate the distanc between the centre and first dark band of diffrac tion pattern obtained on a screen placed at th focal plane of the lens.

2. Two spectral lines of sodium D_1 and D_2 have wave lengths of approximately 5890 Å and 5896 Å A sodium lamp sends incident plane wave onto slit of width 2 micrometre. A screen is located 2 m from the slit. Find the spacing between the firs

Test Your Concepts

These topic based exercise sets are based on simple, single concept classification technique. These are meant for students practice after they study a particular topic and want to practice more on that topic learnt. Finally, in case of any difficulty they can refer to the hints and solutions to these exercise sets given at the end of the book.

Problem Solving Techniques

These techniques ensure that students become capable enough to solve a variety of problems in an easy and quick manner.

Problem Solving Technique(s)

(a) Basic Problems in Optics: Most of the problems asked in optics expect us to find the position and nature of the final image formed by certain optical systems for a given object. The optical system may be just a mirror, or a lens or a combination of several reflecting and refracting surfaces.

(b) Basic Strategy for Solving the Problems: To handle these kinds of problems, first of all, we identify the sequence in which the reflection and refraction are taking place. The several events of reflection or refraction can be named as Event 1, Event 2 and so on following the sequence in which they occur.

Problem Solving Technique(s)

(a) The deviation is maximum for normal incidence i.e., when $i = 0$ then, $\delta = \delta_{max} = 180°$.

(b) The deviation is minimum for grazing incidence i.e., when $i \to \frac{\pi}{2}$, then $\delta = \delta_{min} = 0°$.

(c) While dealing with the case of multiple reflections suffered by a ray, the net deviation suffered by the incident ray is the algebraic sum of deviation due to each single reflection. So,

$$\delta_{total} = \sum \delta_{\substack{individual \\ reflection}}$$

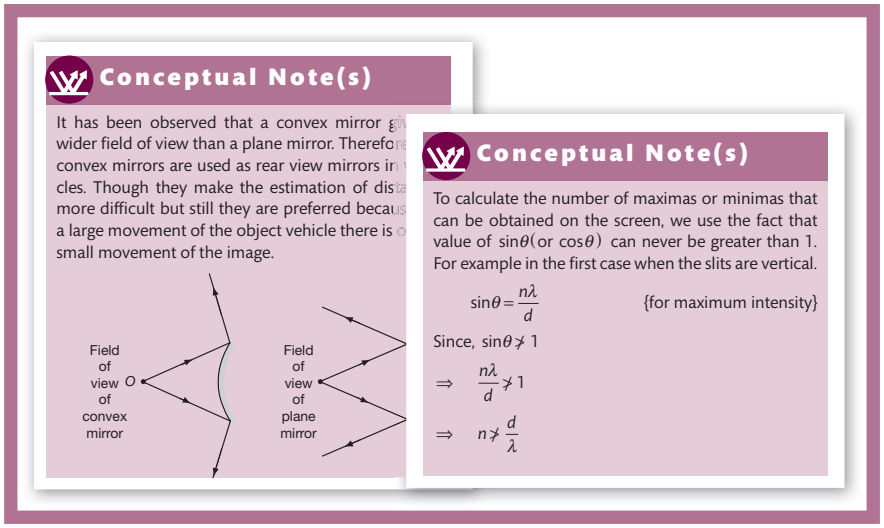
DO NOT FORGET TO TAKE INTO ACCOUNT THE SENSE OF ROTATION WHILE SUMMING UP THE

Problem Solving Technique(s)

(a) Interference occurs due to Law of Conservation of Energy. Actually, redistribution of energy takes place.

(b) If w_1 and w_2 are the widths of the slits and I_1 and I_2 is the intensity of light (with respective amplitudes a_1 and a_2) passing through slits, then

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{w_1}{w_2}$$


Conceptual Note(s)
It has been observed that a convex mirror g
wider field of view than a plane mirror. Therefo
convex mirrors are used as rear view mirrors in
cles. Though they make the estimation of dis
more difficult but still they are preferred becau
a large movement of the object vehicle there is
small movement of the image.
Field of view of convex mirror
O
Field of view of plane mirror
Conceptual Note(s)
To calculate the number of maximas or minimas that can be obtained on the screen, we use the fact that value of sinθ(or cosθ) can never be greater than 1. For example in the first case when the slits are vertical.
sinθ = nλ/d {for maximum intensity}
Since, sinθ ≯ 1
⇒ nλ/d ≯ 1
⇒ n ≯ d/λ

Conceptual Notes

The Conceptual Notes, Remarks, Words of Advice, Misconception Removals provide warnings to the students about common errors and help them avoid falling for conceptual pitfalls.

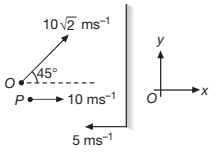
Chapter End Solved Problems

These are based on multiple concept usage in a single problem approach so as to expose a student's brain to the ultimate throttle required to take the JEE examination.

SOLVED PROBLEMS

PROBLEM 1

A plane mirror is moving with a uniform speed of $5\ \text{ms}^{-1}$ along negative x-direction and an observer P is moving with a velocity of $10\ \text{ms}^{-1}$ along $+x$ direction. Calculate the velocity of image of an object O, moving with a velocity of $10\sqrt{2}\ \text{ms}^{-1}$ as shown in the figure, as observed by the observer. Also find its magnitude and direction.



Further, parallel to the mirror, i.e., along y-axis, we have

$$(\vec{v}_I)_y = (\vec{v}_O)_y = 10\hat{j}$$

Since $\vec{v}_I = (\vec{v}_I)_x + (\vec{v}_I)_y$

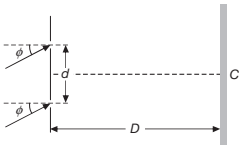
So, absolute velocity of the image is

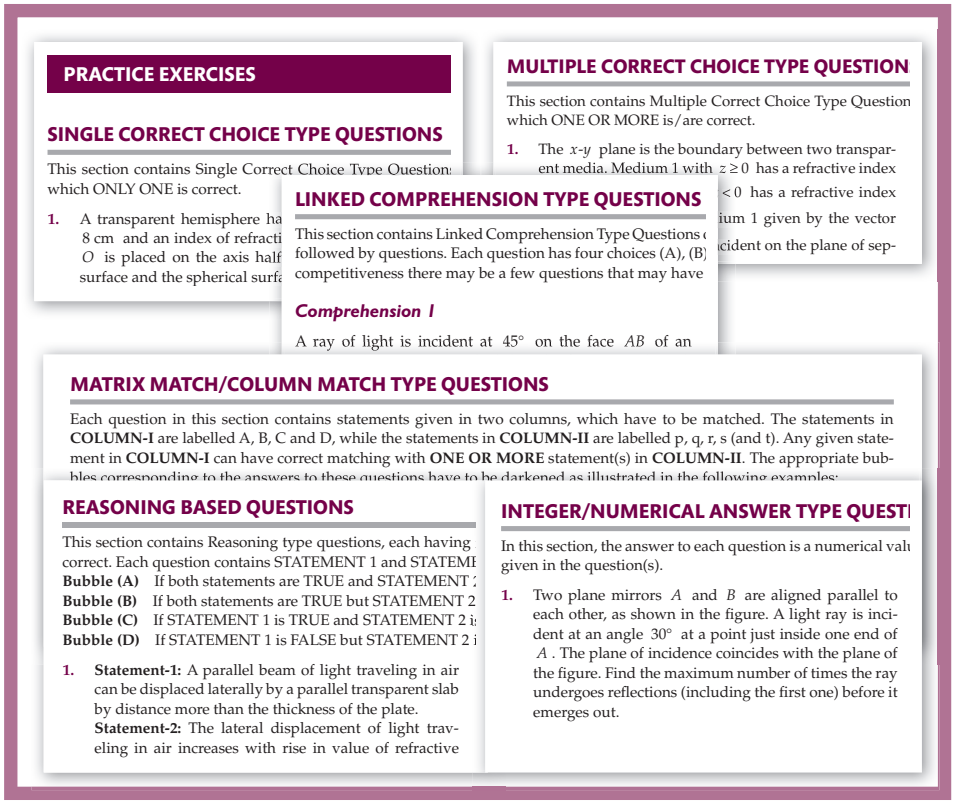
PROBLEM 2

An interference pattern is observed due to two coherent sources S_1 placed at origin and S_2 placed at $(0,\ 3\lambda,\ 0)$, where λ is the wavelength of the sources. A detector D is moved along the positive x-axis. Find the coordinates on the x-axis (excluding

PROBLEM 4

Light of wavelength $\lambda = 500$ nm falls on two narrow slits placed a distance $d = 50\times10^{-4}$ cm apart, at an angle $\phi = 30°$ relative to the slits shown in figure. ON the lower slit a transparent slab of thickness 0.1 nm and refractive index $\frac{3}{2}$ is placed. The interference pattern is observed on a screen at a distance $D = 2$ m from the slits.



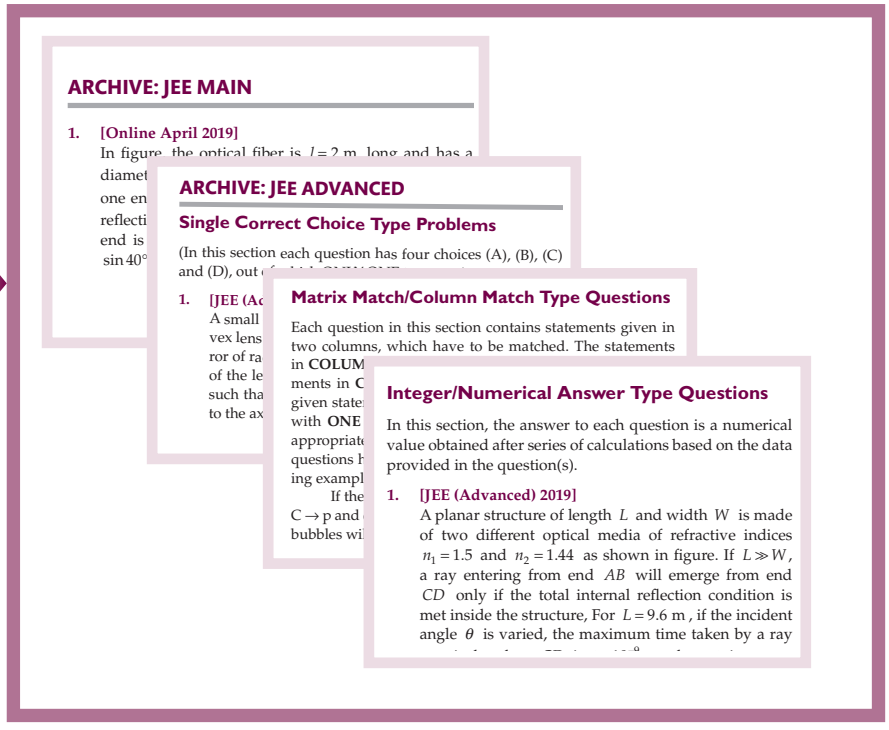

PRACTICE EXERCISES
SINGLE CORRECT CHOICE TYPE QUESTIONS
This section contains Single Correct Choice Type Question
which ONLY ONE is correct.
1. A transparent hemisphere ha
8 cm and an index of refracti
O is placed on the axis half
surface and the spherical surfa
MULTIPLE CORRECT CHOICE TYPE QUESTION
This section contains Multiple Correct Choice Type Question
which ONE OR MORE is/are correct.
1. The x-y plane is the boundary between two transpar-
ent media. Medium 1 with z ≥ 0 has a refractive index
< 0 has a refractive index
ium 1 given by the vector
cident on the plane of sep-
LINKED COMPREHENSION TYPE QUESTIONS
This section contains Linked Comprehension Type Questions
followed by questions. Each question has four choices (A), (B)
competitiveness there may be a few questions that may have
Comprehension I
A ray of light is incident at 45° on the face AB of an
MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS
Each question in this section contains statements given in two columns, which have to be matched. The statements in
COLUMN-I are labelled A, B, C and D, while the statements in COLUMN-II are labelled p, q, r, s (and t). Any given state-
ment in COLUMN-I can have correct matching with ONE OR MORE statement(s) in COLUMN-II. The appropriate bub-
bles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:
REASONING BASED QUESTIONS
This section contains Reasoning type questions, each having
correct. Each question contains STATEMENT 1 and STATEME
Bubble (A) If both statements are TRUE and STATEMENT
Bubble (B) If both statements are TRUE but STATEMENT 2
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 i
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 i
1. Statement-1: A parallel beam of light traveling in air
can be displaced laterally by a parallel transparent slab
by distance more than the thickness of the plate.
Statement-2: The lateral displacement of light trav-
eling in air increases with rise in value of refractive
INTEGER/NUMERICAL ANSWER TYPE QUEST
In this section, the answer to each question is a numerical valu
given in the question(s).
1. Two plane mirrors A and B are aligned parallel to
each other, as shown in the figure. A light ray is inci-
dent at an angle 30° at a point just inside one end of
A . The plane of incidence coincides with the plane of
the figure. Find the maximum number of times the ray
undergoes reflections (including the first one) before it
emerges out.

Practice Exercise

Inclusion of all types of questions asked in JEE Advanced in adequate numbers helps you with enough practice

Archive JEE Main and Advaned

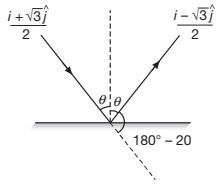
From this fully updated section, students get to know the actual pattern of the problems asked in the past examinations.


ARCHIVE: JEE MAIN
1. [Online April 2019]
In figure the optical fiber is l = 2 m long and has a
diamet
one en
reflecti
end is
sin 40°
ARCHIVE: JEE ADVANCED
Single Correct Choice Type Problems
(In this section each question has four choices (A), (B), (C)
and (D), out
1. [JEE (Ac
A small
vex lens
ror of ra
of the le
such tha
to the ax
Matrix Match/Column Match Type Questions
Each question in this section contains statements given in
two columns, which have to be matched. The statements
in COLUM
ments in C
given state
with ONE
appropriat
questions h
ing exampl
If the
C → p and
bubbles wi
Integer/Numerical Answer Type Questions
In this section, the answer to each question is a numerical
value obtained after series of calculations based on the data
provided in the question(s).
1. [JEE (Advanced) 2019]
A planar structure of length L and width W is made
of two different optical media of refractive indices
n1 = 1.5 and n2 = 1.44 as shown in figure. If L ≫ W,
a ray entering from end AB will emerge from end
CD only if the total internal reflection condition is
met inside the structure, For L = 9.6 m , if the incident
angle θ is varied, the maximum time taken by a ray

CHAPTER 1: RAY OPTICS

Test Your Concepts-I
(Based on Reflection at Plane Surfaces)

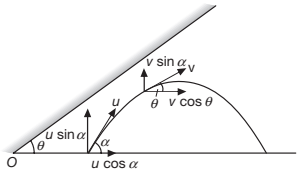
1. The angle between the incident ray and the reflected ray is $180 - 2\theta$, so, we have



$\Rightarrow \quad x = 4$ cm

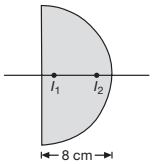
So, point of incidence of light from A should be at 4 c from D on mirror.

3. The image will be momentarily at rest when the pa ticle moves parallel to the mirror. Let at the time t t particle has a velocity v parallel to the mirror.



Single Correct Choice Type Questions

1.



Distance of image from the plane surface is

$$x_1 = \frac{4}{1.6} = 2.5 \text{ cm} \qquad \left\{\because d_{app} = \frac{d_{actual}}{\mu}\right.$$

For the curved surface, we have

$$\frac{1.6}{4} + \frac{1}{x_2} = \frac{1-1.6}{-8}$$

$\Rightarrow \quad x_2 \approx -3$ cm

The minus sign means the image is on the side wher the object lies. So

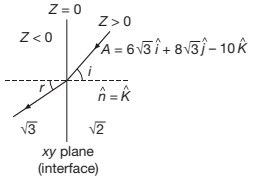
Multiple Correct Choice Type Questions

1. The normal to the interface is along $\hat{k}$.

$$\therefore \quad \cos i = \frac{\vec{A}\cdot(-\hat{k})}{|\vec{A}||\hat{k}|} = \frac{+10}{20} = +\frac{1}{2}$$

$\Rightarrow \quad i = 60°$

{i is the angle which incident ray makes with –Z-axis}



But according to Snell's Law

$$\sqrt{2}\sin 60 = \sqrt{3}\sin r$$

Hints and Explanations

Exhaustive solutions with shortcuts (where ever needed), help students enhance their problem-solving skills.

PREFACE

In the past few years, the IIT-JEE has evolved as an examination designed to check a candidate's true scientific skills. The examination pattern needs one to see those little details which others fail to see. These details tell us how much in-depth we should know to explain a concept in the right direction. Keeping the present-day scenario in mind, this series is written for students, to allow them not only to learn the tools but also to see why they work so nicely in explaining the beauty of ideas behind the subject. The central goal of this series is to help the students develop a thorough understanding of Physics as a subject. This series stresses on building a rock-solid technical knowledge based on firm foundation of the fundamental principles followed by a large collection of formulae. The primary philosophy of this book is to guide the aspirants towards detailed groundwork for strong conceptual understanding and development of problem-solving skills like mature and experienced physicists.

This updated Third Edition of the book will help the aspirants prepare for both Advanced and Main levels of JEE conducted for IITs and other elite engineering institutions in India. This book will also be equally useful for the students preparing for Physics Olympiads.

This book is enriched with detailed exhaustive theory that introduces the concepts of Physics in a clear, concise, thorough and easy-to-understand language. A large collection of relevant problems is provided in eight major categories (including updated archive for JEE Advanced and JEE Main), for which the solutions are demonstrated in a logical and stepwise manner.

We have carefully divided the series into seven parts to make the learning of different topics seamless for the students. These parts are

- Mechanics – I
- Mechanics – II
- Waves and Thermodynamics
- Electrostatics and Current Electricity
- Magnetic Effects of Current and Electromagnetic Induction
- Optics
- Modern Physics

Finally, I would like to thank all my teacher friends who had been a guiding source of light throughout the entire journey of writing this book.

To conclude, I apologise in advance for the errors (if any) that may have inadvertently crept in the text. I would be grateful to the readers who bring errors of any kind to my attention. I truly welcome all comments, critiques and suggestions. I hope this book will nourish you with the concepts involved so that you get a great rank at JEE.

PRAYING TO GOD FOR YOUR SUCCESS AT JEE. GOD BLESS YOU!

Rahul Sardana

ABOUT THE AUTHOR

Rahul Sardana, is a Physics instructor and mentor having rich and vast experience of about 20 years in the field of teaching Physics to JEE Advanced, JEE Main and NEET aspirants. Along with teaching, authoring books for engineering and medical aspirants has been his passion. He authored his first book '*MCQs in Physics*' in 2002 and since then he has authored many books exclusively for JEE Advanced, JEE Main and NEET examinations.

He is also a motivational speaker having skills to motivate students and ignite the spark in them for achieving success in all colours of life. Throughout this journey, by the Grace of God, under his guidance and mentorship, many of his students have become successful engineers and doctors.

CHAPTER

1 Ray Optics

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Reflection for plane and curved surfaces (i.e. for plane and curved mirrors)
(b) Refraction for plane surfaces (i.e. for glass slab and prism)
(c) Refraction for curved surfaces
(d) Lens
(e) Lens Makers Formula
(f) Human eye
(g) Defects in human eye and optical instruments

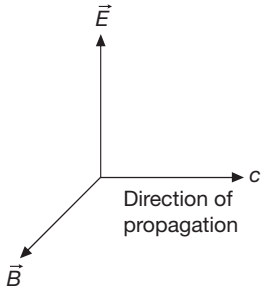
All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

REFLECTION AT PLANE AND CURVED SURFACES

NATURE OF LIGHT: AN INTRODUCTION

Light is a form of energy that makes object visible to our eyes or light is the form of energy that produces in us the sensation of sight. In Seventeenth century **Newton** and **Descartes** believed that light consisted of a stream of particles, called corpuscles. **Huygens** proposed wave theory of light and proposed that light is a disturbance in a medium called **Ether**. This theory could explain the phenomena of interference, diffraction, etc. Thomas Young, through his double slit experiment, measured the wavelength of light.

Maxwell suggested the electromagnetic theory of light. According to this theory, light consists of electric and magnetic fields, in mutually perpendicular directions, and both are perpendicular to the direction of propagation. Heinrich Hertz produced in the laboratory the electromagnetic waves of short wavelengths. He showed that these electromagnetic waves possessed all the properties of light waves.



Light travels in vacuum with a velocity given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ ms}^{-1}$$

where μ_0 and ε_0 are the permeability and permittivity of free space (vacuum).

The magnitudes of electric and magnetic fields are related to the velocity of light by the relation

$$\frac{E}{B} = c$$

In 1905, **Albert Einstein** revived the old corpuscular theory using **Plank's Quantum Hypothesis** and through his photoelectric effect experiment showed that light consists of discrete energy packets, called **photons**. The energy of each photon is

$$E = hf = \frac{hc}{\lambda}$$

So, in view of these developments, light must be regarded to have a dual nature i.e., it exhibits the characteristics of a particle in some situations and that of a wave in other situations. So the question "**Is light a particle or a wave?**" is purely inappropriate to be asked. At present, it is believed that light has **dual nature**, i.e., it has both the characters, wave-like and particle-like.

OPTICS: AN INTRODUCTION

Optics is the study of the properties of light, its propagation through different media and its effects. In most of the situations, the light encounters objects of size much larger than its wavelength. We can assume that light travels in straight lines called **rays**, disregarding its wave nature. This allows us to formulate the rules of optics in the language of **geometry**, as rays of light do not disturb each other on intersection. Such study is called **geometrical** (or **ray**) **optics**. It includes the working of mirrors, lenses, prisms, etc.

When light passes through very narrow slits, or when it passes around very small objects, we have to consider the wave nature of light. This study is called **wave** (or **physical**) **optics**.

DOMAINS OF OPTICS

The study of light can be categorized into three broad domains.

(a) Geometrical Optics (Ray Optics)
(b) Physical Optics (Wave Optics)
(c) Quantum Optics

Please note that these domains are not strictly disjoint as the transitions between them are continuous and not sharp. However for convenience we consider them as distinct. These domains are distinguished as follows.

Geometrical Optics (Ray Optics)

This branch involves the study of propagation of light based on the assumption that light travels in fixed straight line as it passes through a uniform medium and its direction is changed when met by a surface of a different medium or if the optical properties of the medium are non uniform either in time or in space. The ray approximation is valid for the wavelength λ very small compared to the size of the obstacle (d) or the size of the opening through which the ray passes. This approximation $\lambda \ll d$ proves to be very good for the study of mirrors, lenses, prisms and associated optical instruments such as microscope, telescope, cameras etc.

Physical Optics (Wave Optics)

This branch involves the study of propagation of light in the form of a wave and it deals with the phenomenon of interference, diffraction, polarization etc. This nature of light has to be taken when the light passes through very narrow slits or when it goes past very small objects. So this branch works effectively when $\lambda \gg d$.

Quantum Optics

This branch involves the study of propagation of light as a stream of particles called as **Photons**. This concept of light behaving as particles called photons is of utmost importance while studying the origin of spectra, photoelectric effect, concept of radiation pressure, Compton effect etc.

FUNDAMENTAL LAWS OF GEOMETRICAL OPTICS

To a first approximation, we can consider the propagation of light disregarding its wave nature and assuming that light propagates in straight lines called **rays**. This allows us to formulate the laws of optics in the language of **geometry**. Thus, the branch of optics where the wave nature of light is neglected is called **geometrical (or ray) optics**.

Geometrical optics is based on five fundamental laws.

1. **Law of Rectilinear Propagation of Light.** It states that light propagates in straight lines in homogenous media.
2. **Law of Independence of Light Rays.** It states that rays do not disturb each other upon intersection.
3. **The Law of Reversibility of Light.** According to this law, if a ray of light, after suffering a number of reflections and refractions, has its path reversed at any instant, then the ray retraces its path back to the source.
4. **The Laws of Reflection.** The Laws of Reflection govern the bouncing back of the incident ray after striking a surface to the medium from which it was coming.
5. **The Laws of Refraction (discussed later).** The Laws of Refraction govern the bending of light when the light goes from one medium to the other (rarer to denser or denser to rarer) medium.

BASIC TERMS AND DEFINITIONS

Source

A body which emits light is called **source**. The source can be a point one or an extended one. A source is of two types.

(a) **Self luminous:** The source which possess light of its own.
EXAMPLE: sun, electric arc, candle etc.

(b) **Non-luminous:** It is a source of light which does not possess light of its own but acts as source of light by reflecting the light received by it.
EXAMPLE: moon, objects around us, book etc.

Remark(s)

Sources are also classified as isotropic and non-isotropic. Isotropic sources give out light uniformly in all directions whereas non-isotropic sources do not give out light uniformly in all directions.

RAY

The straight line path along which the light travels between two points in a homogeneous medium or in a pair of media is called a Ray. It is represented by an arrow head on a straight line, the arrow head represents the direction of propagation of light. A ray of light will always follow a path along which the time taken is the minimum.

Remark(s)

A single ray cannot be isolated from a source of light.

MEDIUM

Substance through which light propagates or tends to propagate is called a **medium**. It is of following three types.

(a) **Transparent:** It is a medium through which light can be propagated easily.
EXAMPLE: glass, water etc.

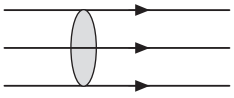
(b) **Translucent:** It is a medium through which light is propagated partially.
EXAMPLE: oil paper, ground glass etc.

(c) **Opaque:** It is a medium through which light cannot be propagated.
EXAMPLE: wood, iron etc.

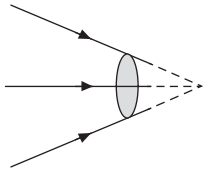
BEAM

A bundle or bunch of rays is called a **beam**. It is of following three types.

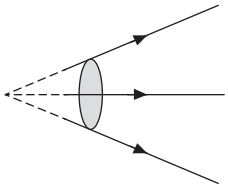
(a) **Parallel beam:** It is a beam in which all the rays constituting the beam move parallel to each other and diameter of beam remains same. A very narrow beam is called a Pencil of Light.



(b) **Convergent beam:** In this case diameter of beam decreases in the direction of ray.



(c) **Divergent beam:** It is a beam in which all the rays meet at a point when produced backward and the diameter of beam goes on increasing as the rays proceed forward.

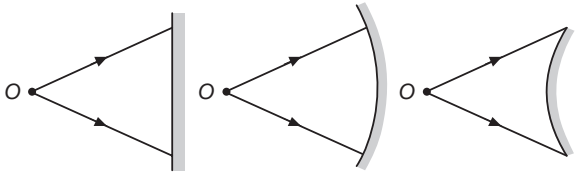


OBJECT(S)

The object for a mirror can be real or virtual. Generally we can define an object as the point where the incident rays intersect (real object) or appear to intersect (virtual object).

Real Object(s)

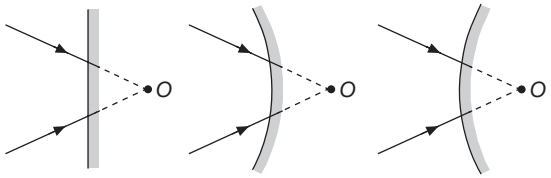
If the rays from a point on an object actually diverge from it and fall on the mirror, the object is said to be real.



In simple language the incident rays are diverging and the point of divergence is the position of the real object. The following diagrams support the arguments given.

Virtual Object(s)

If the rays incident on the mirror appear to converge to a point, then this point is said to be virtual point object for the mirror.



In simple language the incident rays are converging and the point of convergence is the position of the virtual object. The following diagrams support the arguments given.

Conceptual Note(s)

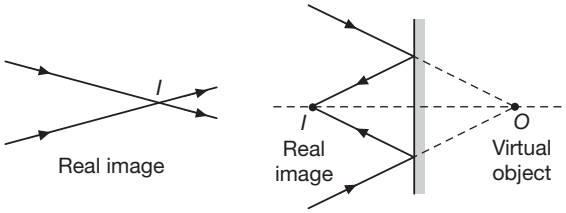
Virtual object cannot be seen by human eye, because for an object or an image to be seen by the eyes, the rays received by the eyes must be diverging.

IMAGE(S)

An **optical image** is a point where reflected or refracted rays of light either intersect or appear to intersect. Thus, the image of an infinite object is actually an assembly of the image points corresponding to various parts or the points of the object. The images formed can again be real or virtual.

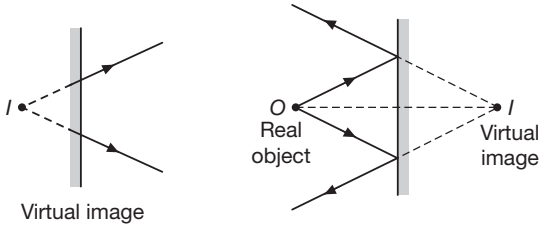
Real Image(s)

If the rays after reflection or refraction actually converge (or meet) at a point then the image is said to be **real** and it can be obtained on a screen.



Virtual Image(s)

However, if the rays do not actually converge but appear to diverge from a point (or appear to meet at a point), then the image so formed is said to be **virtual image.** A virtual image cannot be obtained on a screen.



Conceptual Note(s)

(a) The real images can be obtained on a suitably placed screen, but virtual images cannot be obtained on a screen.

(b) Human eye cannot distinguish between the real image and the virtual image because in both the cases the rays are diverging.

REFLECTION OF LIGHT

When light strikes the surface on an object, some part of the light or the complete light is sent back into the same medium. This phenomenon is called as reflection. The surface, which reflects light, is called mirror. A mirror could be plane or curved.

Conceptual Note(s)

In reflection, the frequency, speed and wavelength remain unchanged, but a phase change may occur depending on the nature of reflecting surface.

The reflection from a denser medium causes an addition phase change of π or a path change of $\frac{\lambda}{2}$ (by Stoke's Law) while reflection from rarer medium does not cause any phase change.

Diffused (irregular) reflection takes place from a rough surface where as **Specular** (regular) reflection takes place from an extraordinarily smooth surface. However, the Laws of Reflection are applicable for both kinds of surfaces.

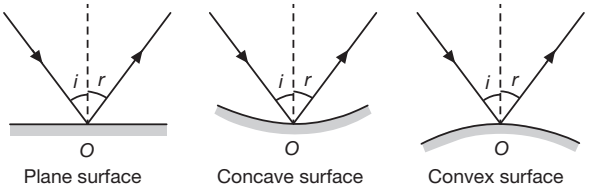
LAWS OF REFLECTION

(a) The incident-ray, the reflected-ray and the normal to the reflecting surface at the point of incidence, all lie in the same plane.

(b) The angle of reflection is equal to the angle of incidence $(i = r)$.

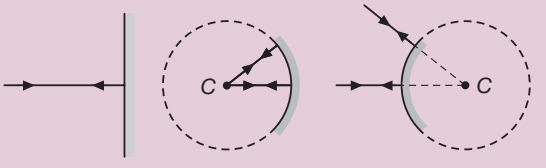
The angle of incidence i is the angle made by the incident ray with the normal.

The angle of reflection r is the angle made by the reflected ray with the normal.



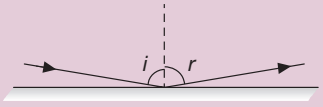
SPECIAL CASES

(a) If $i = 0$, then $r = 0$. It means a ray incident normally on a boundary, after reflection it retraces its path.



(b) The angle made by the incident ray with the plane reflecting surface is called glancing angle. Thus, the glancing angle $= 90° - i$.

(c) **For grazing incidence**, the incident ray grazes the reflecting surface, so $i \to \frac{\pi}{2}$ and hence $r \to \frac{\pi}{2}$ as shown in the figure.



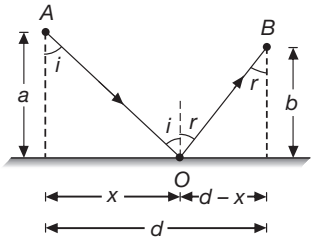
FERMAT'S PRINCIPLE OF LEAST TIME

According to this theorem, the path of a ray of light between any two points is the path along which the time taken is the minimum. This principle is sometimes taken as the definition of a ray of light.

To understand this theorem, let us consider two points A and B in the same medium. Since, we know that between these two points light travels in a straight line, so the time taken by the light to go from A to B must logically be the minimum.

LAWS OF REFLECTION USING FERMAT'S THEOREM

Consider a plane mirror on which light is incident as shown.



Let the incident light start from A, hit the mirror at O and get reflected to point B. Let the points A and B be at perpendicular distances a and b from the mirror and let A and B have a separation d between them as shown in figure. The time taken by the light to go from A to O to B is given by

$$t = t_{A\to O} + t_{O\to B}$$

$$\Rightarrow \quad t = \frac{AO}{c} + \frac{OB}{c}$$

$$\Rightarrow \quad t = \frac{1}{c}\left(\sqrt{a^2+x^2} + \sqrt{b^2+(d-x)^2}\right)$$

Now, according to Fermat's Principle, t is MINIMUM, so

$$\frac{dt}{dx} = 0$$

$$\Rightarrow \quad \frac{d}{dx}\left(\sqrt{a^2+x^2}\right) + \frac{d}{dx}\left(\sqrt{b^2+(d-x)^2}\right) = 0$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{2x}{\sqrt{a^2+x^2}}\right) + \frac{1}{2}\left(\frac{2(d-x)(-1)}{\sqrt{b^2+(d-x)^2}}\right) = 0$$

$$\Rightarrow \quad \frac{x}{\sqrt{a^2+x^2}} = \frac{(d-x)}{\sqrt{b^2+(d-x)^2}}$$

From the figure, we observe that

$$\frac{x}{\sqrt{a^2+x^2}} = \sin i \text{ and } \frac{(d-x)}{\sqrt{b^2+(d-x)^2}} = \sin r$$

$$\Rightarrow \quad \sin i = \sin r$$

$$\Rightarrow \quad i = r \qquad \textbf{\{The Law of Reflection\}}$$

Problem Solving Technique(s)

(a) Basic Problems in Optics: Most of the problems asked in optics expect us to find the position and nature of the final image formed by certain optical systems for a given object. The optical system may be just a mirror, or a lens or a combination of several reflecting and refracting surfaces.

(b) Basic Strategy for Solving the Problems: To handle these kinds of problems, first of all, we identify the sequence in which the reflection and refraction are taking place. The several events of reflection or refraction can be named as Event 1, Event 2 and so on following the sequence in which they occur.

Now, the image of Event 1 would be object for Event 2, image of Event 2 will be object of Event 3 and so on. This way one can proceed to find the final image.

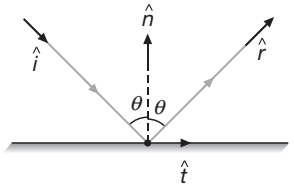
VECTOR FORM OF LAWS OF REFLECTION

Laws of reflection can be redefined with the help of vector algebra by considering unit vectors in the direction of incident rays, reflected rays and normal to the boundary.

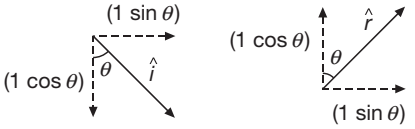
The reflection of a light ray incident on a plane surface is shown in figure. If $\hat{i}$, $\hat{r}$ and $\hat{n}$ are unit vectors along the direction of incident ray, reflected ray and normal to the surface as shown, then first we can write components of $\hat{i}$ and $\hat{r}$ in terms of the unit vectors along the normal and along the surface i.e. tangential to surface. Let $\hat{t}$ be a unit vector tangential to the surface, so we have

$$\hat{i} = (\sin\theta)\hat{t} - (\cos\theta)\hat{n} \qquad \ldots(1)$$

$$\hat{r} = (\sin\theta)\hat{t} + (\cos\theta)\hat{n} \qquad \ldots(2)$$



$$|\hat{i}| = |\hat{r}| = |\hat{n}| = |\hat{t}| = 1$$



Subtracting equation (1) from (2), we get

$$\hat{r} = \hat{i} + (2\cos\theta)\hat{n} \qquad \ldots(3)$$

Also, we know that

$$\hat{i}\cdot\hat{n} = |\hat{i}||\hat{n}|\cos(180-\theta) = -\cos\theta \qquad \ldots(4)$$

Substituting (4) in (3), we get

$$\hat{r} = \hat{i} - 2(\hat{i}\cdot\hat{n})\hat{n}$$

This equation gives us the Laws of Reflection in vector form.

ILLUSTRATION 1

A ray of light is incident on a plane mirror along a vector $\hat{i}+\hat{j}-\hat{k}$. The normal at the point of incidence is along $\hat{i}+\hat{j}$. Find a unit vector along the reflected ray.

SOLUTION

Reflection of a ray of light is just like an elastic collision of a ball with a horizontal ground. The component of the incident ray along the inside normal gets reversed while the component perpendicular to it remains unchanged. So, the component of incident ray vector $\vec{A} = \hat{i}+\hat{j}-\hat{k}$ parallel to normal, i.e., $\hat{i}+\hat{j}$ gets reversed while perpendicular to it, i.e., $-\hat{k}$ remains unchanged. So, the reflected ray is written as,

$$\vec{R} = -\hat{i}-\hat{j}-\hat{k}$$

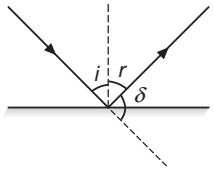
A unit vector along the reflected ray will be,

$$\hat{r} = \frac{\vec{R}}{R} = \frac{-\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$$

$$\Rightarrow \quad \hat{r} = -\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$$

ANGLE OF DEVIATION (δ)

Deviation (δ) is defined as the angle between the initial direction of the incident ray and the final direction of the reflected ray or the emergent ray.

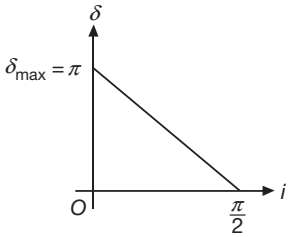


Deviation produced in Reflection is $\delta = 180° - (i+r)$

Since $r = i$

$$\Rightarrow \quad \delta = 180° - 2i$$

The variation of deviation (δ) with the angle of incidence (i) is shown in figure.



Problem Solving Technique(s)

(a) The deviation is maximum for normal incidence i.e., when $i = 0$ then, $\delta = \delta_{max} = 180°$.

(b) The deviation is minimum for grazing incidence i.e., when $i \to \frac{\pi}{2}$, then $\delta = \delta_{min} = 0°$.

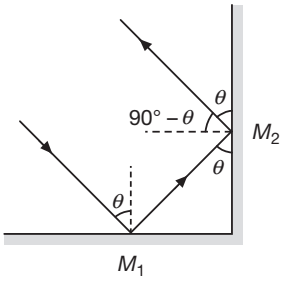
(c) While dealing with the case of multiple reflections suffered by a ray, the net deviation suffered by the incident ray is the algebraic sum of deviation due to each single reflection. So,

$$\delta_{total} = \sum \delta_{\substack{individual \\ reflection}}$$

DO NOT FORGET TO TAKE INTO ACCOUNT THE SENSE OF ROTATION WHILE SUMMING UP THE DEVIATIONS DUE TO SINGLE REFLECTION.

TWO IDENTICAL PERPENDICULAR PLANE MIRRORS

If two plane mirrors are inclined to each other at $90°$, the emergent ray is always antiparallel to the incident ray if it suffers one reflection from each (as shown in figure) whatever be the angle of incidence.



Conceptual Note(s)

The same is found to hold good for three plane mirrors arranged mutually perpendicular to each other thus forming the corner of a cube such that the light incident on this arrangement suffers one reflection from each of the mirrors so as to emerge out anti-parallel to the incident light. This arrangement of three mutually perpendicular plane mirrors forming the corner of a cube is called the CORNER REFLECTOR.

ILLUSTRATION 2

Two plane mirrors are inclined to each other at an angle θ. A ray of light is reflected first at one mirror and then at the other. Find the total deviation suffered by the ray.

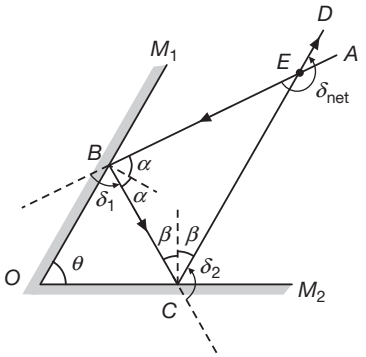
SOLUTION

α be the angle of incidence for mirror M_1

β be the angle of incidence for mirror M_2

δ_1 be the deviation due to mirror M_1 and

δ_2 be the deviation due to mirror M_2



From figure, we observe

$$\delta_1 = \pi - 2\alpha\,,\ \delta_2 = \pi - 2\beta$$

Also ray is rotated in same sense i.e., anticlockwise, so

$$\delta_{\text{net}} = \delta = \text{Total deviation} = \delta_1 + \delta_2$$

$$\Rightarrow \quad \delta = 2\pi - 2(\alpha + \beta)$$

Now in ΔOBC, $\angle OBC + \angle BCO + \angle COB = 180°$

$$\Rightarrow \quad (90° - \alpha) + (90° - \beta) + \theta = 180°$$

$$\Rightarrow \quad \alpha + \beta = \theta$$

$$\Rightarrow \quad \delta = 2\pi - 2\theta = 360° - 2\theta$$

Alternative Method:

$$\delta = \angle BEC + \angle CEA + \angle AED$$

Now, $\angle BEC = \angle AED$ (vertically opposite angle)

$$\Rightarrow \quad \angle BEC = 180° - 2(\alpha + \beta)$$

$$\Rightarrow \quad \angle BEC = 180° - 2\theta \qquad \{\because\ \theta = \alpha + \beta\}$$

Also, $\angle CEA = 2\alpha + 2\beta \ \Rightarrow\ \angle CEA = 2(\alpha + \beta) = 2\theta$

$$\Rightarrow \quad \delta = (180° - 2\theta°) + 2\theta + (180° - 2\theta°)$$

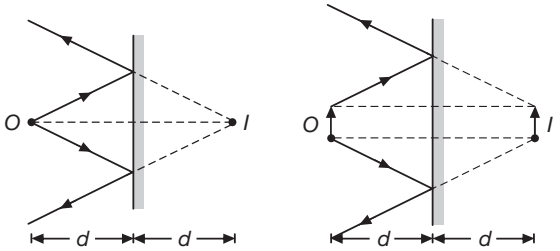
$$\Rightarrow \quad \delta = 360° - 2\theta$$

Conceptual Note(s)

If two mirrors are inclined at $\angle\theta$ then the ray incident on any one mirror will suffer a total deviation $\delta = 2\pi - 2\theta$ after suffering reflection from both of the mirrors.

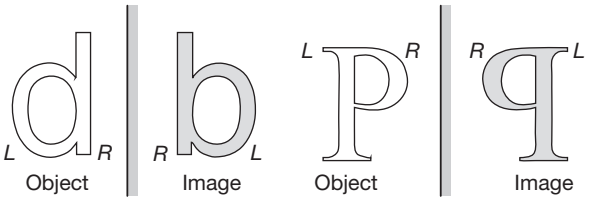
REFLECTION FROM A PLANE SURFACE OR PLANE MIRROR

When a real object is placed in front of a plane mirror, the image is always erect, virtual and of same size as the object. It is at same distance behind the mirror as the object is in front of it.

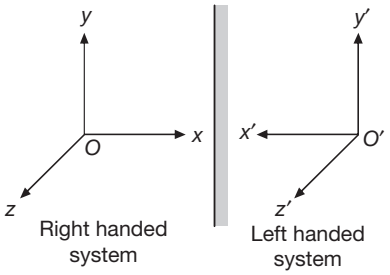


LATERAL INVERSION

The image formed by a plane mirror suffers **lateral-inversion**. That is, in the image the left is turned to the right and vice-versa with respect to object. However, the plane mirror does not turn up and down, as shown in figure.

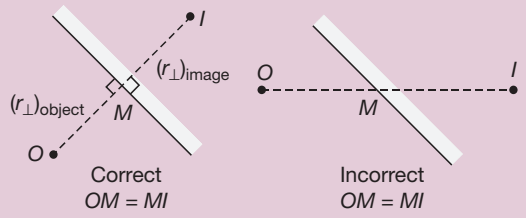


Actually, the plane mirror reverses forward and back in three-dimensions (and not left into right). If we keep a right-handed coordinate system in front of a plane mirror, only the z-axis is reversed. So, a plane mirror changes right-handed co-ordinate system (or screw) to left-handed.



Problem Solving Technique(s)

(a) For finding the location of an image of a point object placed in front of a plane mirror, we must see the perpendicular distance of the object from the mirror.

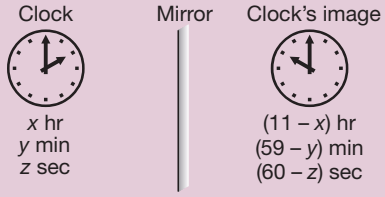


$$\text{i.e., } \begin{pmatrix} \perp \text{ distance of} \\ \text{O from mirror} \end{pmatrix} = \begin{pmatrix} \perp \text{ distance of} \\ \text{I from mirror} \end{pmatrix}$$

(b) When a wall clock is seen in a mirror then

Image Time = 11 hour 60 minute – Actual Time

$$\Rightarrow \quad t_{image} = 11:60 - t_{actual}$$



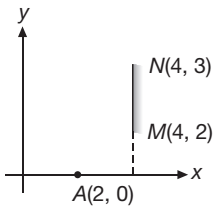
Example 1: If actual time in the clock is 5 : 25, then the image of the clock in the plane mirror will show a time given by

$$t_{image} = 11:60 - 5:25 = 6:35$$

Example 2: If actual time in the clock is 2 hr, 17 min, 25 sec then the image clock will show a time of 9 hr, 42 min, 35 sec.

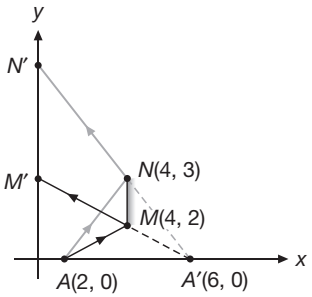
ILLUSTRATION 3

An object is lying at $A(2, 0)$ and MN is a plane mirror, as shown. Find the region on Y-axis in which reflected rays are present.



SOLUTION

The image of point A, in the mirror is at $A'(6, 0)$.



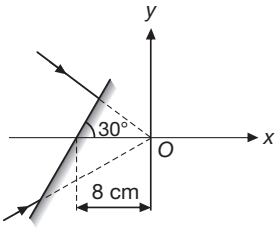
Let us join A' to M and extend it to cut the Y axis at M'. (Ray originating from A which strikes the mirror at M gets reflected as the ray MM' which appears to come from A'). Join $A'N$ and extend to cut Y axis at N' (Ray originating from A which strikes the mirror at N gets reflected as the ray NN' which appears to come from A'). Using Geometry, we get

$$M' = (0,\ 6) \text{ and } N' = (0,\ 9).$$

$M'N'$ is the region on Y axis in which reflected rays are present.

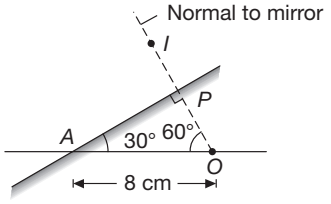
ILLUSTRATION 4

Find the co-ordinates of the location of the image formed for an object kept at origin as shown in figure.



SOLUTION

The first thing we observe is that the object is virtual, because the ray of light is converging on plane mirror. Also, the co-ordinates of object are $(0,\ 0,\ 0)$ and the image co-ordinates are the reflection of object coordinates in the mirror as shown in figure.



The image lies on normal of mirror at I. From ΔAOP, we have

$$\sin(30°) = \frac{PO}{8}$$

$$\Rightarrow \quad PO = 4 \text{ cm}$$

$$\Rightarrow \quad OI = 2(PO) = 8 \text{ cm}$$

So co-ordinates of I are

$$x = -8\cos(60°) = -4 \text{ cm},$$

$$y = 8\sin(60°) = 4\sqrt{3} \text{ cm and}$$

$$z = 0$$

So, the co-ordinates of image are $(-4,\ 4\sqrt{3},\ 0)$

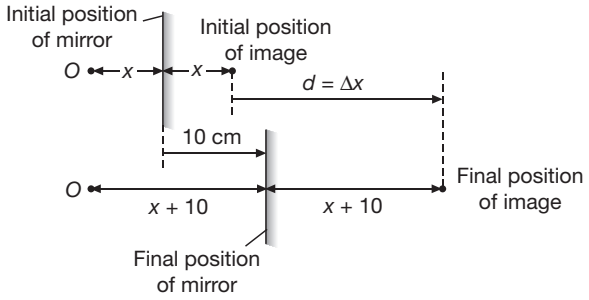
ILLUSTRATION 5

There is a point object and a plane mirror. If the mirror is moved by 10 cm away from the object find the distance which the image will move.

SOLUTION

Since we know that the image distance from the plane mirror is equal to the object distance from the plane mirror.

$$\Rightarrow \quad |x_{im}| = |x_{om}|$$



From figure we observe that

$$2(x+10) = 2x + d$$

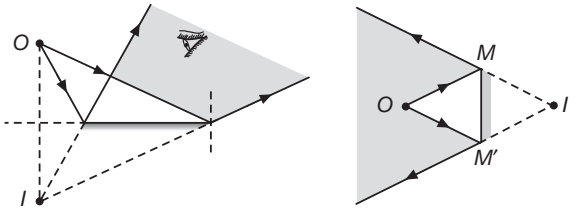
$$\Rightarrow \quad d = \Delta x = 20 \text{ cm}$$

FIELD OF VIEW OF AN OBJECT

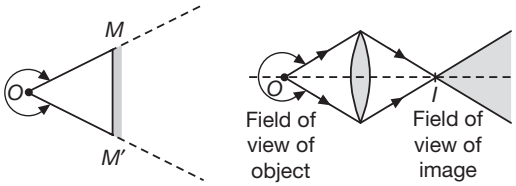
Suppose a point object O is placed in front of a mirror, then a question arises in mind whether this mirror will form the image of this object or not. The answer is yes, it will form. A mirror, irrespective of its size, forms the images of all objects lying in front

of it. But every object has its own field of view for a given mirror.

Field of view is the region where diverging rays from object or image are present. If our eyes are present in field of view then only we can see the object or an image as the case may be. Field of view of image is decided by rays which get reflected or refracted from the extremities or the extreme ends of the mirror or a lens and depends on the location of the object in front of mirror or lens.

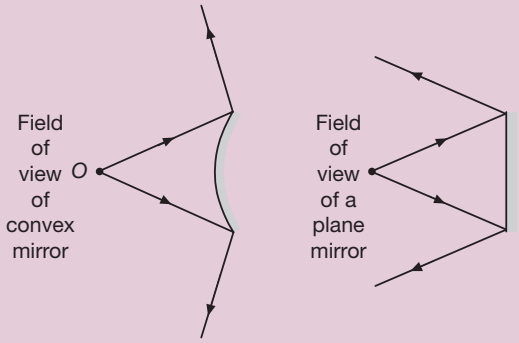


Field of view of image



Conceptual Note(s)

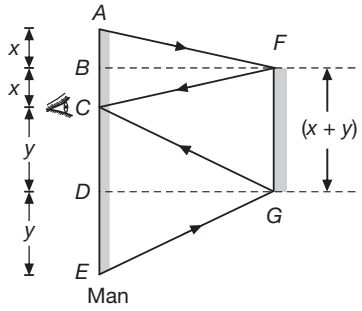
It has been observed that a convex mirror gives a wider field of view than a plane mirror. Therefore, the convex mirrors are used as rear view mirrors in vehicles. Though they make the estimation of distances more difficult but still they are preferred because for a large movement of the object vehicle there is only a small movement of the image.



MINIMUM SIZE OF A PLANE MIRROR TO SEE A COMPLETE IMAGE

CASE-1: To find the minimum size of mirror to see a full image we use the fact that light rays from extreme parts of object should reach eye after reflection from mirror. Let us consider following two situations

(a) The minimum size of mirror to see one's full height is $\frac{H}{2}$ where H is the height of man. To see full image mirror is positioned in such a way so that rays from head and foot reach eye after reflection from mirror, as shown in the figure.



(b) A ray starting from head (A) after reflecting from upper end of the mirror (F) reaches the eye at C. Similarly the ray starting from the foot (E) after reflecting from the lower end (G) also reaches the eye at C. in similar triangles ABF and BFC

$$AB = BC = x \text{ (say)}$$

Similarly in triangles CDG and DGE, we have

$$CD = DE = y \text{ (say)}$$

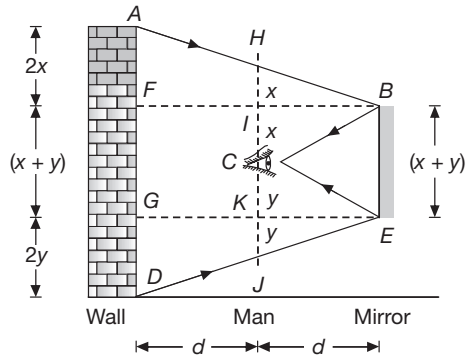
Now, we observe that height of the man is $2(x+y)$ and that the length of mirror is $(x+y)$, i.e., the length of the mirror is half the height of the man. Please note that the mirror can be placed anywhere between the centre line BF (of AC) and DG (of CE).

Conceptual Note(s)

(a) In order to see full image of the man, the mirror is positioned such that the lower edge of mirror is at height half the eye level from the ground.

(b) Minimum size is independent of the distance between man and mirror.

CASE-2: The minimum length of the mirror required to see the full image of a wall behind the man who is standing midway between mirror and the wall is $\frac{H}{3}$, where H is the height of wall. The ray diagram for this situation is shown in figure.



In triangles HBI and IBC let $HI = IC = x$. Now, in triangles HBI and ABF, we have

$$\frac{AF}{HI} = \frac{FB}{BI}$$

$$\Rightarrow \quad \frac{AF}{x} = \frac{2d}{d}$$

$$\Rightarrow \quad AF = 2x$$

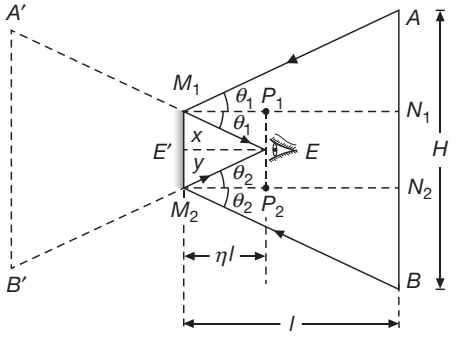
Similarly if, $CK = KJ = y$, then $DG = 2y$. Now, we observe that height of the wall is $3(x+y)$ while that of the mirror is $(x+y)$.

ILLUSTRATION 6

A plane mirror is fitted on a wall and a person gazing at it intends to view the complete image of the rear wall of height H. If η be the fraction of the distance of the person from the mirror, as compared to the length of the room, then calculate the minimum height h of the mirror to do so.

SOLUTION

Let us first draw the ray diagram of the arrangement given in the problem. For this to happen, the rays from the top A and bottom B of wall AB must fall into the eye E of the observer after being reflected from the edges M_1 and M_2 of the mirror. So, in this case height of the mirror will be the minimum.



From similar triangles AM_1N_1 and EM_1P_1

$$\frac{x}{\eta l} = \frac{AN_1}{l}$$

$$\Rightarrow \quad AN_1 = \frac{x}{\eta} \qquad \ldots(1)$$

Similarly, from similar triangle BM_2N_2 and EM_2P_2, we get

$$\frac{y}{\eta l} = \frac{BN_2}{l}$$

$$\Rightarrow \quad BN_2 = \frac{y}{\eta} \qquad \ldots(2)$$

Adding equations (1) and (2), we get

$$AN_1 + BN_2 = \frac{(x+y)}{\eta} \qquad \ldots(3)$$

Now, total height of the room is

$$AB = H$$

$$\Rightarrow \quad AN_1 + N_1N_2 + BN_2 = H$$

$$\Rightarrow \quad AN_1 + (M_1E' + E'M_2) + BN_2 = H$$

$$\Rightarrow \quad \frac{x}{\eta} + (x+y) + \frac{y}{\eta} = H$$

Since, height of the mirror is $h = (x+y)$

$$\Rightarrow \quad h = (x+y) = \frac{\eta H}{(\eta+1)}$$

So, height of mirror is $h = \frac{\eta H}{(\eta+1)}$

Note that the height of mirror obtained above is minimum since, light coming from the extreme edges of the room A and B is just able to enter the person's eye after reflection from the mirror.

Conceptual Note(s)

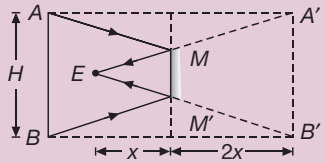
(a) Result is independent of the height or the eye level of the person.

(b) $E'M_2 = EP_2 = y$. But $y + \frac{y}{\eta} = h$, height of eye level above the floor of the room

$$y = \frac{\eta h}{(\eta + 1)}$$

For a person to see the complete image of the rear wall, the lower edge of the mirror, i.e., M_2 should be at a level $\frac{\eta h}{\eta + 1}$ lower than his eye level.

(c) If the person stands at the middle of the room, then $\eta = \frac{1}{2}$.

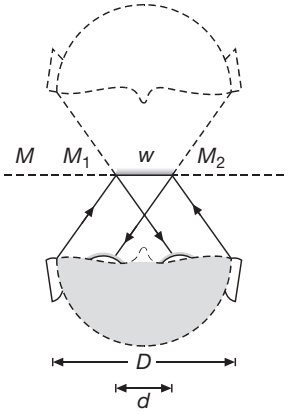


So, minimum height of mirror required is

$$h_{min} = \frac{\left(\frac{1}{2}\right)H}{\left(\frac{1}{2}+1\right)} = \frac{H}{3}$$

REQUIRED MINIMUM WIDTH OF A PLANE MIRROR FOR A PERSON TO SEE THE COMPLETE WIDTH OF HIS FACE

The minimum width of a plane mirror required for a person to see the complete width of his face is $\frac{(D-d)}{2}$ where, D is the width of his face and d is the distance between his two eyes.



$$MM_1 = \frac{1}{2}\left[D - \frac{1}{2}(D-d)\right]$$

$$\Rightarrow \quad MM_1 = \frac{(D+d)}{4} \qquad \ldots(1)$$

and $MM_2 = D - \frac{(D+d)}{4}$

$$\Rightarrow \quad MM_2 = \frac{(3D-d)}{4} \qquad \ldots(2)$$

So, Width of the mirror is $w = M_1M_2$

$$\Rightarrow \quad w = MM_2 - MM_1$$

$$\Rightarrow \quad w = \frac{2D-2d}{4}$$

$$\Rightarrow \quad w = \frac{(D-d)}{2}$$

NUMBER OF IMAGES IN INCLINED MIRRORS

Let θ be the angle between two plane mirrors and n be the number of images formed.

$$\text{Then } n = \begin{cases} \frac{360}{\theta}, & \text{if } \frac{360}{\theta} \text{ is odd} \\ \left(\frac{360}{\theta} - 1\right), & \text{if } \frac{360}{\theta} \text{ is even} \end{cases}$$

Further when $\frac{360}{\theta}$ is odd, then

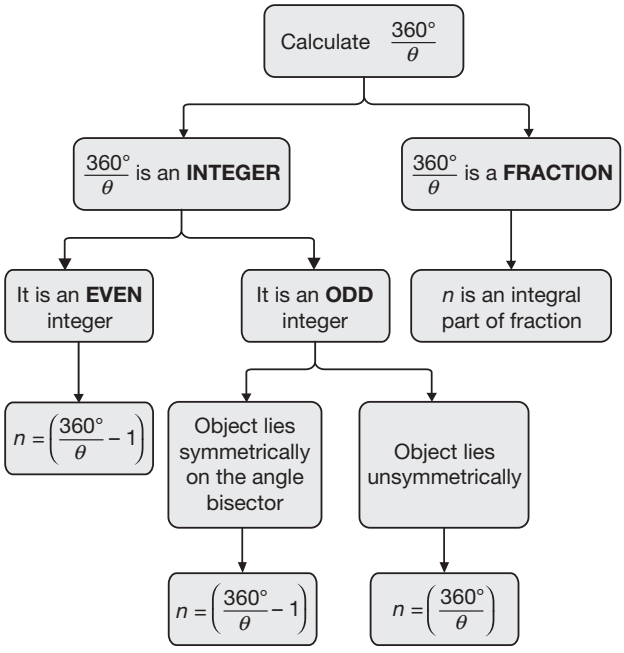
$$n = \begin{cases} \left(\dfrac{360}{\theta} - 1\right), & \text{if object lies symmetrically on the angle bisector of two mirrors} \\ \dfrac{360}{\theta}, & \text{if object lies unsymmetrically} \end{cases}$$

Further if $\dfrac{360}{\theta}$ is a fraction, then the number of images formed will be integral part of the fraction e.g. if $\dfrac{360}{\theta}$ is 4.8, then $n = 4$. Following diagram shows the process to calculate n.

Net deviation produced by two plane mirrors inclined at an angle θ is

$$\delta = 360° - 2\theta$$

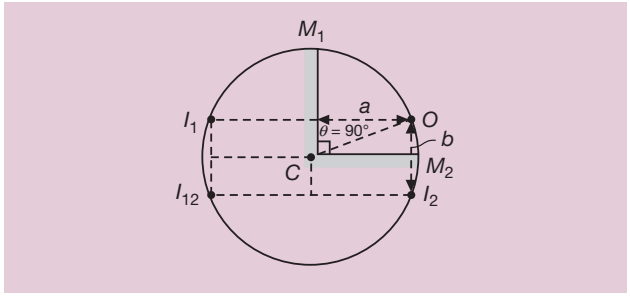
Clearly δ is independent of the angle of incidence of the ray of light.


Calculate 360°/θ
360°/θ is an INTEGER
360°/θ is a FRACTION
It is an EVEN integer
It is an ODD integer
n is an integral part of fraction
n = (360°/θ − 1)
Object lies symmetrically on the angle bisector
Object lies unsymmetrically
n = (360°/θ − 1)
n = (360°/θ)

Conceptual Note(s)

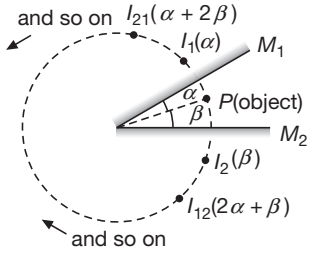
(a) If an object is placed between two parallel mirrors ($\theta = 0°$), the number of images formed will be infinite.

(b) The number of images formed may be different from the number of images seen (which depends on the position of the observer).



LOCATING ALL THE IMAGES FORMED BY TWO PLANE MIRRORS

Consider two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha + \beta$ as shown in figure.



Point P is an object kept such that it makes angle α with mirror M_1 and angle β with mirror M_2. Image of object P formed by M_1, denoted by I_1, will be inclined by angle α on the other side of mirror M_1. This angle is written in bracket in the figure besides I_1. Similarly image of object P formed by M_2, denoted by I_2, will be inclined by angle β on the other side of mirror M_2. This angle is written in bracket in the figure besides I_2.

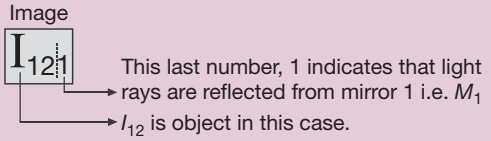
Now I_2 will act as an object for M_1 which is at an angle $(\alpha + 2\beta)$ from M_1. Its image will be formed at an angle $(\alpha + 2\beta)$ on the opposite side of M_1. This image will be denoted as I_{21} and so on. Think when this will process stop.

Conceptual Note(s)

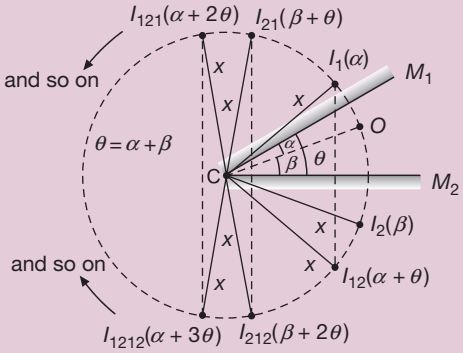
(a) The virtual image formed by a plane mirror must not be in front of the mirror or its extension.

(b) For convenience, we assign symbols to the images formed by mirrors, like

I_1 stands for image of O in M_1
I_{12} stands for image of I_1 in M_2
I_{121} stands for image of I_{12} in M_1
I_{1212} stands for image of I_{121} in M_2
i.e., the last subscript digit in above images tells that reflection is taking place from mirror corresponding to that subscript as shown in the figure.



(c) All the images lie on a circle of radius x where x is the distance between the object O and the point of intersection of the mirrors C.



(d) The angular position of the images formed by mirrors M_1 and M_2 inclined to each other at an angle θ, when an object O is placed between them making an angle α with M_1 and β with M_2 (i.e. $\theta = \alpha + \beta$) can be obtained conveniently by using the following tabular format.

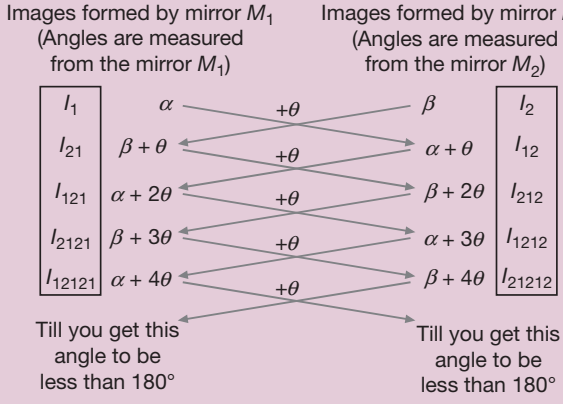

Images formed by mirror M1 (Angles are measured from the mirror M1)
Images formed by mirror M2 (Angles are measured from the mirror M2)
I1 α +θ β I2
I21 β + θ +θ α + θ I12
I121 α + 2θ +θ β + 2θ I212
I2121 β + 3θ +θ α + 3θ I1212
I12121 α + 4θ +θ β + 4θ I21212
Till you get this angle to be less than 180°
Till you get this angle to be less than 180°

ILLUSTRATION 7

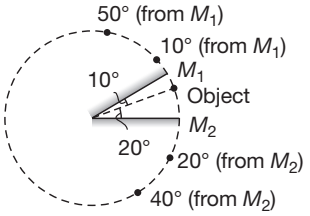
Two mirrors are inclined by an angle $30°$. An object is placed making $10°$ with the mirror M_1. Find the positions of first two images formed by each mirror. Find the total number of images by

(i) counting the images.
(ii) using direct formula.

SOLUTION

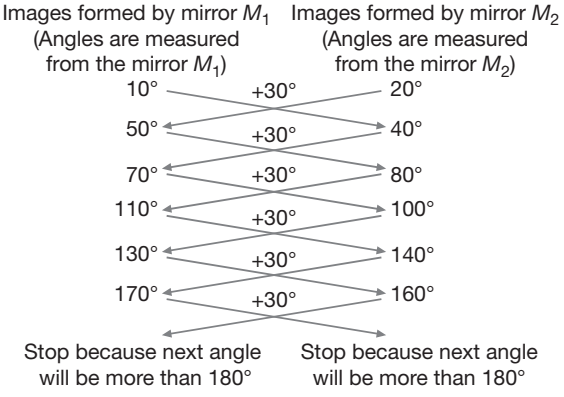
We must understand that the angular position of the image in the mirror is same as the angular position of its object in the same mirror. So, first image of O in mirror M_1 is $10°$ behind mirror M_1. Similarly first image of O in mirror M_2 is $20°$ behind the mirror M_2.

Now the first image of O formed in mirror M_1 acts as an object for the mirror M_2. This image is at an angular position $(10° + 30°) = 40°$ with respect to the mirror M_2 and hence the second image is located at angular position of $40°$ from M_2.



Similarly, the first image of O formed in mirror M_2 acts as an object for the mirror M_1. This image is at an angular position $(20° + 30°) = 50°$ with respect to the mirror M_1 and hence the second image is located at angular position of $50°$ from M_1.

(i) By counting: Let us draw the following table to locate the position of images from the respective mirrors.


Images formed by mirror M1 (Angles are measured from the mirror M1)
Images formed by mirror M2 (Angles are measured from the mirror M2)
10° +30° 20°
50° +30° 40°
70° +30° 80°
110° +30° 100°
130° +30° 140°
170° +30° 160°
Stop because next angle will be more than 180°
Stop because next angle will be more than 180°

To check whether the final images made by the two mirrors coincide or not, proceed as follows. We shall be adding the last obtained angles (i.e. $170°+160°$) and the angle between mirrors (i.e. $30°$). If this sum comes out to be exactly $360°$, then it simply means that the final images formed by the two mirrors coincide. Here last angles made by the mirrors + the angle between the mirrors is $160°+170+30°=360°$. Therefore in this case the last two images coincide due to which they together will be counted as a one single image and hence we shall be subtracting 1 from the total number of images being obtained.

So, the total number of images formed is the sum of the number of images formed by mirrors M_1 (i.e. 6) and M_2 (also 6) minus 1 (as the last images coincide)

$\Rightarrow \quad N=6+6-1=11.$

(ii) Let's first calculate $\dfrac{360°}{30°}=12$ (even number)

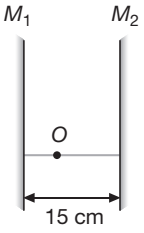
$\Rightarrow$ number of images $=12-1=11$

IMAGES FORMED BY TWO PLANE MIRRORS

Since number of images formed is given by $N=\dfrac{360}{\theta}-1$, so for $\theta=0°$, $N\to\infty$. This is because when rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will function as an object for second mirror, and this process will continue for every successive reflection.

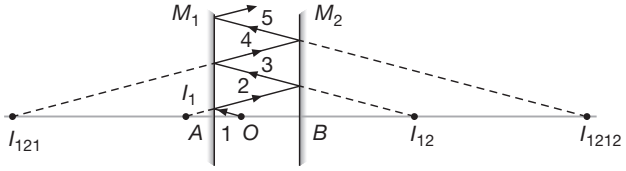
ILLUSTRATION 8

Figure shows a point object O placed between two parallel mirrors separated by $15\ \text{cm}$. O lies at a distance of $5\ \text{cm}$ from M_1. Find the distance of images from the two mirrors considering reflection on mirror M_1 first.



SOLUTION

The following ray diagram helps us to understand the formation of images due to subsequent reflections from mirrors M_1 and M_2.



For convenience, we assign symbols to the images formed by mirrors, like

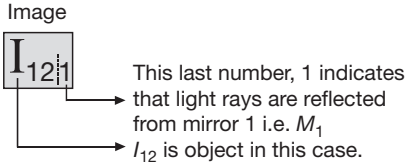
I_1 stands for image of O in M_1

I_{12} stands for image of I_1 in M_2

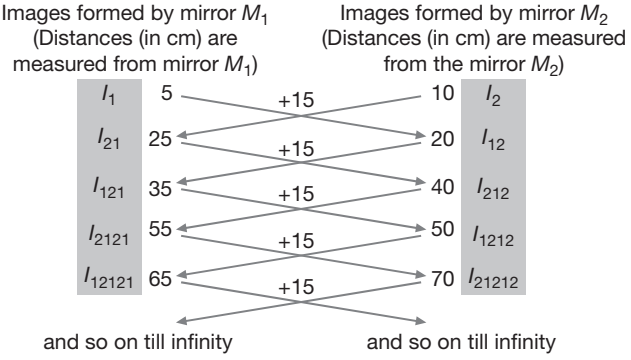
I_{121} stands for image of I_{12} in M_1

I_{1212} stands for image of I_{121} in M_2

i.e., the last subscript digit in above images tells that reflection is taking place from mirror corresponding to that subscript as shown in the figure.

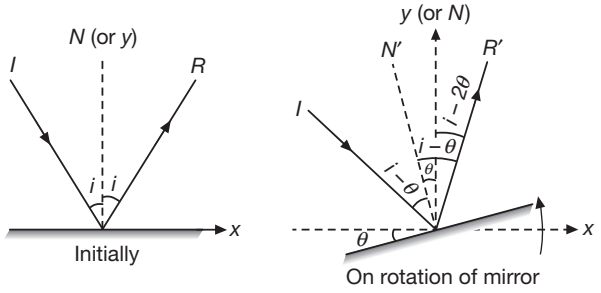


The following figure shows the location of images formed by mirrors M_1 and M_2.



ROTATION OF A PLANE MIRROR

When a mirror is rotated by an angle θ (say anticlockwise), keeping the incident ray fixed, then the reflected ray rotates by 2θ along the same sense, i.e., anticlockwise.



Let I be the incident ray, N the normal and R the reflected ray, then on rotation, I remains as it is, N and R shift to N' and R'.

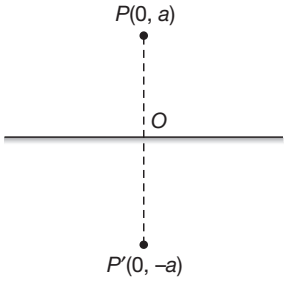
From the two figures we can observe that the reflected ray earlier made an angle i with y-axis while after rotating the mirror it makes the angle $(i-2\theta)$. So, we conclude that the reflected ray has been rotated by an angle 2θ.

Conceptual Note(s)

If a plane mirror rotates with angular velocity ω, then the reflected ray rotates with angular velocity 2ω (excluding rotation of mirror with normal as the axis).

ILLUSTRATION 9

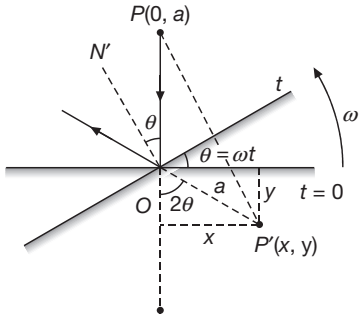
A plane mirror is placed along the xz-plane and an object P is placed at point $(0, a)$. The mirror rotates about z-axis with constant angular velocity ω. Calculate the position and velocity of image as function of time $t\left(<\dfrac{\pi}{2\omega}\right)$.



SOLUTION

When the mirror rotates through an angle $\theta = \omega t$, the reflected ray rotates through an angle 2θ as shown.

Let the new image be now formed at point $P(x, y)$. Since $OP = OP' = a$. So, we have



$$x = a\sin(2\theta) = a\sin(2\omega t) \text{ and}$$

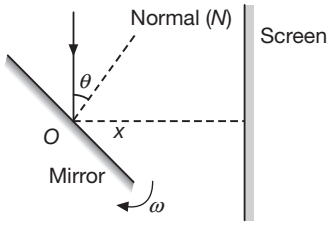
$$y = -a\cos(2\theta) = -a\cos(2\omega t)$$

$$\Rightarrow \quad v_x = \frac{dx}{dt} = 2a\omega\cos(2\omega t) \text{ and}$$

$$v_y = \frac{dy}{dt} = 2a\omega\sin(2\omega t)$$

ILLUSTRATION 10

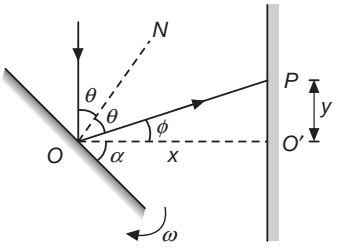
A plane mirror hinged at O is free to rotate in a vertical plane. The point O is at a distance x from a long screen placed in front of the mirror as shown in figure.



A laser beam of light incident vertically downward is reflected by the mirror at O so that a bright spot is formed at the screen. At the instant shown, the angle of incidence is θ and the mirror is rotating clockwise with constant angular velocity ω. Find the speed of the spot at this instant.

SOLUTION

Let P be the bright spot, shown on the screen. Let the distance of point P from O' be y at this instant shown in figure. Then according to the problem we need to calculate $\dfrac{dy}{dt}$



From the figure

$$\theta+\theta+\phi=90^\circ \quad \ldots(1)$$

$$\theta+\phi+\alpha=90^\circ \quad \ldots(2)$$

$$\Rightarrow \quad \alpha=\theta$$

$$\Rightarrow \quad \phi+2\theta=90^\circ$$

$$\Rightarrow \quad \phi+2\alpha=90^\circ$$

$$\Rightarrow \quad \frac{d\phi}{dt}+2\frac{d\alpha}{dt}=0$$

$$\Rightarrow \quad \frac{d\phi}{dt}=-2\frac{d\alpha}{dt}$$

So, the angular speed of the reflected ray is double the angular speed of the mirror.

Since, $y=x\tan\phi$

$$\Rightarrow \quad \frac{dy}{dt}=x\sec^2\phi\frac{d\phi}{dt}$$

Since $\left|\frac{d\phi}{dt}\right|=2\omega$

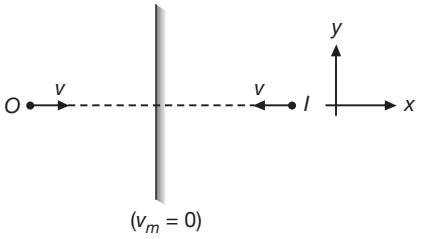
$$\Rightarrow \quad \left|\frac{dy}{dt}\right|=\left(x\sec^2\phi\right)(2\omega)$$

So, the speed of the spot is $\left|\frac{dy}{dt}\right|=2x\omega\sec^2\phi$

VELOCITY OF IMAGE IN A PLANE MIRROR

To understand and interpret the moving images of moving objects in front of plane mirror, we must understand the following cases.

CASE-1: Object moving along the normal to the plane mirror which is at rest. All velocities measured w.r.t. ground frame.



Velocity of object with respect to mirror is

$$\vec{v}_{Om}=v\hat{i}$$

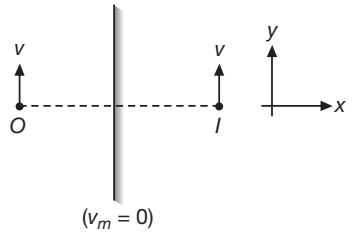
Velocity of image with respect to mirror is

$$\vec{v}_{\mathrm{Im}}=-v\hat{i}$$

Velocity of object with respect to image is

$$\vec{v}_{OI}=\vec{v}_O-\vec{v}_I=(2v)\hat{i}$$

CASE-2: Object moving parallel to the plane of mirror (at rest)



Velocity of object w.r.t. mirror is

$$\vec{v}_{Om}=v\hat{j}$$

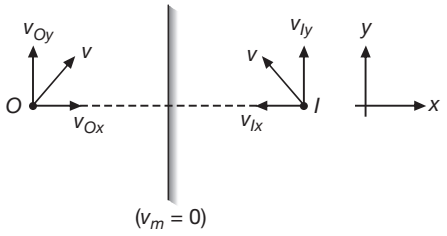
Velocity of image w.r.t. mirror is

$$\vec{v}_{\mathrm{Im}}=v\hat{j}$$

Velocity of object w.r.t. image is

$$\vec{v}_{OI}=\vec{0}$$

CASE-3: Object moving neither along the normal nor along the parallel to the plane mirror (at rest).

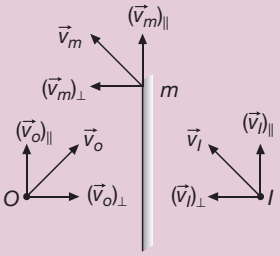


Clearly, we observe this case to be a combination of CASE-1 and CASE-2. So, here

$$(v_{OI})_x = 2v_{Ox} \text{ and } (v_{OI})_y = 0$$

Problem Solving Technique(s)

While solving problems that involve the calculation of image of an object w.r.t. any observer, then



STEP-1: Firstly, calculate the velocity of image w.r.t. mirror keeping in mind that

$$(\vec{v}_{Im})_{\text{along mirror}} = (\vec{v}_{Om})_{\text{along mirror}}$$

$$\Rightarrow (\vec{v}_{Im})_{\|} = (\vec{v}_{Om})_{\|}$$

Since, both the object and the image approach the mirror with equal and opposite speed, so we have

$$(\vec{v}_{Im})_{\text{normal to mirror}} = -(\vec{v}_{Om})_{\text{normal to mirror}}$$

$$\Rightarrow (\vec{v}_{Im})_{\perp} = -(\vec{v}_{Om})_{\perp}$$

$$\Rightarrow \vec{v}_I - \vec{v}_m = -(\vec{v}_O - \vec{v}_m)$$

$$\Rightarrow \vec{v}_I = 2\vec{v}_m - \vec{v}_O$$

STEP-2: Then the velocity of image w.r.t. mirror is

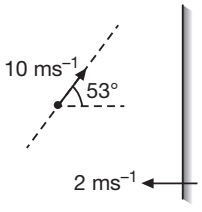
$$\vec{v}_{Im} = (\vec{v}_{Im})_{\|} + (\vec{v}_{Im})_{\perp}$$

However, velocity of image w.r.t. any other observer, say *A* is then given by

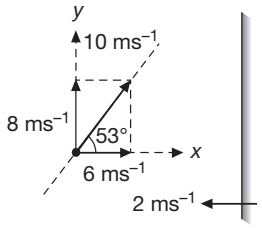
$$\vec{v}_{IA} = \vec{v}_I - \vec{v}_A$$

ILLUSTRATION 11

Find the velocity of image of a moving particle shown in figure.



SOLUTION



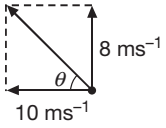
The component of velocity of image perpendicular to mirror is

$$\vec{V}_I = 2\vec{V}_m - \vec{V}_O$$

$$\Rightarrow (\vec{V}_I)_{\perp} = 2(-2) - (6) = -10 \text{ ms}^{-1}$$

For component of velocity of image parallel to the mirror

$$(V_I)_{\|} = 8 \text{ ms}^{-1}$$

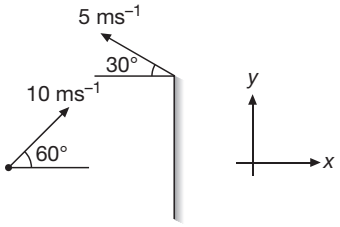


$$\therefore \quad \text{Velocity of image } (V_I) = \sqrt{(V_I)_{\perp}^2 + (V_I)_{\|}^2}$$

$$\Rightarrow V_I = \sqrt{100+64} = \sqrt{164} \text{ ms}^{-1} \text{ and } \theta = \tan^{-1}\left(\frac{4}{5}\right)$$

ILLUSTRATION 12

In the situation shown in figure, find the velocity of image.



SOLUTION

Along x direction, applying $v_i = v_m = -(v_0 - v_m)$

$$v_i - (-5\cos 30°) = -(10\cos 60° - (-5\cos 30°))$$

$\Rightarrow \quad v_i = -5\left(1+\sqrt{3}\right)\ \text{ms}^{-1}$

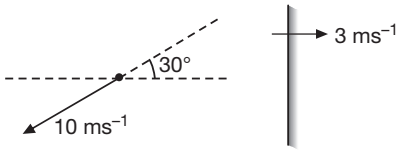
Along y direction $v_0 = v_i$

$\Rightarrow \quad v_i = 10\sin 60° = 5\sqrt{3}\ \text{ms}^{-1}$

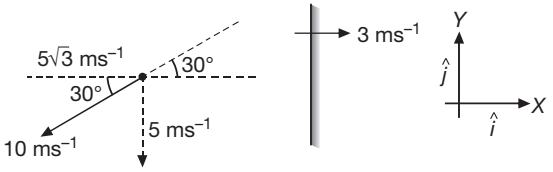
$\Rightarrow$ Velocity of the image $= -5\left(1+\sqrt{3}\right)\hat{i} + 5\sqrt{3}\hat{j}\ \text{ms}^{-1}$.

ILLUSTRATION 13

A point object is moving with a speed of $10\ \text{ms}^{-1}$ in front of a mirror moving with a speed of $3\ \text{ms}^{-1}$ as shown in figure. Find the velocity of image of the object with respect to mirror, object and ground.



SOLUTION



Velocity of object, $\vec{v}_O = \left(-5\sqrt{3}\hat{i} - 5\hat{j}\right)\ \text{ms}^{-1}$

Velocity of mirror, $\vec{v}_M = 3\hat{i}\ \text{ms}^{-1}$

For component of velocity perpendicular to mirror, we have

$$\left(\vec{v}_{IM}\right)_\perp = -\left(\vec{v}_{OM}\right)_\perp = -\left(\vec{v}_O - \vec{v}_M\right)$$

$\Rightarrow \quad \left(\vec{v}_{IM}\right)_\perp = -\left(-5\sqrt{3}\hat{i} - 3\hat{i}\right) = \left(5\sqrt{3}+3\right)\hat{i}\ \text{ms}^{-1}$

For component of velocity parallel to mirror, we have

$$\left(\vec{v}_{IM}\right)_\parallel = \left(\vec{v}_{OM}\right)_\parallel = \vec{v}_O - \vec{v}_M = -5\hat{j} - 0 = -5\hat{j}$$

Since, $\left(\vec{v}_{IM}\right) = \left(\vec{v}_{IM}\right)_\perp + \left(\vec{v}_{IM}\right)_\parallel$

$\Rightarrow \quad \left(\vec{v}_{IM}\right) = \left(5\sqrt{3}+3\right)\hat{i} - 5\hat{j}$

Also, $\vec{v}_{IM} = \vec{v}_I - \vec{v}_M$

$\Rightarrow \quad \left(5\sqrt{3}+3\right)\hat{i} - 5\hat{j} = \vec{v}_I - \left(3\hat{i}\right)$

$\Rightarrow \quad \left(5\sqrt{3}+3\right)\hat{i} + 3\hat{i} - 5\hat{j} = \vec{v}_I$

$\Rightarrow \quad \vec{v}_I = \left[\left(5\sqrt{3}+6\right)\hat{i} - 5\hat{j}\right]\ \text{ms}^{-1}$

Further $\vec{v}_{IO} = \vec{v}_I - \vec{v}_O$

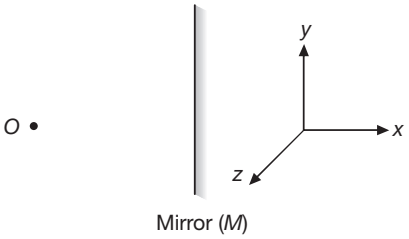
$\Rightarrow \quad \vec{v}_{IO} = \left(5\sqrt{3}+6\right)\hat{i} - 5\hat{j} - \left(-5\sqrt{3}\hat{i} - 5\hat{j}\right)$

$\Rightarrow \quad \vec{v}_{IO} = \left(5\sqrt{3}+6+5\sqrt{3}\right)\hat{i} + (5-5)\hat{j}$

$\Rightarrow \quad \vec{v}_{IO} = \left(10\sqrt{3}+6\right)\hat{i}\ \text{ms}^{-1}$

ILLUSTRATION 14

A plane mirror in y-z plane moves with a velocity $-3\hat{i}$ as shown in figure. An object O starts moving with a velocity $4\hat{i} + \hat{j} - 4\hat{k}$. Find the velocity of the image.



SOLUTION

Since the mirror is placed in y-z plane, so the y and z components of the velocity of the image remain the same as that of the object. However, **perpendicular to the mirror**, the velocity of approach of object towards the mirror is always equal and opposite to the velocity of approach of the image towards the mirror, so, we have

$$\left(\vec{v}_{OM}\right)_x = -\left(\vec{v}_{IM}\right)_x$$

$\Rightarrow \quad \left(\vec{v}_O\right)_x - \left(\vec{v}_M\right)_x = -\left(\vec{v}_I\right)_x + \left(\vec{v}_M\right)_x$

$\Rightarrow \quad \left(\vec{v}_I\right)_x = 2\left(\vec{v}_M\right)_x - \left(\vec{v}_O\right)_x$

$\Rightarrow \quad \left(\vec{v}_I\right)_x = 2\left(-3\hat{i}\right) - 4\hat{i} = -10\hat{i}$

So, $\vec{v}_I = -10\hat{i} + \hat{j} - 4\hat{k}$

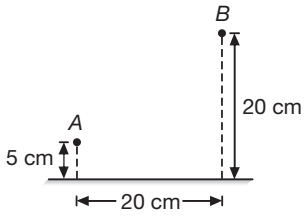
Test Your Concepts-I

Based on Reflection at Plane Surfaces

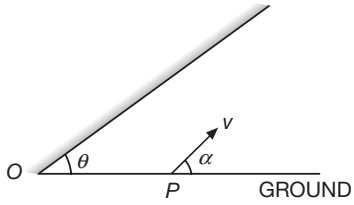
(Solutions on page H.3)

1. A ray of light travelling in the direction $\frac{1}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$. Find the angle of incidence.

2. A ray of light travels from point *A* to a point *B* after being reflected from a plane mirror as shown in figure. From where should it strike the mirror?

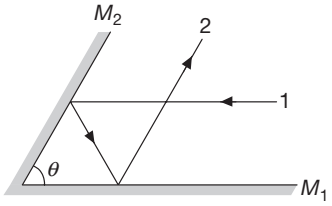


3. A plane mirror is inclined at an angle $\theta = 60°$ with horizontal surface. A particle is projected from point *P* on the ground (see figure) at $t = 0$ with a velocity *v* at an angle α with horizontal. The image of the particle is observed from the frame of the particle projected. Assuming the particle does not collide the mirror. Find the time when image will come momentarily at rest with respect to particle.

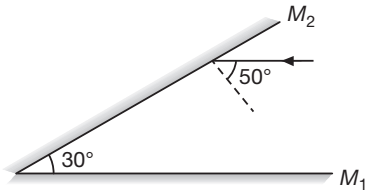


4. Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror and parallel to the second is reflected from the second mirror parallel to the first mirror.
- **(a)** Find the angle between the two mirrors.
- **(b)** Also calculate the total deviation produced in the incident ray due to the two reflections.

5. Two plane mirrors M_1 and M_2 are inclined at angle θ as shown in figure. A ray of light 1, which is parallel to M_1 strikes M_2 and after two reflections, the ray 2 becomes parallel to M_2. Find the angle θ.

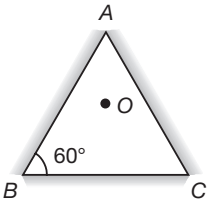


6. Calculate the deviation suffered by an incident ray in the situation shown in figure after it suffers three successive reflections.

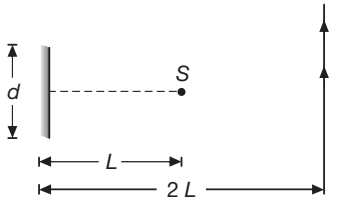


7. Two plane mirrors are placed parallel to each other and 40 cm apart. An object is placed 10 cm from one mirror. Find the distance from the object to the respective image for each of the five images that are closest to the object.

8. Find the number of images formed of an object *O* enclosed by three mirrors *AB*, *BC*, *AC* having equal lengths in situation shown in figure.

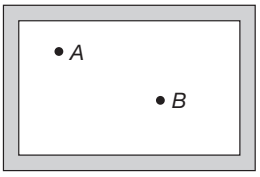


9. A point source of light *S*, placed at a distance *L* in front of the centre of a mirror of width *d*, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 2*L* from it as shown. Find the greatest distance over which he can see the image of the light source in the mirror.

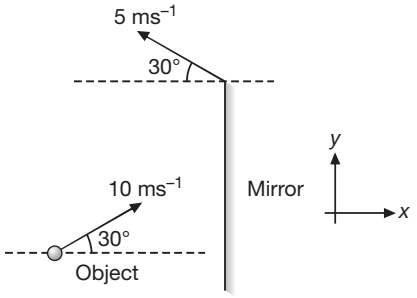


10. Find the smallest size of a looking glass which a man with a face 24 cm × 16 cm should purchase that will enable him to see his whole face completely, if the
(a) man is one eyed.
(b) man is two eyed.
Given that the separation between his eyes is 8 cm.

11. In what direction should *A* beam of light is to be sent from point *A* (shown in figure) contained in a mirror box for it to fall onto point *B* after being reflected once from each of the four walls. If the points *A* and *B* are in one plane perpendicular to the walls of the box (i.e., in the plane of the drawing) then in what direction should the beam be sent from *B* to *A*?



12. The object and the mirror move with velocity shown in figure. Calculate the velocity of the image.



13. A ray of light is incident on an arrangement of two plane mirrors inclined at an angle θ with each other. It suffers two reflections one from each mirror and finally moves in a direction making angle α with the incident ray (α is acute). Find the angle α and show that it is independent of angle of incidence.

14. A ray of light is incident at an angle of 30° with the horizontal. At what angle with horizontal must a plane mirror be placed in its path so that it becomes vertically upwards after reflection?

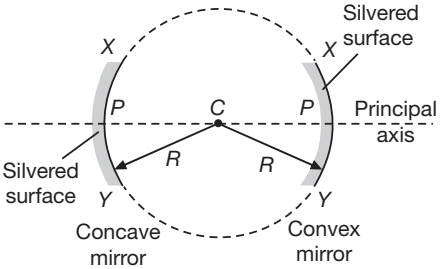
15. Two plane mirrors are inclined to each other at an angle of 70°. A ray is incident on one mirror at an angle θ. The ray reflected from this mirror falls on the second mirror from where it is reflected parallel to the first mirror. Find the value θ.

REFLECTION FROM CURVED SURFACES

A small curved reflecting surface can be considered to be a part of a sphere. Hence, such surfaces are called spherical mirrors. Depending upon the surface silvered, these are of two types—concave and convex, as shown in figure. Some important terms are described below.

(a) Pole or Vertex: Centre P of the surface of the mirror.
(b) Centre of Curvature: Centre C of the sphere.
(c) Radius of Curvature: Radius R of the sphere.
(d) Principal Axis: Line PC, joining the pole and the centre.
(e) Linear Aperture: Distance XY between the extremities of the mirror surface.

Note that since lenses are also made of spherical surfaces, the above terms also apply to lenses, except that the pole is replaced by a new term called as Optical Centre.



Important Terms and Definitions

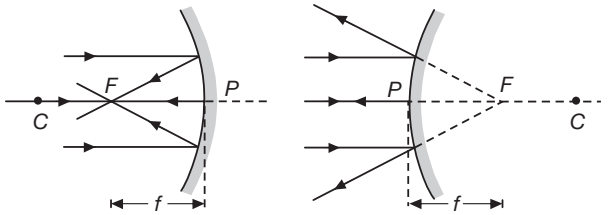
(a) Centre of curvature: It is the centre of the sphere of which the mirror/lens is a part.
(b) Radius of curvature: It is the radius of the sphere of which the mirror/lens is a part.
(c) Pole: It is the geometrical centre of the spherical reflecting surface of which the mirror/lens is a part.
(d) Principal axis (for a spherical mirror): It is the straight line joining the centre of curvature to the pole.
(e) Focus: When a narrow beam of rays of light, parallel to the principal axis and close to it, is incident on the surface of a mirror (lens), the reflected (refracted) beam either converges to a point or appears to diverge from a point on the principal axis. This point is called the focus (F).
(f) Focal length (for a mirror): It is the distance between pole and the principal focus (F).
(g) Real image: If reflected (or refracted) rays converge to a point (i.e. intersect there), then the point is a real image.
(h) Virtual image: If reflected (or refracted) rays appear to diverge from a point, then the point is a virtual image.
(i) Real object: If the incident rays diverge from a point, then the point is a real object.
(j) Virtual object: If incident rays converge and appear to intersect at a point behind the mirror (or lens), then the point is a virtual object.

PARAXIAL RAYS

Paraxial rays are the rays which are either parallel to the principal axis or make small angles with it i.e., these rays are nearly parallel to the principal axis. Our treatment for the spherical mirrors has been restricted to these rays and due to this we shall be considering the **curved mirrors** that have **smaller aperture**. However, for the sake of convenience, comfort and clarity, we shall be drawing the diagrams of larger size.

FOCUS, FOCAL LENGTH AND POWER OF A MIRROR

When a narrow beam of light, parallel to the principal axis and close to it, is incident on the surface of a mirror (lens), the reflected (refracted) beam is found to converge to or appears to diverge from a point on the principal axis. This point is the focus also called Principal Focus in case of mirror(s). The plane passing through the focus and perpendicular to the principal axis is called **focal plane**.



Concave mirror Convex mirror

Focal length (f) is the distance of focus (F) from the pole (P) of the mirror or the optical centre for a lens.

The focal length of a mirror does not change when it is immersed completely in a liquid, i.e. the focal length of the mirror is independent in the medium surrounding it.

Power of a mirror P is defined as the negative reciprocal of the focal length f of the mirror (taken in metre), so

$$P = -\frac{1}{f\,(\text{in metre})}$$

SIGN CONVENTIONS FOR MIRRORS

While solving problems, we must follow a set of sign conventions given for convenience. According to this sign convention

(a) Origin is placed at the pole (P).
(b) All distances are to be measured from the pole (P).
(c) Distances measured in the direction of incident rays are taken as positive.
(d) Distances measured in a direction opposite to that of the incident rays are taken as negative.
(e) Distances above the principal axis are taken as positive.
(f) Distances below the principal axis are taken as negative.
(g) This sign convention is used to find the position and nature (virtual or real, erect or inverted) of the image formed by the mirror (or lens).
(h) Object distance is denoted by u, image distance by v, focal length by f and radius of curvature by R.

(i) Note that generally we keep the object to the left of the mirror (or lens), so that the ray of light starting from object must go from left to the right i.e., towards positive direction of x-axis. Now since the distances have to be measured from the pole consequently,

u must always be negative,

v is positive (for a virtual image) and negative (for a real image).

f is positive (for a convex mirror) and negative (for a concave mirror).

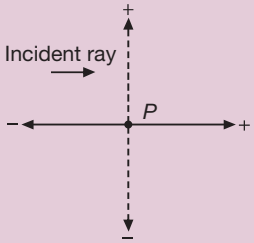
For both the mirrors and lenses.

Magnification for a real image is negative i.e., $m_{\text{real}} = \ominus$

Magnification for a virtual image is positive i.e., $m_{\text{virtual}} = \oplus$

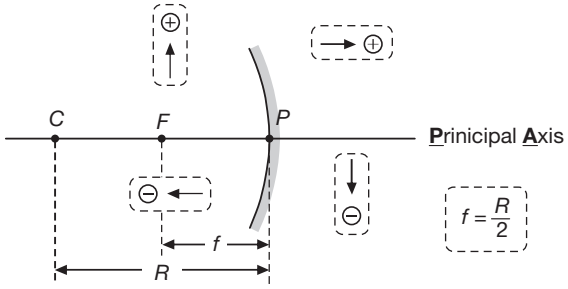
Conceptual Note(s)

The convention that all distances measured along the ray of light are positive and all distances measured opposite to the ray of light are negative matches exactly with the **Cartesian coordinate system**, where we can simply place the origin at the pole P and say that all distances to the left of the pole are negative, all distances to the right of the pole are positive, all distances above the pole are positive and all distances below the pole are negative.

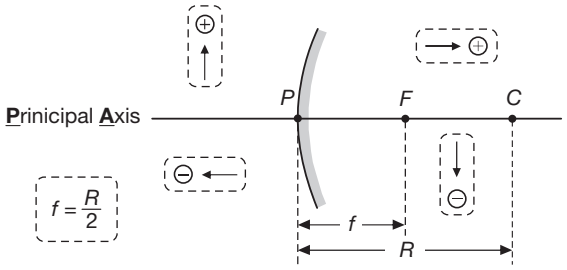


(j) For solving problems in which any of u, v, f (or R) is to be found, we must make sure that no convention should be applied on the quantity to be found. The unknown quantity will automatically take up its sign from which we shall make obvious conclusion.

(k) The diagrams show the application of sign convention to curved mirrors.



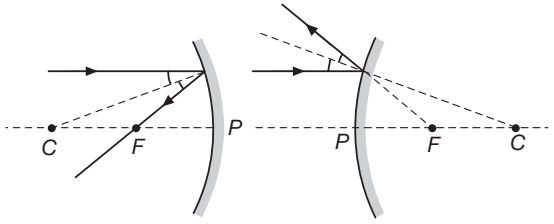
P is pole, **F** is focus and **C** is centre of curvature



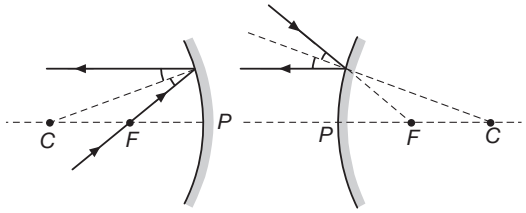
RULES FOR OBTAINING IMAGE BY RAY TRACING

These rules are based on the laws of reflection, i.e. the angle of incidence equals the angle of reflection, $i = r$ and are used to find the location, nature (real or virtual, inverted or erect) and size of the image formed by a spherical mirror. Take any two rays coming from any given point on the object. Find out at which point these rays actually meet (or appear to meet) after reflection from the mirror. This point is the real (or virtual) image. In this way, taking one point after another on the object, the entire image can be constructed.

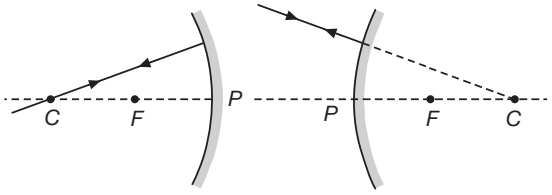
(a) A ray of light **coming parallel to principal axis,** after reflection passes through the focus (in case of concave mirror) or appears to come from the focus (in case of convex mirror).



(b) A ray of light **passing through the focus** (in case of concave mirror) or appearing to pass through the focus (in case convex mirror) is reflected parallel to the principle axis.



(c) A ray of light **passing through the centre** of curvature falls normally on the mirror and is therefore reflected back along the same path i.e., retraces its path.



(d) Incident and reflected rays at the pole of a mirror are symmetrical about the principal axis. (Because for the pole principle axis acts as normal and by Laws of Reflection $i = r$). So by observing the size of erect image in a mirror we can decide the nature of the mirror i.e., whether it is convex, concave or a plane mirror.

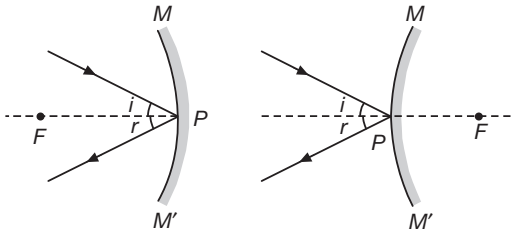
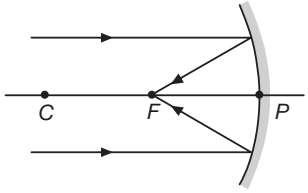
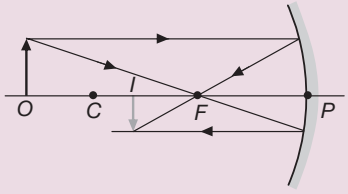
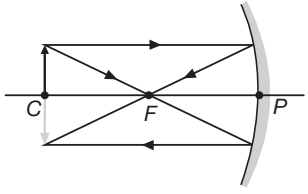


IMAGE FORMATION BY CONCAVE MIRROR

Object Position	Diagram	Position of Image	Nature of Image
At infinity		At the principal focus (F) or in the focal plane	Real, inverted and extremely diminished
Beyond C		Between F and C	Real, inverted and diminished
At C		At C	Real, inverted and of same size as the object

(*Continued*)

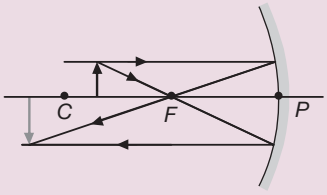
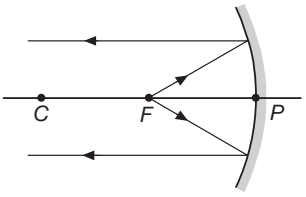
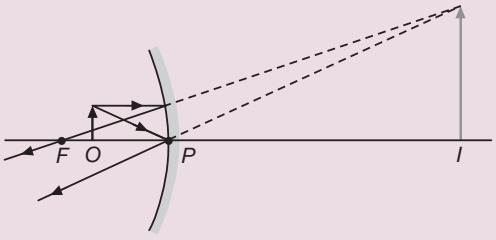
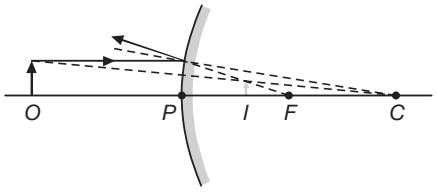
Object Position	Diagram	Position of Image	Nature of Image
Between F and C		Beyond C	Real, inverted and magnified
At F or in the focal plane		At infinity	Real, inverted and highly magnified
Between F and P		Behind the mirror	Virtual, erect and magnified

IMAGE FORMATION BY CONVEX MIRROR

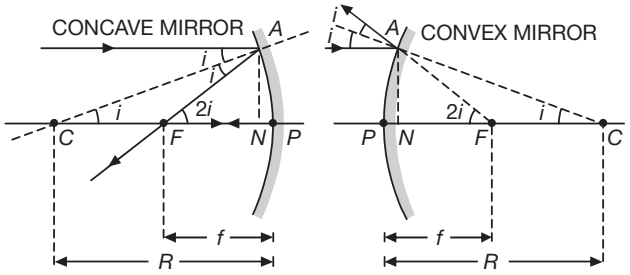
Object Position	Diagram	Position of Image	Nature and Size of Image
For all positions of object		Images formed between the Pole and the focus (F).	Always forms a Virtual, Erect and Diminished Image

RELATION BETWEEN FOCAL LENGTH (f) AND RADIUS OF CURVATURE (R)

A ray parallel to the principal axis passes through the focus (as in concave mirror) or appears to pass through the focus (as in convex mirror). The normal to the mirror(s) at the point of reflection i.e., A must pass through the centre of curvature. In triangle CAN, we have $\tan i = \frac{AN}{NC}$

For paraxial rays and mirrors of small aperture, we have

$$\tan i \cong i = \frac{AN}{NC} \qquad \ldots(1)$$



In triangle FAN, we have

$$\tan(2i) = \frac{AN}{NF}$$

Again for paraxial rays and mirror of small aperture, we have

$$\tan(2i) \cong 2i = \frac{AN}{NF} \quad \ldots(2)$$

From (1) and (2), we get

$$2\left(\frac{AN}{NC}\right) = \frac{AN}{NF}$$

$$\Rightarrow \quad \frac{2}{NC} = \frac{1}{NF} \quad \ldots(3)$$

Since, aperture is small, so N coincides with P, so we have

$$NC \cong PC \text{ and } NF \cong PF$$

For convex mirror, we have

$$PC = +R \text{ and } PF = +f$$

$$\Rightarrow \quad f = \frac{R}{2}$$

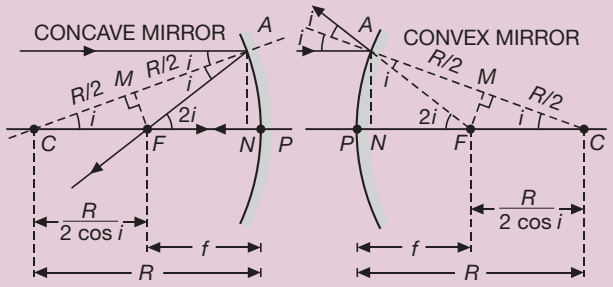
For concave mirror, we have

$$PC = -R \text{ and } PF = -f$$

$$f = \frac{R}{2}$$

So, for a curved mirror of small aperture, focal length is half the radius of curvature.

Conceptual Note(s)



If paraxial rays are not taken into account, then we have

$$f = R - \frac{R}{2\cos i}$$

Since, we see that $CM = MA = \frac{R}{2}$

Also, in triangle CFM, we have

$$\cos i = \frac{R/2}{FC}$$

$$\Rightarrow \quad FC = \frac{R}{2\cos i}$$

Since $PF = PC - FC$

$$\Rightarrow \quad f = R - \frac{R}{2\cos i}$$

For paraxial rays $i \to 0$, so $\cos i \to 1$

$$\Rightarrow \quad f = \frac{R}{2}$$

MIRROR FORMULA

For Concave Mirror

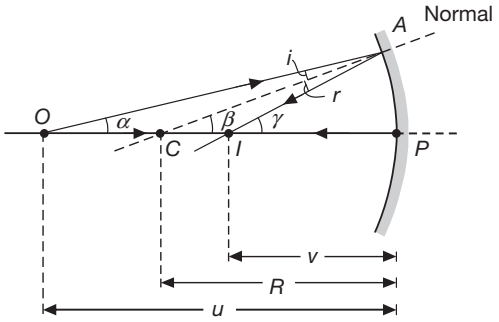
Consider a point object O placed on the principal axis of a concave mirror. A ray of light, incident on the point A at an angle of incidence i on the mirror makes an angle r with the normal as shown in the figure. From the Laws of Reflection we know that $i = r$. Further to find the location of the image let us take another ray along the principal axis so that it hits

the mirror normally at the point P to reverse its path and meet the other ray at I. This point of intersection of the two rays happens to be the place where the image is formed.

Since from geometry we know that in a triangle, external angle equals sum of internal opposite angles, so for triangle CAO and triangle CAI, we have

$$\beta = \alpha + i$$

and $$\gamma = r + \beta$$



Since by Laws of Reflection, we have

$$i = r$$

$$\Rightarrow \quad \alpha + \gamma = 2\beta$$

Applying paraxial ray approximation, we get

$$\tan\alpha \approx \alpha = \frac{AP}{PO},\ \tan\beta \approx \beta = \frac{AP}{PC} \text{ and } \tan\gamma = \gamma = \frac{AP}{PI}$$

$$\Rightarrow \quad \tan\alpha + \tan\gamma = 2\tan\beta$$

$$\Rightarrow \quad \frac{AP}{PO} + \frac{AP}{PI} = 2\left(\frac{AP}{PC}\right)$$

Using sign conventions, we have

$$PO = -u,\ PI = -v \text{ and } PC = -R$$

$$\Rightarrow \quad \frac{1}{(-u)} + \frac{1}{(-v)} = \frac{2}{(-R)}$$

Since we know that $f = \frac{R}{2}$, so we get

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R} = \frac{1}{f} \qquad \{\text{Mirror Formula}\}$$

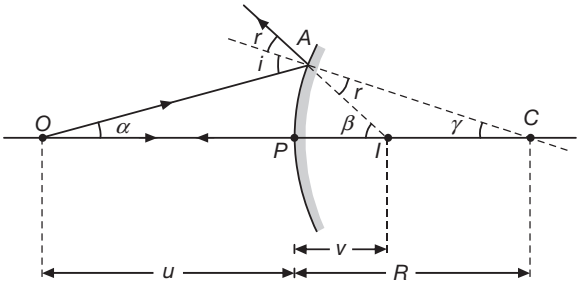
FOR CONVEX MIRROR

Similarly we can drive a formula for a convex mirror.

Since from geometry we know that in a triangle, external angle equals sum of internal opposite angles, so for triangle CAO and triangle CAI, we have

$$i = \alpha + \gamma$$

and $$\beta = r + \gamma$$



Since $i = r$, so we get

$$-\alpha + \beta = 2\gamma$$

Applying paraxial ray approximation, we get

$$-\tan\alpha + \tan\beta = 2\tan\gamma$$

$$\Rightarrow \quad -\frac{AP}{PO} + \frac{AP}{PI} = 2\left(\frac{AP}{PC}\right)$$

Using sign conventions, we have

$$PO = -u,\ PI = +v \text{ and } PC = +R$$

$$\Rightarrow \quad -\frac{1}{(-u)} + \frac{1}{v} = \frac{2}{R}$$

$$\Rightarrow \quad \frac{1}{u} + \frac{1}{v} = \frac{2}{R} = \frac{1}{f} \qquad \{\text{Mirror Formula}\}$$

Interestingly, the mirror formula is the same irrespective of the mirror used.

NEWTON'S FORMULA

If instead of measuring the object distance and the image distance from the pole, the distances are measured from the focus, then we get a modified mirror formula. This modified mirror formula is called the **Newton's Formula**. Let

x_1 be the distance of object from focus and

x_2 be the distance of image from focus, then

$$u = f + x_1$$

and $$v = f + x_2$$

According to the mirror formula, we have

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{(f+x_1)}+\frac{1}{(f+x_2)}=\frac{1}{f}$$

$$\Rightarrow \quad (2f+x_1+x_2)f=(f+x_1)(f+x_2)$$

$$\Rightarrow \quad 2f^2+(x_1+x_2)f=f^2+(x_1+x_2)f+x_1x_2$$

$$\Rightarrow \quad x_1x_2=f^2$$

This is known as Newton's formula.
This formula is applicable to real object and real images.

LINEAR MAGNIFICATION OR LATERAL MAGNIFICATION OR TRANSVERSE MAGNIFICATION

To have an idea of the relative size of the image and the object, we define linear magnification also called as lateral magnification as

$$m=\frac{\text{size of the image}}{\text{size of the object}}=\frac{h_2}{h_1}=\frac{h_i}{h_0}$$

For both concave and convex mirrors, it can be shown that

$$m=-\frac{v}{u}$$

Since we know that $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ so we get

$$m=-\frac{v}{u}=\frac{f}{f-u}=\frac{f-v}{f}$$

For spherical mirrors positive value of m means v and u are having opposite signs i.e. when u is negative v is positive and vice versa.

So for a real object if the image formed is virtual, erect and three times the size of the real object then, we have $m = +3$.

Similarly for a real object if the image formed is real, inverted and one third the size of the real object then $m=-\frac{1}{3}$.

ILLUSTRATION 15

An object is placed at a distance of 15 cm from a concave mirror of focal length 10 cm. Describe the size, nature and position of the image formed.

SOLUTION

The rough figure indicating the pole of the mirror, focus, and the given distances is shown. The sign convention is also given.

Since, $u=-15$ cm (negative since it lies to the left of O)

$f=-10$ cm (negative since it lies to the left of O)

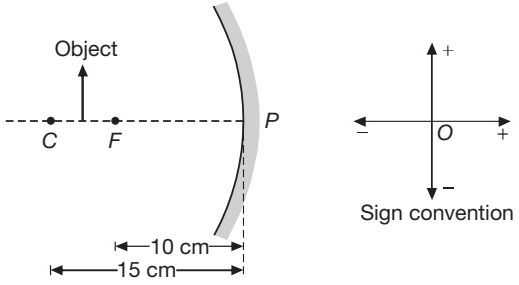
Since we have, from mirror formula that

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{(-10)}-\frac{1}{(-15)}=-\frac{1}{10}+\frac{1}{15}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{-15+10}{150}=-\frac{5}{150}$$

$$\Rightarrow \quad v=-\frac{150}{5}=-30 \text{ cm}$$



The negative sign for v shows that the image lies to the left of O.
Now, the magnification is given by

$$m=-\frac{v}{u}=-\frac{-30}{-15}=-2$$

The negative sign for m indicates that the image is inverted, and hence real and is double the size of the object.

Thus, we find that the image is real, inverted, twice the size of the object, and is formed 30 cm in front of the mirror. The ray diagram is shown in figure.

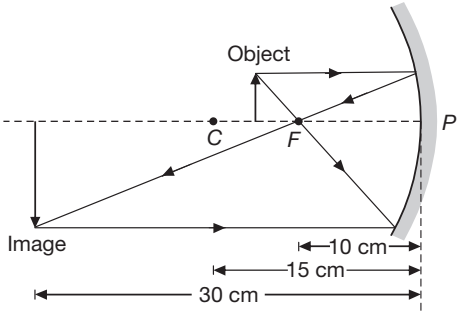


ILLUSTRATION 16

A beam of light converges towards a point O, behind a convex mirror of focal length 20 cm. Find the nature and position of image if the point O is

(a) **10** cm behind the mirror

(b) **30** cm behind the mirror

SOLUTION

(a) Here, in this case the object is virtual. So, for this we have

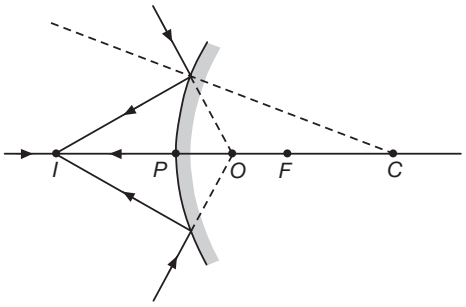
$u = +10$ cm, $f = +20$ cm

Using mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{20}-\frac{1}{+10}=\frac{1-2}{20}=\frac{-1}{20}$$

$\Rightarrow \quad v = -20$ cm

Magnification, $m=-\frac{v}{u}=-\frac{(-20)}{10}=2$



Hence, the image formed will be real, erect and enlarged, and at a distance of 20 cm in front of the mirror.

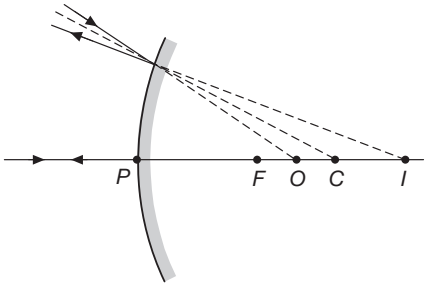
(b) Again, in this case too the object is virtual. So, we have

$u = +30$ cm, $f = +20$ cm

Using mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{20}-\frac{1}{30}=\frac{3-2}{60}=\frac{1}{60}$$

$\Rightarrow \quad v = 60$ cm



Magnification, $m=-\frac{v}{u}=-\frac{60}{30}=-2$

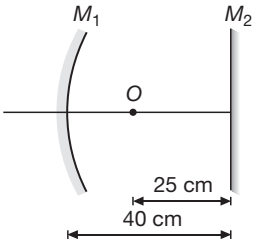
Hence, the image formed will be virtual, inverted and enlarged, and at a distance of 60 cm behind the mirror.

Conceptual Note(s)

Note that for the real objects, a convex mirror always gives virtual and diminished image, but for virtual objects it gives real image if $u < f$ and virtual image if $u > f$.

ILLUSTRATION 17

A concave mirror M_1 of radius of curvature 20 cm and a plane mirror M_2 are placed 40 cm apart as shown. An object O is placed 25 cm in front of the plane mirror. Find the position of final image formed after three successive reflections, assuming that the first reflection takes place from the curved mirror.



SOLUTION

Since $R = -20$ cm, so focal length of the mirror is given by $f=\frac{R}{2}=-10$ cm

For 1st reflection at M_1

$$u_1 = -15 \text{ cm}, \; v_1 = ?, \; f = -10 \text{ cm}$$

Since, $\frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f}$

$\Rightarrow \quad v_1 = -30$ cm

For 2nd reflection at plane mirror M_2

$$u_2 = -(40 - 30) = -10 \text{ cm}$$

Since, for a plane mirror the image distance from the plane mirror behind it is equal to the object distance from the plane mirror.

$\Rightarrow \quad v_2 = 10$ cm

For 3rd reflection at the curved mirror M_2

$$u_3 = -(40 + 10) = -50 \text{ cm}, \; v_3 = ?, \; f = -10 \text{ cm}$$

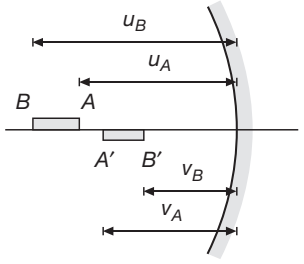
Since, $\frac{1}{v_3} + \frac{1}{u_3} = \frac{1}{f}$

$\Rightarrow \quad v = -12.5$ cm

LONGITUDINAL MAGNIFICATION OR AXIAL MAGNIFICATION

When an object of finite length is placed along the principal axis, then instead of defining the linear magnification we define the axial magnification. Mathematically we define axial magnification, for small objects as

$$m_{\text{axial}} = \frac{\text{Size of image along principal axis}}{\text{Size of object along principal axis}}$$



$$m_{\text{axial}} = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) = -\frac{\Delta v}{\Delta u}$$

where, u_1, u_2 and v_1, v_2 are the respective object and image distances of the ends of the object from the pole, such that

$$\frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f} \text{ and } \frac{1}{v_2} + \frac{1}{u_2} = \frac{1}{f}$$

However, if the object has infinitesimal size du and the corresponding image size is dv, then we have

$$m_{\text{axial}} = -\frac{dv}{du}$$

Further since we know that $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ i.e., $v^{-1} + u^{-1} = f^{-1}$. Taking the derivative of this equation with respect to u, we get

$$\frac{d}{du}\left(v^{-1}\right) + \frac{d}{du}\left(u^{-1}\right) = \frac{d}{du}\left(f^{-1}\right)$$

$$\Rightarrow \quad -v^{-2}\frac{dv}{du} - u^{-2} = 0$$

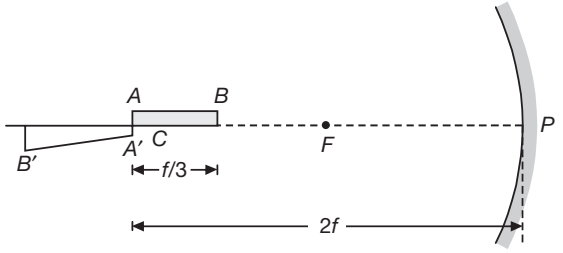
$$\Rightarrow \quad m_{\text{axial}} = \frac{dv}{du} = -\frac{v^2}{u^2} = -\left(\frac{f}{f-u}\right)^2 = -\left(\frac{f-v}{f}\right)^2$$

ILLUSTRATION 18

A thin rod of length $\frac{f}{3}$ is placed along the principal axis of a concave mirror of focal length f such that its image, which is real and elongated, just touches the rod. What is the magnification ?

SOLUTION

According to the problem, the image is real and enlarged, the object must have been placed between C and F. Since one end of the image just touches one end of the object so, this end must lie on C. Let AB be the object and $A'B'$ be its image, such that A and A' both lie at C, as shown in figure.



Now, as the length of the object AB is $\frac{f}{3}$, so the distance of end B of the object from the pole P is

$$u_B = -\left(PA - \frac{f}{3}\right) = -\left(2f - \frac{f}{3}\right) = -\left(\frac{5}{3}\right)f$$

The distance of the image of end B, v_B, is calculated by using the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v_B} + \frac{1}{-\frac{5}{3}f} = \frac{1}{f}$$

$$\Rightarrow \quad v_B = -\frac{5}{2}f$$

Therefore, the size of the image $A'B'$ is

$$A'B' = |v_B| - |v_A| = \frac{5}{2}f - 2f = \frac{1}{2}f$$

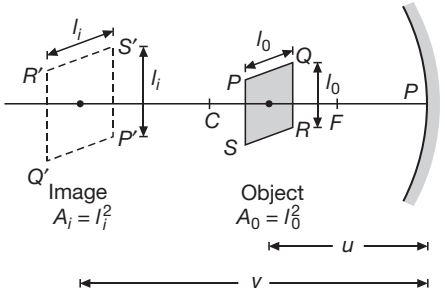
Now, magnification, $m = -\frac{A'B'}{AB} = -\frac{\left(\frac{f}{2}\right)}{\left(\frac{f}{3}\right)} = -\frac{3}{2}$

SUPERFICIAL OR AREAL MAGNIFICATION BY A SPHERICAL MIRROR

Let us find the magnification in area for the image produced by a spherical mirror. There are two cases in which area of image is calculated. In both of these cases, the object size is considered very small compared to the distance of object from the mirror.

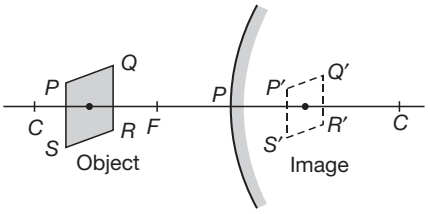
CASE-1: Object is Placed Normal to Principal Axis

Consider a square object $PQRS$ of area $A_0 = l_0^2$ which is placed at a point between F and C of a concave mirror at a distance u from the mirror. The image is produced as $P'Q'R'S'$ at a distance v from the mirror as shown. The image will also be of square shape because both edges of the object are perpendicular to principal axis of mirror. So, for both the edges we use the concept of lateral magnification and hence size of both will be same.



(For concave mirror)

A similar treatment can also be extended to an object placed in front of a convex mirror as shown.



(For convex mirror)

The image edge length l_i can be obtained by using the concept of lateral magnification.

Since $m = \frac{h_i}{h_0} = \frac{l_i}{l_0}$

$$\Rightarrow \quad l_i = ml_0$$

for both the edges. So final image produced for a concave mirror is a magnified square real image and similar to this for convex mirror, the final image will be a diminished square virtual image.
The area of image produced is given by

$$A_i = l_i^2 = m^2 l_0^2 = m^2 A_0$$

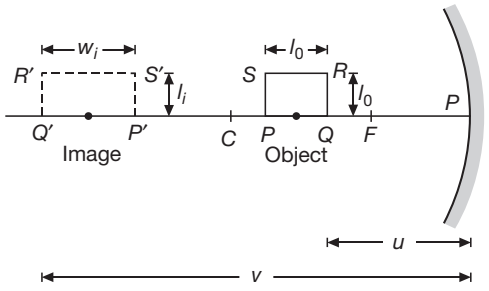
So, in this arrangement we can simply define areal magnification as the ratio of area of image to the area of object and hence

$$m_{\text{areal}} = \frac{\text{Area of image}}{\text{Area of object}} = \frac{A_I}{A_O} = \frac{v^2}{u^2}$$

$$\Rightarrow \quad m_{\text{areal}} = \left(\frac{f}{f-u}\right)^2 = \left(\frac{f-v}{f}\right)^2$$

CASE-2: Object is Placed with One Edge on the Principal Axis

Consider a small square object $PQRS$ of edge $l_0\,(l_0 \ll f)$ area $A_0 = l_0^2$, which is placed at a point between F and C of a concave mirror at a distance u from the mirror. The image $A'B'C'D'$ is produced at a distance v from the mirror as shown.



(For concave mirror)

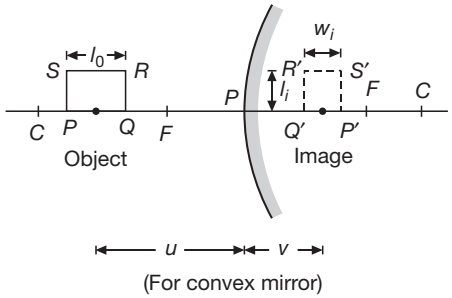
In this case the image produced will be in shape of a rectangle, because for the edges QR and PS of the object (which are perpendicular to the principal axis) we use the concept of lateral magnification and for the edges PQ and RS (which are along the principal axis) we use the concept of longitudinal magnification.

So in this case, the image height l_i and width w_i are given by

$$l_i = ml_0 \text{ and}$$

$$w_i = m^2 l_0$$

where m is the lateral magnification given as $m = -\dfrac{v}{u}$



(For convex mirror)

RELATION BETWEEN OBJECT AND IMAGE VELOCITY FOR CURVED MIRRORS

According to the mirror formula, we have

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

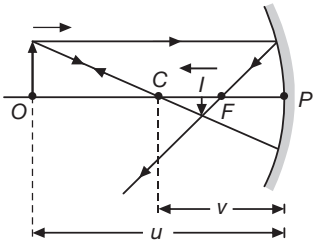
$$\Rightarrow \quad v^{-1} + u^{-1} = f^{-1} = \text{ constant}$$

Differentiating with respect to time, we get

$$-v^{-2}\frac{dv}{dt} - u^{-2}\frac{du}{dt} = 0$$

$$\Rightarrow \quad \frac{dv}{dt} = -\left(\frac{v^2}{u^2}\right)\frac{du}{dt} \qquad \ldots(1)$$

Here $\dfrac{du}{dt}$ is the rate at which the object distance u is changing i.e., it is the object speed if the mirror is stationary. Similarly, $\dfrac{dv}{dt}$ is the rate at which v (distance between image and mirror) is changing i.e., it is image speed if the mirror is stationary. So if at a known values of v and u, the object speed is given, we can find the image speed from the above formula.



Let us take the example for a concave mirror.

Suppose the object is moved from infinity towards focus, then since u is decreasing therefore,

$$-\left(\frac{du}{dt}\right) = \text{rate of decrease of } u \qquad \{\text{object speed}\}$$

$$\Rightarrow \quad \left(\frac{dv}{dt}\right) = \text{rate of increase of } v \qquad \{\text{image speed}\}$$

Further, when the object lies between ∞ and C, then $v < u$,

$$\Rightarrow \quad \left(\frac{dv}{dt}\right) < \left(-\frac{du}{dt}\right) \qquad \{\text{from equation (1)}\}$$

Hence, when the object is moved towards the mirror, its image (which is real) will recede from the mirror with speed less than the speed of object.

When the object is at C, image is also at C

$$\Rightarrow \quad v = u$$

$$\Rightarrow \quad \left(\frac{dv}{dt}\right) = \left(-\frac{du}{dt}\right)$$

Hence, when the object is at C speed of image is equal to the speed of object.

When the object lies between C and F then $v > u$ so, the image speed is more than the object speed.

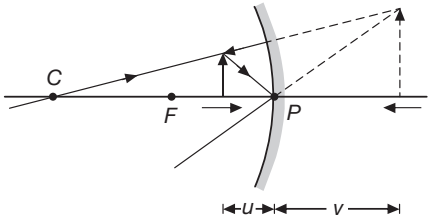
When the object lies between F and P, then the image becomes virtual i.e., u and f are negative while v is positive. So from mirror formula we get,

$$\frac{1}{v} + \frac{1}{(-u)} = \frac{1}{(-f)} \quad \Rightarrow \quad \frac{1}{u} - \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \quad -u^{-2}\left(\frac{du}{dt}\right) - v^{-2}\left(\frac{dv}{dt}\right) = 0$$

$$\left(-\frac{dv}{dt}\right) = \left(\frac{v^2}{u^2}\right)\left(-\frac{du}{dt}\right)$$

Now, when u is further decreased, v also decreases to keep $\frac{1}{f}$ constant. So, $-\frac{du}{dt}$ is the rate at which object is approaching towards mirror and $\left(-\frac{dv}{dt}\right)$ is rate at which the image is approaching towards the mirror.



Further in this case we observe that the image is always enlarged i.e., $v > u$. Therefore, image speed is more than the object speed. Thus, the above entire discussion can simply be concluded as follows.

CONCLUSION

(a) When an object is moved from $-\infty$ to F, the image (real) moves from F to $-\infty$ and then when the object is further moved from F to P image (now virtual) moves from $+\infty$ to P.

(b) Therefore for a real image formed by a curved mirror we have

$$\frac{dv}{dt} = -\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$$

and for a virtual image formed by a curved mirror we have

$$\frac{dv}{dt} = +\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$$

(c) When the object is either at centre of curvature C or at pole P, the two speeds are equal. However, when the object is at pole, then due to the small aperture of the mirror, it appears as if the image is being formed by a plane mirror.

ILLUSTRATION 19

An object approaches a convex mirror of focal length 25 cm with speed 10 ms^{-1}. Calculate the velocity of the image when object is 25 cm from the mirror?

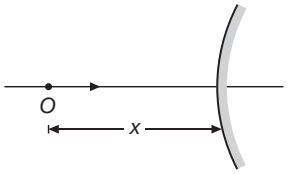
SOLUTION

Let at any instant t, the object be at a distance x from mirror and is moving towards it. Then,

$$u = -x$$

$$f = +25 \text{ cm}$$

Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$



Let the image be formed at a distance y from pole of mirror, then

$$\frac{1}{y} = \frac{1}{25} + \frac{1}{x}$$

$$\Rightarrow \quad y = \frac{25x}{25+x}$$

Differentiating both sides w.r.t. time, we get

$$\frac{dy}{dt} = \frac{625}{(25+x)^2}\frac{dx}{dt}$$

Since object is approaching towards mirror, x decreases as t increases

$$\Rightarrow \quad \frac{dx}{dt} = -10 \text{ ms}^{-1}$$

$$\Rightarrow \quad \frac{dy}{dt} = \frac{625}{(25+25)^2}(-10)$$

$$\Rightarrow \quad \frac{dy}{dt} = -2.5 \text{ ms}^{-1}$$

So velocity of image has a magnitude 2.5 ms^{-1} and negative sign indicates that y will be decreasing as t increases i.e., image is moving towards pole of mirror.

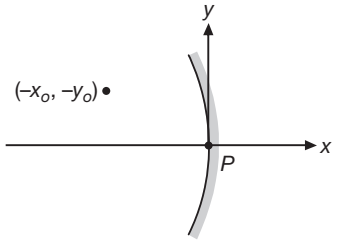
FINDING COORDINATES OF IMAGE OF A POINT

If the coordinates of a point object $(-x_O, -y_O)$ with respect to the coordinate axes shown in figure are known to us and the coordinates of image be (x_i, y_i) then for finding the x-coordinate, we use the mirror formula, according to which

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{x_i}+\frac{1}{x_O}=\frac{1}{f} \qquad \{\because\ u = -(-x_O) = x_O\}$$

$$\Rightarrow \quad x_i = \frac{fx_O}{x_O - f}$$



For finding the y-coordinate, we apply the concept of magnification (m), according to which, we have

$$m = \frac{y_i}{y_O} = -\frac{v}{u} = -\frac{x_i}{x_O}$$

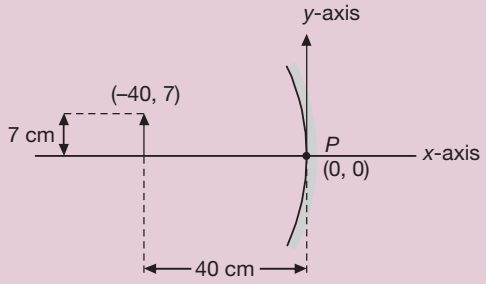
$$\Rightarrow \quad y_i = \frac{fy_O}{f - x_O}$$

EXAMPLE

A point object is placed at (–40, 7) cm in front of a concave mirror of focal length 5 cm having its pole at origin (0, 0). Assuming the principal axis to be along x-axis, find the position of the image formed.

SOLUTION

The situation discussed in the problem is shown in figure.



Since, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v}+\frac{1}{(-40)}=\frac{1}{(-5)}$$

$$\Rightarrow \quad v = -\frac{40}{7} \text{ cm}$$

Since, $m = \frac{h_I}{h_O} = -\frac{v}{u}$

$$\Rightarrow \quad \frac{h_I}{h_O} = -\frac{\left(-\frac{40}{7}\right)}{-40}$$

$$\Rightarrow \quad \frac{h_I}{h_O} = -\frac{1}{7}$$

But $h_O = 7$ cm

$$\Rightarrow \quad h_I = -1 \text{ cm}$$

So, image coordinates are $\left(-\frac{40}{7}, -1\right)$ cm

Problem Solving Technique(s)

(a) Place the object to the left of the mirror (or lens), so that sign convention matches with the familiar sign convention in the coordinate geometry.

(b) Both for concave as well as convex mirrors, use the same mirror formula i.e.

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \text{and} \quad m=-\frac{v}{u}$$

(c) Substitute the numerical values of the given quantities with proper sign (+ve or –ve) as per sign convention.

(d) Though the SI unit of distance is metre, it may be more convenient in some problems to take the given distances in cm rather than in m. But then your answer too will be in cm.

(e) Do not give any sign to the quantity to be determined. In your answer, the unknown quantity will be obtained with its proper sign.

In addition to the above hints, if you remember the following facts, it will help you.

(a) Since the object is generally placed to the left of the mirror so, u is negative.

(b) For a concave mirror, f is negative.

(c) For a convex mirror, f is positive.

(d) A real image is formed in front of the mirror, so for a real image v is negative.

(e) A virtual image is formed behind the mirror, so for a virtual image v is positive.

(f) A real image is always inverted, so for a real image h is negative.

(g) A virtual image is always erect, so for a virtual image h is positive.

(h) For the real image of a real object and the virtual image of a virtual object, m is negative.

(i) For the virtual image of a real object and the real image of a virtual object, m is positive.

GRAPH OF $\frac{1}{v}$ VERSUS $\frac{1}{u}$

Let us first take the case of a concave mirror. Here, two cases are possible.

CASE-1: When the Image formed is Real.

When the image is real, i.e., object lies between F and infinity. In such a situation u, v and f are negative. Hence, the mirror formula i.e., $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ becomes

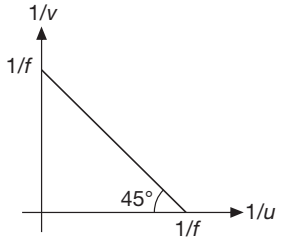
$$-\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}=-\frac{1}{u}+\frac{1}{f}$$

Comparing with $y=mx+c$, the desired graph will be a straight line with slope -1 and intercept on y-axis is equal to $\frac{1}{f}$.

Do not confuse here, the slope m with magnification.

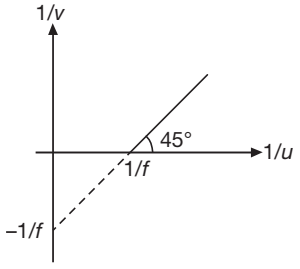


CASE-2: When the Image formed is Virtual.

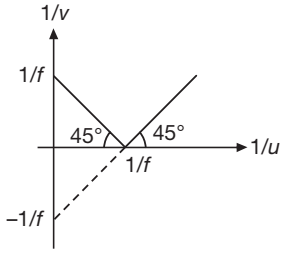
When the image is virtual, i.e., object lies between F and P. Under such situation u and f are negative while v is positive. The mirror formula thus becomes

$$\frac{1}{v}=\frac{1}{u}-\frac{1}{f}$$

Comparing it with $y=mx+c$ the desired graph is a straight line with slope $m=1$ and intercept on y-axis is equal to $-\frac{1}{f}$.



The graph is thus shown in figure. The two graphs can be drawn in one single graph as in figure.



Conceptual Note(s)

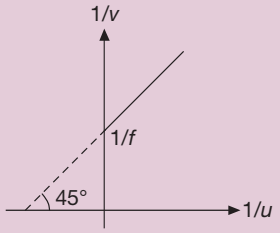
Please note that $\frac{1}{u}$ and $\frac{1}{v}$ are actually the magnitudes of $\frac{1}{u}$ and $\frac{1}{v}$ (i.e., without sign)

For a convex mirror, the image formed is always virtual, i.e., u is always negative while v and f are always positive. Hence, the mirror formula becomes,

$$\frac{1}{v}+\frac{1}{(-u)}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{u}+\frac{1}{f}$$

Comparing with $y=mx+c$, the desired graph is a straight line of slope $m = 1$ and intercept on y-axis equal to $\frac{1}{f}$. The graph is thus shown in figure.



GRAPH OF *v* VERSUS *u*

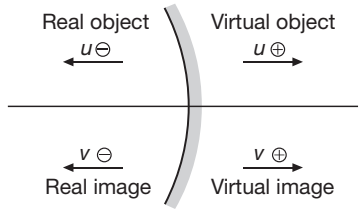
For Concave Mirror

For a spherical mirror, we have $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad v=\frac{fu}{u-f}$$

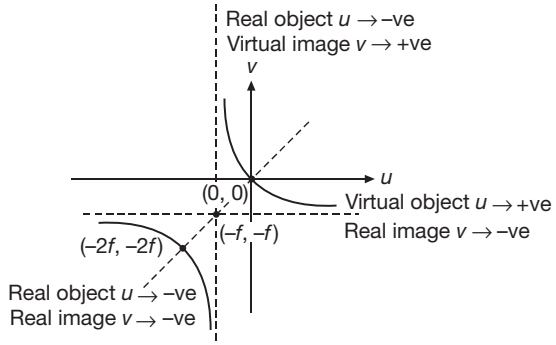
For concave mirror of focal length f, we have $f=-f$

$$\Rightarrow \quad v=-\frac{fu}{u+f} \qquad \ldots(1)$$



Substituting the following values of u in equation (1) to get the corresponding values of v for purpose of plotting the u-v graph.

u	$-\infty$	$-2f$	$-f$	0	$+f$	$+2f$	$+\infty$
v	$-f$	$-2f$	$\pm\infty$	0	$-\frac{f}{2}$	$-\frac{2f}{3}$	$+f$

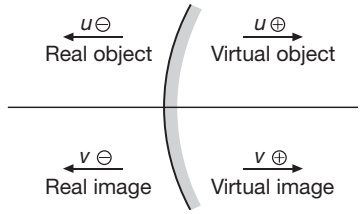


For Convex Mirror

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

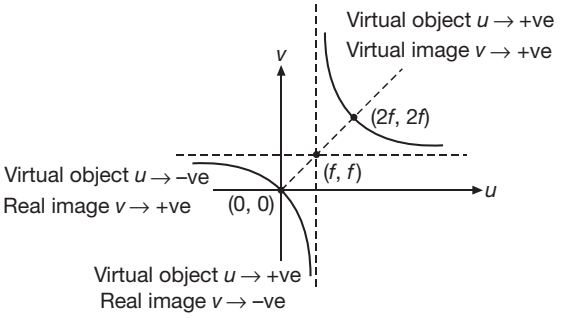
For concave mirror of focal length f, we have $f=+f$

$$\Rightarrow \quad v=\frac{fu}{u-f} \qquad \ldots(1)$$



Substituting the following values of u in equation (1) to get the corresponding values of v for purpose of plotting the u-v graph.

u	$-\infty$	$-2f$	$-f$	0	$+f$	$+2f$	$+\infty$
v	$+f$	$-\frac{2f}{3}$	$-\frac{f}{2}$	0	$\pm\infty$	$+2f$	$+f$



Problem Solving Technique(s)

(a) As focal-length of a spherical mirror $f = \frac{R}{2}$ depends only on the radius of mirror and is independent of wavelength of light and refractive index of medium so the focal length of a spherical mirror in air or water and for red or blue light is same. This is also why the image formed by mirrors do not show chromatic aberration.

(b) In case of spherical mirror if $R \to \infty$ (i.e., it becomes plane), so, $f = \frac{R}{2} \to \infty$. The mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ reduces to

$$\frac{1}{v}+\frac{1}{u}=0 \text{ i.e., } v=-u$$

i.e., image is at same distance behind the mirror as the object is in front of it. This in turn verifies the correctness of mirror formula.

(c) Every part of a mirror forms complete image. If some portion of a mirror is obstructed (say covered with black paper), then complete image will be formed but intensity will be reduced.

(d) In case of concave spherical mirrors if a real object is placed at a distance x_1 from the focus and a real image is formed at a distance x_2 from the focus (instead of pole), then

$$u=-(f+x_1) \text{ and } v=-(f+x_2)$$

Since, we know that $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

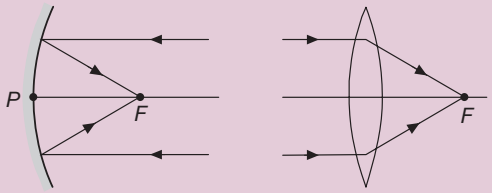
$$\Rightarrow \quad \frac{1}{-(f+x_2)}+\frac{1}{-(f+x_1)}=\frac{1}{-f}$$

$$\Rightarrow \quad x_1x_2=f^2$$

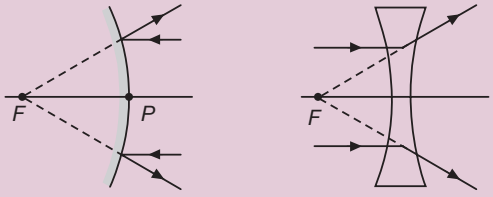
This result is called 'Newton's formula'.

(e) If an object is moved at constant speed towards a concave mirror from infinity to focus, the image will move (slower in the beginning and faster later on) away from the mirror. This is because, during the time the object moves from ∞ to C the image will move from F to C and when the object moves from C to F the image will move from C to ∞. At C the speed of object and image will be equal.

(f) Concave mirror behaves as convex lens (both convergent) while convex mirror behaves as concave lens (both divergent). This is shown in figure.



(a) Convergent behaviour

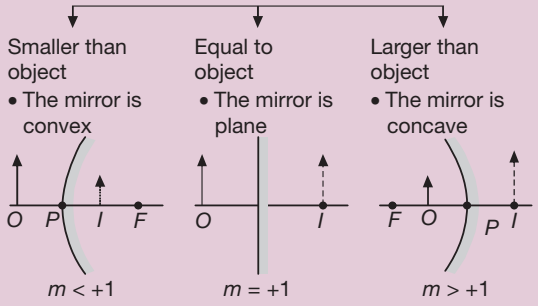


(b) Divergent behaviour

(g) As convex mirror gives erect, virtual and diminished image, field of view is increased. This is why it is used as rear-view mirror in vehicles. Concave mirrors give enlarged erect and virtual image (if object is between F and P) so are used

by dentists for examining teeth. Further due to their converging property concave mirrors are also used as reflectors in automobiles head lights and search lights and by ENT surgeons in ophthalmoscope.

(h) For real extended objects, if the image formed by a single mirror is erect it is always virtual and in this situation if the size of the image is



(i) For real extended objects, if the image formed by a single mirror is inverted, it is always real (i.e., m is –ve) and the mirror is concave. In this situation if the size of image is

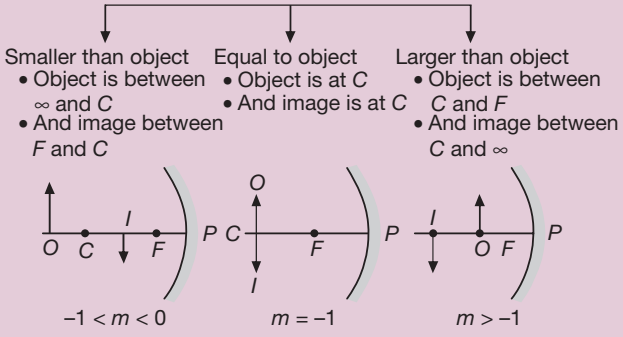

Smaller than object
• Object is between ∞ and C
• And image between F and C
Equal to object
• Object is at C
• And image is at C
Larger than object
• Object is between C and F
• And image between C and ∞
O C I F P
O I F P
I O F P
–1 < m < 0
m = –1
m > –1

ILLUSTRATION 20

A gun of mass m_1 fires a bullet of mass m_2 with a horizontal speed v_0. The gun is fitted with a concave mirror of focal length f facing towards the receding bullet. Find the speed of separations of the bullet and the image at the instant just after the bullet is fired from the gun.

SOLUTION

Let v_1 be the speed of gun (or mirror) just after the firing of bullet. By Law of Conservation of Linear Momentum, we have

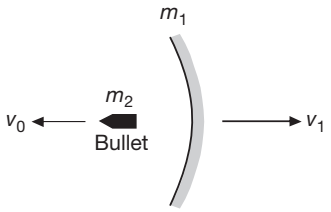
$$m_2 v_0 = m_1 v_1$$

$$\Rightarrow \quad v_1 = \frac{m_2 v_0}{m_1} \qquad \ldots(1)$$

Now, $\frac{du}{dt}$ is the rate at which distance between mirror and bullet is increasing, so

$$\frac{du}{dt} = v_1 + v_0 \qquad \ldots(2)$$

Since, we know that $\left|\frac{dv}{dt}\right| = \left(\frac{f}{f-u}\right)^2 \left|\frac{du}{dt}\right|$ $\ldots(3)$



Since, at the instant just after the bullet is fired from the gun, the bullet is actually very close to the pole of the mirror, so $u \to 0$ and hence we get at that instant

$$\frac{v^2}{u^2} = \left(\frac{f}{f-u}\right)^2 = m^2 = 1$$

So, from (2) and (3), we get

$$\frac{dv}{dt} = \frac{du}{dt} = v_1 + v_0 \qquad \ldots(4)$$

where $\frac{dv}{dt}$ is the rate at which distance between image (of bullet) and mirror is increasing.

Therefore, speed of separation of bullet and image will be,

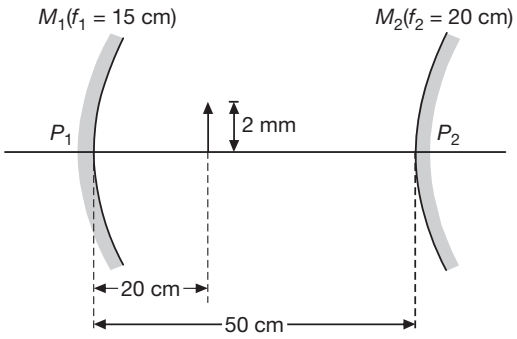
$$v_r = 2(v_1 + v_0)$$

Substituting value of v_1 from equation (1) we get

$$v_r = 2\left(1 + \frac{m_2}{m_1}\right) v_0$$

ILLUSTRATION 21

Find the location, size and the nature of the image of an object of height 2 mm kept between two mirrors (as shown in figure) after two successive reflections, considering the first reflection at the concave mirror and then at the convex mirror.



SOLUTION

As asked in the problem, let us first consider the reflection at mirror M_1. Before executing the mirror formula, we must keep two things in mind.

1. The incident light must go from the object to the mirror and we preferably take it parallel to the principal axis.
2. All distances have to be measured from the pole of the respective mirror for which reflection is being considered.
3. All distances measured along the incident ray are positive and all distances measured opposite to the incident ray are negative.

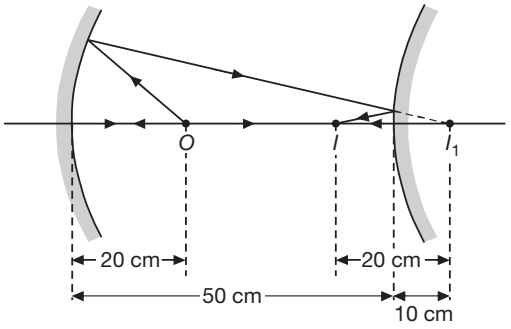


Figure is just representative and not to scale

Now, for reflection at concave mirror M_1, the incident ray from the object goes to left of object and object distance is measured towards right of pole P_1, so

$$u = -20 \text{ cm}$$

Similarly, $f_1 = -15$ cm

Now, according to mirror formula, we have

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}+\frac{1}{(-20)}=\frac{1}{(-15)}$$

$$\Rightarrow \quad v = -60 \text{ cm}$$

Negative sign with v means that it is formed to right of pole P_1 at a distance of 60 cm from P_1 (10 cm behind M_2).

$$\Rightarrow \quad m_1 = -\frac{v}{u} = -\frac{(-60)}{-20} = -3$$

So, image (I_1) formed is real, inverted and three times size of object i.e., 6 mm.

This image (I_1) formed now acts as object for the convex mirror. Further, this image formed is 10 cm to the left of P_2 and the incident ray from the original object goes to the right for reflection at M_2 to take place, so

$$u = +10 \text{ cm}$$

Similarly, $f = +20$ cm

Applying the mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{v}+\frac{1}{10}=\frac{1}{20}$$

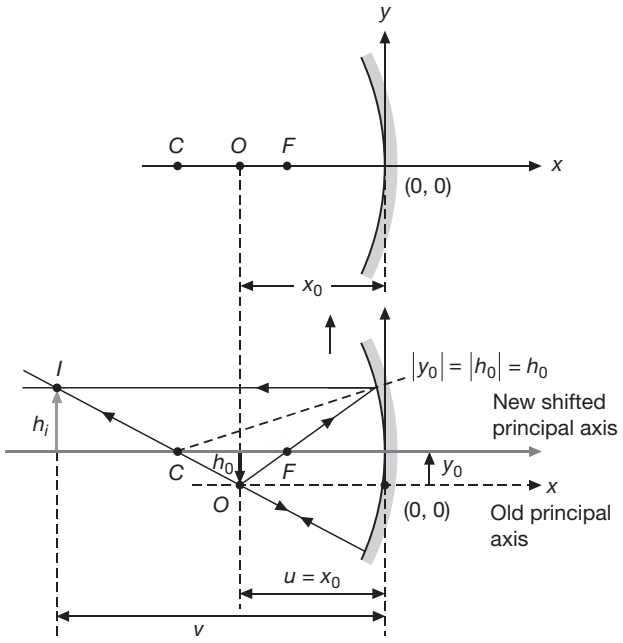
$$\Rightarrow \quad v = -20 \text{ cm}$$

$$\Rightarrow \quad m_2 = -\frac{v}{u} = -\frac{(-20)}{10} = 2$$

So, image (I) formed is virtual, erect and two times the size of object (here I_1). Hence the size of I is 12 mm. So, finally I is formed at 20 cm in front of convex mirror M_2, with size 12 mm, virtual and erect.

EFFECT OF SHIFTING THE PRINCIPAL AXIS OF A SPHERICAL MIRROR

Let a point object O, be placed on the principal axis of a concave mirror as shown. Correspondingly, the image of this point object will also suitably lie on the principal axis. Let the mirror be now displaced by a small distance y_0 (say) perpendicular to the principal axis. Then, obviously the principal axis also shifts along the mirror by a distance y_0.



To find the location of image, we use mirror formula.

So we use $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{x_i}-\frac{1}{(-x_0)}=\frac{1}{-f}$$

$$\Rightarrow \quad \frac{1}{x_i}=-\left(\frac{1}{x_0}+\frac{1}{f}\right)$$

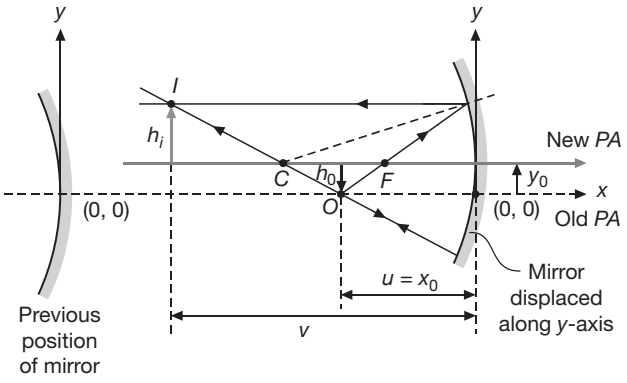
$$\Rightarrow \quad x_i=-\left(\frac{x_0 f}{x_0+f}\right)$$

However, we must note that the object (O) now lies at a distance y_0 below the new shifted principal axis. To calculate the location of I from this new shifted principal axis, we simply use the concept of magnification, according to which

$$m=\frac{h_i}{h_0}=\left|-\frac{v}{u}\right|$$

$$\Rightarrow \quad h_i = mh_0 = my_0$$

So, after shifting the mirror, the image is formed mh_0 above the new principal axis or formed $mh_0 + h_0 = (m+1)h_0$ above the old principal axis.



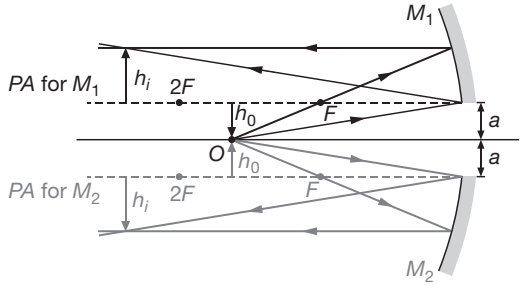
Perpendicular separation between the tip of object and tip of image is $(OI)_\perp = h_i + h_0 = mh_0 + h_0 = (m+1)h_0$

Also, the coordinates of the image I w.r.t. (0, 0) are $\left(-\frac{x_0 f}{x_0+f},\ (m+1)y_0\right)$

SPLITTING OF A MIRROR

Let a concave mirror of focal length f be cut into two parts M_1 and M_2 at the pole and then each part is displaced perpendicular to the principal axis through A. Due to this the point object O lies at a distance a from the new principal axis for each of the mirrors M_1 and M_2.

CASE-1: When the object O lies between F and $2F$, then the ray diagram for this arrangement is shown in figure.



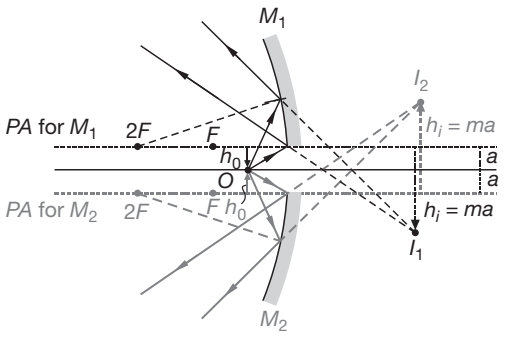
Since the magnification $m=\frac{h_i}{h_o}=\frac{h_i}{a}$

$$\Rightarrow \quad h_i = ma \text{, where } m=\frac{h_i}{h_o}=-\frac{v}{u}$$

The separation I_1I_2 between the tips of images is

$$I_1I_2 = 2h_i + 2a = 2ma + 2a = 2a(m+1)$$

CASE-2: When the object O lies between F and P, then the ray diagram for this arrangement is shown in figure.



Since the magnification $m = \dfrac{h_i}{h_o} = \dfrac{h_i}{a}$

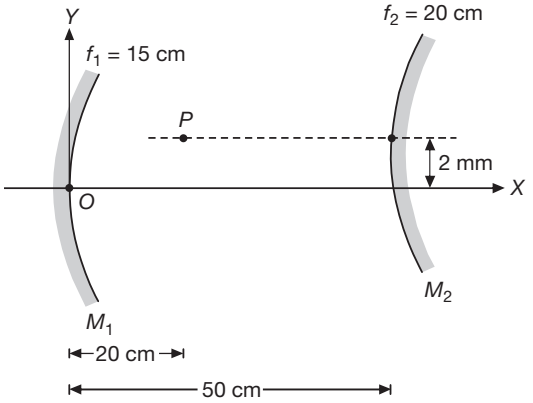
$\Rightarrow \quad h_i = ma$, where $m = \dfrac{h_i}{h_o} = -\dfrac{v}{u}$

The separation I_1I_2 between the tips of images is

$$I_1I_2 = 2h_i - 2a = 2ma - 2a = 2a(m-1)$$

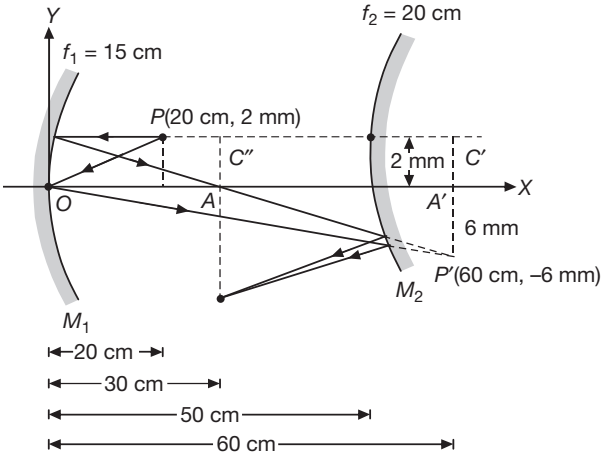
ILLUSTRATION 22

Find the co-ordinates of image of point object P formed after two successive reflections in figure, considering the first reflection at concave mirror and then at convex mirror.



SOLUTION

The ray diagram for the situation is drawn in figure (but not to scale).



For reflection at concave mirror M_1, we have

$u = -20$ cm

$f_1 = -15$ cm

Since, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$\Rightarrow \quad \dfrac{1}{v_1} + \dfrac{1}{(-20)} = \dfrac{1}{(-15)}$

$\Rightarrow \quad v_1 = -60$ cm

So, magnification $m_1 = -\dfrac{v_1}{u} = -\dfrac{-60}{-20} = -3$ (Inverted)

$\Rightarrow \quad A'P' = m_1(AP) = 3 \times 2 = 6$ mm

For reflection at convex mirror M_2, we have

$u = +10$ cm

$f_2 = +20$ cm

Since $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$\Rightarrow \quad \dfrac{1}{v_2} + \dfrac{1}{10} = \dfrac{1}{20}$

$\Rightarrow \quad v_2 = -20$ cm

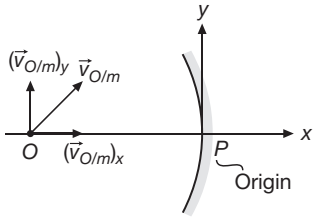
Again, magnification, $m_2 = -\dfrac{v_2}{u} = -\dfrac{(-20)}{10} = 2$

$$\Rightarrow \quad C''P'' = m_2 (C'P') = 2 \times 8 = 16 \text{ mm}$$

So, the co-ordinate of image of point object P as measured from the origin O is $(30 \text{ cm}, -14 \text{ mm})$

VELOCITY OF IMAGE IN SPHERICAL MIRROR

Let pole of mirror be origin of co-ordinate system and x-axis be the principal axis of mirror and y-axis is perpendicular to principal axis. Further object is placed such that incident rays travel along +ve x-axis.



From mirror equation, we have

$$\frac{1}{x_{I/m}} + \frac{1}{x_{O/m}} = \frac{1}{f}$$

Differentiating both sides w.r.t. t we get

$$-\frac{1}{x_{I/m}^2}\frac{d}{dt}\left(x_{I/m}\right) - \frac{1}{x_{O/m}^2}\frac{d}{dt}\left(x_{O/m}\right) = 0$$

$$\Rightarrow \quad \frac{d}{dt}\left(x_{I/m}\right) = -\left(\frac{x_{I/m}}{x_{O/m}}\right)^2 \frac{d}{dt}\left(x_{O/m}\right)$$

$$\Rightarrow \quad \left(V_{I/m}\right)_x = -m^2 \left(V_{O/m}\right)_x$$

We know that,

$$m = \frac{f}{f-u} = \frac{\text{Height of Image}}{\text{Height of Object}}$$

$$\Rightarrow \quad y_{I/m} = \left(\frac{f}{f-u}\right)\left(y_{O/m}\right)$$

Differentiating w.r.t. t we get

$$\frac{d}{dt}\left(y_{I/m}\right) = \frac{d}{dt}\left[\left(\frac{f}{f-u}\right)\left(y_{O/m}\right)\right]$$

$$\Rightarrow \quad \left(V_{I/m}\right)_y = \left(\frac{f}{f-u}\right)\frac{d}{dt}\left(y_{O/m}\right) + \left(y_{O/m}\right)\frac{d}{dt}\left(\frac{f}{f-u}\right)$$

(Using product rule)

$$\Rightarrow \quad \left(V_{I/m}\right)_y = \left(\frac{f}{f-u}\right)\left(V_{O/m}\right)_y + \left(y_{O/m}\right)\frac{f}{(f-u)^2}\frac{du}{dt}$$

CASE-1: If object is on principal axis, then $y_{O/m} = 0$

$$\Rightarrow \quad \left(V_{I/m}\right)_y = \left(\frac{f}{f-u}\right)\left(V_{O/m}\right)_y$$

CASE-2: If object is not on principal axis but moving parallel to principal axis then $\left(V_{O/m}\right)_y = 0$

$$\Rightarrow \quad V_{I/m} = \left(V_{O/m}\right)\frac{f}{(f-u)^2}\frac{du}{dt}$$

Note that $\frac{du}{dt}$ is negative if u is decreasing with time and it is taken positive if u is increasing with time.

CASE-3: If object is on principal axis and moving along it then $y_{O/m} = 0$ and $\left(V_{O/m}\right)_y = 0$

$$\Rightarrow \quad \left(V_{I/m}\right)_y = 0$$

ILLUSTRATION 23

A thief is driving away on a road in a car with velocity of 20 ms^{-1}. A police jeep is chasing him, which is sighted by thief in his rear view mirror, which is a convex mirror of focal length 10 m. He observes that the image of the jeep is moving towards him with a velocity of 1 cms^{-1}. If the magnification of the mirror for the jeep at that time is $\frac{1}{10}$. Find

(a) The actual speed of the jeep.
(b) The rate at which magnification is changing.

Assume that police jeep is on axis of the mirror.

SOLUTION

(a) The velocity of image with respect to mirror is related to velocity of object with respect to mirror is given as

$$\left(V_{I/m}\right)_{\perp} = -m^2 \left(V_{O/m}\right)$$

$$\Rightarrow \quad -1\times10^{-2} = -\frac{1}{10^2}\left(V_{O/m}\right)$$

$$\Rightarrow \quad \left(V_{O/m}\right) = +1 \text{ ms}^{-1} = +1\hat{i}$$

Velocity of object with respect to ground is given as

$$\vec{V}_{O/G} = \vec{V}_{O/m} + \vec{V}_{m/G}$$

$$\Rightarrow \quad \vec{V}_{O/G} = 1 + 20 = \left(+21 \text{ ms}^{-1}\right)\hat{i}$$

(b) The magnification produced by the mirror is

$$m = \frac{f}{f-u} = \frac{1}{10}$$

If police jeep is at a distance d behind the thief's car then we can use $u = -d$ so we have

$$\frac{10}{10-(-d)} = \frac{1}{10}$$

$$\Rightarrow \quad d = 90 \text{ m}$$

Thus distance of image from mirror is

$$v = -mu = -\frac{1}{10}\times -90 = 9 \text{ m}$$

Now rate at which magnification is changing is given as

$$\frac{dm}{dt} = \frac{u\dfrac{dv}{dt} - v\dfrac{du}{dt}}{u^2}$$

$$\Rightarrow \quad \frac{dm}{dt} = \left[\frac{(-90)\left(-1\times10^{-2}\right)-9(1)}{90^2}\right] \text{s}^{-1}$$

$$\Rightarrow \quad \frac{dm}{dt} = -\left[\frac{81}{10\times8100}\right] = +1\times10^{-3} \text{ s}^{-1}$$

Test Your Concepts-II

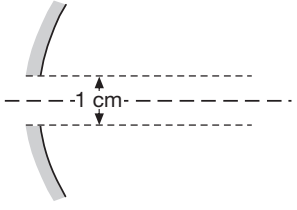
Based on Reflection at Curved Surfaces

(Solutions on page H.6)

1. An object of height 2.5 cm is placed at a 1.5 f from a concave mirror, where f is the magnitude of the focal length of the mirror. The height of the object is perpendicular to the principal axis. Find the height of the image. Is the image erect or inverted?

2. A mirror (in a laughing gallery) forms an erect image four times enlarged, of a boy standing 2.5 m away. Is the mirror concave or convex? What is its radius of curvature?

3. A concave mirror forms the real image of a point source lying on the optical axis at a distance of 50 cm from the mirror. The focal length of the mirror is 25 cm. The mirror is cut in two halves and these halves are drawn apart at a distance of 1 cm in a direction perpendicular to the optical axis. How will the images formed by the halves of the mirror be arranged?



4. Find the distance of object from a concave mirror of focal length 10 cm so that image size is four times the size of the object.

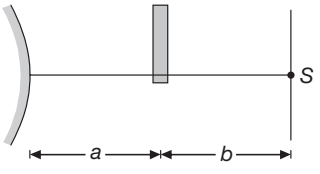
5. A concave mirror has a radius of curvature of 24 cm. How far is an object from the mirror when the image formed is

(a) virtual and 3 times the size of the object.

(b) real and 3 times the size of the object and

(c) real and $\frac{1}{3}$ the size of the object?

6. A thin flat glass plate is placed in front of a convex mirror. At what distance b from the plate should a point source of light S be placed so that its image produced by the rays reflected from the front surface of the plate coincides with the image formed by the rays reflected from the mirror? The focal length of the mirror is $f = 20$ cm and the distance from the plate to the mirror $a = 5$ cm. How can the coincidence of the images be established by direct observation?

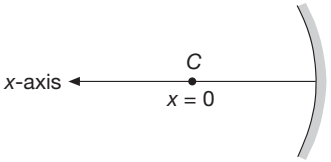


7. A ball swings back and forth in front of a concave mirror. The motion of the ball is described approximately by the equation $x = f\cos(\omega t)$, where f is the focal length of the mirror and x is measured along the axis of mirror. The origin is taken at the centre of curvature of the mirror.

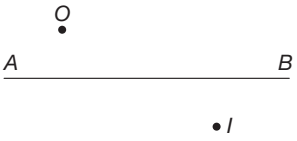
(a) Derive an expression for the distance from the mirror to the image of the swinging ball.

(b) At what point does the ball appear to coincide with its image.

(c) What will be the lateral magnification of the image of the ball at time $t = \frac{T}{2}$, where T is time period of oscillation?



8. An image I is formed of a point object O by a mirror whose principal axis is AB as shown in figure.



(a) State whether it is a convex mirror or a concave mirror.

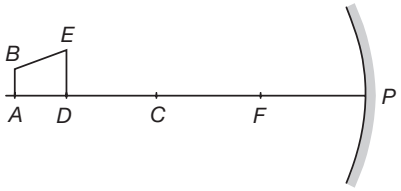
(b) Draw a ray diagram to locate the mirror and its focus. Write down the steps of construction of the ray diagram. Consider the possible two cases:

(i) When distance of I from AB is more than the distance of O from AB and

(ii) When distance of O from AB is more than the distance of I from AB

9. Convex and concave mirrors have the same radii of curvature R. The distance between the mirrors is $2R$. At what point on the common optical axis of the mirrors should a point source of light A be placed for the rays to converge at the point A after being reflected first on the convex and then on the concave mirror?

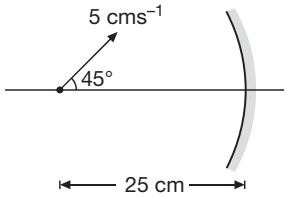
10. An object $ABED$ is placed in front of a concave mirror beyond centre of curvature C as shown in figure. State the shape of the image.



11. An object is 30 cm from a spherical mirror, along the central axis. The absolute value of lateral magnification of an inverted image is $\frac{1}{2}$. Find the focal length of the mirror?

12. A thin rod of length $\frac{f}{3}$ is placed along the principal axis of a concave mirror of focal length f such that its image just touches the rod. Calculate magnification.

13. A point object on the principal axis of a concave mirror of focal length 20 cm is moving at a speed of $5\ \text{cms}^{-1}$ making an angle of 45° with the principal axis as shown in figure.

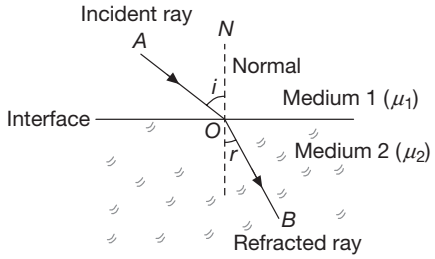


Initially object is located at a distance of 25 cm from the pole of mirror. Calculate the velocity components of image along and normal to principal axis at this instant.

REFRACTION AT PLANE SURFACES

REFRACTION OF LIGHT AT PLANE SURFACES

The phenomenon of the bending of light rays as they travel from one medium to the other is called Refraction. The surface separating two media is called an Interface. In other words, the phenomenon of bending of light rays at the boundary between two media is called refraction.



LAWS OF REFRACTION

(a) The incident ray, the refracted ray and normal at the point of incidence to the surface separating the two media all lie in the same plane.

(b) **Snell's Law**

For two media, **the ratio of sine of angle of incidence i to the sine of the angle of refraction r is constant** (for a beam of particular wavelength). For a given set of media this constant is called the **refractive index** of the medium 2 with respect to medium 1 (represented as ${}^1\mu_2$) i.e.,

$$\frac{\sin i}{\sin r} = \text{constant} = \frac{\mu_2}{\mu_1} = {}^1\mu_2 \quad \text{(SNELL'S LAW)}$$

OR $\mu_1 \sin i = \mu_2 \sin r$

where μ_1 and μ_2 are Absolute Refractive Indices of Medium 1 and 2 respectively and ${}^1\mu_2$ is the refractive index of medium 2 with respect to medium 1. If medium 1 happens to be the vacuum, then the constant is simply called as the **Absolute Refractive Index** of medium 2, expressed as μ_2 or simply μ.

REFRACTIVE INDEX (RI)

The refractive index of a medium is not determined by its density. It is governed by the velocity of light in the medium. The lesser the value of the velocity of light, the more is the refractive index of the medium, and the denser is the medium. A medium having greater refractive index is called denser medium whereas the other medium is called **rarer medium**.

ABSOLUTE REFRACTIVE INDEX

The absolute refractive index of a medium is defined as the ratio of the speed of light in vacuum to the speed of light in the medium,

$$\mu = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}} = \frac{c}{v} > 1$$

Absolute refractive index is more than one because the speed of light is maximum in vacuum/air.

RELATIVE REFRACTIVE INDEX

The relative refractive index of medium 2 with respect to medium 1 is denoted by ${}^1\mu_2$ and is given by

$${}^1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\left(\frac{c}{v_2}\right)}{\left(\frac{c}{v_1}\right)} = \frac{v_1}{v_2}$$

The relative refractive index of medium 1 with respect to medium 2 is denoted by ${}^2\mu_1$ and is given by

$${}^2\mu_1 = \frac{\mu_1}{\mu_2} = \frac{\left(\frac{c}{v_1}\right)}{\left(\frac{c}{v_2}\right)} = \frac{v_2}{v_1}$$

Conceptual Note(s)

(a) The velocity of light in air is not much different from that in vacuum. Hence, while defining the refractive index of a medium we often take velocity of light in air rather than that in vacuum.

Medium	Refractive Index (μ)
Water	$\frac{4}{3}=1.33$
Glass	$\frac{3}{2}=1.50$

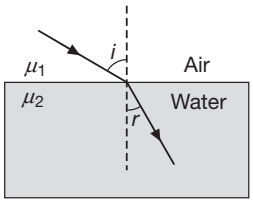
(b) Relative refractive index can be less than one. If we calculate the refractive index of water with respect to glass, then

$$^{g}\mu_{w}=\frac{\mu_{w}}{\mu_{g}}=\frac{\left(\frac{4}{3}\right)}{\left(\frac{3}{2}\right)}=\frac{8}{9}<1$$

(c) Refractive index is different for different wavelengths for a pair of media, because $\mu_1\lambda_1=\mu_2\lambda_2$.

BENDING OF A LIGHT RAY

(a) When light passes from rarer to denser medium it bends towards the normal i.e., a light ray passing from air to water bends towards the normal as shown in the figure.



According to Snell's Law

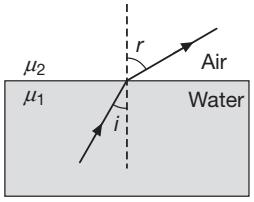
$$\mu_1\sin i=\mu_2\sin r$$

$$\Rightarrow \frac{\sin i}{\sin r}=\frac{\mu_2}{\mu_1}>1$$

$$\Rightarrow \sin i>\sin r$$

$$\Rightarrow i>r$$

(b) When light passes from denser to rarer medium it bends away from the normal as i.e., a light ray passing from water to air bends away from the normal shown in the figure.



According to Snell's Law

$$\mu_1\sin i=\mu_2\sin r$$

$$\Rightarrow \frac{\sin i}{\sin r}=\frac{\mu_2}{\mu_1}<1$$

$$\Rightarrow \sin i<\sin r$$

$$\Rightarrow i<r$$

REFRACTION: IMPORTANT POINTS

(a) Whenever light goes from one medium to another, the frequency of light (f) remains unchanged. Since

$$\mu=\frac{c}{v}=\frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$$

$$\Rightarrow \mu=\frac{f\lambda_{\text{air}}}{f\lambda_{\text{medium}}}=\frac{\lambda_{\text{air}}}{\lambda_{\text{medium}}}$$

where λ_{air} and λ_{medium} being wavelengths of light in air and medium respectively.

$$\Rightarrow \frac{\sin i}{\sin r}=\frac{\mu_2}{\mu_1}=\frac{\frac{c}{v_2}}{\frac{c}{v_1}}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}$$

(MODIFIED FORM OF SNELL'S LAW)

From above we conclude that

$$\mu_1\lambda_1=\mu_2\lambda_2$$

$$\Rightarrow \mu\lambda=\text{constant}$$

Also, we conclude that

$$\mu_1 v_1=\mu_2 v_2$$

$$\Rightarrow \mu v=\text{constant}$$

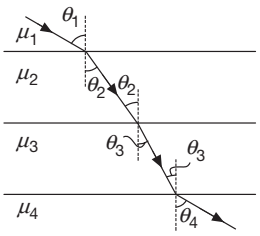
(b) $^{2}\mu_{1}\times{}^{1}\mu_{2}=1$

$$\Rightarrow {}^{2}\mu_{1}=\frac{1}{{}^{1}\mu_{2}}$$

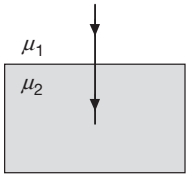
(c) When light propagates through a series of parallel layers of different medium as shown in the figure, then the Snell's Law may be written as

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 = \mu_3 \sin\theta_3 = \mu_4 \sin\theta_4 = \text{constant}$$

In general, $\mu \sin\theta = \text{constant}$



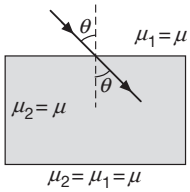
(d) If light is incident normal to a boundary (i.e. $i = 0$), then, it passes undeviated from the boundary as shown in the figure.



$\mu_2 \neq \mu_1$

Condition for no refraction

(e) If the refractive indices of the two media are equal as shown in figure, then also the light ray is not refracted and the boundary between the two media is not visible. This is why a transparent solid is invisible in a liquid of same refractive index.



$\mu_2 = \mu_1 = \mu$

Condition for no refraction

(f) Note that for sound waves,

speed in air, $v_1 = 330\ \text{ms}^{-1}$

speed in water, $v_w = 1500\ \text{ms}^{-1}$

Therefore, the refractive index of water with respect to air, for sound waves is

$${}^a\mu_w = \frac{v_a}{v_w} = \frac{330}{1500} = 0.22$$

Thus, we find that for the refraction of sound waves, water is rarer than air.

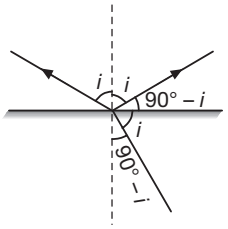
ILLUSTRATION 24

A ray of light falls on a glass plate of refractive index $n = \sqrt{3}$. What is the angle of incidence of the ray if the angle between the reflected and refracted rays is $90°$?

SOLUTION

According to Snell's Law

$$n = \frac{\sin i}{\sin r}$$



Since $i + r = 90°$

$$\Rightarrow \quad r = 90 - i$$

$$\Rightarrow \quad \sqrt{3} = \frac{\sin i}{\sin(90 - i)} = \tan i$$

$$\Rightarrow \quad i = \tan^{-1}\left(\sqrt{3}\right) = 60°$$

ILLUSTRATION 25

A ray of light passes through a medium whose refractive index varies with distance as $n = n_0\left(1 + \frac{x}{2a}\right)$. If ray enters the medium parallel to x-axis, what will be the time taken for ray to travel between $x = 0$ and $x = a$?

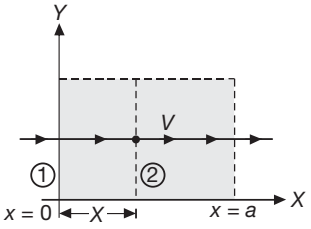
SOLUTION

Since, we know that $\mu = \frac{c}{v}$

$$\Rightarrow \quad v = \frac{c}{\mu}$$

So, if v be the speed at a distance x from y-axis, then

$$v = \frac{c}{n_0\left(1 + \frac{x}{2a}\right)}$$



Since, $v = \frac{dx}{dt}$, so we have

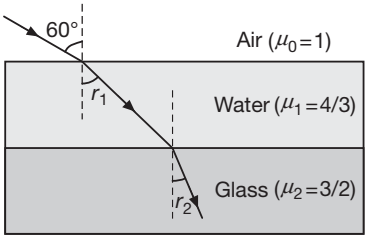
$$\frac{dx}{dt} = \frac{c}{n_0\left(1+\frac{x}{2a}\right)}$$

$$\Rightarrow \quad \int_0^t dt = \frac{n_0}{c}\int_0^a \left(1+\frac{x}{2a}\right)dx$$

$$\Rightarrow \quad t = \frac{5n_0 a}{4c}$$

ILLUSTRATION 26

For the arrangement shown in the figure, a light ray is incident at an angle of $60°$ on the layer of water. Find the angle between this ray and the normal to the glass.



SOLUTION

According to Snell's Law, we have

$$\mu_0 \sin(60°) = \mu_1 \sin r_1 = \mu_2 \sin r_2$$

$$\Rightarrow \quad \sin r_2 = \frac{\mu_0 \sin(60°)}{\mu_2} = \frac{1\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{3}{2}\right)} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad r_2 = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 35°$$

ILLUSTRATION 27

A ray of light goes from air to medium of refractive index μ. If i be the angle of incidence, r be the angle of refraction and δ be the angle of deviation, then prove that $\tan\left(\frac{\delta}{2}\right) = \left(\frac{\mu-1}{\mu+1}\right)\tan\left(\frac{i+r}{2}\right)$.

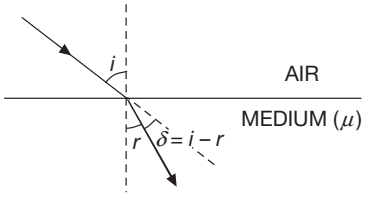
SOLUTION

The angle of deviation δ is given by

$$\delta = i - r \qquad \ldots(1)$$

According to Snell's Law,

$$\mu = \frac{\sin i}{\sin r}$$



Applying componendo and dividendo, we get

$$\frac{\mu-1}{\mu+1} = \frac{\sin i - \sin r}{\sin i + \sin r}$$

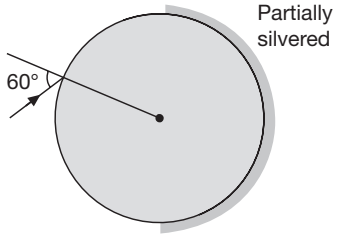
$$\Rightarrow \quad \frac{\mu-1}{\mu+1} = \frac{2\cos\left(\frac{i+r}{2}\right)\sin\left(\frac{i-r}{2}\right)}{2\sin\left(\frac{i+r}{2}\right)\cos\left(\frac{i-r}{2}\right)}$$

$$\Rightarrow \quad \frac{\mu-1}{\mu+1} = \frac{\tan\left(\frac{\delta}{2}\right)}{\tan\left(\frac{i+r}{2}\right)}$$

$$\Rightarrow \quad \tan\left(\frac{\delta}{2}\right) = \left(\frac{\mu-1}{\mu+1}\right)\tan\left(\frac{i+r}{2}\right)$$

ILLUSTRATION 28

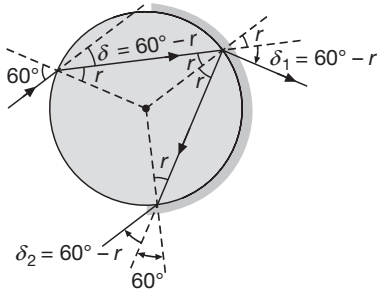
A ray is incident on a glass sphere as shown in figure. The opposite surface of the sphere is partially silvered. If the net deviation of the ray transmitted at the partially silvered surface is one third of the net deviation suffered by the ray reflected at the partially silvered surface (after emerging out of the sphere), find the refractive index of the sphere.



SOLUTION

Since, the distance of all the points lying on the sphere is constant from the centre, all the angles of refraction are same. Here we consider δ_1 is the deviation of the light ray at first refraction, δ_2 is the deviation of the transmitted ray through partially polished surface and δ_3 is the deviation of the light ray emerging out of the sphere at final refraction. Figure shows the ray diagram of the given situation then according to the given condition, we have

$$(60-r)+(60-r)=\frac{1}{3}((60-r)+(60-r)+(180-2r))$$



$$\Rightarrow \quad 120-2r=\frac{1}{3}(300-4r)$$

$$\Rightarrow \quad 360-6r=300-4r$$

$$\Rightarrow \quad 60=2r$$

$$\Rightarrow \quad r=30^\circ$$

Now using Snell's law, we have

$$1\sin(60^\circ)=\mu\sin r$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2}=\mu\times\frac{1}{2}$$

$$\Rightarrow \quad \mu=\sqrt{3}$$

LIGHT INCIDENT ON A MEDIUM HAVING VARIABLE REFRACTIVE INDEX

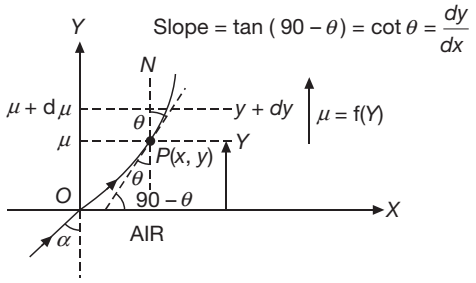
Let us find the mathematical expression for the equation of a ray in the medium when the medium is of variable refractive index. Consider a ray of light to be incident at an angle α at air-medium interface as shown. Now, two cases arise i.e. refractive index is varying either as function of y or function of x.

CASE-1: Refractive index μ varies with y i.e. $\mu=f(y)$

At point $P(x, y)$, let the angle of incidence be θ and refractive index be $f(y)$.

From Snell's Law, we have $\mu\sin\theta=$ constant

$$\Rightarrow \quad 1\times\sin\alpha=f(y)\sin\theta \qquad \ldots(1)$$



Snell's law at $O'A'$ interface.

Slope of curve at A is

$$\frac{dy}{dx}=\tan(90-\theta)$$

$$\Rightarrow \quad \cot\theta=\frac{dy}{dx}$$

So, from equation (1), we get

$$\frac{dy}{dx}=\frac{\sqrt{\mu^2-\sin^2\alpha}}{\sin\alpha}=\frac{\sqrt{[f(y)]^2-\sin^2\alpha}}{\sin\alpha}$$

CASE-2: Refractive index μ varies as function of x i.e. $\mu=f(x)$

According to Snell's law applied at interface $O'A'$, we have $\mu\sin\theta=$ constant

For initial refraction at the air-medium interface,

$$1\times\sin\alpha=\mu_0\sin(90-r)$$

$$\Rightarrow \quad \sin\alpha=\mu_0\cos r$$

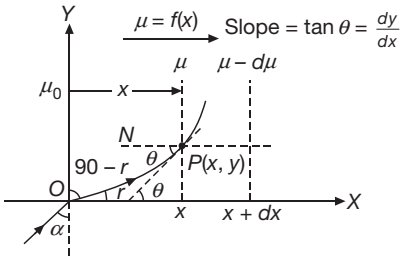
where r is angle of refracting ray at point O with line OX

$$\Rightarrow \quad \cos r = \frac{\sin\alpha}{\mu_0}$$

$$\Rightarrow \quad \sin r = \sqrt{1-\frac{\sin^2\alpha}{\mu_0^2}}$$

Applying Snell's law at P, we get

$$\mu \sin\theta = \mu_0 \sin r$$



$$\Rightarrow \quad \sin\theta = \frac{\mu_0}{f(x)}\sqrt{1-\frac{\sin^2\alpha}{\mu_0^2}} \qquad \{\because\ \mu = f(x)\}$$

$$\Rightarrow \quad \sin\theta = \frac{\sqrt{\mu_0^2-\sin^2\alpha}}{f(x)}$$

Now slope of tangent at P is

$$\frac{dy}{dx} = \tan\theta$$

$$\Rightarrow \quad \frac{dy}{dx} = \sqrt{\frac{\mu_0^2-\sin^2\alpha}{[f(x)]^2-\mu_0^2+\sin^2\alpha}}$$

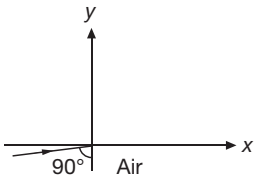
Conceptual Note(s)

(a) For Case-1: When refractive index varies along the y-axis, then normal is taken along the y-axis.

(b) For Case-2: When refractive index varies along the y-axis, then normal is taken along the x-axis.

ILLUSTRATION 29

Find the variation of Refractive index assuming it to be a function of y such that a ray entering origin at grazing incident follows a parabolic path $y = x^2$ as shown in figure.

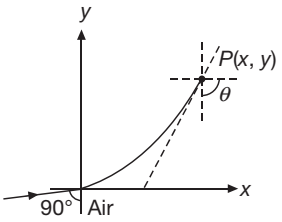


SOLUTION

We draw a tangent at any point (x, y) which makes an angle θ with optical normal parallel to y axis. From the Snell's law, we have

$$1\sin(90) = \mu\sin\theta$$

$$\Rightarrow \quad \sin\theta = \frac{1}{\mu} \qquad \ldots(1)$$



Slope of tangent is

$$\frac{dy}{dx} = \tan(90-\theta) = \cot\theta \qquad \ldots(2)$$

The trajectory of the light ray is

$$y = x^2$$

$$\Rightarrow \quad \frac{dy}{dx} = 2x$$

From equation (2), we have

$$\cot\theta = 2x$$

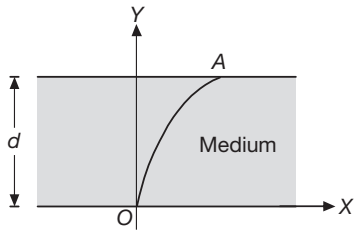
This gives $\mu = \dfrac{1}{\sin\theta} = \operatorname{cosec}\theta$

$$\Rightarrow \quad \mu = \sqrt{1+\cot^2\theta} = \sqrt{1+4x^2} = \sqrt{1+4y}$$

ILLUSTRATION 30

A long rectangular slab of transparent medium of thickness d is placed on a table with length parallel to the x-axis and width parallel to the y-axis. A ray of light is travelling along y-axis at origin. The refractive

index μ of the medium varies as $\mu = \dfrac{\mu_0}{1-\left(\dfrac{x}{a}\right)}$, where μ_0 and $a(>1)$ are constants. The refractive index of air is 1.



(a) Determine the x-coordinate of the point A, where the ray intersects the upper surface of the slab-air boundary.

(b) Write down the refractive index of the medium at A.

(c) Indicate the subsequent path of the ray in air.

SOLUTION

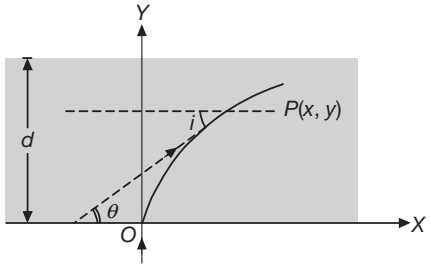
(a) Refractive index is a function of x, i.e., the plane separating the two media is parallel to y-z plane or normal to this plane at any point is parallel to x-axis.

Further refractive index increases as x is increases. So, the ray of light will bend towards normal and the path is shown in figure. Let at the point $P(x, y)$ the angle of incidence be i. Then

$$\theta = i$$

$$\Rightarrow \tan\theta = \tan i$$

$$\Rightarrow \frac{dy}{dx} = \tan i \qquad \ldots(1)$$



Applying Snell's Law at O and P, we get

$$\mu_0 \sin i_0 = \mu_P \sin i_P$$

$$\Rightarrow (1)(\sin 90^\circ) = \left(\frac{\mu_0}{1-\dfrac{x}{a}}\right)\sin i$$

$$\Rightarrow \sin i = \left(\frac{1-\dfrac{x}{a}}{\mu_0}\right)$$

$$\Rightarrow \tan i = \frac{\left(1-\dfrac{x}{a}\right)}{\sqrt{\mu_0^2 - \left(1-\dfrac{x}{a}\right)^2}} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$dy = \frac{\left(1-\dfrac{x}{a}\right)}{\sqrt{\mu_0^2 - \left(1-\dfrac{x}{a}\right)^2}}dx$$

Integrating, we get

$$\int_0^d dy = \int_0^x \frac{1-\dfrac{x}{a}}{\sqrt{\mu_0^2 - \left(1-\dfrac{x}{a}\right)^2}}dx$$

$$\Rightarrow x = a\left[1 - \sqrt{\mu_0^2 - \left(\frac{d}{a} + \sqrt{\mu_0^2 - 1}\right)^2}\right]$$

(b) At point A, $1-\dfrac{x}{a} = \sqrt{\mu_0^2 - \left(\dfrac{d}{a} + \sqrt{\mu_0^2 - 1}\right)^2}$

$$\Rightarrow \mu = \frac{\mu_0}{\sqrt{\mu_0^2 - \left(\dfrac{d}{a} + \sqrt{\mu_0^2 - 1}\right)^2}} \qquad \left\{\because \mu = \frac{\mu_0}{1-\dfrac{x}{a}}\right\}$$

(c) After A, medium is again air. Hence, from Snell's Law, angle of incidence will again become 90° or it will move parallel to y-axis as shown.

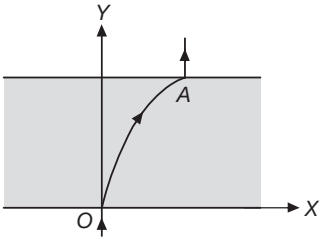
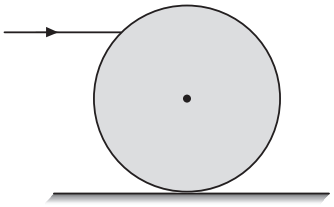


ILLUSTRATION 31

A cylindrical glass rod of radius 0.1 m and refractive index $\sqrt{3}$ lies on a horizontal plane mirror. A horizontal ray of light moving perpendicular to the axis of the rod is incident on it.

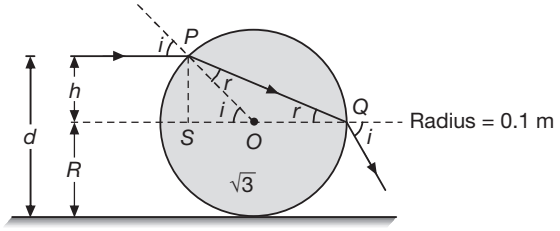
(a) At what height from the mirror should the ray be incident so that it leaves the rod at a height of 0.1 m above the plane mirror?

(b) At what distance a second similar rod, parallel to the first, be placed on the mirror, such that the emergent ray from the second rod is in line with the incident ray on the first rod?



SOLUTION

Let us first draw the ray diagram for the situation.



(a) **Since,** $PO = OQ$

$\Rightarrow \angle OPQ = \angle OQP = r$ (say)

Also, $i = r + r = 2r$

In ΔPOS, we have

$h = OP \sin i = 0.1 \sin i$

$\Rightarrow h = 0.1 \sin 2r$

$\Rightarrow h = 0.2 \sin r \cos r$ …(1)

Applying Snell's Law at P, we get

$$\sqrt{3} = \frac{\sin i}{\sin r} = \frac{2 \sin r \cos r}{\sin r} = 2 \cos r$$

$\Rightarrow r = 30°$

Substituting in equation (1), we get

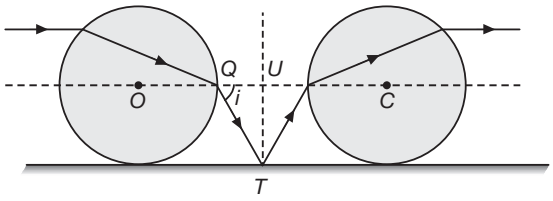
$$h = (0.2)\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = 0.086 \text{ m}$$

Hence, height from the mirror is

$$d = h + R = 0.1 + 0.086 = 0.186 \text{ m}$$

(b) Using the principle of reversibility of light, we get

$i = 2r = 60°$



Now, $\frac{QU}{TU} = \cot i = \cot(60°) = \frac{1}{\sqrt{3}}$

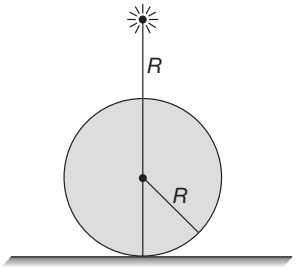
$$\Rightarrow QU = \frac{TU}{\sqrt{3}} = \frac{0.1}{\sqrt{3}}$$

So, the desired distance is

$$OC = 2(0.1) + 2\left(\frac{0.1}{\sqrt{3}}\right) = 0.315 \text{ m}$$

ILLUSTRATION 32

An opaque sphere of radius R lies on a horizontal plane. On the perpendicular through the point of contact there is a point source of light a distance R above the sphere.

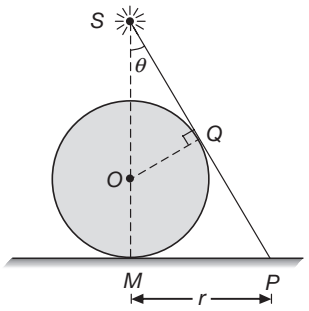


(a) Find the area of the shadow on the plane.

(b) A transparent liquid of refractive index $\sqrt{3}$ is filled above the plane such that the sphere is just covered with the liquid. Show that new area of the shadow.

SOLUTION

(a) The situation is shown in the figure



Since, we observe that

$$\sin\theta = \frac{OQ}{OS} = \frac{R}{2R} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30^\circ$$

Further, radius of shadow is given by

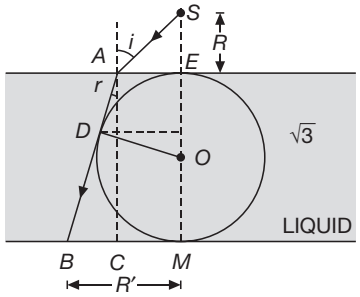
$$r = MP = MS\tan 30^\circ$$

$$\Rightarrow \quad r = (3R)\left(\frac{1}{\sqrt{3}}\right) = \sqrt{3}R$$

So, area of the shadow is

$$A = \pi\left(\sqrt{3}R\right)^2 = 3\pi R^2$$

(b) The situation is again shown in the figure.



$$AB = AD + BD$$

Since, by geometry, we have $AD = AE$ and $BD = BM$

$$\Rightarrow \quad AB = AE + BM$$

$$\Rightarrow \quad AB = R\tan i + R'$$

$$\Rightarrow \quad \cos r = \frac{AC}{AB} = \frac{2R}{R' + R\tan i}$$

$$\Rightarrow \quad R' = 2R\sec r - R\tan i \qquad \ldots(1)$$

Also, we observe that

$$R' = BC + CM = 2R\tan r + R\tan i \qquad \ldots(2)$$

From equations (1) and (2), we get

$$2\sec r - \tan i = 2\tan r + \tan i$$

$$\Rightarrow \quad \sec r - \tan r = \tan i$$

Using the concepts of trigonometry, we get

$$\tan i = \frac{1-\sin r}{\cos r} = \frac{\left(\cos\frac{r}{2} - \sin\frac{r}{2}\right)^2}{\cos^2\frac{r}{2} - \sin^2\frac{r}{2}}$$

$$\Rightarrow \quad \tan i = \frac{\cos\left(\frac{r}{2}\right) - \sin\left(\frac{r}{2}\right)}{\cos\left(\frac{r}{2}\right) + \sin\left(\frac{r}{2}\right)} = \frac{1-\tan\left(\frac{r}{2}\right)}{1+\tan\left(\frac{r}{2}\right)}$$

$$\Rightarrow \quad \tan i = \tan\left(\frac{\pi}{4} - \frac{r}{2}\right)$$

$$\Rightarrow \quad 2i = \frac{\pi}{2} - \frac{r}{2} \qquad \ldots(3)$$

Also, from Snell's Law, we get

$$\sin i = \sqrt{3}\sin r \qquad \ldots(4)$$

Solving the above equations, we get

$$R' = \sqrt{2}R$$

So the new area of shadow is

$$A' = \pi R'^2 = 2\pi R^2$$

CONCEPT OF OPTICAL PATH LENGTH (OPL) AND REDUCED THICKNESS

If a distance L separates two buildings, then the measured distance has nothing to do with the medium between the buildings. If this separation is filled with water, then too the distance between the buildings is L. However the time taken by the light to travel between the buildings is different for different media between the buildings. This time difference is due to the interaction of the light with the molecules of the medium which impede (slow down) the light's velocity and this causes the light to take more time to travel the same physical distance for different media.

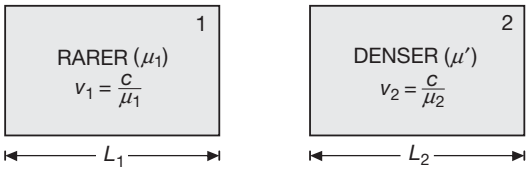
Due to this, a new concept of distance needs to be introduced that accounts for the delay in the

travelling time of the light in water (or a denser medium) in comparison to air (or a rarer medium). This new distance is called the **Optical Path Length (OPL) or Optical Path** and takes into account the slower velocity of light within a denser medium and it is simply the product of the distance with the refractive index i.e.,

$$OPL = \mu L$$

Thus, light passing through a denser medium seems to travel a longer distance than the light propagating in free space/vacuum, during the same time intervals for both the media.

Let me illustrate this thing to you. For that let me take two media, one rarer of length L_1, refractive index μ_1 and other denser of length L_2 and refractive index μ_2, as shown.



Time taken by light to travel a distance L_1 in rarer medium with speed $v_1 = \dfrac{c}{\mu_1}$ is

$$t_1 = \frac{L_1}{v_1} = \frac{L_1}{\left(\frac{c}{\mu_1}\right)} = \frac{\mu_1 L_1}{c} \qquad \ldots(1)$$

The time taken by light to travel a distance L_2 in denser medium with a speed $v_2 = \dfrac{c}{\mu_2}$ is

$$t_2 = \frac{L_2}{v_2} = \frac{L_2}{\left(\frac{c}{\mu_2}\right)} = \frac{\mu_2 L_2}{c} \qquad \ldots(2)$$

Now if both times are equal, as said above, then

$$t_1 = t_2$$

$$\Rightarrow \quad \mu_1 L_1 = \mu_2 L_2 \qquad \ldots(3)$$

Since, Optical Path Length (OPL) is the distance travelled by light in vacuum/air/rarer medium during the same time it travels a distance L_2 in medium. So, from (3), we get

$$\mu_1 L_1 = \mu_{\text{air}} L_{\text{air}} = \mu_{\text{medium}} L_{\text{medium}} = \mu_2 L_2$$

Since, $\mu_{\text{air}} = 1$, so, we get

$$OPL = L_{\text{air}} = \mu_{\text{medium}} L_{\text{medium}}$$

{for same time in air and medium}

Since light always travels slower in denser medium, so the OPL (the distance in air corresponding to same time in both) is always longer than the actual thickness L of the medium.

Conceptual Note(s)

Also, note that $\mu_1 l_1$ is OPL in air/rarer medium and $\mu_2 l_2$ is OPL in denser medium. However for standard purposes OPL is the distance travelled by light in vacuum/air to travel a distance L in a medium during the same time in either air or medium.

So, from equation (3), we conclude that for a pair of media,

$$\begin{pmatrix}\text{Optical Path}\\ \text{Length in}\\ \text{Air/Rarer Medium}\end{pmatrix} = \begin{pmatrix}\text{Optical Path}\\ \text{Length in}\\ \text{Denser Medium}\end{pmatrix}$$

$$\Rightarrow \quad \mu_1 L_1 = \mu_2 L_2$$

$$\Rightarrow \quad L_2 = \frac{\mu_1}{\mu_2} L_1$$

Since $\mu_1 < \mu_2$, so we get

$$L_2 < L_1$$

Due to this reason, L_2 is also called the **Reduced Thickness**. So, in general, we get

$$\begin{pmatrix}\text{Reduced}\\ \text{Thickness}\end{pmatrix} = \left(\frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}}\right) L_{\text{rarer}} = \frac{OPL \text{ in air}}{\mu_{\text{denser}}}$$

ILLUSTRATION 33

A light beam of wavelength 600 nm in air passes firstly through film 1 of thickness 1 μm and refractive index $n_1 = 1.2$, then through an air film 2 of thickness 1.5 μm and finally through film 3 of thickness 1 μm and refractive index $n_3 = 1.8$.

(a) Which film does the light cross in the least time and what is that least time?

(b) Calculate the total number of wavelengths (at any instant) across all three films together.

SOLUTION

(a) Since, $t_1 = \dfrac{d_1}{v_1} = \dfrac{10^{-6}}{\dfrac{(3\times10^8)}{1.2}} = 4\times10^{-15}$ s $\quad \left\{v = \dfrac{c}{n}\right\}$

Similarly, $t_2 = \dfrac{d_2}{c} = \dfrac{1.5\times10^{-6}}{3\times10^8} = 2\times10^{-15}$ s

and $t_3 = \dfrac{d_3}{v_3} = \dfrac{10^{-6}}{\dfrac{(3\times10^8)}{1.8}} = 6\times10^{-15}$ s

So, $t_{\min} = 2\times10^{-15}$ s

(b) The total number of wavelengths in a film of refractive index μ, thickness d is

$$n = \frac{\text{Optical Path Length}}{\text{Wavelength of Light}}$$

$$\Rightarrow \quad n = \frac{\mu d}{\lambda}$$

So, total number of wavelengths, is

$$n = \frac{\mu_1 d_1}{\lambda} + \frac{\mu_2 d_2}{\lambda} + \frac{\mu_3 d_3}{\lambda}$$

$$\Rightarrow \quad n = \frac{1}{\lambda}(\mu_1 d_1 + \mu_2 d_2 + \mu_3 d_3)$$

$$\Rightarrow \quad n = \frac{10^{-6}}{600\times10^{-9}}((1.2)(1)+(1)(1.5)+(1.8)(1))$$

$$\Rightarrow \quad n = \frac{1000}{600}(4.5) = \frac{4500}{600} = 7.5$$

ILLUSTRATION 34

A light ray enters the atmosphere of a planet and descends vertically 20 km to the surface. The index of refraction where the light enters the atmosphere is 1 and it increases linearly to the surface where it has a value 1.005. How long does it take the ray to traverse this path.

SOLUTION

Since variation is linear, so we have

$$\frac{x-0}{\mu-1} = \frac{2\times10^4-0}{1.005-1}$$

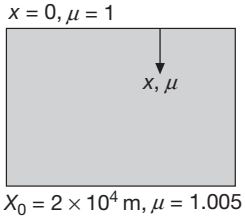
$$\Rightarrow \quad \mu = 1+\frac{0.005x}{2\times10^4}$$

Now, by definition, we have

$$\mu(x) = \frac{c}{v(x)}$$

$$\Rightarrow \quad v(x) = \frac{c}{\mu(x)}$$

where $\mu(x) = \mu = 1+\dfrac{0.005x}{2\times10^4}$ and $c = 3\times10^8$ ms^{-1}



$$\Rightarrow \quad v(x) = \frac{c}{\mu(x)} = \frac{3\times10^8}{1+\dfrac{0.005}{2\times10^4}x} = \frac{3\times10^8}{1+2.5\times10^{-7}x}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{3\times10^8}{1+2.5\times10^{-7}x}$$

$$\Rightarrow \quad dt = \frac{1}{3\times10^8}\left[(1+2.5\times10^{-7}x)dx\right]$$

$$\Rightarrow \quad \int_0^t dt = \frac{1}{3\times10^8}\int_0^{2\times10^4}(1+2.5\times10^{-7}x)dx$$

$$\Rightarrow \quad t = \frac{1}{3\times10^8}\left[2\times10^4 + \frac{(2\times10^4)^2(2.5\times10^{-7})}{2}\right]$$

$$\Rightarrow \quad t = 6.68\times10^{-5} \text{ s}$$

LAWS OF REFRACTION USING FERMAT'S PRINCIPLE

Consider a refracting surface/interface separating medium 1 from medium 2. Let the incident light start from A, in medium 1, hit the surface at O and get refracted to a point B, in medium 2. Let the points A and B be at perpendicular distances a and b from the interface. Further, let A and B be at a separation d as shown in figure. The time taken by the light to go from A to O to B is

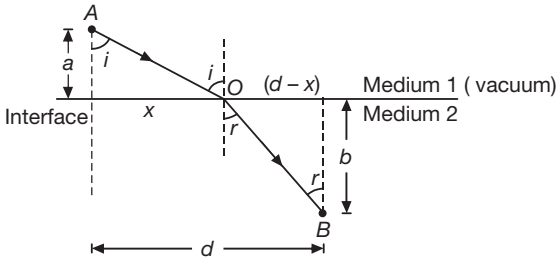
$$t = t_{A\to O} + t_{O\to B}$$

$$\Rightarrow \quad t = \frac{AO}{c} + \frac{OB}{v}$$

$$\Rightarrow \quad t = \frac{\sqrt{a^2+x^2}}{c} + \frac{\sqrt{b^2+(d-x)^2}}{v}$$

Now, according to Fermat's Theorem, t is MINIMUM, so

$$\frac{dt}{dx} = 0$$



$$\Rightarrow \quad \frac{1}{c}\frac{d}{dt}\left(\sqrt{a^2+x^2}\right) + \frac{1}{v}\frac{d}{dt}\left(\sqrt{b^2+(d-x)^2}\right)$$

$$\Rightarrow \quad \frac{1}{2c}\frac{2x}{\sqrt{a^2+x^2}} + \frac{1}{2v}\left(\frac{2(d-x)(-1)}{\sqrt{b^2+(d-x)^2}}\right) = 0$$

$$\Rightarrow \quad \frac{1}{c}\frac{x}{\sqrt{a^2+x^2}} = \frac{1}{v}\frac{(d-x)}{\sqrt{b^2+(d-x)^2}}$$

From the figure, we observe that

$$\frac{x}{\sqrt{a^2+x^2}} = \sin i \text{ and } \frac{d-x}{\sqrt{b^2+(d-x)^2}} = \sin i$$

$$\Rightarrow \quad \frac{1}{c}\sin i = \frac{1}{v}\sin r$$

$$\Rightarrow \quad \frac{\sin i}{\sin r} = \frac{c}{v} = \mu \qquad \{\text{The Law of Refraction}\}$$

VECTOR FORM OF SNELL'S LAW

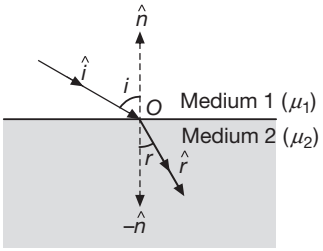
According to Snell's Law, we have

$$\mu_1 \sin i = \mu_2 \sin r$$

Vectorially, Snell's Law can be written as

$$\mu_1(\hat{i}\times\hat{n}) = \mu_2(\hat{r}\times\hat{n})$$

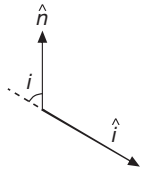
Let $\hat{i}$ be the unit vector along the incident ray, $\hat{r}$ be the unit vector along the refracted ray and $\hat{n}$ be a unit vector along the normal as shown.

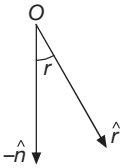


Then, $|\hat{i}| = |\hat{n}| = |\hat{r}| = 1$

Using our knowledge of cross product of vectors, we have

$$\hat{i}\times\hat{n} = (1)(1)\sin i\,,\ \odot \text{ outwards}$$





and $-\hat{n}\times\hat{r} = \hat{r}\times\hat{n} = (1)(1)\sin r\,,\ \odot$ outwards

So, in vector form, the Snell's Law can be expressed as

$$\mu_1(\hat{i}\times\hat{n}) = \mu_2(\hat{r}\times\hat{n})$$

REFRACTION THROUGH A COMPOSITE SLAB

Consider the refraction of light ray through a series of media as shown in figure. The ray AB is incident on interface X_1Y_1 at an angle i. The ray is deviated in medium 2 along BC towards the normal. Then it falls on interface X_2Y_2 and is again deviated towards normal along CD. If the last medium is again Medium 1, the ray emerges parallel to the incident ray. Let r_1 and r_2 be angles of refraction in Medium 2 and Medium 3 respectively. Then from Snell's Law,

$$\frac{\sin i}{\sin r_1} = \frac{\mu_2}{\mu_1} = {}^1\mu_2 \quad \ldots(1)$$

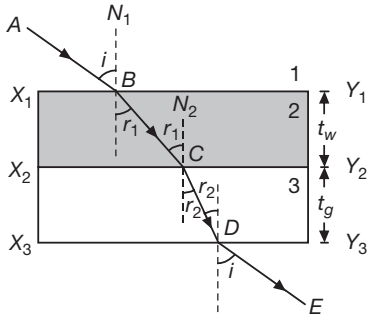
$$\frac{\sin r_1}{\sin r_2} = \frac{\mu_3}{\mu_2} = {}^2\mu_3 \quad \ldots(2)$$

$$\frac{\sin r_2}{\sin i} = \frac{\mu_1}{\mu_3} = {}^3\mu_1 \quad \ldots(3)$$

μ_1 = refractive index of medium 1

μ_2 = refractive index of medium 2

μ_3 = refractive index of medium 3



Multiplying (1), (2) and (3), we get

$${}^1\mu_2 \times {}^2\mu_3 \times {}^3\mu_1 = 1$$

$$\Rightarrow \quad {}^1\mu_2 \times {}^2\mu_3 = {}^1\mu_3$$

In general if a ray passes through a number of composite parallel plate glass slabs, then

$${}^1\mu_2 \times {}^2\mu_3 \times {}^3\mu_4 \times {}^4\mu_5 = {}^1\mu_5$$

ILLUSTRATION 35

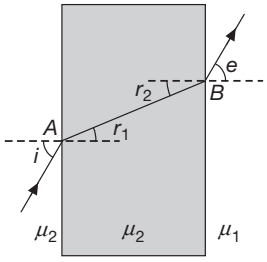
A light beam passes from a parallel plate glass slab of refractive index μ_2 placed in a medium of refractive index μ_1. Show that the emerging beam is parallel to the incident beam.

SOLUTION

Applying Snell's Law at A, we get

$$\mu_1 \sin i = \mu_2 \sin r_1$$

$$\Rightarrow \quad \frac{\mu_1}{\mu_2} = \frac{\sin r_1}{\sin i} \quad \ldots(1)$$



Applying Snell's Law at B, we get

$$\mu_2 \sin r_2 = \mu_1 \sin e$$

$$\Rightarrow \quad \frac{\mu_1}{\mu_2} = \frac{\sin r_2}{\sin e} \quad \ldots(2)$$

From equation (1) and (2), we get

$$i = e$$

i.e., the emergent ray is parallel to incident ray.

ILLUSTRATION 36

Refractive index of glass with respect to water is $\frac{9}{8}$. Refractive index of glass with respect to air is $\frac{3}{2}$. Find the refractive index of water with respect to air.

SOLUTION

Given, ${}^w\mu_g = \frac{9}{8}$ and ${}^a\mu_g = \frac{3}{2}$

Since, ${}^a\mu_g \times {}^g\mu_w = {}^a\mu_w$

$$\Rightarrow \quad {}^a\mu_w = \frac{{}^a\mu_g}{{}^w\mu_g} \qquad \left\{\because {}^g\mu_w = \frac{1}{{}^w\mu_g}\right\}$$

$$\Rightarrow \quad {}^a\mu_w = \frac{\left(\frac{3}{2}\right)}{\left(\frac{9}{8}\right)} = \frac{4}{3}$$

LATERAL SHIFT ON PASSING THROUGH A GLASS SLAB

Consider a ray AO incident on the slab at an angle of incidence i through the glass slab $EFGH$ of thickness t. After refraction the ray emerges parallel to the incident ray.

Let PQ be perpendicular dropped from P on incident ray produced as OQ.

The lateral displacement caused by plate,

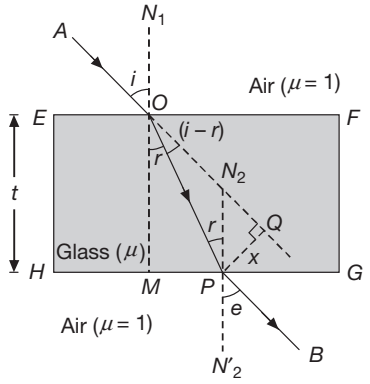
$$x = PQ = OP\sin(i-r)$$

$$x = \frac{OM}{\cos r}\sin(i-r) \qquad \left\{\because OP = \frac{OM}{\cos r}\right\}$$

$$\Rightarrow \quad x = \frac{t}{\cos r}\sin(i-r)$$

$$\Rightarrow \quad x = \frac{t}{\cos r}(\sin i\cos r - \cos i\sin r)$$

$$\Rightarrow \quad x = t(\sin i - \cos i\tan r)$$



Since $\mu = \dfrac{\sin i}{\sin r}$

$$\Rightarrow \quad \sin r = \frac{\sin i}{\mu}$$

$$\Rightarrow \quad \tan r = \frac{\sin i}{\sqrt{\mu^2 - \sin^2 i}}$$

$$\Rightarrow \quad y = t\sin i\left(1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right)$$

If i is very small then r is also very small, hence

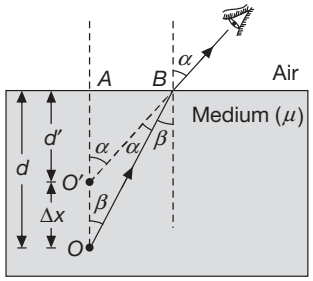
$$\sin i \to i, \text{ and } \cos i \to 1$$

Then expression for lateral displacement takes the form

$$x = ti\left(1 - \frac{1}{\mu}\right)$$

APPARENT DEPTH

An object O placed in a medium of refractive index μ is observed from air at a small angle α to the normal to the interface (in figure, angle α is shown exaggerated for clarity) i.e., for near normal incidence.



If the object O is at a real depth d from the interface, its apparent depth d' can be calculated. From Δs ABO and ABO',

$$\frac{\tan\alpha}{\tan\beta} = \frac{d}{d'}$$

Since angles α and β are small, so $\sin\alpha \approx \tan\alpha$ and $\sin\beta \approx \tan\beta$.

Therefore, from Snell's Law, we get

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin\alpha}{\sin\beta} \approx \frac{\tan\alpha}{\tan\beta} = \frac{d}{d'}$$

$$\Rightarrow \quad \text{Apparent depth, } d' = \frac{d}{\mu}$$

The **apparent shift** in normal direction (or the **normal shift**) in the position of the object is

$$\Delta x = d - d' = d\left(1 - \frac{1}{\mu}\right)$$

In case the object is seen through n number of slabs with different refractive indices, the total apparent shift is simply the sum of individual shifts, so

$$\Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3 + ... + \Delta x_n$$

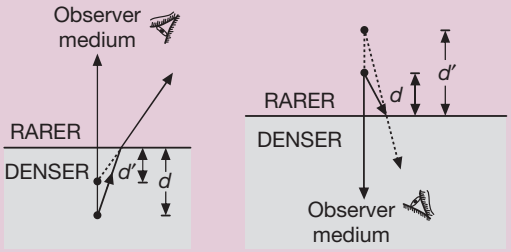
$$\Rightarrow \quad \Delta x = d_1\left(1 - \frac{1}{\mu_1}\right) + d_2\left(1 - \frac{1}{\mu_2}\right) + d_3\left(1 - \frac{1}{\mu_3}\right) + .. + d_n\left(1 - \frac{1}{\mu_n}\right)$$

Conceptual Note(s)

(a) If the medium in which the object is placed is rarer ($\mu < 1$) and it is seen from the denser medium, the apparent shift calculated will be **negative**. It means that the object apparently shifts away from the observer.

If the shift comes out to be positive, the image of the object shifts towards the observer.

(b) At near normal incidence (small angle of incidence i) apparent depth (d') is given by



$$d' = \frac{d}{\mu_{\text{relative}}} \text{ and } v' = \frac{v}{\mu_{\text{relative}}}$$

where

$$\mu_{\text{relative}} = \frac{\text{RI of medium of incidence object}}{\text{RI of medium of refraction observer}} = \frac{\mu_{\text{object}}}{\mu_{\text{observer}}}$$

d = distance of object from the interface = real depth.

d' = distance of image from the interface = apparent depth.

v = velocity of object perpendicular to interface relative to surface.

v' = velocity of image perpendicular to interface relative to surface.

ILLUSTRATION 37

A fish is rising vertically to the surface of water in a lake at a uniform speed of $3\ \text{ms}^{-1}$. It observes that a bird is diving vertically towards the water at a uniform speed of $9\ \text{ms}^{-1}$. If the refractive index of water is $\frac{4}{3}$, find the actual speed of dive of the bird.

SOLUTION

Let x be the depth of the fish F below the surface of water, and y be the height of the bird B above the surface at an instant.

To the fish, the bird will appear to be farther away, at an apparent height y' given by

$$y' = \frac{y}{\mu_{\text{relative}}} = \frac{y}{\left(\frac{\mu_{\text{object medium}}}{\mu_{\text{observer medium}}}\right)}$$

$$\Rightarrow \quad y' = \frac{y}{\left(\frac{\mu_{\text{bird medium}}}{\mu_{\text{fish medium}}}\right)}$$

$$\Rightarrow \quad y' = \frac{y}{\left(\frac{1}{\mu}\right)} = \mu y$$

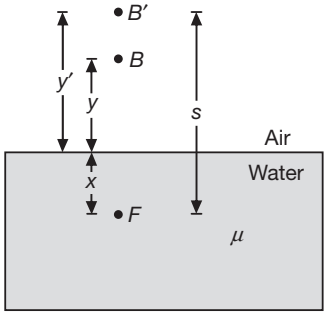
The total apparent distance of the bird from the fish is

$$s = x + y'$$

$$\Rightarrow \quad s = x + \mu y$$

Differentiating w.r.t. time t, we get

$$\frac{ds}{dt} = \frac{dx}{dt} + \mu \frac{dy}{dt}$$



Substituting the values, we get

$$9 = 3 + \left(\frac{4}{3}\right)\frac{dy}{dt}$$

Therefore, the actual speed of dive of the bird is given by

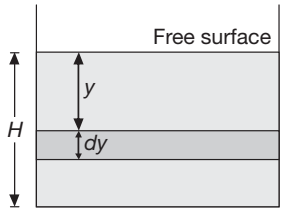
$$\frac{dy}{dt} = (9-3)\left(\frac{3}{4}\right) = 4.5\ \text{ms}^{-1}$$

ILLUSTRATION 38

A vessel is filled with a non-homogeneous liquid whose refractive index varies with the depth y from the free surface of liquid as $\mu = \left(1 + \frac{y}{H}\right)$. Calculate the apparent depth as seen by an observer from above, if H is the height to which the liquid is filled in the vessel.

SOLUTION

Let us consider a thin layer of liquid of thickness dy at a distance y below the free surface of liquid. The apparent depth of this layer having real depth dy is $dH' = \frac{dy}{\mu}$.



$$\Rightarrow \quad dH' = \frac{dy}{\mu}$$

$$\Rightarrow \quad dH' = \frac{dy}{\left(1 + \frac{y}{H}\right)} \qquad \left\{\because \ \mu = 1 + \frac{y}{H}\right\}$$

Total apparent depth is obtained by integrating this expression within appropriate limits. So,

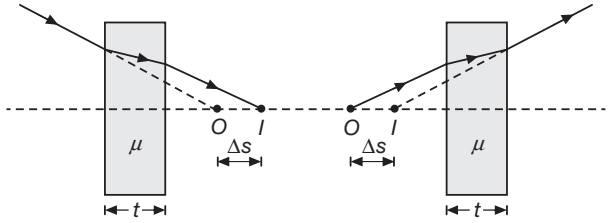
$$H' = \int dH' = \int_0^H \frac{dy}{\left(1 + \frac{y}{H}\right)} = H \log_e (H + y)\Big|_0^H$$

$$\Rightarrow \quad H' = H \log_e 2$$

SHIFT OF POINT OF CONVERGENCE OR DIVERGENCE

If a glass slab of thickness t, refractive index μ is placed in the path of a convergent (or divergent) beam of light, the point of convergence (or divergence) gets shifted by

$$\Delta s = t\left(1 - \frac{1}{\mu}\right)$$

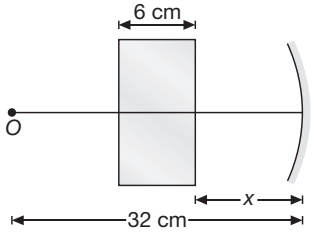


(A) Convergent beam (B) Divergent beam

ILLUSTRATION 39

A point object O is placed in front of a concave mirror of focal length 10 cm. A glass slab of refractive index $\mu = \frac{3}{2}$ and thickness 6 cm is inserted between object and mirror. Find the position of final image when the distance x shown in figure is

(a) 5 cm
(b) 20 cm



SOLUTION

The normal shift produced by a glass slab is given by

$$\Delta x = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{2}{3}\right)(6) = 2 \text{ cm}$$

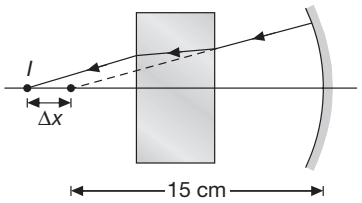
i.e., for the mirror the object is placed at a distance $(32 - \Delta x) = 30$ cm from it. Applying mirror formula i.e. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v} - \frac{1}{30} = -\frac{1}{10}$$

$$\Rightarrow \quad v = -15 \text{ cm}$$

(a) When x = 5 cm
The light falls on the slab after being reflected from the mirror as shown. But the slab will again shift it by a distance $\Delta x = 2$ cm. Hence,

the final real image is formed at a distance $(15+2)=17$ cm from the mirror.



(b) When x = 20 cm

This time too the final image is at a distance 17 cm from the mirror but it is virtual as shown.

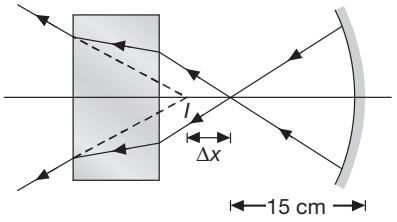
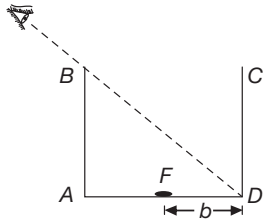


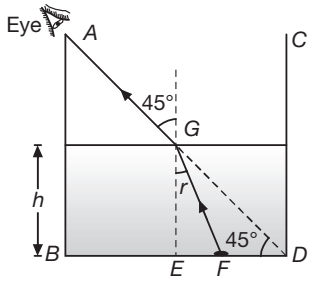
ILLUSTRATION 40

A cubical vessel with non-transparent walls is so located that the eye of an observer does not see its bottom but sees all of the wall CD. To what height should water be poured into the vessel for the observer to see an object F arranged at a distance of $b=10$ cm from corner D? The face of the vessel is $a=40$ cm and refractive index of water is $\frac{4}{3}$.



SOLUTION

Since, the vessel is cubical, $\angle GDE = 45°$ and $GE = ED = h$ (say) then $EF = ED - FD$



But $\tan(45°)=1=\frac{GE}{ED}$

$\Rightarrow \quad ED = GE = h$

$\Rightarrow \quad EF = ED - FD = h - 10$

Further, $\frac{4}{3}=\frac{\sin(45°)}{\sin r}$

$\Rightarrow \quad r = 32°$

Now $\frac{EF}{GE}=\tan r=\tan(32°)$

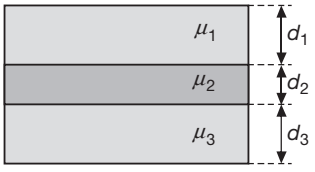
$\Rightarrow \quad \frac{h-10}{h}=0.62$

Solving this, we get

$h = 26.65$ cm

MULTISLABS

If a number of slabs (or immiscible liquids) of depth d_1, d_2, d_3,.... and refractive index μ_1, μ_2, μ_3, ... are placed one over the other as shown.



Then the real depth is

$$d = d_1 + d_2 + d_3 + ...$$

The apparent depth is given as

$$d' = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3} + ...$$

Therefore, for the combination, the effective μ is

$$\mu = \frac{d}{d'} = \frac{d_1 + d_2 + d_3 + \ldots}{\left(\frac{d_1}{\mu_1}\right) + \left(\frac{d_2}{\mu_2}\right) + \left(\frac{d_3}{\mu_3}\right) + \ldots} = \frac{\Sigma d_i}{\Sigma\left(\frac{d_i}{\mu_i}\right)}$$

If there are only two slabs, of equal thickness, $d_1 = d_2 = d$, $\mu = \dfrac{d+d}{\left(\frac{d}{\mu_1}\right)+\left(\frac{d}{\mu_2}\right)} = \dfrac{2\mu_1\mu_2}{\mu_1+\mu_2} =$ harmonic mean of μ_1 and μ_2

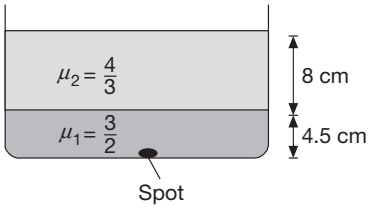
ILLUSTRATION 41

The bottom of a tub has a black spot. A glass slab of thickness 4.5 cm is placed over it and then water is filled to the height of 8 cm above the glass slab. Looking from top, what shall be the apparent depth of the spot below the water surface? Also find the effective refractive index of the combination of glass slab and water layer. (Refractive index of glass is $\frac{3}{2}$ and of water is $\frac{4}{3}$).

SOLUTION

The apparent depth is given as

$$d_a = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} = \frac{4.5}{\left(\frac{3}{2}\right)} + \frac{8}{\left(\frac{4}{3}\right)} = 3 + 6 = 9 \text{ cm}$$



The effective refractive index is given as

$$\mu_{\text{effective}} = \frac{\text{Real Depth}}{\text{Apparent Depth}}$$

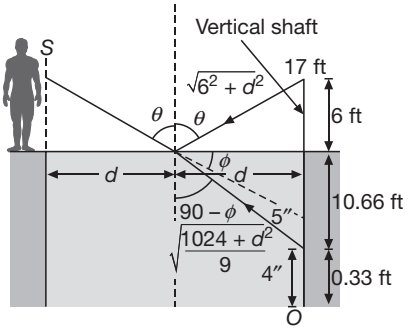
$$\Rightarrow \quad \mu_{\text{effective}} = \frac{d_r}{d_a} = \frac{d_1 + d_2}{d_a} = \frac{4.5+8}{9} = \frac{12.5}{9} = 1.39$$

ILLUSTRATION 42

A surveyor on one bank of a canal observes the image of the 4 inch mark and 17 ft mark on a vertical staff, which is partially immersed in the water and held against the bank directly opposite to him. He sees that the reflected and refracted rays come from the same point which is the centre of the canal. If the 17 ft mark and the surveyor's eye are both 6 ft above the water level, estimate the width of the canal (in foot), assuming that the refractive index of the water is $\frac{4}{3}$. Zero mark is at the bottom of the canal.

SOLUTION

Figure below shows the ray diagram of image produced at 5 ft mark for both the marks – One at 4 inch by refraction and other at 17 ft by reflection.



By using Snell's law we have

$$1 \times \sin\theta = \frac{4}{3}\sin(90 - \phi)$$

$$\Rightarrow \quad 1 \times \frac{d}{\sqrt{36 + d^2}} = \frac{4}{3} \times \frac{d}{\sqrt{\left(\frac{32}{3}\right)^2 + d^2}}$$

$$\Rightarrow \quad \sqrt{1024 + 9d^2} = 4\sqrt{36 + d^2}$$

On squaring both sides, we get

$$1024 + 9d^2 = 16(36 + d^2)$$

$$\Rightarrow \quad 1024 + 9d^2 = 576 + 16d^2$$

$$\Rightarrow \quad 7d^2 = 448$$

$$\Rightarrow \quad d^2 = \frac{448}{7} = 64$$

$$\Rightarrow \quad d = 8 \text{ foot}$$

$$\Rightarrow \quad \text{Width of canal is } 2d = 16 \text{ foot}.$$

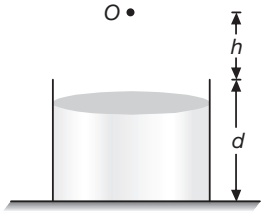
ILLUSTRATION 43

A point source of light is arranged at a height h above the surface of water. Where will the image of this source in the flat mirror-like bottom of a vessel be if the depth of the vessel full of water is d? Refractive index of water is $n = \frac{4}{3}$. Consider only two steps.

SOLUTION

When we consider only two steps, then the ray of light starting from object O first gets refracted and then reflected. Distance of image I_1 formed after refraction from the plane surface is given by

$$x = nh + d = \frac{4}{3}h + d$$



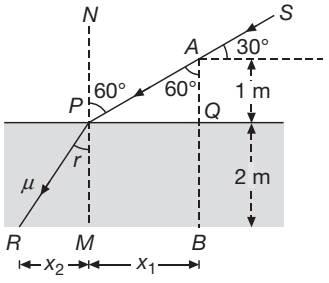
Therefore, distance of image I_2 formed by plane mirror will be $\frac{4}{3}h + d$

ILLUSTRATION 44

In a river 2 m deep, a water level measuring post embedded into the river stands vertically with 1 m of it above the water surface. If the angle of inclination of sun above the horizon is $30°$, calculate the length of the post on the bottom surface of the river. $\left(\mu \text{ for water} = \frac{4}{3}\right)$

SOLUTION

The situation is shown in figure.



The length of shadow of pole AB at the bottom of river is given as

$$L = x_1 + x_2 \quad \ldots(1)$$

From ΔAPQ, we have

$$x_1 = AQ\tan(60°) = \sqrt{3} \text{ meter} \quad \ldots(2)$$

From ΔRPM, we have

$$x_2 = PM\tan r = 2\tan r \quad \ldots(3)$$

From Snell's law, at point P, we have

$$\frac{\sin 60}{\sin r} = \mu = \frac{4}{3}$$

$$\Rightarrow \quad \sin r = \frac{3}{4}\sin(60°) = \frac{3\sqrt{3}}{8}$$

$$\Rightarrow \quad \cos r = \sqrt{1 - \sin^2 r} = \frac{\sqrt{37}}{8}$$

We substitute the value of r in Equation (3)

$$x_2 = 2\tan r$$

$$\Rightarrow \quad x_2 = 2 \times \frac{\frac{3\sqrt{3}}{8}}{\frac{\sqrt{37}}{8}} = \frac{6\sqrt{3}}{\sqrt{37}}$$

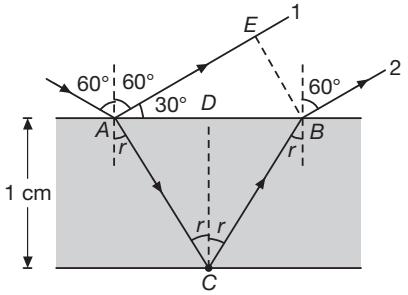
ILLUSTRATION 45

A ray of light falls onto a plane-parallel glass plate 1 cm thick at an angle of $60°$. The refractive index of the glass is $\sqrt{3}$. Some of the light is reflected and the rest, being refracted, passes into the glass is reflected from the bottom of the plate, refracted a second time and emerges back into the air parallel to the first reflected ray. Determine the distance l between the rays.

SOLUTION

From Snell's Law, we have $\sqrt{3} = \frac{\sin(60°)}{\sin r}$

$$\Rightarrow \quad r = 30°$$



Since, $AB = 2(AD) = 2(DC\tan r)$

$$\Rightarrow \quad AB = (2)(1)\left(\frac{1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}} \text{ cm}$$

So, the distance between rays 1 and 2 is given by

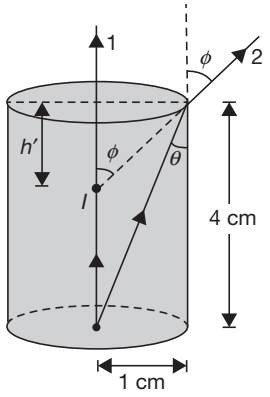
$$BE = AB\sin(30°) = \frac{1}{\sqrt{3}} \text{ cm}$$

ILLUSTRATION 46

A small object is kept at the centre of bottom of a cylindrical beaker of diameter 6 cm and height 4 cm filled completely with water $\left(\mu = \frac{4}{3}\right)$. Consider the light ray from an object leaving the beaker through a corner. If this ray and the ray along the axis of beaker is used to locate the image, find the apparent depth in this case.

SOLUTION

Figure shows the ray diagram of the image formation as described in the given condition.



By using Snell's law, we have

$$\mu_W \sin\theta = 1\sin\phi$$

$$\Rightarrow \quad \frac{4}{3}\times\frac{3}{5} = \sin\phi$$

$$\Rightarrow \quad \phi = 53°$$

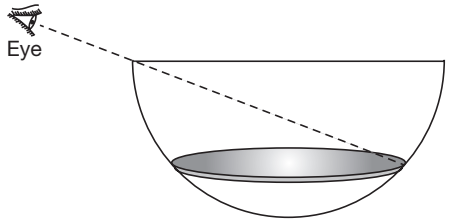
Here we have $\tan\phi = \frac{3}{h'}$

$$\Rightarrow \quad h' = \frac{3}{\frac{4}{3}} = \frac{9}{4} = 2.25 \text{ cm}$$

ILLUSTRATION 47

A circular disc of diameter d lies horizontally inside a metallic hemispherical bowl of radius a. The disc is just visible to an eye looking over the edge. The bowl is now filled with a liquid of refractive index μ. Now, the whole of the disc is just visible to the eye in the same position. Show that

$$d = 2a\left(\frac{\mu^2 - 1}{\mu^2 + 1}\right)$$

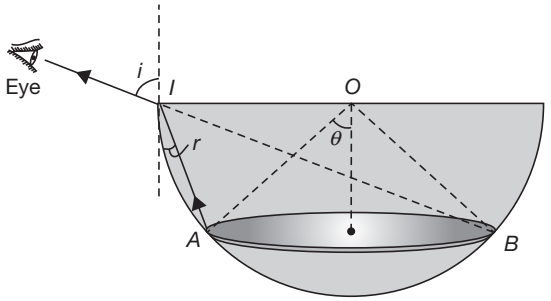


SOLUTION

In the figure, let AB be the disc and O be the centre of the bowl.

Let $\angle AOM = \theta$

Then by symmetry, $\angle AIB = \theta$



Now from geometry of figure, we have

$$\angle AOI = 2\angle IBA$$

$$\Rightarrow \quad \angle IBA = \frac{1}{2}(90° - \theta)$$

$$\Rightarrow \quad \angle r = \angle IBA = \frac{90° - \theta}{2} \qquad \ldots(1)$$

$$\text{Also } \angle i = \frac{90° - \theta}{2} + \theta = \frac{90° + \theta}{2} \qquad \ldots(2)$$

Now, Snell's Law gives

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{90° + \theta}{2}\right)}{\sin\left(\frac{90° - \theta}{2}\right)}$$

$$\Rightarrow \quad \left(\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right)^2=\mu^2$$

$$\Rightarrow \quad \frac{1+\sin\theta}{1-\sin\theta}=\mu^2$$

$$\Rightarrow \quad \sin\theta=\frac{\mu^2-1}{\mu^2+1}$$

Since, $\sin\theta=\frac{d}{2a}$

$$\Rightarrow \quad d=2a\left(\frac{\mu^2-1}{\mu^2+1}\right)$$

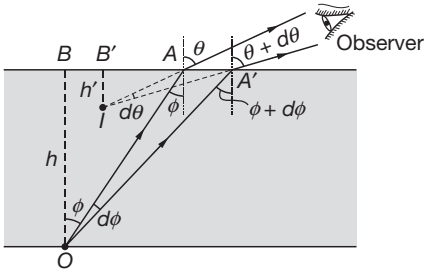
ILLUSTRATION 48

A man standing on the edge of a swimming pool looks at a stone lying at the bottom of the pool. The depth of the swimming pool is h. At what distance from the surface of water is the image of the stone formed if the line of sight makes an angle θ with the normal to the surface?

SOLUTION

Please note that, this problem is not the case of normal viewing (as discussed earlier), in which the apparent depth of the object lying at the bottom of the pool was $h'=\frac{h}{\mu}$.

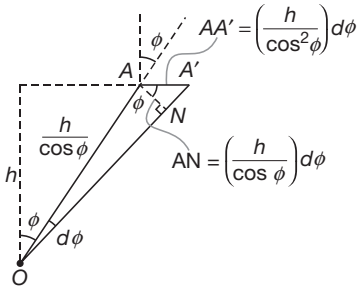
However, in this case, the observer is seeing the stone standing at the edge of the pool and the observer has line of sight angle θ. Let us draw two rays OA and OA' very close to each other. Out of these two rays, one ray is passing through the edge of the pool. The rays incident on the interface in the region AA', after refraction will reach the eye of the observer, so that he sees the object O as shown in figure.



Let us now produce the refracted rays backwards so that their point of intersection (I) is the place where the image of the object O is formed. So $OB=h$ is the real depth and $IB'=h'$ is the apparent depth.

To calculate h', let us consider triangles OAA' and IAA'. From triangle OAA', we calculate AA' and also from triangle IAA' we again calculate AA' (common side to both triangles) and equate both.

From A, drop a perpendicular on OA' at the point N so that AN is an arc of a circular portion of radius $OA=\frac{h}{\cos\phi}$ as shown in figure.



Since $AN=(OA)\,d\phi=\left(\frac{h}{\cos\phi}\right)d\phi$

In triangle $AA'N$, we have

$$\cos\phi=\frac{AN}{AA'}$$

$$\Rightarrow \quad AA'=\frac{AN}{\cos\phi}=\left(\frac{h}{\cos^2\phi}\right)d\phi \qquad \ldots(1)$$

Similarly, from triangle IAA', we get

$$AA'=\left(\frac{h'}{\cos^2\theta}\right)d\theta \qquad \ldots(2)$$

Equating equations (1) and (2), we get

$$\left(\frac{h}{\cos^2\phi}\right)d\phi=\left(\frac{h'}{\cos^2\theta}\right)d\theta$$

$$\Rightarrow \quad h'=h\left(\frac{\cos^2\theta}{\cos^2\phi}\right)\left(\frac{d\phi}{d\theta}\right) \qquad \ldots(3)$$

Now applying Snell's Law at A, we get

$$\mu=\frac{\sin\theta}{\sin\phi} \qquad \ldots(4)$$

$$\Rightarrow \quad \sin\theta=\mu\sin\phi$$

Differentiating both sides, we get

$$\cos\theta d\theta = \mu\cos\phi d\phi$$

$$\Rightarrow \quad \frac{d\phi}{d\theta} = \frac{\cos\theta}{\mu\cos\phi} \qquad \ldots(5)$$

Put equation (5) in (1), we get

$$h' = \frac{h}{\mu}\left(\frac{\cos^3\theta}{\cos^3\phi}\right) \qquad \ldots(6)$$

Further from equation (4), we get $\sin\phi = \dfrac{\sin\theta}{\mu}$

$$\Rightarrow \quad \cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\frac{\sin^2\theta}{\mu^2}}$$

$$\Rightarrow \quad \cos^3\phi = \left(1-\frac{\sin^2\theta}{\mu^2}\right)^{\frac{3}{2}} = \frac{\left(\mu^2-\sin^2\theta\right)^{\frac{3}{2}}}{\mu^3} \qquad \ldots(7)$$

Substituting equation (7) in (6), we get

$$h' = \frac{\mu^2 h\cos^3\theta}{\left(\mu^2-\sin^2\theta\right)^{\frac{3}{4}}}$$

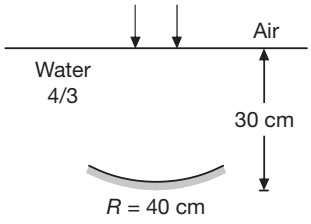
Test Your Concepts-III

Based on General Refraction

(Solutions on page H.9)

1. An object lies 100 cm inside water. It is viewed from air nearly normally. Find the apparent depth of the object.

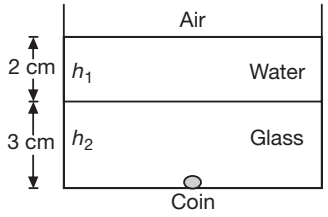
2. A concave mirror is placed inside water with its shining surface upwards and principal axis vertical as shown. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image.



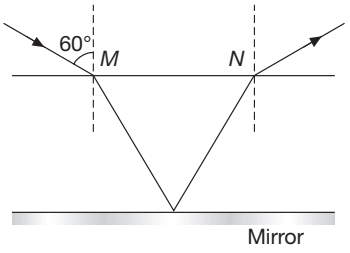
3. A small object is placed on the principal axis of a concave spherical mirror of radius 20 cm at a distance of 30 cm. By how much will the position and size of the image alter, when a parallel-sided slab of glass of thickness 6 cm and refractive index 1.5 is introduced between the centre of curvature and the object? The parallel sides are perpendicular to the principal axis.

4. The velocity of light in air is 3×10^8 ms^{-1}. If yellow light of wavelength 6000 Å is passed from air to glass of refractive index 1.5, determine the velocity, the wavelength and the colour of light in glass.

5. A 2 cm thick layer of water covers a 3 cm thick glass slab. A coin is placed at the bottom of the slab and is being observed from the air side along the normal to the surface. Find the apparent position of the coin from the surface.



6. A plate with plane parallel faces having refractive index 1.8 rests on a plane mirror. A light ray is incident on the upper face of the plate at 60°. How far from the entry point will the ray emerge after reflection by the mirror of the plate is 6 cm thick?



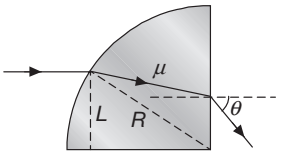
7. A pole 4 m high is driven into the bottom of a lake and happens to be 1 m above the water. Determine the length of the shadow of the pole at the bottom of the lake if the sunrays make an angle of 45° with the water surface. The refractive index of water is $\frac{4}{3}$.

8. A ray of light is refracted through a sphere whose material has a refractive index μ in such a way that it passes through the extremities of two radii which make an angle β with each other. prove that if α is the deviation of the ray caused by its passage through the sphere,

$$\cos\left(\frac{\beta-\alpha}{2}\right)=\mu\cos\left(\frac{\beta}{2}\right)$$

9. A vertical beam of light of cross-sectional radius r is incident symmetrically on the curved surface of a glass hemisphere $\left(\mu=\frac{3}{2}\right)$ of radius $2r$ placed with its base on a horizontal table. Find the radius of the luminous spot formed on the table.

10. A material having an index of refraction μ is surrounded by vacuum and is in the shape of a quarter circle of radius R. A light ray parallel to the base of the material is incident from the left at a distance of L above the base and emerges out of the material at an angle θ. Determine an expression for θ.



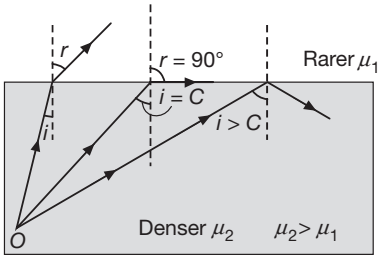
11. How much water should be filled in a container of height 21 cm so that it will appear half filled when viewed along normal to water surface. Take refractive index of water $\mu_W=\frac{4}{3}$.

12. A ray of light is incident on a glass slab at grazing incidence. The refractive index of the material of the slab is given by $\mu=\sqrt{1+y}$. If the thickness of the slab is d, determine the equation of the trajectory of the ray inside the slab and the coordinates of the point where the ray exits from the slab. Take the origin to be at the point of entry of the ray.

TOTAL INTERNAL REFLECTION (TIR)

When a ray of light goes from a denser to a rarer medium, it bends away from the normal. If the angle of incidence in the denser medium is increased the angle of refraction in the rarer medium also increases. At a particular angle of incidence in the denser medium (called as the Critical angle C), the angle of refraction in the rarer medium is 90° (i.e., the refracted ray grazes the interface). **This angle of refraction in the denser medium for which the refracted ray grazes the interface is called the critical angle for the pair of interface**.

Please note that for small angles of incidence, both reflection and refraction occur, however we shall be neglecting the reflection at the interface as most of the light is refracted. However, when $i>C$, no part of light is refracted and the entire light is reflected back to the denser medium itself. This phenomenon is called total internal reflection (TIR) and was first noted by Kepler in 1604.



Images formed by TIR are much brighter than those formed by the mirrors (or lenses). Some loss of intensity always takes place, when light is reflected from a mirror (or refracted through a lens).

CRITICAL ANGLE

According to Snell's Law, we have

$$\frac{\sin i}{\sin r} = {}^{d}\mu_{r}$$

$$\Rightarrow \quad \frac{\sin C}{\sin 90} = \frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}}$$

$$\Rightarrow \quad \sin C = \frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}} = \frac{1}{\mu_{\text{denser}}}$$

$$\text{Thus } C = \sin^{-1}\left(\frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}}\right) = \sin^{-1}\left(\frac{1}{\mu_{\text{denser}}}\right)$$

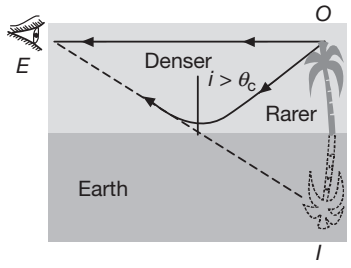
where μ_{denser} is the refractive index of the denser medium w.r.t. the rarer medium. The lesser the value of μ_{denser}, the greater is the critical angle C.

For a given pair of media, since μ depends on the wavelength of light the critical angle also depends on the wavelength. The greater the wavelength, the greater will be the critical angle.

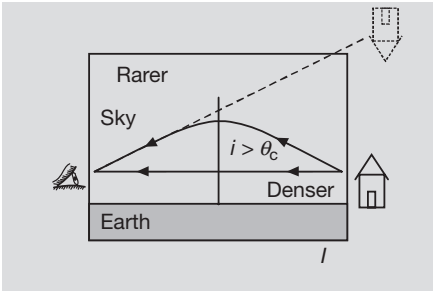
Media Pair	μ_{denser}	Critical angle $C = \sin^{-1}\left(\frac{1}{\mu_{\text{denser}}}\right)$
Water-Air	$\mu_d = \frac{\mu_w}{\mu_a} = \frac{4/3}{1} = \frac{4}{3}$	49°
Glass-Air	$\mu_d = \frac{\mu_g}{\mu_a} = \frac{3/2}{1} = \frac{3}{2}$	42°
Glass-Water	$\mu_d = \frac{\mu_g}{\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$	63°

EXAMPLES OF TOTAL INTERNAL REFLECTION

(a) Mirage: Mirage is an optical illusion observed in deserts and roads on a hot day. When the air near the ground is hotter (and hence rarer) than the air above, there occurs a continuous decrease of refractive index of air towards the ground.



(b) Looming: Similarly, in extremely cold regions (near polar regions), the refractive index decreases with height. Due to TIR (shown in figure), the image of a hut appears hanging in the air. This is called looming.

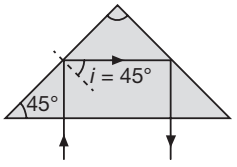


(c) The μ of diamond is 2.5, for which C is only $24°$. Diamonds are cut such that $i > C$, so TIR takes place again and again inside it. The light coming out from few meticulously cut surfaces makes it sparkle.

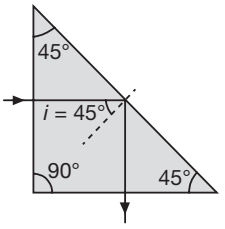
(d) Air bubbles in water shine due to TIR.

(e) The working of an optical fibre is due to multiple TIR inside it.

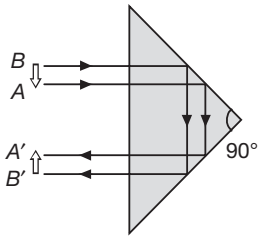
(f) Porro prisms used in periscopes or binoculars bend the ray due to TIR. Some examples are shown in figure.



(a) Bending of rays by 180°



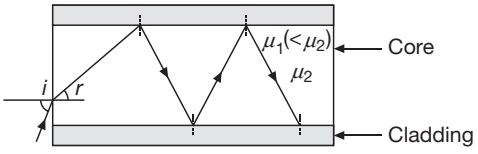
(b) Bending of rays by 90°



(c) Erecting of image

OPTICAL FIBRE

An optical fibre is a transmission medium to carry the optical signal without any appreciable loss. It is a device based on total internal reflection by which signals can be transmitted from one location to another. The optical fibre works even if it is bent or twisted.
The structure of optical fibre consists of a core surrounded by a cladding.

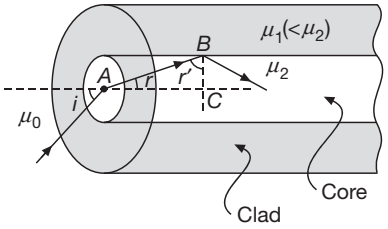


The core is denser medium of refractive index μ_2 and cladding is relatively rarer medium of refractive index μ_1 such that $\mu_1 < \mu_2$. The light incident at one dent on the interface between the core and cladding at an angle greater than the critical angle is continuously reflected in the core.

It is a thin fibre of plastic or specially coated glass in which light enters at one end and leaves it at the other end suffering a number of total internal reflections with little loss of energy. The optical fibre works even if it is bent or twisted.
The thickness of the fibre is of the order of human hair $(10^{-6}\ \text{m})$.

ANGLE OF ACCEPTANCE

Maximum angle at which the ray should enter into the core for the transmission through optical fibre is called angle of acceptance $(i_{\max})$. Suppose the surrounding medium has refractive index μ_0 and core and clad have refractive indices μ_2 and μ_1 respectively $(\mu_2 > \mu_1)$.



The ray of light enters into the core at an angle of incidence i as shown
From Snell's Law

$$\mu_0 \sin i = \mu_2 \sin r \qquad \ldots(1)$$

From ΔABC,

$$r = 90 - r' \qquad \ldots(2)$$

From equation (1) and (2)

$$\mu_0 \sin i = \mu_2 \sin(90 - r')$$

$$\Rightarrow \quad \mu_0 \sin i = \mu_2 \cos r'$$

$$\Rightarrow \quad \mu_0 \sin i = \mu_2 \sqrt{1 - \sin^2 r'}$$

$$\Rightarrow \quad \mu_2^2 (1 - \sin^2 r') = \mu_0^2 \sin^2 i$$

$$\Rightarrow \quad \sin^2 r' = \frac{\mu_2^2 - \mu_0^2 \sin^2 i}{\mu_2^2}$$

$$\Rightarrow \quad \sin r' = \frac{\sqrt{\mu_2^2 - \mu_0^2 \sin^2 i}}{\mu_2}$$

Now for total internal reflection at B
$r' \geq C$, where C is critical angle for a ray coming from core to clad

$$\Rightarrow \quad \sin r' \geq \sin C$$

$$\Rightarrow \quad \sin r' \geq \frac{\mu_1}{\mu_2}$$

$$\Rightarrow \quad \frac{\sqrt{\mu_2^2 - \mu_0^2 \sin^2 i}}{\mu_2} \geq \frac{\mu_1}{\mu_2}$$

$$\Rightarrow \quad \mu_2^2 - \mu_0^2 \sin^2 i \geq \mu_1^2$$

$$\Rightarrow \quad \sin^2 i \leq \frac{\mu_2^2 - \mu_1^2}{\mu_0^2}$$

$$\Rightarrow \quad \sin i \leq \frac{\sqrt{\mu_2^2 - \mu_1^2}}{\mu_0}$$

$$\Rightarrow \quad i \le \sin^{-1}\left(\frac{\sqrt{\mu_2^2-\mu_1^2}}{\mu_0}\right)$$

$$\Rightarrow \quad i_{max} = \sin^{-1}\left(\frac{\sqrt{\mu_2^2-\mu_1^2}}{\mu_0}\right)$$

FIELD OF VISION OF A FISH

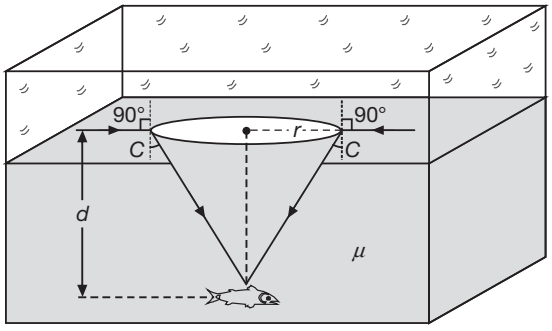
A fish inside a pond does not see the outside world through the entire surface of water. The light from outside can reach the fish only through a circular patch, which forms a cone of half angle equal to the critical angle.

If r is the radius of the circular patch, d is the depth of the fish and μ is the refractive index of water, then

$$r = d\tan C = d\frac{\sin C}{\cos C} = d\frac{\sin C}{\sqrt{1-\sin^2 C}}$$

Since, $\sin C = \frac{1}{\mu}$

$$\Rightarrow \quad r = \frac{d}{\sqrt{\mu^2-1}}$$

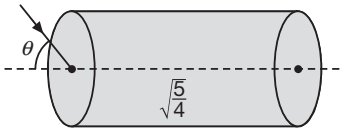


Fish in glass tank

Similarly, if a source of light is kept in a pond, its light will come out only through a circular region. For any incident angle i greater than C, the light will be totally reflected back into the water, making corresponding region on the surface of water appear dark.

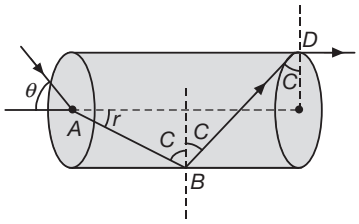
ILLUSTRATION 49

Light is incident making an angle θ with the axis of a transparent cylindrical fiber of refractive index $n = \sqrt{\frac{5}{4}}$ as shown in figure. Determine the maximum value of θ so that the light entering the cylinder does not come out of the curved surface.



SOLUTION

The ray of light is incident at A and it just gets reflected totally at B. Therefore incident angle at B is equal to the critical angle given as $C = \sin^{-1}\left(\frac{1}{n}\right)$



Snell's Law of refraction at A gives

$$\frac{\sin\theta}{\sin r} = n$$

$$\Rightarrow \quad \sin r = \frac{\sin\theta}{n} \qquad \ldots(1)$$

Since $r + C = 90°$

$$\Rightarrow \quad \sin r = \sin(90° - C) = \cos C$$

For a ray not to come through the curved surface,

$$r \le 90 - C$$

$$\Rightarrow \quad \sin r \le \sqrt{1-\sin^2 C} \le \sqrt{1-\frac{1}{n^2}} \qquad \ldots(2)$$

Eliminating $\sin r$ from (1) and (2), we get

$$\frac{\sin\theta}{n} \le \sqrt{1-\frac{1}{n^2}}$$

$$\Rightarrow \quad \sin\theta \le \sqrt{n^2-1}$$

$$\Rightarrow \quad \sin^2\theta \le 1.25 - 1$$

$\Rightarrow \quad \sin^2\theta \le 0.25$

$\Rightarrow \quad \sin\theta \le \frac{1}{2}$

$\Rightarrow \quad \theta \le 30^\circ$

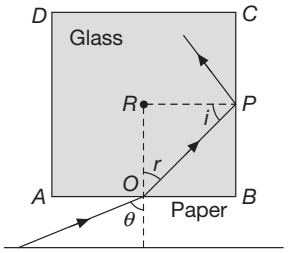
$\Rightarrow \quad \theta_{max} = 30^\circ$

ILLUSTRATION 50

A rectangular block of glass is placed on a printed page lying on a horizontal surface. Find the minimum value of the refractive index of glass for which the letters on the page are not visible from any of the vertical faces of the block.

SOLUTION

Light will not emerge out from the vertical face BC, when



$i >$ Critical Angle (C)

$\Rightarrow \quad \sin i > \sin C$

$\Rightarrow \quad \sin i > \frac{1}{\mu} \qquad \left\{\because \sin C = \frac{1}{\mu}\right\}$

Applying Snell's Law at O, we get

$1\sin\theta = \mu \sin r$

$\Rightarrow \quad \sin\theta = \mu\sin(90^\circ - i) = \mu\cos i$

$\Rightarrow \quad \cos i = \frac{\sin\theta}{\mu}$

$\Rightarrow \quad \sin i = \sqrt{1-\cos^2 i} = \sqrt{1-\frac{\sin^2\theta}{\mu^2}}$

Therefore, the condition for no light to emerge from vertical face BC becomes,

$$\sqrt{\frac{\mu^2 - \sin^2\theta}{\mu^2}} > \frac{1}{\mu}$$

$\Rightarrow \quad \mu^2 > 1 + \sin^2\theta$

$\Rightarrow \quad \mu^2 > 1 + 1 \quad \{\because$ maximum value of θ can be $90^\circ\}$

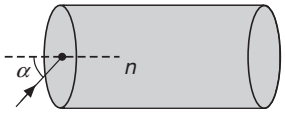
$\Rightarrow \quad \mu > \sqrt{2}$

So, the minimum value of refractive index is

$\mu_{min} = \sqrt{2}$

ILLUSTRATION 51

Light is incident at an angle α on one planar end of a transparent cylindrical rod of refractive index n. Determine the least value of n so that the light entering the rod does not emerge from the curved surface of the rod irrespective of the value of α.

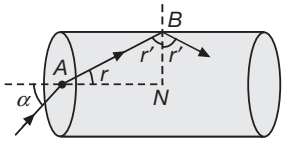


SOLUTION

Since, $\sin C = \frac{1}{n}$

For TIR at B, $(r')_{min} > C$

In triangle ABN, $r' + r + 90^\circ = 180^\circ$



$\Rightarrow \quad r' = 90 - r$

$\Rightarrow \quad (r')_{min} = 90^\circ - (r)_{max}$

and $\quad n = \frac{\sin(i)_{max}}{\sin(r)_{max}} = \frac{\sin 90^\circ}{\sin(r)_{max}} \left(i_{max} = 90^\circ\right)$

Then, $\sin(r)_{max} = \frac{1}{n} = \sin C$

$(r)_{max} = C$

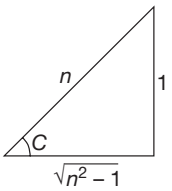
$\Rightarrow \quad (r')_{min} = (90^\circ - C)$

Now, if minimum value of r' i.e., $90^\circ - \theta_c$ is greater than θ_c, then obviously all values of r' will be greater than θ_c i.e., total internal reflection will take

place at face AB in all conditions. Therefore, the necessary condition is

$$(r')_{\min} > C$$

$$\Rightarrow \quad (90^\circ - C) > C$$



$$\Rightarrow \quad \sin(90^\circ - C) > \sin C$$

$$\Rightarrow \quad \cot C > \sin C$$

$$\Rightarrow \quad \cos C > 1$$

$$\Rightarrow \quad \sqrt{n^2-1} > 1$$

$$\Rightarrow \quad n^2 > 2$$

$$\Rightarrow \quad n > \sqrt{2}$$

Therefore, minimum value of n is $\sqrt{2}$

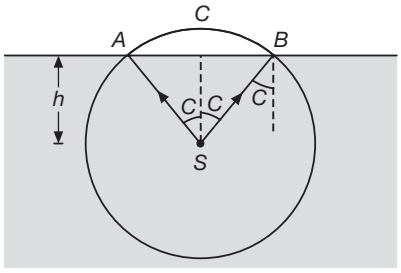
ILLUSTRATION 52

A point source of light is placed at a distance h below the surface of a large and deep lake. Show that the fraction f of light that escapes directly from water surface is independent of h and is given by,

$$f = \frac{\left[1 - \sqrt{1 - \frac{1}{\mu^2}}\right]}{2}.$$

SOLUTION

Due to TIR, light will be reflected back into the water for $i > C$. So only that portion of incident light will escape which passes through the cone of angle $\theta = 2C$.



So, the fraction of light escaping is given by

$$f = \frac{\text{Area of Surface } ACB}{\text{Total Area of Sphere}}$$

$$\Rightarrow \quad f = \frac{2\pi R^2(1 - \cos C)}{4\pi R^2} = \frac{1 - \cos C}{2}$$

Now, as f depends on C, which depends only on μ, hence f is independent of h.

Since, we know that

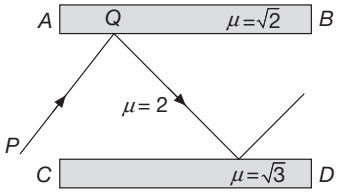
$$\sin C = \frac{1}{\mu}$$

$$\Rightarrow \quad \cos C = \frac{\sqrt{\mu^2 - 1}}{\mu} = \sqrt{1 - \frac{1}{\mu^2}}$$

$$\Rightarrow \quad f = \frac{1}{2}\left(1 - \sqrt{1 - \frac{1}{\mu^2}}\right)$$

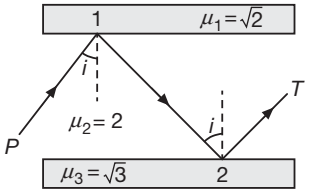
ILLUSTRATION 53

AB and CD are two slabs. The medium between the slabs has refractive index 2. Find the minimum angle of incidence of Q, so that the ray is totally reflected by both the slabs.



SOLUTION

Let the critical angles at 1 and 2 be C_1 and C_2 respectively. Then



$$C_1 = \sin^{-1}\left(\frac{\mu_1}{\mu_2}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

and $$C_2 = \sin^{-1}\left(\frac{\mu_3}{\mu_2}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$$

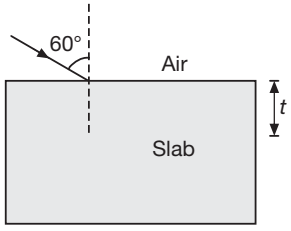
For TIR, $i > C_2$

Therefore, minimum angle of incidence, for total internal reflection to take place on both slabs must be $60°$.

$$i_{min} = 60°$$

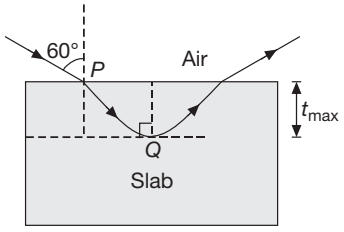
ILLUSTRATION 54

A ray of light enters into a glass slab from air as shown in figure. If refractive of glass slab varies with t, the thickness of the slab measured from the top as $\mu = A - Bt$ where A and B are constants. Find the maximum depth travelled by ray in the slab. Assume thickness of slab to be sufficiently large.



SOLUTION

The path of ray is curved as shown in figure. As it travels successively into denser layers, it bends away from normal and TIR takes place at depth where angle of incidence approaches $\frac{\pi}{2}$.



Applying Snell's Law at interfaces P and Q, we get

$$1\sin(60°) = \mu_B \sin\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = (A - Bt_{max})$$

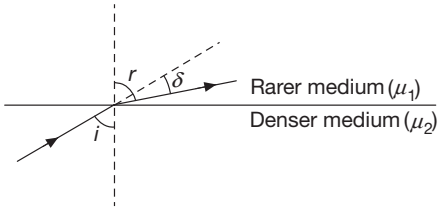
$$\Rightarrow \quad t_{max} = \frac{1}{B}\left(A - \frac{\sqrt{3}}{2}\right)$$

ILLUSTRATION 55

Plot the deviation (δ) versus the angle of incidence (i) graph for a ray travelling from denser to rarer medium.

SOLUTION

CASE-1: When angle of incidence (i) is less than critical angle C i.e., $i < C$



Since, $\delta = \text{Deviation} = r - i$ …(1)

From Snell's Law, we get

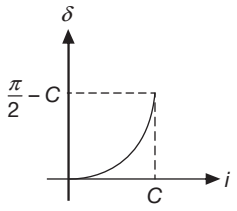
$$\mu_1 \sin i = \mu_2 \sin r$$

$$\Rightarrow \quad r = \sin^{-1}\left(\frac{\sin i}{{}^1\mu_2}\right)$$

Substituting in equation (1), we get

$$\delta = \sin^{-1}\left(\frac{\sin i}{{}^1\mu_2}\right) - i$$

This is a non-linear function and graph is given below

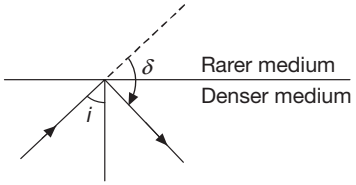


Deviation versus angle of incidence graph when TIR is not taking place.

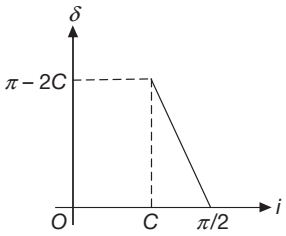
CASE-2: When the angle of incidence i is greater than the critical angle C, i.e., $i > C$

In this case TIR will take place as shown, so deviation is

$$\delta = \pi - 2i \qquad \ldots(2)$$



This is a linear function and so the graph is given below



Deviation versus angle of incidence graph when TIR is taking place

Conceptual Note(s)

When ray is travelling from rarer to denser medium then deviation is given by

$$\delta = i - \sin^{-1}\left({}^{1}\mu_2 \sin i\right)$$

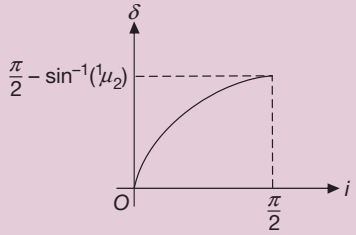
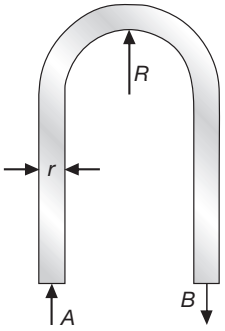


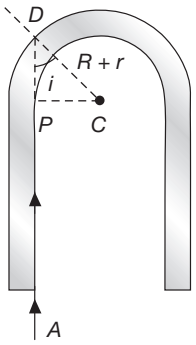
ILLUSTRATION 56

Find the maximum value of $\frac{r}{R}$, so that the beam of light incident normally at the face A of a U shaped glass tube emerges through B as shown in the figure. The refractive index of glass is $\mu = \frac{3}{2}$.



SOLUTION

Incident angle i is least for ray AP and this angle should be greater than the critical angle C



i.e., $i > C$

$\Rightarrow \quad \sin i > \sin C$

$\Rightarrow \quad \frac{R}{R+r} > \frac{1}{\mu}$

$\Rightarrow \quad \frac{R}{R+r} > \frac{2}{3}$

$3R > 2R + 2r$

$\Rightarrow \quad R > 2r$

$\Rightarrow \quad \frac{r}{R} < \frac{1}{2}$

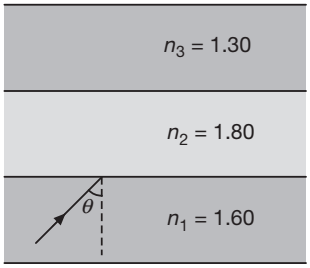
Hence, $\left(\frac{r}{R}\right)_{\max} = \frac{1}{2}$

Test Your Concepts-IV

Based on Total Internal Reflection (TIR)

(Solutions on page H.12)

1. Light refracts from medium 1 into a thin layer of medium 2, crosses that layer and then is incident at the critical angle on the interface between media 2 and 3 as shown in figure.

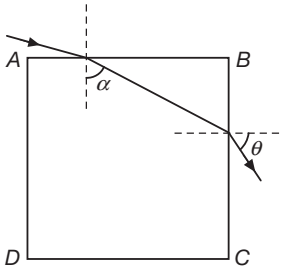


(a) Find the angle θ.
(b) If θ is decreased, will the light be refracted to medium 3?

2. A container contains water upto a height of 20 cm and there is a point source of light at the centre of the bottom of the container. A rubber ring of radius a floats centrally on the water. The ceiling of the room is 2 cm above the water surface.
(a) Find the radius of the shadow of the ring formed on the ceiling if $a = 15$ cm.
(b) Find the maximum value of a for which the shadow of the ring is formed on the ceiling.

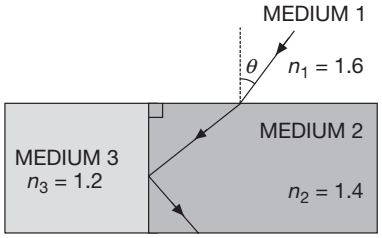
Refractive index of water $= \frac{4}{3}$.

3. *ABCD* is the plane of glass cube of refractive index μ. A horizontal beam of light enters the face *AB* at the grazing incidence.
(a) Show that the angle θ which any ray emerging from *BC* would make with normal to *BC* is given by $\sin\theta = \cot\alpha$ where α is the critical angle.



(b) What is the greatest value that the refraction index of glass may have if any of the light is to emerge from *BC*?

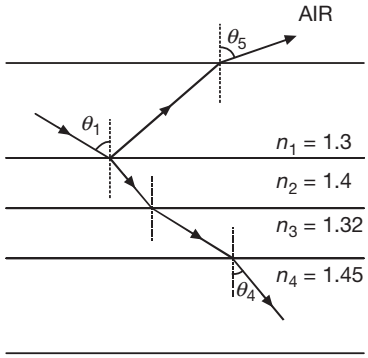
4. In figure, light refracts into material 2, crosses that material and is then incident at the critical angle on the interface between materials 2 and 3.



(a) What is angle θ?
(b) If θ is increased, is there refraction of light into material 3?

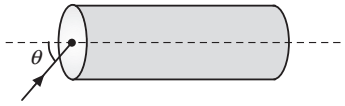
5. An isotropic point source is placed at a depth h below the water surface. An opaque disc capable of floating on water surface is placed on the surface of water so that the bulb is not visible from the surface. Find the minimum radius of the disc for the bulb not to be visible. Take refractive index of water $= \mu$.

6. In figure, light begins from medium of refractive index $n_1 = 1.3$, undergoes three refractions as it heads downward and a reflection and then a refraction to reach the air. The initial angle $\theta_1 = 30°$. Find the values of the angles



(a) θ_5 and
(b) θ_4.

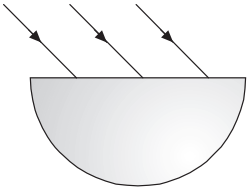
7. Determine the maximum angle θ for which the light ray incident on the end of pipe shown in figure are subject to TIR along the walls of the pipe. Assume that the pipe has an index of refraction of 1.36 and the outside medium is air.



8. A point source of light S is placed at the bottom of a vessel containing a liquid of refractive index $\frac{5}{3}$. A person is viewing the source from above the surface. There is an opaque disc of radius 1 cm floating on the surface. The center of the disc lies vertically above the source S. The liquid from the vessel is gradually drained out through a tap. What is the maximum height of the liquid for which the source cannot at all seen from above.

9. A rectangular glass block is placed on top of a sheet of paper on which there is a small cross. When the paper is soaked in alcohol and a sodium lamp is placed opposite to one vertical of the block the cross can be seen through the opposite vertical face up to a point where the angle of emergence of the light is 30°. If the refractive index of the glass is 1.5, find the refractive index of alcohol. Why can't the black cross be seen through the face when the paper is dry.

10. Rays of light fall on the plane surface of a half cylinder at an angle 45° in the plane perpendicular to the axis (see figure). Refractive index of glass is $\sqrt{2}$. Discuss the condition that the rays do not suffer total internal reflection.

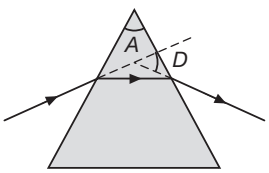


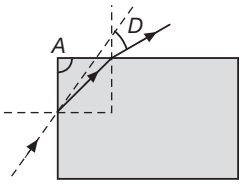
PRISM

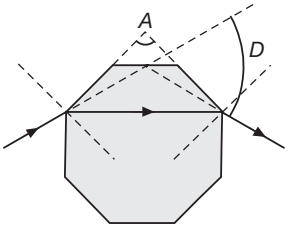
Prism is a transparent medium bounded by any number of surfaces in such a way that the surface on which light is incident and the surface from which light emerges are **plane** and **non-parallel**.

Refracting angle of prism, or simply the **angle of prism** is the angle between the faces on which light is incident and from which light emerges. In all the prisms shown in figure above, angle A is the angle of prism.

Angle of deviation (D) is the angle between the incident ray and the emergent ray. Sometimes the angle of deviation is also denoted by δ.





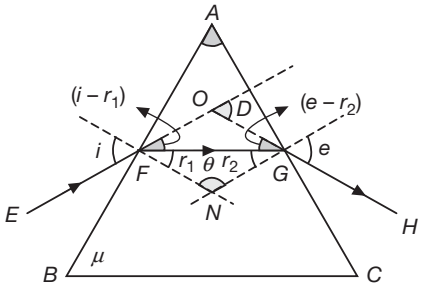


Please note that, for a glass-slab, the angle of prism is zero, and the incident ray emerges parallel to itself, i.e., there is no deviation. If μ of the prism material is same as that of its surroundings, no refraction takes place and light passes through undeviated.

REFRACTION THROUGH A PRISM

Consider a monochromatic ray EF to be incident on the face AB of prism ABC of refracting angle A at angle of incidence i. The ray is refracted along FG, r_1 being angle of refraction. The ray FG is incident on

the face AC at angle of incidence r_2 and is refracted in air along GH. Thus GH is the emergent ray and e is the angle of emergence. The angle between incident ray EF and emergent ray GH (produced backwards) is called angle of deviation D.



In triangle OFG,

$$D=(i-r_1)+(e-r_2)$$

$$\Rightarrow \quad D=(i+e)-(r_1+r_2) \qquad \ldots(1)$$

Also in quadrilateral $AFNG$,

$$A+90°+\theta+90°=360°$$

$$\Rightarrow \quad A+\theta=180° \qquad \ldots(2)$$

And in triangle FGN,

$$r_1+r_2+\theta=180° \qquad \ldots(3)$$

Comparing equations (2) and (3), we get

$$A=r_1+r_2 \qquad \ldots(4)$$

From (1), we get

$$D=i+e-A$$

$$\Rightarrow \quad i+e=A+D \qquad \ldots(5)$$

If μ is the refractive index of material of prism, then from Snell's Law

$$\mu=\frac{\sin i}{\sin r_1}=\frac{\sin e}{\sin r_2} \qquad \ldots(6)$$

Since, $D=i+e-A$, where $\sin e=\mu\sin r_2=\mu\sin(A-r_1)$

$$\Rightarrow \quad \sin e=\mu(\sin A\cos r_1-\sin r_1\cos A), \text{ where}$$

$$\sin r_1=\frac{\sin i}{\mu}$$

$$\Rightarrow \quad \sin e=\sin A\sqrt{\mu^2-\sin^2 i}-\sin i\cos A$$

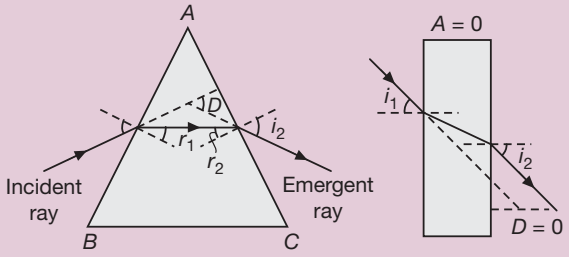
$$\Rightarrow \quad D=i+\sin^{-1}\left(\sin A\sqrt{\mu^2-\sin^2 i}-\sin i\cos A\right)$$

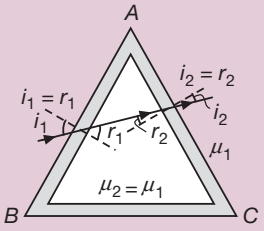
For a prism with small refracting angle, we have

$$D=(\mu-1)A \ .$$

Conceptual Note(s)

(a) Angle of deviation (D) means the angle between emergent and incident rays i.e., the angle through which incident ray turns in passing through a prism. It is represented by D and is shown in figure.





(b) If the faces of a prism on which light is incident and from which it emerges becomes parallel (as in figure), angle of prism will be zero and as incident ray will emerge parallel to itself, deviation will also be zero i.e., the prism will act as a slab.

(c) If μ of the material of the prism becomes equal to that of surroundings, no refraction at its faces will take place and light will pass through it undeviated. So, deviation is zero.
i.e., $D = 0$

CONDITION OF NO EMERGENCE

The light entering the prism at surface AB, will not be able to come out from the surface AC, if TIR takes place at this surface. For any angle of incidence,

this condition will be satisfied, provided we have at surface AC,

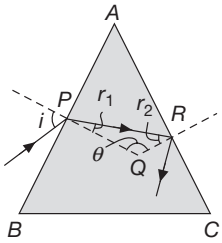
$$(r_2)_{min} > C \quad \ldots(1)$$

Since, $r_1 + r_2 = A$

$\Rightarrow \quad r_2 = A - r_1$

So, r_2 is minimum, when r_1 is maximum, because A is constant.

$$\Rightarrow \quad (r_2)_{min} = A - (r_1)_{max} \quad \ldots(2)$$



But $(r_1)_{max}$ is possible when $i = i_{max} = 90°$ i.e., incident ray grazes the interface AB.
Now, applying Snell's Law at AB,

$$1 \times \sin i = \mu \sin r_1$$

$$\Rightarrow \quad \sin(90°) = \mu \sin r_1$$

$$\Rightarrow \quad r_1 = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\Rightarrow \quad r_1 = C \quad \ldots(3)$$

From equations (1), (2) and (3), we get

$$A - C > C \quad \ldots(4)$$

Therefore, the condition becomes

$$A > 2C \text{ where } \sin C = \frac{1}{\mu}$$

$$\Rightarrow \quad \sin\left(\frac{A}{2}\right) > \sin C$$

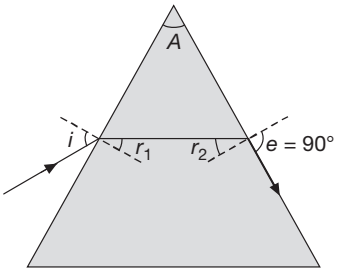
$$\Rightarrow \quad \sin\left(\frac{A}{2}\right) > \frac{1}{\mu}$$

$$\Rightarrow \quad \mu > \text{cosec}\left(\frac{A}{2}\right)$$

Thus, a ray of light will not emerge out of a prism (whatever be the angle of incidence) if $A > 2C$, that is, if $\mu > \text{cosec}\left(\frac{A}{2}\right)$.

CONDITION FOR GRAZING EMERGENCE

A ray can enter a prism in such a way that the angle of emergence, $e = 90°$, as shown in the figure.



We can determine the angle of incidence i for such grazing emergence. We should have

$$r_2 = C$$

Since, for a prism, $r_1 + r_2 = A$

$\Rightarrow \quad r_1 = A - r_2 = A - C$

Using Snell's Law,

$$1 \sin i = \mu \sin r_1 = \mu \sin(A - C)$$

$$\Rightarrow \quad \sin i = \mu(\sin A \cos C - \cos A \sin C)$$

$$\Rightarrow \quad \sin i = \mu\left[(\sin A)\sqrt{1 - \sin^2 C} - (\cos A)(\sin C)\right]$$

$$\Rightarrow \quad \sin i = \mu\left[\sin A \sqrt{1 - \frac{1}{\mu^2}} - \cos A\left(\frac{1}{\mu}\right)\right]$$

$$\Rightarrow \quad \sin i = \sin A\sqrt{\mu^2 - 1} - \cos A$$

$$\Rightarrow \quad i = \sin^{-1}\left(\sin A\sqrt{\mu^2 - 1} - \cos A\right)$$

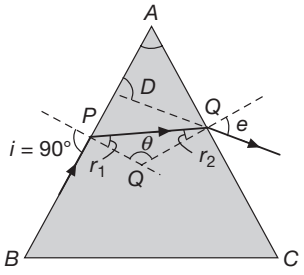
The light will emerge out of the prism only if the angle of incidence i is greater than the above value.

MAXIMUM DEVIATION

The angle of deviation D is maximum when the angle i is maximum, i.e., $i = 90°$.

$$D_{max} = (i + e) - A = (90° + e) - A$$

Under such conditions of grazing incidence, $r_1 = C$



And at the second surface,

$$\mu \sin r_2 = 1 \sin e$$

$$\Rightarrow \quad \sin e = \mu \sin r_2$$

Since $r_1 + r_2 = A$

$$\Rightarrow \quad \sin e = \mu \sin(A - r_1) = \mu \sin(A - C)$$

$$\Rightarrow \quad e = \sin^{-1}(\mu \sin(A - C)) = \sin^{-1}\left[\frac{\sin(A-C)}{\sin C}\right]$$

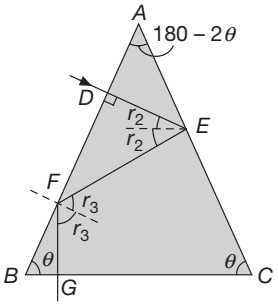
ILLUSTRATION 57

An isosceles glass prism has one of its faces coated with silver. A ray of light is incident normally on the other face (which is equal to the silvered face). The ray of light is reflected twice on the same sized faces and then emerges through the base of the prism perpendicularly. Find angles of prism.

SOLUTION

As the ray is incident normally at the face AB, so

$$r_1 = 0$$



Since, we know that $r_1 + r_2 = A$, so we get

$$r_2 = A = 180° - 2\theta \qquad \ldots(1)$$

Now, $\angle DFE = 180° - 90° - 2r_2$

$$\Rightarrow \quad \angle DFE = 180° - 90° - 360° + 4\theta \quad \{\because r_2 = 180 - 2\theta\}$$

$$\Rightarrow \quad \angle DFE = 4\theta - 270° \qquad \ldots(2)$$

Since, $r_3 = 90° - \angle DFE \qquad \ldots(3)$

$$\Rightarrow \quad r_3 = 360° - 4\theta$$

Again $\angle BFG = 90° - \theta = 90° - r_3$

$$\Rightarrow \quad r_3 = \theta \qquad \ldots(4)$$

From equations (3) and (4), we get

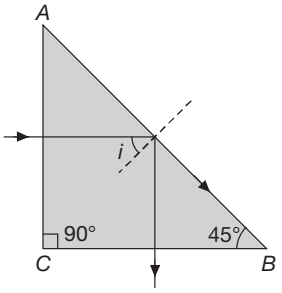
$$5\theta = 360°$$

$$\Rightarrow \quad \theta = 72° \text{ and } 180° - 2\theta = 36°$$

So, the angles of prism are $72°$, $72°$ and $36°$.

ILLUSTRATION 58

A ray of light incident normally on one of the faces of a right angle isosceles prism is found to be totally reflected as shown. What is the minimum value of the refractive index of the material of the prism? When prism is immersed in water $(\mu = 1.33)$ trace the path of the emergent ray for the same incident ray, indicating the values of all the angles.



SOLUTION

For total internal reflection to take place at surface AB, we have

$$i > C$$

$$\Rightarrow \quad \sin i > \sin C$$

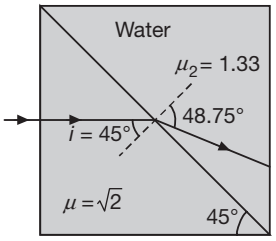
Since, $\sin C = \frac{1}{\mu}$

$$\Rightarrow \quad \sin 45° > \left(\frac{1}{\mu}\right)$$

$$\Rightarrow \quad \mu > \sqrt{2}$$

$$\Rightarrow \quad \mu_{\min} = \sqrt{2}$$

When the prism is immersed in water, the boundary AB now separates glass from water.



$$\Rightarrow \quad C = \sin^{-1}\left(\frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}}\right) = \sin^{-1}\left(\frac{1.33}{\sqrt{2}}\right)$$

$$\Rightarrow \quad C = 70.12°$$

Since $i = 45°$ and also, we observe that $i < C$
Hence, TIR will not take place.
From Snell's Law, we get

$$\mu_1 \sin i = \mu_2 \sin r$$

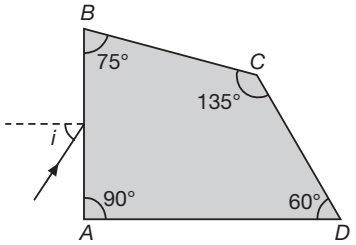
$$\Rightarrow \quad \sqrt{2}\sin(45°) = 1.33 \sin r$$

$$\Rightarrow \quad \sin r = \frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)}{1.33} = 0.752$$

$$\Rightarrow \quad r = \sin^{-1}(0.752) = 48.75°$$

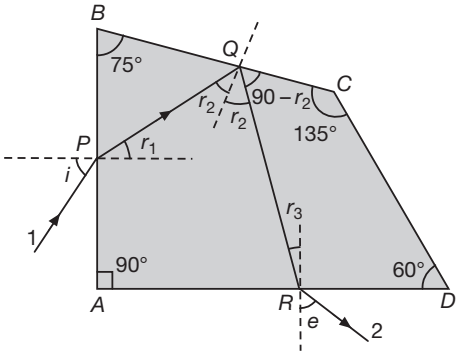
ILLUSTRATION 59

A ray of light is falling on face AB of a tetrahedral of refractive index μ at angle of incidence i. The ray after getting internally reflected on face BC emerges from AD perpendicularly to the incident beam. Find the range of μ and i.

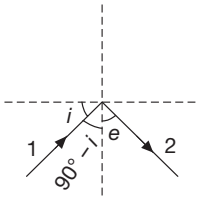


SOLUTION

Since, $r_1 + r_2 = \angle B = 75°$ …(1)



From figure, we observe that $e = i$, because 1 and 2 are perpendicular



In quadrilateral $QCDR$, we have

$$(90° - r_2) + (90° + r_3) + 60° + 135° = 360°$$

$$r_3 = 360° - 60° - 135° - (90° - r_2) - 90° \qquad \ldots(2)$$

$$r_3 = r_2 - 15°$$

Further, $\mu = \dfrac{\sin i}{\sin r_1} = \dfrac{\sin e}{\sin r_3}$

$$\Rightarrow \quad r_3 = r_1 \qquad \{\text{because } i = e\} \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$r_2 = 45° \text{ and } r_1 = 30°$$

Now, for TIR (total internal reflection) to take place at the face BC; we have

$$r_2 > C$$

$$\Rightarrow \quad \sin r_2 > \sin C$$

$$\Rightarrow \quad \sin(45°) > \frac{1}{\mu}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} > \frac{1}{\mu} \qquad \left\{\because \sin C = \frac{1}{\mu}\right\}$$

$\Rightarrow \quad \mu > \sqrt{2}$

Further, we have $\mu = \dfrac{\sin i}{\sin r_1} = \dfrac{\sin i}{\sin(30°)} = 2\sin i$

Since, $\mu > \sqrt{2}$

$\Rightarrow \quad 2\sin i > \sqrt{2}$

$\Rightarrow \quad \sin i > \dfrac{1}{\sqrt{2}}$

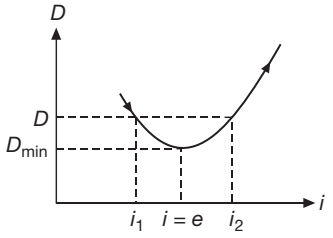
$\Rightarrow \quad i > 45°$

MINIMUM DEVIATION

As discussed and derived already we know that the angle of deviation D is given by

$$D = i + \sin^{-1}\left(\sin A\sqrt{\mu^2 - \sin^2 i} - \sin i \cos A\right)$$

The above function of deviation D, when plotted against i the angle of incidence gives a plot that is unsymmetrical as shown in the figure. It must be observed that for two different angles of incidence, we have the same deviation.



It is found that D is minimum when $i = e$. Thus,

$$D_{min} = (i + e) - A = 2i - A$$

Using Snell's Law,

$$1\sin i = \mu \sin r_1$$

and $\quad \mu \sin r_2 = 1\sin e = \sin i$

$\Rightarrow \quad \mu \sin r_1 = \mu \sin r_2$

$\Rightarrow \quad r_1 = r_2 = r$ (say)

Since, $r_1 + r_2 = A$

$\Rightarrow \quad r = \dfrac{A}{2}$

$\Rightarrow \quad \mu = \dfrac{\sin i}{\sin r_1} = \dfrac{\sin\left(\dfrac{A + D_{min}}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

Note that if the prism is equilateral or isosceles, then the ray inside the prism is parallel to its base.

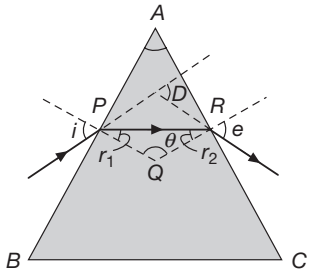


ILLUSTRATION 60

The angle of minimum deviation for a glass prism with refractive index $\sqrt{3}$ equals the refracting angle of the prism. What is the angle of the prism?

SOLUTION

Since we know that

$$\mu = \frac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$$

Since $\delta_{minimum} = \delta_{min} = A$, so we get

$$\sqrt{3} = \frac{\sin A}{\sin\left(\dfrac{A}{2}\right)}$$

$\Rightarrow \quad \sqrt{3} = 2\cos\left(\dfrac{A}{2}\right)$

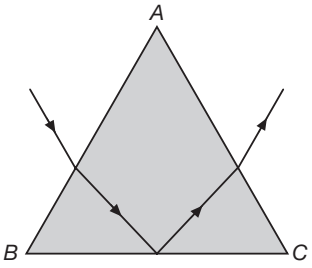
$\Rightarrow \quad \cos\left(\dfrac{A}{2}\right) = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \dfrac{A}{2} = 30°$

$\Rightarrow \quad A = 60°$

ILLUSTRATION 61

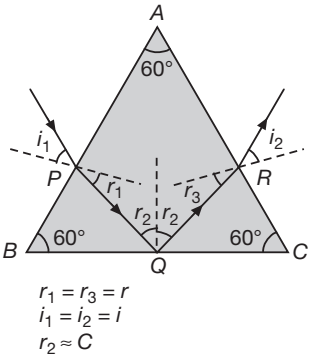
The path of a ray of light passing through an equilateral glass prism ABC is shown in the figure.



The ray of light is incident on face BC at an angle just greater than the critical angle for total internal reflection to take place. The total angle of deviation after the refraction at face AC is $120°$. Calculate the refractive index of the glass.

SOLUTION

The ray diagram is drawn for the sake of convenience.



Since, $r_1 + r_2 = r_2 + r_3 = 60°$

$\Rightarrow \quad r_1 = r_3 = r$ (say)

Similarly by symmetry, we have $i_1 = i_2 = i$ (say)

Also, $r_2 \approx C$

$\Rightarrow \quad r = 60° - C$

Given, $\delta_{\text{Total}} = 120°$

$\Rightarrow \quad \delta_P + \delta_Q + \delta_R = 120°$

$\Rightarrow \quad (i - r) + (180 - 2C) + (i - r) = 120°$

$\Rightarrow \quad 2i - 2(60° - C) + 180° - 2C = 120°$

$\Rightarrow \quad 2i = 60°$

$\Rightarrow \quad i = 30°$

Now, according to Snell's Law, we have

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin i}{\sin r}$$

But $r_1 + r_2 = 60°$

$\Rightarrow \quad r + C = 60°$

$\Rightarrow \quad r = 60° - C$

$$\Rightarrow \quad \mu = \frac{\sin(30°)}{\sin(60° - C)}$$

$$\Rightarrow \quad \mu = \frac{0.5}{\frac{\sqrt{3}}{2}\cos C - \frac{1}{2}\sin C}$$

But $\sin C = \frac{1}{\mu}$

$$\Rightarrow \quad \mu = \frac{0.5}{0.5\left(\sqrt{3}\sqrt{1 - \frac{1}{\mu^2}} - \frac{1}{\mu}\right)}$$

$$\Rightarrow \quad \mu = \frac{\mu}{\sqrt{3}\sqrt{\mu^2 - 1} - 1}$$

$$\Rightarrow \quad \sqrt{3}\sqrt{\mu^2 - 1} = 2$$

$$\Rightarrow \quad 3(\mu^2 - 1) = 4$$

$$\Rightarrow \quad 3\mu^2 = 7$$

$$\Rightarrow \quad \mu = \sqrt{\frac{7}{3}}$$

$$\Rightarrow \quad \mu = 1.52$$

WHITE LIGHT

White light consists of infinite number of continuous wavelengths (colours) ranging from 4000 Å to 7800 Å. For convenience it is divided into seven colours.
Violet, Indigo, Blue, Green, Yellow, Orange, Red called as 'VIBGYOR' pattern.

The Violet having least wavelength (maximum frequency) and Red having maximum wavelength (minimum frequency).

VARIATION OF REFRACTIVE INDEX WITH COLOUR (CAUCHY'S FORMULA)

The refractive index (μ) of a medium varies with wavelength (λ) according to Cauchy's formula

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \ldots$$

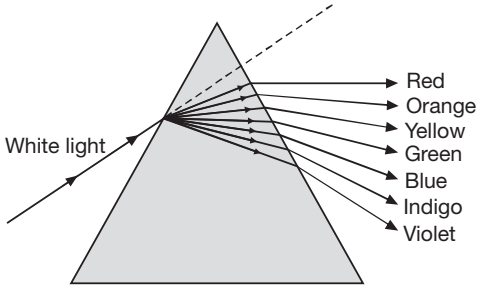
where A, B and C are constants.

From above we observe that refractive index decreases with increase of wavelength. It is maximum for violet and minimum for red colour and due to this variation of the refractive index with the wavelength or the colour, a composite beam of light entering a prism splits into constituent colours.

DISPERSION

It has been observed that when a beam of composite light (consisting of several wavelengths) passes though a prism, it splits into its constituent colours. This phenomenon is called **dispersion**. The band of colours thus obtained on a screen is called the **spectrum**.

If white light is used, seven colours are obtained as shown in the figure. The sequence of colours is VIBGYOR, from bottom to top.



The dispersion of light takes place because the refractive index μ of the medium depends on the wavelength of light as given by **Cauchy's formula**, according to which

$$\mu = A + \frac{B}{\lambda^2}$$

where A and B are constants. The smaller the value of λ, the larger is the value of μ. Thus, μ is maximum for violet colour and minimum for red. The deviation of a ray depends on μ it is larger for higher μ. Hence, violet suffers the maximum deviation and red the minimum.

If light from sodium lamp falls on a prism then it disperses (breaks) into two lines called $D_1\ (5890\ \text{Å})$ and $D_2\ (5896\ \text{Å})$ lines. **Thus we observe that a prism causes deviation as well as dispersion.**

If D_V, D_R and D_Y are the deviations caused by prism for violet, red and mean yellow rays, then for prism with small refracting angle (A), we have

$$\text{Angular Dispersion } D = D_V - D_R = (\mu_V - \mu_R)A$$

DISPERSIVE POWER OF A PRISM

The ratio of angular dispersion to the mean deviation is called dispersive power, so Dispersive Power is

$$\omega = \frac{\text{Angular Dispersion}}{\text{Mean Deviation}} = \frac{D}{D_Y} = \frac{D_V - D_R}{D_Y}$$

where D_Y is the deviation of mean light i.e., yellow light, whose wavelength is considered as mean of all the wavelengths present. Further for a prism of small refracting angle A, we have

$$D = (\mu - 1)A$$

So, we have

$$D_V = (\mu_V - 1)A,\ D_R = (\mu_R - 1)A \text{ and } D_Y = (\mu_Y - 1)A$$

So the dispersive power ω becomes

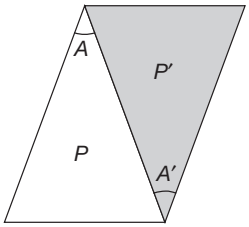
$$\omega = \frac{(\mu_V - \mu_R)A}{(\mu_Y - 1)A} = \frac{(\mu_V - \mu_R)}{\mu_Y - 1} = \frac{d\mu}{\mu - 1}$$

where $d\mu = \mu_V - \mu_R$ and $\mu = \mu_Y$

The dispersive power ω has no units and no dimensions. Its value depends on the material of the prism.

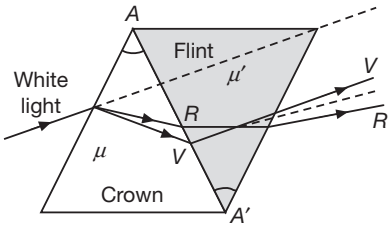
COMBINATION OF TWO PRISMS

From a single prism, it is not possible to get deviation without dispersion, or to get dispersion without deviation. However, two small angled prisms may be combined to produce Dispersion without Deviation or Deviation without Dispersion. The prism placement for both is shown here. The placement remains the same. It is just that we are to decide the relation between their refractive indices such that required condition may be achieved.



Dispersion Without Deviation (Chromatic Combination)

Two prisms can be combined in such a way that the deviation of the mean ray produced by one is equal and opposite to that produced by the other. Such a combination is called a **direct vision prism**.



So, in this arrangement of prisms, the mean deviation (D) caused by one prism is cancelled by the mean deviation (D') caused by the other prism i.e.

$$D - D' = 0$$

$$\Rightarrow \quad (\mu - 1)A - (\mu' - 1)A' = 0$$

$$\text{or} \quad A' = \left(\frac{\mu - 1}{\mu' - 1}\right)A .$$

The net dispersion produced is

$$D_{\text{net}} = (D_V - D_R) - (D'_V - D'_R)$$

$$\Rightarrow \quad D_{\text{net}} = (\mu_V - \mu_R)A - (\mu'_V - \mu'_R)A'$$

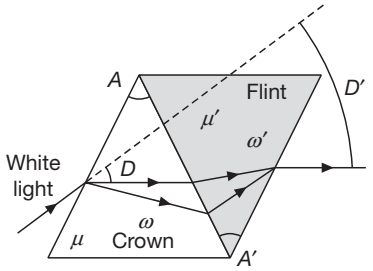
$$\Rightarrow \quad D_{\text{net}} = \left(\frac{\mu_V - \mu_R}{\mu - 1}\right)(\mu - 1)A - \left(\frac{\mu'_V - \mu'_R}{\mu' - 1}\right)(\mu' - 1)A'$$

$$\Rightarrow \quad D_{\text{net}} = \omega D - \omega' D'$$

where ω and ω' are dispersive powers of prisms P and P'.

Deviation Without Dispersion (Achromatic Prism)

It is possible to combine two prisms of different materials in such a way that each cancels the dispersion due to the other. Thus, the net dispersion is zero but a deviation is produced. So, in this arrangement of prisms, the dispersion $(D_V - D_R)$ caused by one prism is cancelled by dispersion $(D'_V - D'_R)$ produced by the other prism.



i.e., $\quad (D_V - D_R) - (D'_V - D'_R) = 0$

or $\quad (\mu_V - \mu_R)A - (\mu'_V - \mu'_R)A' = 0 \qquad \ldots(1)$

This gives

$$A' = \left(\frac{\mu_V - \mu_R}{\mu'_V - \mu'_R}\right)A$$

Also from (1) we get

$$(\mu_V - \mu_R)A = (\mu'_V - \mu'_R)A'$$

$$\Rightarrow \quad \left(\frac{\mu_V - \mu_R}{\mu - 1}\right)(\mu - 1)A = \left(\frac{\mu'_V - \mu'_R}{\mu - 1}\right)(\mu' - 1)A'$$

$$\Rightarrow \quad \omega D = \omega' D'$$

is the condition for Deviation without Dispersion.
The net mean deviation is

$$D - D' = (\mu - 1)A - (\mu' - 1)A'$$

ILLUSTRATION 62

The refractive indices of the crown glass for blue and red light are 1.51 and 1.49 respectively and those of the flint glass are 1.77 and 1.73 respectively. An isosceles prism of angle 6° is made of crown glass. A beam of white light is incident at a small angle on this prism. The other flint glass isosceles prism is combined with the crown glass prism such that there is no deviation of the incident prism.

(a) Determine the angle of the flint glass prism.
(b) Calculate the net dispersion of the combined system.

SOLUTION

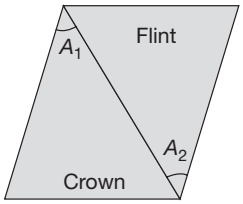
(a) When angle of prism is small and angle of incidence is also small, the deviation is given by $\delta = (\mu - 1)A$

Net deviation by the two prisms is zero, when deviation due to one cancels the deviation due to the other. So,

$$\delta_1 - \delta_2 = 0$$

$$\Rightarrow (\mu_1 - 1)A_1 - (\mu_2 - 1)A_2 = 0 \quad \ldots(1)$$

Here, μ_1 and μ_2 are the refractive indices for crown and flint glasses respectively, where



$$\mu_1 = \frac{1.51 + 1.49}{2} = 1.5 \text{ and } \mu_2 = \frac{1.77 + 1.73}{2} = 1.75$$

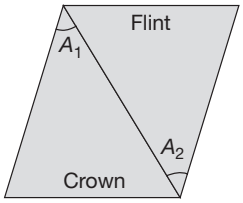
Angle of prism for crown glass is $A_1 = 6°$

Substituting this values in equation (1), we get

$$(1.5 - 1)(6°) - (1.75 - 1)A_2 = 0$$

This gives $A_2 = 4°$

Hence, angle of flint glass prism is $4°$



$\delta_1 + \delta_2 = 0$

(b) Net dispersion due to the two prisms is given by

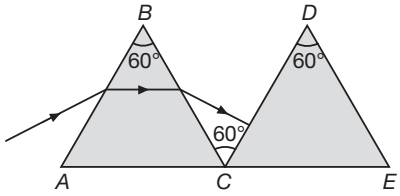
$$\text{Net Dispersion} = (\mu_{b_1} - \mu_{r_1})A_1 - (\mu_{b_2} - \mu_{r_2})A_2$$

$$\Rightarrow \text{Net Dispersion} = (1.51 - 1.49)(6°) - (1.77 - 1.73)(4°) = -0.04°$$

$$\Rightarrow \text{Net dispersion} = -0.04°$$

ILLUSTRATION 63

A ray of light is incident on a prism ABC of refractive index $\sqrt{3}$ as shown in figure.



(a) Find the angle of incidence for which the deviation of light ray by the prism ABC is minimum.
(b) By what angle the second prism must be rotated, so that the final ray suffer net minimum deviation.

SOLUTION

(a) At minimum deviation, we have $r_1 = r_2 = 30°$

According to Snell's Law, we have

$$\mu = \frac{\sin i_1}{\sin r_1}$$

$$\Rightarrow \sqrt{3} = \frac{\sin i_1}{\sin(30°)}$$

$$\Rightarrow \sin i_1 = \frac{\sqrt{3}}{2}$$

$$\Rightarrow i_1 = 60°$$

(b) In the position shown, net deviation suffered by the ray of light should be minimum. Therefore, the second prism should be rotated by $60°$ (anticlockwise).

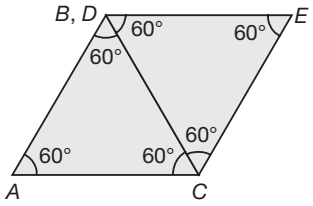


ILLUSTRATION 64

A beam of light enters a glass prism at an angle α and emerges into the air at an angle β. Having passed through the prism, the beam is deflected from the original direction by an angle δ. Find the refracting angle of the prism and the refractive index of the material of the prism.

SOLUTION

$$r_1 + r_2 = A$$

Since $i + e = A + \delta$

$$\Rightarrow \quad \delta = \alpha + \beta - A$$

Further applying Snell's Law at incident surface and emergent surface, we get

$$\mu = \frac{\sin\alpha}{\sin r_1} \text{ and } \frac{\sin r_2}{\sin\beta} = \frac{1}{\mu}$$

$$\Rightarrow \quad \frac{\sin\alpha}{\sin r_1} = \frac{\sin\beta}{\sin r_2}$$

$$\Rightarrow \quad \frac{\sin\alpha}{\sin(A - r_2)} = \frac{\sin\beta}{\sin r_2}$$

$$\frac{\sin(A - r_2)}{\sin r_2} = \frac{\sin\alpha}{\sin\beta}$$

$$\Rightarrow \quad \frac{\sin A\cos r_2}{\sin r_2} - \frac{\cos A\sin r_2}{\sin r_2} = \frac{\sin\alpha}{\sin\beta}$$

$$\Rightarrow \quad \sin A\cot r_2 = \frac{\sin\alpha}{\sin\beta} + \cos A$$

$$\Rightarrow \quad \cot r_2 = \frac{\sin\alpha}{\sin\beta\sin A} + \cot A$$

Since $\mu = \frac{\sin\beta}{\sin r_2} = \sin\beta\,\text{cosec}\, r_2$

$$\Rightarrow \quad \mu = \sin\beta\sqrt{1 + \cot^2 r_2}$$

$$\Rightarrow \quad \mu = \sin\beta\sqrt{1 + \left(\frac{\sin\alpha}{\sin\beta\sin A} + \cot A\right)^2}$$

Since $A = \alpha + \beta - \delta$, so μ is given by

$$\mu = \sin\beta\sqrt{1 + \left(\frac{\sin\alpha}{\sin\beta\sin(\alpha + \beta - \delta)} + \cot(\alpha + \beta - \delta)\right)^2}$$

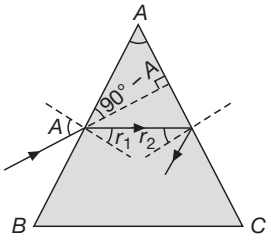
ILLUSTRATION 65

A ray of light is incident upon one face of a prism (angle of prism $< \frac{\pi}{2}$) in a direction perpendicular to the other face. Prove that the ray will fail to emerge from the other face if $\cot A < \cot C - 1$, where C is critical angle for the material of prism.

SOLUTION

From geometry, we observe that the angle of incidence at the face AB is A. Applying Snell's Law at face AB, we get

$$\mu = \frac{\sin A}{\sin r_1} \qquad \ldots(1)$$



If C is the critical angle of the prism, then

$$\mu = \frac{1}{\sin C} \qquad \ldots(2)$$

From (1) and (2), we get

$$\frac{\sin A}{\sin r_1} = \frac{1}{\sin C}$$

$$\Rightarrow \quad \sin r_1 = \sin A\sin C \qquad \ldots(3)$$

The ray does not emerge from the other face AC, when

$$r_2 > C$$

Since, $r_1 + r_2 = A$

$$\Rightarrow \quad A - r_1 > C$$

$$\Rightarrow \quad r_1 < A - C$$

$$\Rightarrow \quad \sin r_1 < \sin(A - C)$$

$$\Rightarrow \quad \sin A\sin C < \sin A\cos C - \sin C\cos A$$

$$\Rightarrow \quad 1 < \cot C - \cot A$$

$$\Rightarrow \quad \cot A < \cot C - 1$$

COLOURS OF OBJECTS AND COLOUR TRIANGLE

The colours of objects are due to a number of phenomena.

The colours of opaque bodies are due to **Selective Reflection**. For example grass appears green because when white light is incident on grass,

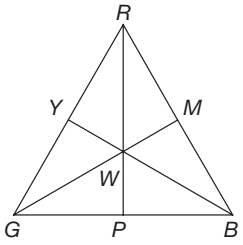
it absorbs all colours except green which is reflected. Black appears black because it absorbs all colours falling an it an reflects nothing. Similarly white appears white because it reflects all colours falling on it and absorbs nothing.

The colours of transparent bodies are due to **Selective Transmission**. For example a glass appears blue, because it absorbs all colours except blue, which it transmits.

The colours of sky, rising and setting sun are due to **scattering** while the colours of soap bubble and kerosene oil film are due to **interference**.

COLOUR TRIANGLE

If, Red (R), Green (G) and Blue (B) are primary colours. If P denotes Peacock Blue also called Cyan, M denotes Magenta, Y denotes Yellow and W denotes White, then from colour triangle we observe that



$$R+G+B=W$$
$$R+G=Y$$
$$G+B=P$$
$$R+B=M$$
$$B+Y=W$$
$$R+P=W$$
$$G+M=W$$

RAYLEIGH LAW

According to Lord Rayleigh, intensity (I) of scattered light is inversely proportional to the fourth power of the wavelength λ. So,

$$I \propto \frac{1}{\lambda^4}$$

It can also be concluded that the amplitude (a) of the scattered light is inversely proportional to the square of the wavelength.

So, $a \propto \frac{1}{\lambda^2}$ $\quad \{\because I \propto a^2\}$

COLOUR OF THE SKY

When light from the sun travels through earth's atmosphere, it gets scattered by the large number of molecules of various gases. It is found that the amount of scattering by molecules, called Rayleigh scattering, is inversely proportional to the fourth power of the wavelength. Thus light of shorter wavelength is scattered much more than the light of longer wavelength. Since blue colour has relatively shorter wavelength, it predominates the sky and hence sky appears bluish.

COLOUR OF CLOUDS

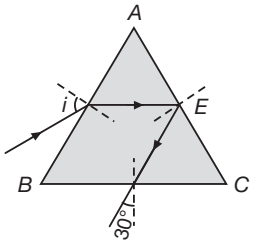
Large particles like water droplets and dust do not have this selective scattering power. They scatter all wavelengths almost equally. Hence clouds appear to the white.

Test Your Concepts-V

Based on Prism

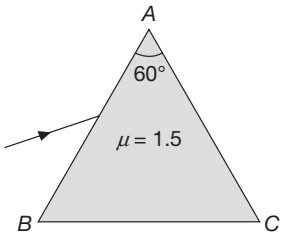
(Solutions on page H.15)

1. The path of a ray undergoing refraction in an equilateral prism is shown in figure. The ray suffers refraction at the face AB and the refracted ray is incident on the face AC at an angle slightly greater than the critical angle and hence, totally reflected. After refraction at the face BC the emergent ray makes an angle of $30°$ with normal at BC at the point of emergence. Find the



(a) corresponding angle of incidence *i*.
(b) refractive index of the prism.

2. In a prism of refractive index $\mu = 1.5$ and refracting angle 60°, the condition for minimum deviation is fulfilled. If face *AC* is polished



(a) Find the net deviation.
(b) If the system is placed in water what will be the net deviation? Refractive index of water $= \frac{4}{3}$

3. A ray of light incident on the face of a prism is refracted and escapes through an adjacent face. What is the maximum permissible angle of refraction of the prism, if it is made of glass with a refractive index of $\mu = 1.5$.

4. In an isosceles prism of angle 45°, it is found that when the angle of incidence is same as the prism angle and the emergent ray grazes the emergent surface.
(a) Find the refractive index of the material of the prism.
(b) For what angle of incidence the angle of deviation will be minimum?

5. A prism of flint glass $\left(\mu_g = \frac{3}{2}\right)$ with an angle of refraction 30° is placed inside water $\left(\mu_w = \frac{4}{3}\right)$.
(a) At what angle should a ray of light fall on the face of the prism so that inside the prism the ray is perpendicular to the bisector of the angle of the prism.
(b) Through what angle will the ray turn after passing through both faces of the prism?

6. Light rays from a source are incident on a glass prism of index of refraction μ and angle of prism α. At near normal incidence, calculate the angle of deviation of the emerging rays.

7. One face of a prism with a refractive angle of 30° is coated with silver. A ray incident on another face at an angle of 45° is refracted and reflected from the silver coated face and retraces its path. What is the refractive index of the prism?

8. A ray of light is incident at an angle of 60° at one face of a prism having refracting angle 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. Find the angle of emergence and calculate the refractive index of the material of the prism.

9. The index of refraction for violet light in silica flint glass is 1.66 and that for red light is 1.62. Find the angular dispersion of visible light passing through a prism of apex angle 60°, if the angle of incidence is 50°.

10. A light ray is passing through a prism with refracting angle $A = 90°$ and refractive index $\mu = 1.3$. Find the minimum and maximum angle of deviation.

11. A ray of light is incident at an angle of 60° on the face of a prism having refracting angle 30°. The ray emerging out of the prism makes an angle 30° with the incident ray. Find the angle of emergence of the ray.

12. The refracting angle of a glass prism is 30°. A ray is incident onto one of the faces perpendicular to it. Find the angle δ between the incident ray and the ray that leaves the prism. The refractive index of glass is $n = 1.5$.

13. The refractive index of the material of a prism is 1.6 for a certain monochromatic ray. What should be the maximum angle of incidence of this ray on the prism so that no total internal reflection occurs when the ray leaves the prism? The refracting angle of the prism is 45°.

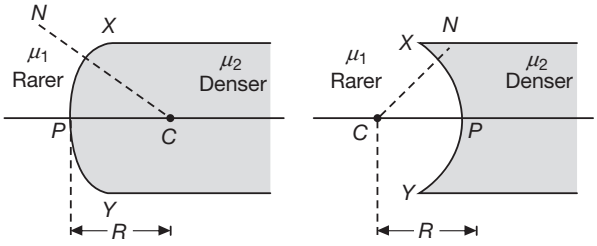
14. A ray of white light falls onto the side surface of an isosceles prism at such an angle that the red ray leaves the prism normally to the second face. Find the deflection of the red and violet rays from the initial direction if the refraction angle of the prism is 45°. The refractive indices of the prism material for red and violet rays are 1.37 and 1.42, respectively.

15. A parallel beam of light falls normally on the first face of a prism of small refracting angle. At the second face it is partly transmitted and partly reflected, the reflected beam striking at the first face again and emerging from it in a direction making an angle of 4° with the reversed direction of the incident beam. The refracted beam is found to have undergone a deviation of 1° from the original direction. Calculate the refractive index of the glass and the angle of the prism.

REFRACTION AT CURVED SURFACES AND LENS

SINGLE REFRACTING SURFACE

A spherical surface which separates two media of different refractive index is called a single refracting surface. The convexity or concavity of the surface is decided by looking at it from rarer medium as shown in figure.



Convex Refracting Surface Concave Refracting Surface

Some Terms Connected with Single Refracting Surface

1. **Pole (*P*):** It is a point which bulges out most (in case of convex surface) or is depressed most (in case of concave surface) as seen from the rarer medium.
2. **Centre of Curvature (*C*):** It is the centre of the sphere of which the surface forms a part.
3. **Radius of Curvature (*R*):** It is the radius of the sphere of which the surface forms a part.
4. **Aperture (*XY*):** The diameter of the refracting surface is called the aperture of the surface.
5. **Principal axis:** The line joining the pole and centre of curvature and extended on either side of the surface is called the principal axis.

SIGN CONVENTIONS

Following sign conventions must be used while dealing with ray diagrams.

(a) All the distances will be measured from the pole of the surface.
(b) The distances measured against the incident ray are taken as negative.
(c) The distances measured along the incident ray are taken as positive.
(d) All transverse measurements done above the principal axis are taken as positive while the ones done below the principal axis are taken as negative.

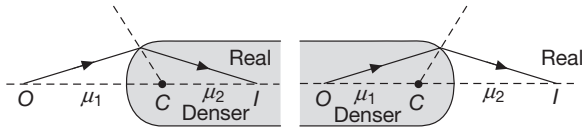
ASSUMPTIONS

While obtaining some relations, in ray optics, we make some assumptions given below. All those formulae will hold good only if these conditions are satisfied.

(a) The object/source is considered to be point object/source placed on principal axis.
(b) The aperture of the surface/lens is small.
(c) Rays of light make smaller angles with the principal axis i.e., are paraxial in nature.

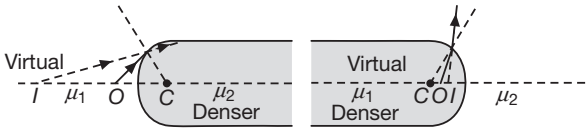
REFRACTION OF LIGHT AT CURVED SURFACES

For the curved surfaces the same law of refraction are applicable. When a light-ray enters a denser medium, it bends towards the normal. The figures show six situations. The shaded region is denser.



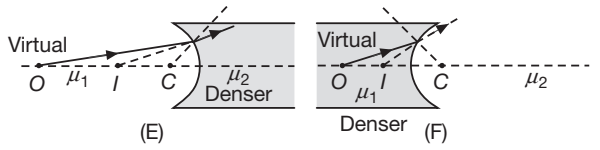
(A) (B)

In Figs. (*A*) and (*B*), the object *O* is kept relatively far from the refracting surface, and the image formed is real.



(C) (D)

In Figs. (*C*) and (*D*), the object is nearer the refracting surface, and the image is virtual.



In Figs. (*E*) and (*F*), the refraction always directs the ray away from the central axis, and hence virtual images are formed.

Note the major difference from the images formed due to reflection from a spherical mirror. Here, real images are formed on the other side of the refracting surface, and virtual images are formed on the same side as the object.

REFRACTION AT CONVEX SURFACE

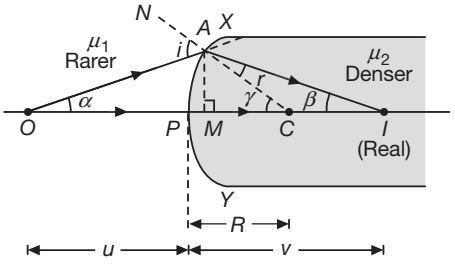
CASE-1: When the object lies in the rarer medium and the image formed is real.

Consider a spherical surface of radius R separating the two media 1 and 2 $(\mu_2 > \mu_1)$. A point object O is placed on the principal axis to the left of the pole P at a considerable distance from it. The incident ray from O falls on point A and is refracted according to

$$\mu_1 \sin i = \mu_2 \sin r \qquad \ldots(1)$$

Since the rays are paraxial, so the angle α is small and hence the angles i and r will also be small. Thus, applying such paraxial approximation, then $\sin i \cong i$ and $\sin r \cong r$, so we get from (1)

$$\mu_1 i = \mu_2 r \qquad \ldots(2)$$



Using the geometrical property that an exterior angle of a triangle is equal to the sum of the two internal opposite angles, we get from triangles AOC and AIC,

$$i = \alpha + \gamma \qquad \ldots(3)$$

and $\gamma = r + \beta$ …(4)

Substituting the value of i and r from Equations (3) and (4) in Equation (2), we get

$$\mu_1(\alpha + \gamma) = \mu_2(\gamma - \beta)$$

$$\Rightarrow \quad \mu_1\alpha + \mu_2\beta = (\mu_2 - \mu_1)\gamma \qquad \ldots(5)$$

Now, since the aperture of the refracting surface is small, so M and P are very close to each other and hence, we have

$$\alpha \cong \tan\alpha = \frac{AM}{MO} \cong \frac{AM}{PO},$$

$$\beta \cong \tan\beta = \frac{AM}{MI} \cong \frac{AM}{PI} \text{ and}$$

$$\gamma \cong \tan\gamma = \frac{AM}{MC} \cong \frac{AM}{PC}$$

Therefore (5) becomes

$$\Rightarrow \quad \mu_1\left(\frac{AM}{PO}\right) + \mu_2\left(\frac{AM}{PI}\right) = (\mu_2 - \mu_1)\left(\frac{AM}{PC}\right)$$

$$\Rightarrow \quad \frac{\mu_1}{PO} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

Since $PO = -u$, $PI = +v$, $PC = +R$ so we get

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

If the object is in air, then $\mu_1 = 1$ and $\mu_2 = \mu$, so we get

$$\frac{1}{-u} + \frac{\mu}{v} = \frac{\mu - 1}{R}$$

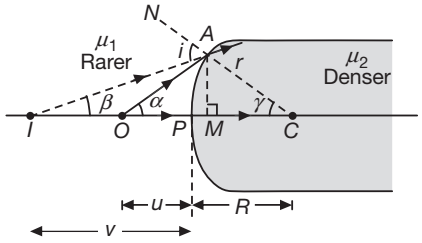
CASE-2: When the object lies in the rarer medium and the image formed is virtual.

Consider a spherical surface of radius R separating the two media 1 and 2 $(\mu_2 - \mu_1)$. A point object O is placed on the principal axis to the left of the pole P. The incident ray from O falls on point A and is refracted according to

$$\mu_1 \sin i = \mu_2 \sin r \qquad \ldots(1)$$

Since the rays are paraxial, so the angle α is small and hence the angles i and r will also be small. Thus, applying such paraxial approximation, then $\sin i \cong i$ and $\sin r \cong r$, so from (1), we have

$$\mu_1 i = \mu_2 r \qquad \ldots(2)$$



Using the geometrical property that an exterior angle of a triangle is equal to the sum of the two internal opposite angles, we get from triangles AOC and AIC,

$$i = \alpha + \gamma \qquad \ldots(3)$$

and $$r = \beta + \gamma \qquad \ldots(4)$$

Substituting the value of i and r from Equations (3) and (4) in Equation (2), we get

$$\mu_1(\alpha+\gamma) = \mu_2(\beta+\gamma)$$

$$\Rightarrow \quad \mu_1\alpha - \mu_2\beta = (\mu_2 - \mu_1)\gamma \qquad \ldots(5)$$

Now, since the aperture of the refracting surface is small, so M and P are very close to each other and hence we have

$$\alpha \cong \tan\alpha = \frac{AM}{MO} \cong \frac{AM}{PO},$$

$$\beta \cong \tan\beta = \frac{AM}{MI} \cong \frac{AM}{PI} \text{ and}$$

$$\gamma \cong \tan\gamma = \frac{AM}{MC} \cong \frac{AM}{PC}$$

$$\Rightarrow \quad \mu_1\left(\frac{AM}{PO}\right) - \mu_2\left(\frac{AM}{PI}\right) = (\mu_2 - \mu_1)\left(\frac{AM}{PC}\right)$$

$$\Rightarrow \quad \frac{\mu_1}{PO} - \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

Since $PO = -u$, $PI = -v$, $PC = +R$ so we get

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

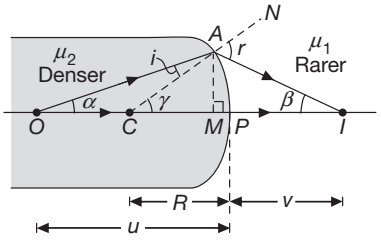
CASE-3: When the object lies in the denser medium and the image formed is real.

Consider a spherical surface of radius R separating the two media 1 and 2 $(\mu_2 > \mu_1)$. A point object O is placed on the principal axis to the left of the pole P. The incident ray from O falls on point A and is refracted according to

$$\mu_2 \sin i = \mu_1 \sin r \qquad \ldots(1)$$

Since the rays are paraxial, so the angle α is small and hence the angles i and r will also be small. Thus, applying such paraxial approximation, then $\sin i \cong i$ and $\sin r \cong r$, so from (1), we have

$$\mu_2 i = \mu_1 r \qquad \ldots(2)$$



Using the geometrical property that an exterior angle of a triangle is equal to the sum of the two internal opposite angles, we get from triangles AOC and AIC

$$\gamma = \alpha + i \qquad \ldots(3)$$

and $$r = \beta + \gamma \qquad \ldots(4)$$

Substituting the value of i and r from Equations (3) and (4) in Equation (2), we get

$$\mu_2(\gamma - \alpha) = \mu_1(\beta + \gamma)$$

$$\Rightarrow \quad \mu_2\alpha + \mu_1\beta = (\mu_2 - \mu_1)\gamma \qquad \ldots(5)$$

Now, since the aperture of the refracting surface is small, so M and P are very close to each other and hence we have

$$\alpha \cong \tan\alpha = \frac{AM}{MO} \cong \frac{AM}{PO},$$

$$\beta \cong \tan\beta = \frac{AM}{MI} \cong \frac{AM}{PI} \text{ and}$$

$$\gamma \cong \tan\gamma = \frac{AM}{MC} \cong \frac{AM}{PC}$$

$$\Rightarrow \quad \mu_2\left(\frac{AM}{PO}\right) + \mu_1\left(\frac{AM}{PI}\right) = (\mu_2 - \mu_1)\left(\frac{AM}{PC}\right)$$

$$\Rightarrow \quad \frac{\mu_2}{PO} + \frac{\mu_1}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

Since $PO = -u$, $PI = +v$, $PC = -R$ so we get

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Simply replace subscript 2 by 1 and 1 by 2 in the formula derived in CASE-1 or CASE-2.

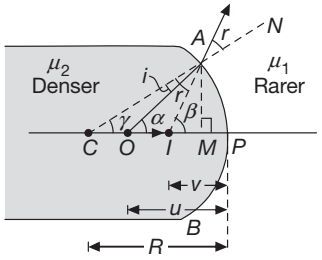
CASE-4: When the object lies in the denser medium and the image formed is virtual.

Consider a spherical surface of radius R separating the two media 1 and 2 $(\mu_2 > \mu_1)$. A point object O is placed on the principal axis to the left of the pole P. The incident ray from O falls on point A and is refracted according to

$$\mu_2 \sin i = \mu_1 \sin r \quad \ldots(1)$$

Since the rays are paraxial, so the angle α is small and hence the angles i and r will also be small. Thus, applying such paraxial approximation, then $\sin i \cong i$ and $\sin r \cong r$, so from (1), we have

$$\mu_2 i = \mu_1 r \quad \ldots(2)$$



Using the geometrical property that an exterior angle of a triangle is equal to the sum of the two internal opposite angles, we get from triangles AOC and AIC

$$\alpha = i + \gamma \quad \ldots(3)$$

and $$\beta = r + \gamma \quad \ldots(4)$$

Substituting the value of i and r from Equations (3) and (4) in Equation (2), we get

$$\mu_2(\alpha - \gamma) = \mu_1(\beta - \gamma)$$

$$\Rightarrow \quad \mu_2\alpha - \mu_1\beta = (\mu_2 - \mu_1)\gamma \quad \ldots(5)$$

Now, since the aperture of the refracting surface is small, so M and P are very close to each other and hence we have

$$\alpha \cong \tan\alpha = \frac{AM}{MO} \cong \frac{AM}{PO},$$

$$\beta \cong \tan\beta = \frac{AM}{MI} \cong \frac{AM}{PI} \text{ and}$$

$$\gamma \cong \tan\gamma = \frac{AM}{MC} \cong \frac{AM}{PC}$$

$$\Rightarrow \quad \mu_2\left(\frac{AM}{PO}\right) - \mu_1\left(\frac{AM}{PI}\right) = (\mu_2 - \mu_1)\left(\frac{AM}{PC}\right)$$

$$\Rightarrow \quad \frac{\mu_2}{PO} - \frac{\mu_1}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

Since $PO = -u$, $PI = -v$, $PC = -R$ so we get

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

REFRACTION AT CONCAVE SURFACE

For a concave refracting surface the image formed is always virtual irrespective of the placement of the object.

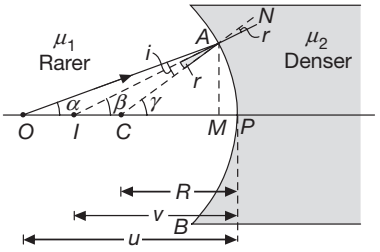
CASE-1: When the object lies in the rarer medium.

Consider a spherical surface of radius R separating the two media 1 and 2 $(\mu_2 > \mu_1)$. A point object O is placed on the principal axis to the left of the pole P. The incident ray from O falls on point A and is refracted according to

$$\mu_1 \sin i = \mu_2 \sin r \quad \ldots(1)$$

Since the rays are paraxial, so the angle α is small and hence the angles i and r will also be small. Thus, applying such paraxial approximation, then $\sin i \cong i$ and $\sin r \cong r$, so from (1), we have

$$\mu_1 i = \mu_2 r \quad \ldots(2)$$



Using the geometrical property that an exterior angle of a triangle is equal to the sum of the two internal opposite angles, we get from triangles AOC and AIC

$$\gamma = \alpha + i \quad \ldots(3)$$

and $$\gamma = \beta + r \quad \ldots(4)$$

Substituting the value of i and r from Equations (3) and (4) in Equation (2), we get

$$\mu_1(\gamma - \alpha) = \mu_2(\gamma - \beta)$$

$$\Rightarrow \quad \mu_1\alpha - \mu_2\beta = (\mu_1 - \mu_2)\gamma \qquad \ldots(5)$$

Now, since the aperture of the refracting surface is small, so M and P are very close to each other and hence we have

$$\alpha \cong \tan\alpha = \frac{AM}{MO} \cong \frac{AM}{PO},$$

$$\beta \cong \tan\beta = \frac{AM}{MI} \cong \frac{AM}{PI} \text{ and}$$

$$\gamma \cong \tan\gamma = \frac{AM}{MC} \cong \frac{AM}{PC}$$

$$\Rightarrow \quad \mu_1\left(\frac{AM}{PO}\right) - \mu_2\left(\frac{AM}{PI}\right) = (\mu_1 - \mu_2)\left(\frac{AM}{PC}\right)$$

$$\Rightarrow \quad \frac{\mu_1}{PO} - \frac{\mu_2}{PI} = \frac{\mu_1 - \mu_2}{PC}$$

Since $PO = -u$, $PI = -v$, $PC = -R$ so we get

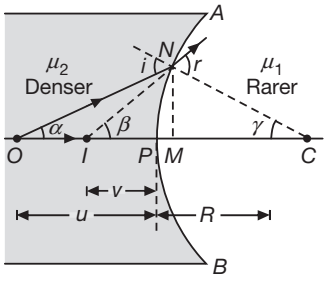
$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

CASE-2: When the object lies in the denser medium. Consider a spherical surface of radius R separating the two media 1 and 2 $(\mu_2 > \mu_1)$. A point object O is placed on the principal axis to the left of the pole P. The incident ray from O falls on point A and is refracted according to

$$\mu_2 \sin i = \mu_1 \sin r \qquad \ldots(1)$$

Since the rays are paraxial, so the angle α is small and hence the angles i and r will also be small. Thus, applying such paraxial approximation, then $\sin i \cong i$ and $\sin r \cong r$, so from (1), we have

$$\mu_2 i = \mu_1 r \qquad \ldots(2)$$



Using the geometrical property that an exterior angle of a triangle is equal to the sum of the two internal opposite angles, we get from triangles AOC and AIC

$$i = \alpha + \gamma \qquad \ldots(3)$$

and $$r = \beta + \gamma \qquad \ldots(4)$$

Substituting the value of i and r from Equations (3) and (4) in Equation (2), we get

$$\mu_2(\alpha + \gamma) = \mu_1(\beta + \gamma)$$

$$\Rightarrow \quad \mu_2\alpha - \mu_1\beta = (\mu_1 - \mu_2)\gamma \qquad \ldots(5)$$

Now, since the aperture of the refracting surface is small, so M and P are very close to each other and hence we have

$$\alpha \cong \tan\alpha = \frac{AM}{MO} \cong \frac{AM}{PO},$$

$$\beta \cong \tan\beta = \frac{AM}{MI} \cong \frac{AM}{PI} \text{ and}$$

$$\gamma \cong \tan\gamma = \frac{AM}{MC} \cong \frac{AM}{PC}$$

$$\Rightarrow \quad \mu_2\left(\frac{AM}{PO}\right) - \mu_1\left(\frac{AM}{PI}\right) = (\mu_1 - \mu_2)\left(\frac{AM}{PC}\right)$$

$$\Rightarrow \quad \frac{\mu_2}{PO} - \frac{\mu_1}{PI} = \frac{\mu_1 - \mu_2}{PC}$$

Since $PO = -u$, $PI = -v$, $PC = +R$ so we get

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Conceptual Note(s)

(a) For both convex and concave spherical surfaces, the refraction formulae are same, only proper signs of u, v and R are to be used.

(b) For refraction from rarer to denser medium, the refraction formula is

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

(c) For refraction from denser to rarer medium, we inter-change μ_1 and μ_2 and obtain the refraction formula,

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

(d) If the rarer medium is air $(\mu_1 = 1)$ and the denser medium has refractive index μ (i.e., $\mu_2 = \mu$), then

(i) for refraction from air to medium, we have

$$\frac{1}{-u} + \frac{\mu}{v} = \frac{\mu - 1}{R}$$

(ii) for refraction from medium to air, we have

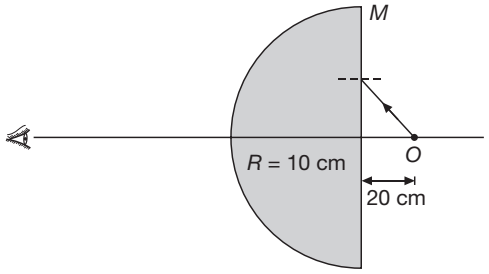
$$\frac{1}{-v}+\frac{\mu}{u}=\frac{\mu-1}{R}$$

(e) The factor $\frac{\mu_2-\mu_1}{R}$ is called power of the spherical refracting surface. It gives a measure of the degree to which the refracting surface can converge or diverge the rays of light passing through it. For air-medium interface, the power is

$$P=\frac{\mu-1}{R}$$

ILLUSTRATION 66

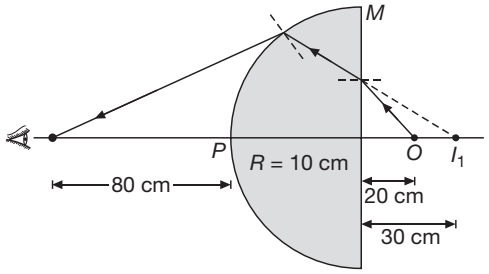
A glass hemisphere M of $\mu=\frac{3}{2}$ and radius 10 cm has a point object O placed at a distance 20 cm behind the flat face which is viewed by an observer from the curved side as shown. Find the location of final image after two refractions as seen by observer.



SOLUTION

After first refraction at flat surface image is produced at a distance given by

$$\mu h=\frac{3}{2}\times 20=30 \text{ cm}$$



For second refraction at spherical surface, for refraction formula we use

$$u=+40 \text{ cm};\ R=+10 \text{ cm};\ \mu_1=\frac{3}{2};\ \mu_2=1$$

Substituting values in refraction formula, we get

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\frac{1}{v}-\frac{3}{2\times 40}=\frac{1-\frac{3}{2}}{10}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{3}{90}-\frac{1}{20}=-\frac{1}{80}$$

$$\Rightarrow \quad v=-80 \text{ cm}$$

Thus final image is seen by observer at a distance 80 cm from the pole P of curved surface and it is a real image.

Conceptual Note(s)

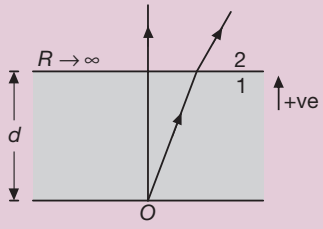
The refraction formula $\frac{\mu_1}{-u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$ is equally applicable to plane refracting surfaces i.e., surfaces for which $R\to\infty$. Let us derive $\mu=\frac{\text{Real Depth}}{\text{Apparent Depth}}$ using this.

Applying $\frac{\mu_1}{-u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$

with proper sign conventions and values, we get

$$\frac{\mu}{-(-d)}+\frac{1}{v}=\frac{1-\mu}{\infty}=0$$

$$\Rightarrow \quad v=-\frac{d}{\mu}$$



i.e., image of object O is formed at a distance $\frac{d}{\mu}$ on same side.

So, $d_{app} = \dfrac{d_{actual}}{\mu}$

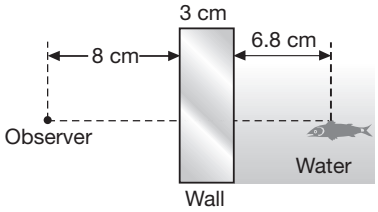
$\Rightarrow \mu = \dfrac{\text{Real Depth}}{\text{Apparent Depth}}$

ILLUSTRATION 67

In figure, a fish watcher watches a fish through a 3 cm thick glass wall of a fish tank. The watcher is in level with the fish; the index of refraction of the glass is $\frac{8}{5}$ and that of the water is $\frac{4}{3}$.

(a) To the fish, how far away does the watcher appear to be?

(b) To the watcher, how far away does the fish appear to be?

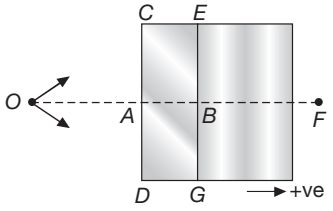


SOLUTION

(a) $OA = 3$ cm

So, $AI_1 = (n_g)(OA)$

$\Rightarrow AI_1 = \left(\frac{8}{5}\right)(3) = 4.8$ cm



For refraction at $EG\,(R \to \infty)$, using

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\Rightarrow \frac{\frac{4}{3}}{BI_2} - \frac{\frac{8}{5}}{-(4.8+3)} = 0$$

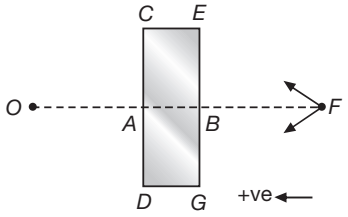
$$\Rightarrow BI_2 = -(7.8)\left(\frac{4}{3}\right)\left(\frac{5}{8}\right) = -6.5 \text{ cm}$$

So, $FI_2 = 6.5 + 6.8 = 13.3$ cm

(b) For face EF, we have

$$\frac{\frac{8}{5}}{BI_1} - \frac{\frac{4}{3}}{-6.8} = 0 \qquad \{\because R \to \infty\}$$

$$\Rightarrow BI_1 = -(6.8)\left(\frac{8}{5}\right)\left(\frac{3}{4}\right) = -8.16 \text{ cm}$$



For face CD, we have

$$\frac{1}{AI_2} - \frac{\frac{8}{5}}{-11.16} = 0 \qquad \{\because R \to \infty\}$$

$$\Rightarrow AI_2 = -(11.16)\left(\frac{5}{8}\right) = -6.975 \text{ cm}$$

$$\Rightarrow FI_2 = 8 + 6.975$$

$$\Rightarrow FI_2 = 14.975 \text{ cm}$$

ILLUSTRATION 68

There are two objects O_1 and O_2 at an identical distance of 20 cm on the two sides of the pole of a spherical concave refracting boundary of radius 60 cm. The indices of refraction of the media on two sides of the boundary are 1 and $\left(\frac{4}{3}\right)$ respectively. Find the location of the object

(a) O_1 when seen from O_2.

(b) O_2 when seen from O_1.

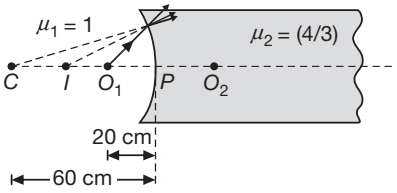
SOLUTION

The formula for refraction from a curved boundary is

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

(a) From the ray diagram drawn, we get

$$u_1 = -20 \text{ cm}, \; R = -60 \text{ cm}, \; \mu_1 = 1, \; \mu_2 = \frac{4}{3}$$



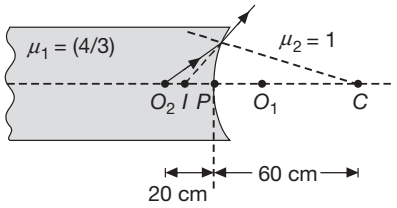
$$\Rightarrow \quad \frac{\left(\frac{4}{3}\right)}{v} - \frac{1}{(-20)} = \frac{\left(\frac{4}{3}\right)-1}{(-60)}$$

$$\Rightarrow \quad v = -24 \text{ cm}$$

Thus, the object O_1, will appear at a distance of 24 cm from P towards C.

(b) Keeping the object O_2 on the left of the pole P as shown, here, we get

$$u_1 = -20 \text{ cm}, \; R = +60 \text{ cm}, \; \mu_1 = \frac{4}{3}, \; \mu_2 = 1$$



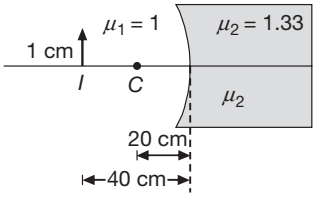
$$\Rightarrow \quad \frac{1}{v} - \frac{\left(\frac{4}{3}\right)}{(-20)} = \frac{1-\left(\frac{4}{3}\right)}{60}$$

$$\Rightarrow \quad v = -16.36 \text{ cm}$$

Thus, the object O_2 will appear at a distance of 16.36 cm from P towards O_2.

ILLUSTRATION 69

An object of height 1 cm is kept at a distance of 40 cm from a concave spherical surface having radius of curvature $R = 20$ cm, separating air and glass having refractive index $\mu = 1.33$. Find the location, height and the nature of the image formed.



SOLUTION

According to Cartesian sign convention,

$$u = -40 \text{ cm}, \; R = -20 \text{ cm}$$

Applying the formula $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, we get

$$\frac{1.33}{v} - \frac{1}{(-40)} = \frac{1.33-1}{(-20)}$$

$$\Rightarrow \quad v = -32 \text{ cm}$$

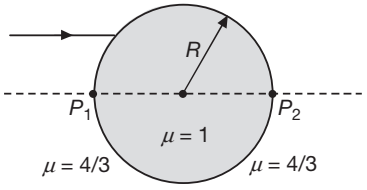
Magnification, $m = \frac{h_2}{h_1} = \frac{\mu_1 v}{\mu_2 u} = \frac{1\times(-32)}{1.33\times(-40)} = 0.60$

So, height of image, $h_2 = mh_1 = 0.6\times1 = 0.6$ cm

The positive sign of magnification indicates that the image is virtual and erect.

ILLUSTRATION 70

A parallel beam of light travelling in water $\left(\mu = \frac{4}{3}\right)$ is refracted by a spherical air bubble of radius R situated in water.



(a) Find the position of the image due to refraction at the first surface and the position of the final image.

(b) Draw the ray diagram showing the position of the two images.

SOLUTION

(a) Applying the formula for the refraction at the curved boundary i.e.,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

For, refraction at the first surface, the pole is P_1 and we observe that

$$u \to -\infty,\ R = +R,\ \mu_1 = \frac{4}{3} \text{ and } \mu_2 = 1$$

$$\Rightarrow \frac{1}{v} - \frac{\left(\frac{4}{3}\right)}{(-\infty)} = \frac{1-\left(\frac{4}{3}\right)}{(+R)}$$

$$\Rightarrow v = -3R$$

Thus, the first image I_1 is formed at a distance of $3R$ to the left of pole P_1.

This image acts as an object for the refraction at the second surface, with pole P_2. For this refraction, we have

$$u = -(3R + 2R) = -5R,\ R = -R,\ \mu_1 = 1,\ \mu_2 = \frac{4}{3}$$

$$\Rightarrow \frac{\frac{4}{3}}{v} - \frac{1}{-5R} = \frac{\frac{4}{3}-1}{-R}$$

$$\Rightarrow v = -\frac{5}{2}R$$

Thus, the final image I_2 is at a distance $\frac{5R}{2}$ from P_2 towards left.

(b) The ray diagram is shown in figure.

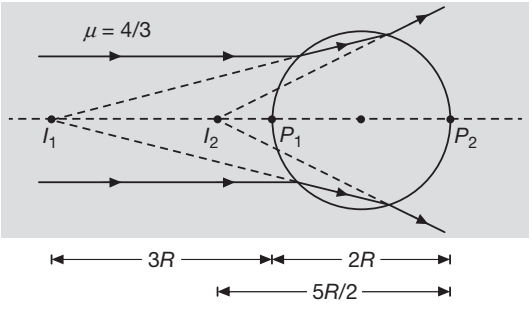
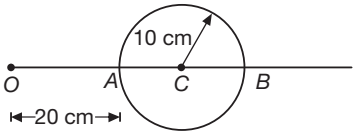


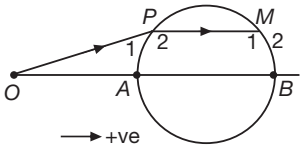
ILLUSTRATION 71

A glass sphere of radius $R = 10$ cm having refractive index $\mu_g = \frac{3}{2}$ is kept inside water. A point object O is placed at 20 cm from A as shown in figure. Find the position and nature of the image when seen from other side of the sphere. Also draw the ray diagram. Given refractive index of water is $\mu_w = \frac{4}{3}$.



SOLUTION

A ray of light starting from O gets refracted twice. The ray of light is travelling in a direction from left to right. Hence, the distances measured in this direction are taken positive. Applying $\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$, twice with appropriate signs at the two refracting surfaces, we get



$$\frac{\left(\frac{4}{3}\right)}{-(-20)} + \frac{\left(\frac{3}{2}\right)}{AI_1} = \frac{\left(\frac{3}{2}-\frac{4}{3}\right)}{10}$$

$$\Rightarrow AI_1 = -30 \text{ cm}$$

Now, the first image I_1, acts as an object for the second surface, so, we have

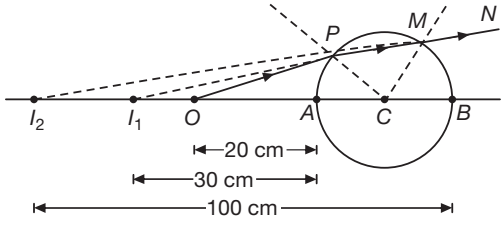
$$BI_1 = u = -(30 + 20) = -50 \text{ cm}$$

Again applying $\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$, we get

$$\frac{\left(\frac{3}{2}\right)}{-(-50)} + \frac{\left(\frac{4}{3}\right)}{BI_2} = \frac{\frac{4}{3}-\frac{3}{2}}{-10}$$

$$\Rightarrow BI_2 = -100 \text{ cm}$$

i.e., the final image I_2 is virtual and is formed at a distance 100 cm (towards left) from B. The ray diagram is as shown.



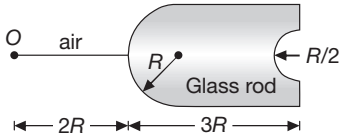
Following points should be kept in mind while drawing the ray diagram.

(i) At P the ray travels from rarer to a denser medium. Hence, it will bend towards normal PC. At M, it travels from a denser to a rarer medium, hence, moves away from the normal MC.

(ii) The ray PM when extended backwards meets the principal axis at I_1 and the ray MN when extended meets the principal axis at I_2.

ILLUSTRATION 72

A glass rod has ends as shown in figure. The refractive index of glass is μ. The object O is at a distance $2R$ from the surface of larger radius of curvature. The distance between apexes of ends is $3R$.



(a) Find the distance of image formed of the point object from right hand vertex.

(b) What is the condition to be satisfied if the image is to be real?

SOLUTION

For refraction at curved surface S_1,

$$\frac{\mu}{v_1}-\frac{1}{(-2R)}=\frac{(\mu-1)}{R}$$

$$\Rightarrow \quad \frac{\mu}{v_1}=\frac{2\mu-3}{2R}$$

$$\Rightarrow \quad v_1=\frac{2\mu R}{2\mu-3} \qquad \ldots(1)$$

The first image acts as object for refraction at second surface S_2. The origin of our Cartesian coordinate system is now at vertex/pole of surface S_2. Object distance for second refraction is

$$u_2=-(3R-v_1)$$

$$\Rightarrow \quad u_2=-\left(3R-\frac{2\mu R}{(2\mu-3)}\right)$$

$$\Rightarrow \quad u_2=-\left(\frac{4\mu-9}{2\mu-3}\right)R$$

For refraction at curved surface S_2, we have

$$\frac{1}{v_2}-\frac{\mu}{-\left(\frac{4\mu-9}{2\mu-3}\right)R}=\frac{(1-\mu)}{\left(\frac{R}{2}\right)} \qquad \ldots(2)$$

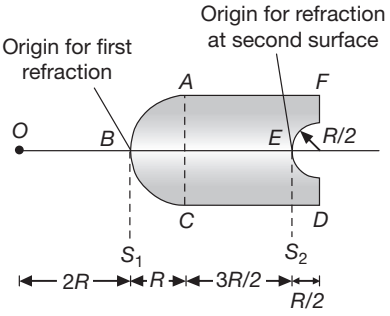
On solving the above expression for v_2, we get

$$v_2=\frac{(4\mu-9)}{(10\mu-9)(\mu-2)} \qquad \ldots(3)$$

The image will be real if v_2 is positive, i.e.,

$$\frac{(4\mu-9)}{(10\mu-9)(\mu-2)}>0 \qquad \ldots(4)$$

The equation (4) is satisfied when



CASE-1:

$$(4\mu-9)>0$$

$$\Rightarrow \quad \mu>\frac{9}{4}$$

and $\quad (10\mu-9)(\mu-2)>0$

$$\Rightarrow \quad 0.9<\mu<2$$

there is no common solution for this condition and hence this is rejected.

CASE-2:

$$4\mu-9<0$$

$$\Rightarrow \quad \mu<\frac{9}{4}$$

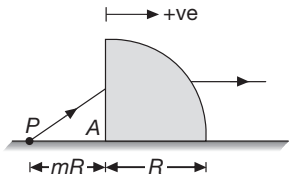
$$(10\mu-9)(\mu-2)<0$$

$$\Rightarrow \quad \mu>2 \text{ OR } \mu<0.9$$

Hence the common result is $2<\mu<2.25$

ILLUSTRATION 73

A quarter cylinder of radius R and refractive index 1.5 is placed on a table. A point object P is kept at a distance of mR from it. Find the value of m for which a ray from P will emerge parallel to the table as shown in figure.



SOLUTION

Applying $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$, firstly at the plane surface and then at the curved surface.

For the plane surface, we get

$$\frac{1.5}{AI_1}-\frac{1}{(-mR)}=\frac{1.5-1}{\infty}=0 \qquad \{\because R\to\infty\}$$

$$\Rightarrow \quad AI_1=-(1.5mR)$$

For the curved surface, since the final image is formed at infinity, so we get

$$\frac{1}{\infty}-\frac{1.5}{-(1.5mR+R)}=\frac{1-1.5}{-R}$$

$$\Rightarrow \quad \frac{1.5}{(1.5m+1)R}=\frac{0.5}{R}$$

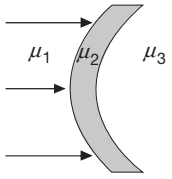
$$\Rightarrow \quad 3=1.5m+1$$

$$\Rightarrow \quad \frac{3}{2}m=2$$

$$\Rightarrow \quad m=\frac{4}{3}$$

ILLUSTRATION 74

In the figure, light is incident on the thin lens as shown. The radius of curvature for both the surface is R. Determine the focal length of this system.

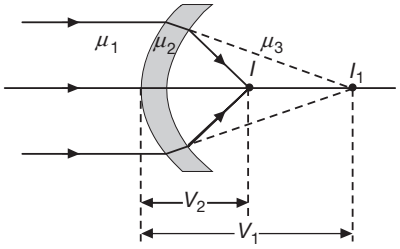


SOLUTION

Since parallel rays after passing through a lens must converge (or appear to converge) at the point. So this point is the place where focus is located and the final image is also formed at the focus.

For refraction at first surface, we get

$$\frac{\mu_2}{v_1}-\frac{\mu_1}{-\infty}=\frac{\mu_2-\mu_1}{+R} \qquad \ldots(1)$$



For refraction at 2nd surface, we get

$$\frac{\mu_3}{v_2}-\frac{\mu_2}{v_1}=\frac{\mu_3-\mu_2}{+R} \qquad \ldots(2)$$

Adding equations (1) and (2), we get

$$\frac{\mu_3}{v_2}=\frac{\mu_3-\mu_1}{R}$$

$$\Rightarrow \quad v_2=\frac{\mu_3 R}{\mu_3-\mu_1}$$

Hence, focal length of the given lens system is

$$f=\frac{\mu_3 R}{\mu_3-\mu_1}$$

ILLUSTRATION 75

A parallel beam of light travelling in water having refractive index $\frac{4}{3}$ is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial.

(i) Find the position of the image due to refraction at the first surface and the position of the final image.

(ii) Draw a ray diagram showing the positions of both the images.

SOLUTION

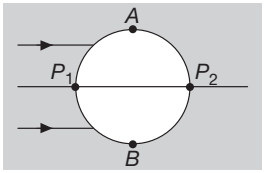
(i) To get the desired result(s), we shall be applying $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$, one by one on two spherical surfaces.

For first refraction at AP_1B, we have

$$\frac{1}{v_1} - \frac{\frac{4}{3}}{\infty} = \frac{1-\frac{4}{3}}{+2}$$

$$\Rightarrow \quad \frac{1}{v_1} = -\frac{1}{6}$$

$$\Rightarrow \quad v_1 = -6 \text{ mm}$$



So, the first image will be formed at 6 mm towards left of P_1

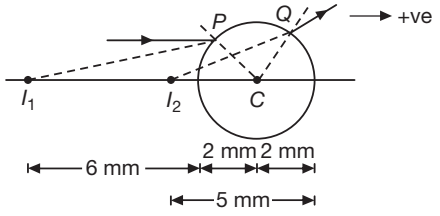
For second refraction at AP_2B, the distance of first image I_1 from P_2 will be 6 mm + 4 mm = 10 mm (towards left). So, we get

$$\frac{\frac{4}{3}}{v_2} - \frac{1}{-10} = \frac{\frac{4}{3}-1}{-2}$$

$$\Rightarrow \quad \frac{4}{3v_2} = -\frac{1}{6} - \frac{1}{10} = -\frac{4}{15}$$

$$\Rightarrow \quad v_2 = -5 \text{ mm}$$

(ii) The ray diagram is shown in figure



Conceptual Note(s)

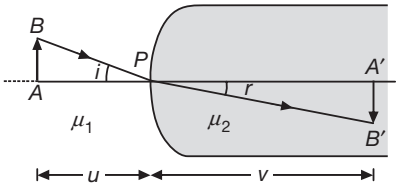
(a) At P and Q both normal will pass through C

(b) At P ray of light is travelling from a denser medium (water) to rarer medium (air) therefore, ray of light will bend away from the normal and on extending meet at I_1. Similarly at Q ray of light bends towards the normal.

(c) Both the images I_1 and I_2 are virtual.

LATERAL OR TRANSVERSE MAGNIFICATION

Instead of a point object O let us now, keep an extended object AB at point O such that its image $A'B'$ will be formed at point I. The distance $x(=-u)$ and $y(=v)$ are related by the above formula.



A ray from point B of the object is incident at point P and is refracted, in accordance with Snell's Law, such that

$$\mu_1 \sin i = \mu_2 \sin r$$

$$\Rightarrow \quad \mu_1 i = \mu_2 r \qquad \{\text{applying paraxial approximation}\}$$

$$\Rightarrow \quad \frac{r}{i} = \frac{\mu_1}{\mu_2} \qquad \ldots(1)$$

Now, in ΔABP and $\Delta A'B'P$, we have

$$AB = u \tan i \cong ui \qquad \ldots(2)$$

and $$A'B' = v \tan r \cong vr \qquad \ldots(3)$$

The magnification is defined as

$$m = \frac{\text{height of image}}{\text{height of object}} = \frac{A'B'}{AB}$$

Using Equations (1), (2) & (3), we get

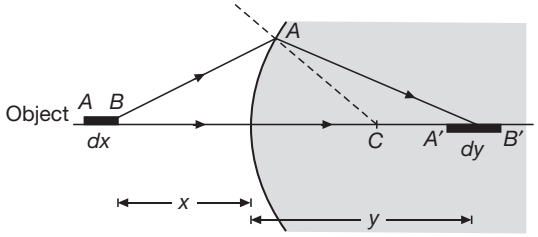
$$m = \frac{A'B'}{AB} = \frac{vr}{ur} = \left(\frac{v}{u}\right)\left(\frac{\mu_1}{\mu_2}\right)$$

$$\Rightarrow \quad m = \frac{v}{u}\,\frac{\mu_1}{\mu_2}$$

If m is positive, the image is erect and virtual.
If m is negative, the image is inverted and real.

LONGITUDINAL OR AXIAL MAGNIFICATION OF IMAGE

An object of width dx which is placed on principal axis of the refracting surface S at a distance x from the pole P. After refraction its image I is produced at a distance y from the pole and is of width dy as shown in figure.



The distances of object and image from the pole of the surface are related by the refraction formula given as

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

Here we use $u=-x$, $R=+R$ and $v=+y$ which gives

$$\frac{\mu_2}{y}+\frac{\mu_1}{x}=\frac{\mu_2-\mu_1}{R} \quad \ldots(1)$$

Differentiating the above equation we get

$$-\frac{\mu_2}{y^2}-dy-\frac{\mu_1}{x^2}dx=0 \quad \ldots(2)$$

From above Equation (2), we get axial magnification as

$$m_{\text{axial}}=\frac{dy}{dx}=-\frac{\mu_1 y^2}{\mu_2 x^2} \quad \ldots(3)$$

As already discussed that above relation given by Equation (3) is only valid for paraxial rays. Here negative sign shows the lateral inversion of the image.

EFFECT OF MOTION OF OBJECT OR REFRACTING SURFACE ON IMAGE

As already discussed in case of spherical mirrors, for small velocities of the object or refracting surface, the velocity magnification along the principal axis (i.e. $v_{\|}$) and perpendicular to the principal axis (i.e. $v_{\perp}$) can be given by the expressions of lateral and longitudinal magnifications.

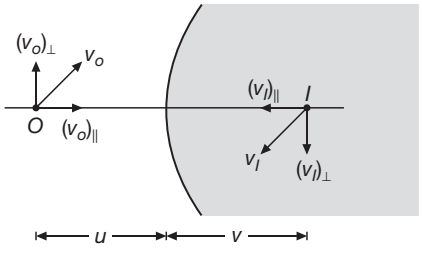


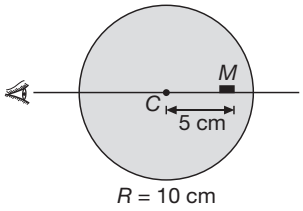
Figure shows an object moving with velocity v_O as shown. Its velocity component parallel to the principal axis with respect to the refracting surface is $(v_O)_{\|}$ and in direction perpendicular to the principal axis is $(v_O)_{\perp}$, then the image velocity components can be directly given by

$$(v_I)_{\perp}=m_{\text{lateral}}(v_O)_{\perp}$$

$$(v_I)_{\|}=m_{\text{axial}}(v_O)_{\|}, \text{ where } m_{\text{axial}}=-\frac{\mu_1 v^2}{\mu_2 u^2}$$

ILLUSTRATION 76

Figure shows a small object M of length 1 mm which lies along a diametrical line of a glass sphere of radius 10 cm and $\mu=\frac{3}{2}$ which is viewed by an observer as shown. Find the size of object as seen by the observer.



SOLUTION

For refraction at glass-air interface, we use

$$u=+15 \text{ cm}, \; R=+10 \text{ cm}, \; \mu_1=\frac{3}{2} \text{ and } \mu_2=1$$

Substituting values in refraction formula, we get

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\Rightarrow \quad \frac{1}{v}-\frac{\frac{3}{2}}{15}=\frac{1-\frac{3}{2}}{10}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{10}-\frac{1}{20}=\frac{1}{20}$$

$$\Rightarrow \quad v=+20 \text{ cm}$$

Axial magnification for refraction is given by

$$m=\frac{l_i}{l_o}=\frac{\mu_1 v^2}{\mu_2 u^2}=\frac{\frac{3}{2}\times(20)^2}{1\times(15)^2}=\frac{1.5\times 400}{225}=\frac{8}{3}$$

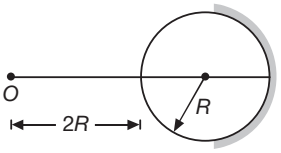
$$\Rightarrow \quad l_i=\frac{8}{3}\times 1=\frac{8}{3} \text{ mm}$$

ILLUSTRATION 77

A solid glass with radius R and an index of refraction 1.5 is silvered over one hemisphere. A small object is located on the axis of the sphere at a distance $2R$ to the left of the vertex of the unsilvered hemisphere. Find the position of final image after all refractions and reflections have taken place.

SOLUTION

The ray of light first gets refracted then reflected and then again refracted. For first refraction and then reflection the ray of light travels from left to right while for the last refraction it travels from right to left. Hence, the sign convention will change accordingly.



First Refraction:

Using $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ with appropriate sign conventions, we get

$$\frac{1.5}{v_1}-\frac{1}{(-2R)}=\frac{1.5-1}{+R}$$

$$\Rightarrow \quad v_1 \to \infty$$

Second Reflection:

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{R}$ with appropriate sign conventions, we get

$$\frac{1}{v_2}+\frac{1}{\infty}=-\frac{2}{R}$$

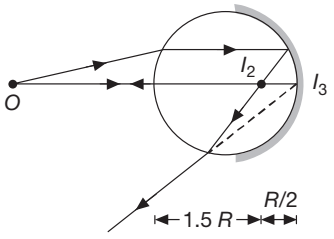
$$\Rightarrow \quad v_2=-\frac{R}{2}$$

Third Reflection:

Again using $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ with reversed sign convention, we get

$$\frac{1}{v_3}-\frac{1.5}{-1.5R}=\frac{1-1.5}{-R}$$

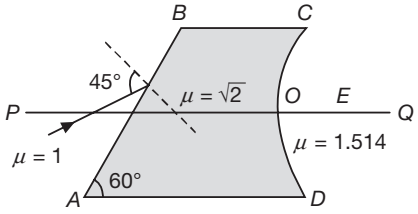
$$\Rightarrow \quad v_3=-2R$$



i.e., final image is formed at the vertex of the silvered face i.e., at the pole of silvered/curved surface.

ILLUSTRATION 78

Figure shows an irregular block of material of refractive index $\sqrt{2}$. A ray of light strikes the face AB as shown in the figure. After refraction it is incident on a spherical surface CD of radius of curvature 0.4 m and enters a medium of refractive index 1.514 to meet PQ at E. Find the distance OE.



SOLUTION

Applying Snell's Law at face AB, we get

$$(1)\sin 45^\circ=(\sqrt{2})\sin r$$

$$\Rightarrow \quad \sin r=\frac{1}{2}$$

$$\Rightarrow \quad r=30^\circ$$

i.e., ray becomes parallel to AD inside the block.

Now applying, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ on face CD, we get

$$\frac{1.514}{OE}-\frac{\sqrt{2}}{\infty}=\frac{1.514-\sqrt{2}}{0.4}$$

Solving this equation, we get

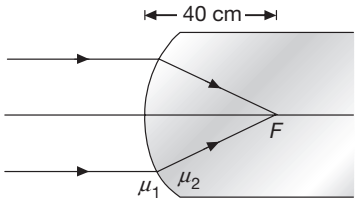
$$OE \approx 6 \text{ m}$$

Test Your Concepts-VI

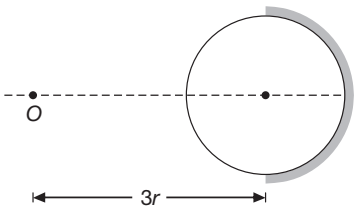
Based on Refraction at Curved Surfaces

(Solutions on page H.18)

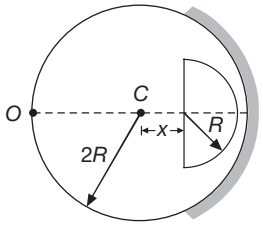
1. A spherical convex surface separates object and image space of refractive index 1 and $\frac{4}{3}$. If radius of curvature of the surface is 10 cm, find its power.



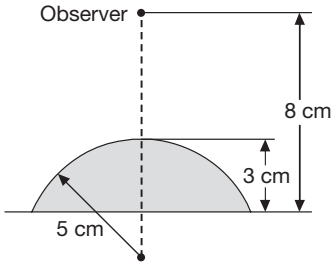
2. A hemispherical portion of the surface of a solid glass sphere of refractive index 1.5 and of radius r is silvered to make the inner side reflecting. An object is placed at the axis of the sphere at a distance $3r$ from the centre of the sphere. The light from the object is refracted at the unsilvered part, then reflected from the silvered part and again refracted at the unsilvered part. Locate the final image formed.



3. Consider the figure shown. A hemispherical cavity of radius R is carved out from a sphere (μ = 1.5) of radius $2R$ such that principal axis of cavity coincides with that of sphere. One side of sphere is silvered as shown. Find the value of x for which the image of an object at O is formed at O itself.



4. A glass sphere has a radius of 5 cm and a refractive index of 1.6. A paperweight is constructed by slicing through the sphere on a plane that is 2 cm from the centre of the sphere and perpendicular to a radius of the sphere that passes through the centre of the circle formed by the intersection of the plane and the sphere. The paperweight is placed on a table and viewed from directly above by an observer who is 8 cm from the table top, as shown in figure. When viewed through the paperweight, how far away does the table top appear to the observer?



5. One end of a long glass rod having refractive index μ = 1.5 is formed into the shape of a convex surface of radius 6 cm. An object is located in air along the axis of the rod, at a distance of 10 cm from the end of the rod.

(a) How far apart are the object and the image formed by the glass rod?

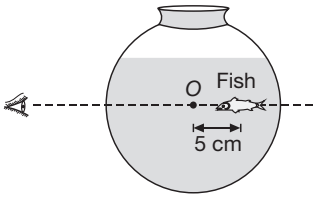
(b) For what range of distances from the end of the rod must the object be located in order to produce a virtual image?

6. An object is at a distance of d = 2.5 cm from the surface of a glass sphere with a radius R = 10 cm. Find the position of the image produced by the sphere. The refractive index of the glass is μ = 1.5.

7. A glass hemisphere of radius 10 cm and refractive index μ = 1.5 is silvered over its curved surface. There is an air bubble in the glass 5 cms from the plane surface along the axis. Find the position of the images of this bubble seen by observer looking along the axis into the flat surface of the hemisphere.

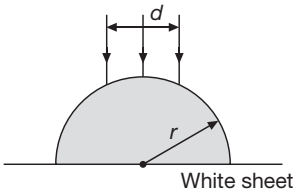
8. A hollow sphere of glass of refractive index μ has a small mark on its interior surface which is observed from a point outside the sphere on the side opposite the centre. The inner cavity is concentric with external surface and the thickness of the glass is everywhere equal to the radius of the inner surface. Prove that the mark will appear nearer than it really is, by a distance $\frac{(\mu-1)R}{(3\mu-1)}$, where R is the radius of the inner surface.

9. Figure shows a fish bowl of radius 10 cm in which along a diametrical line a fish F is moving at speed 2 mms^{-1}. Find the speed of fish as observed by an observer from outside along same line when fish is at a distance 5 cm from the centre of bowl to right of it as shown in figure.

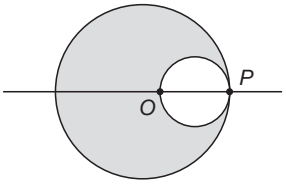


10. A parallel incident beam falls on a solid glass sphere at near normal incidence. Calculate the image distance in terms of refractive index μ of the sphere and its radius R.

11. Figure shows a glass hemisphere placed on a white horizontal sheet. A vertical paraxial light beam of diameter d incident on the curved surface of hemisphere as shown. Find the diameter of the light spot formed on sheet after refraction.



12. A transparent sphere of radius R has a cavity of radius $\frac{R}{2}$ as shown in figure. Find the refractive index of the sphere if a parallel beam of light falling on left surface focuses at point P.



13. A glass sphere ($\mu = 1.5$) with a radius of 15.0 cm has a tiny air bubble 5 cm above its centre. The sphere is viewed looking down along the extended radius containing the bubble. What is the apparent depth of the bubble below the surface of the sphere?

THIN SPHERICAL LENSES

A **lens** is a piece of transparent material with two refracting surfaces, at least one of them being curved. It may have one surface plane.

A **spherical lens** has spherical surfaces as bounds. If the thickness of the lens is small (compared to the radius of curvature of spherical surfaces, the object distance, the image distance, etc.), it is said to be **thin**.

There are two types of lenses:

(a) convex or converging lenses,
(b) concave or diverging lenses.

NAMING CONVENTION FOR LENSES

While naming a lens, the surface with larger radius of curvature is named first. The lens has a nature of the surface that has the smaller radius of curvature.

EXAMPLE:

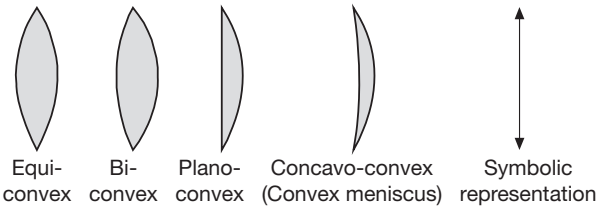
A lens with one surface plane and the other surface convex will be named as Plano-Convex irrespective of its placement and this lens will have converging nature (the same as the nature of the surface having smaller radius of curvature).

Similarly a Convexo-Concave lens will have a diverging nature and Concavo-Convex lens will have a converging nature.

To summarise, we can say that the first name of a bifocal lens is derived from the name of the surface with bigger radius of curvature and the last name of the lens is derived from the nature of the lens.

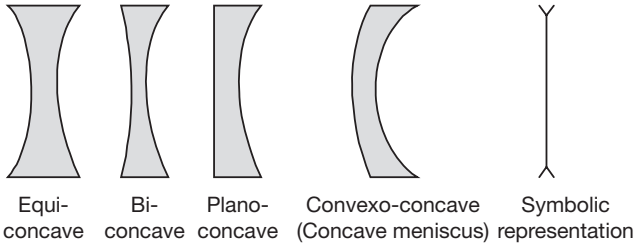
CONVEX OR CONVERGING LENSES

Converging lenses convert a parallel beam of incident rays into a convergent beam. Converging lenses are convex, i.e. such that the thickness at the middle is larger than the thickness of edges. A convex lens is thicker in the centre than at its edges. They include convexo-convex, plano-convex, and cancavo-convex lenses. Sometimes, a converging lens is represented symbolically by a double headed arrow as shown.



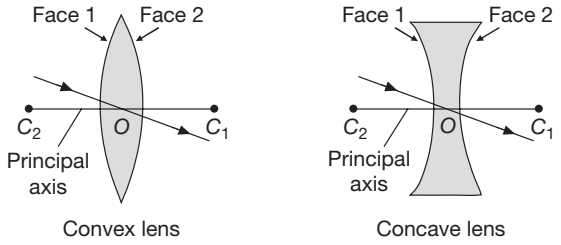
CONCAVE OR DIVERGING LENSES

Diverging lenses convert a parallel beam of rays into a divergent beam. Diverging lenses are concave, i.e. such that the thickness at their edges is larger than the thickness at the middle. A concave lens is thinner at the centre. They include concavo-concave, plano-concave and convexo-concave lenses. Sometimes, a diverging lens is represented symbolically by a line with inverted arrows at its two ends.



OPTICAL CENTRE OF LENS

The central portion of a lens (both convex and concave) behaves as a flat slab. Optical centre O is a point through which any ray passes undeviated.

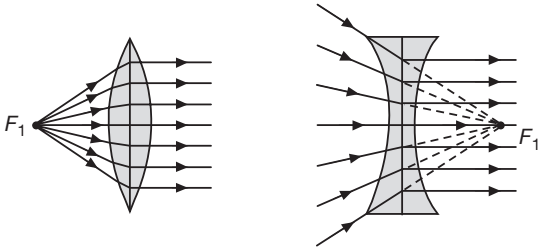


PRINCIPAL AXIS OF A LENS

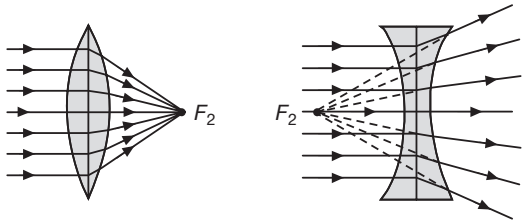
The line joining the centres of curvature C_1C_2 is called the **principal axis** of the lens.

PRINCIPAL FOCUS

A lens has two focal points. The **first focal point** F_1 is a point object on the principal axis for which the image is at infinity.



The **second focal point** F_2 is a point image on the principal axis for which the object is at infinity.

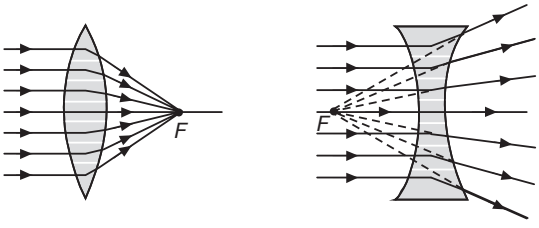


Focal length (f) is the distance between O and the second focus F_2.

Aperture is the effective diameter of the light transmitting area of the lens. The intensity of the image formed by the lens,

$$I \propto (\text{Aperture})^2$$

Converging and diverging action of a lens is due to the fact that a lens may be thought of a combination of small prisms, as shown in figure. A parallel beam of light, when incident on a convex lens, converges to a point called **focus** F.



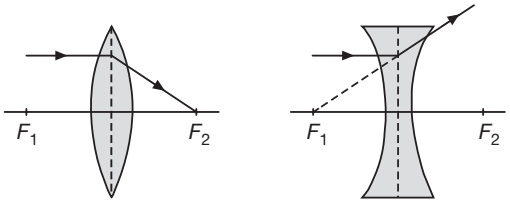
Converging action Diverging action

A concave lens diverges a parallel beam of light. It appears to be diverging from a point F, called **focus**.

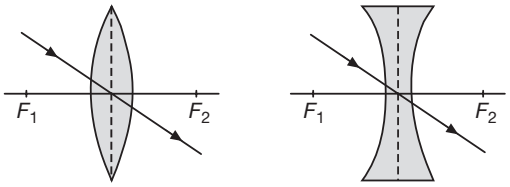
For thin lenses, we need not consider refraction of light at the two surfaces separately. Instead, we say that the light-ray is bent (towards the principal axis in case of convex lens, and away from the principal axis in case of concave lens) when it passes through a thin lens.

RULES FOR OBTAINING IMAGES IN LENSES

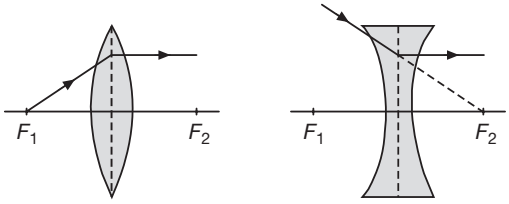
1. A **ray parallel to the principal axis**, after refraction through the lens, converges to the focus (in case of a convex lens) or appears to diverge from the focus (in case of a concave lens).



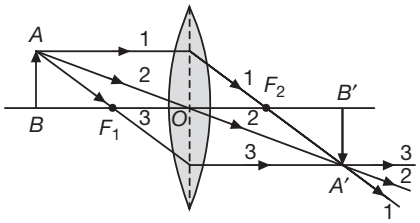
2. A **ray passing through the optical centre** goes through the lens undeviated.



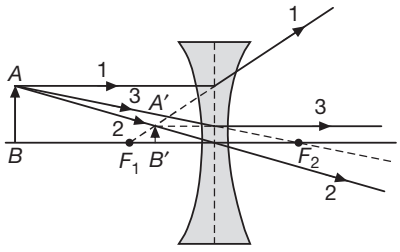
3. A **ray passing through the focus** (in case of a convex lens) or appearing to pass through the focus (in case of a concave lens) is rendered parallel to the principal axis after refraction through the lens.



Any two of the above three rays can be used to obtain the location of the image.



Convex lens



Concave lens

THIN LENS FORMULA FOR A CONVEX LENS

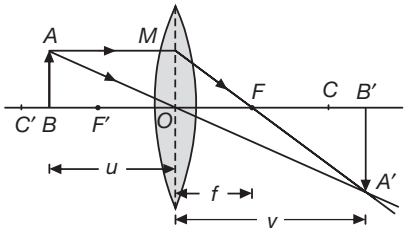
Assumptions used in the derivation of lens formula

(a) The lens used is thin.
(b) The aperture of the lens is small.
(c) The incident and refracted rays make small angles with the principal axis.
(d) The object is a small object placed on the principal axis.

CASE-1: When a real image is formed

Consider an object AB placed perpendicular to the principal axis of a thin convex lens between its F' and C'. A real, inverted and magnified image $A'B'$

is formed beyond C on the other side of the lens, as shown in figure.



Since, $\Delta A'B'O$ and ΔABO are similar,

$$\Rightarrow \quad \frac{A'B'}{AB} = \frac{OB'}{OB} \qquad \ldots(1)$$

Also $\Delta A'B'F$ and ΔMOF are similar,

$$\Rightarrow \quad \frac{A'B'}{MO} = \frac{FB'}{OF}$$

Since $MO = AB$,

$$\Rightarrow \quad \frac{A'B'}{AB} = \frac{FB'}{OF} \qquad \ldots(2)$$

From (1) and (2), we get

$$\frac{OB'}{OB} = \frac{FB'}{OF} = \frac{OB' - OF}{OF}$$

Using new Cartesian sign convention, we get

Object distance, $OB = -u$

Image distance, $OB' = +v$

Focal length, $OF = +f$

$$\Rightarrow \quad \frac{v}{-u} = \frac{v - f}{f}$$

$$\Rightarrow \quad vf = -uv + uf$$

$$\Rightarrow \quad uv = uf - vf$$

Dividing both sides by uvf, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

CASE-2: When a virtual image is formed

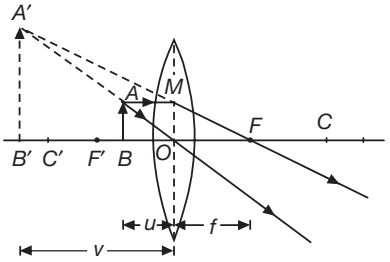
When an object AB is placed between the optical centre O and the focus F of a convex lens, the image $A'B'$ formed by the convex lens is virtual, erect and magnified as shown in figure.

Since, triangles $A'B'O$ and ABO are similar, so we have

$$\frac{A'B'}{AB} = \frac{OB'}{OB} \qquad \ldots(1)$$

Also, triangles $A'B'F$ and MOF are similar, so we have

$$\frac{A'B'}{OM} = \frac{B'F}{OF}$$



Since $OM = AB$

$$\Rightarrow \quad \frac{A'B'}{AB} = \frac{B'F}{OF} \qquad \ldots(2)$$

From (1) and (2), we get

$$\frac{OB'}{OB} = \frac{B'F}{OF} = \frac{OB' + OF}{OF}$$

Using new Cartesian sign convention,

Object distance $BO = -u$

Image distance $OB' = -v$

Focal length $OF = +f$

$$\Rightarrow \quad \frac{-v}{-u} = \frac{-v + f}{f}$$

$$\Rightarrow \quad -vf = uv - uf$$

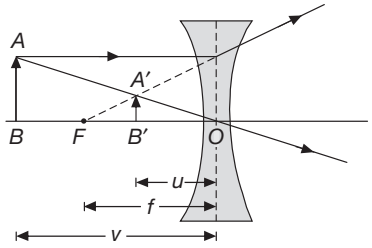
$$\Rightarrow \quad uv = uf - vf$$

Dividing both sides by uvf, we again get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

THIN LENS FORMULA FOR A CONCAVE LENS

Let O be the optical centre and F be the principal focus of concave lens of focal length f. AB is an object placed perpendicular to its principal axis. A virtual, erect and diminished image $A'B'$ is formed due to refraction through the lens.



Since, $\Delta A'B'O$ and ΔABO are similar

So, $\frac{A'B'}{AB} = \frac{OB'}{OB}$ …(1)

Also, $\Delta A'B'F$ and ΔMOF are similar

So, $\frac{A'B'}{OM} = \frac{FB'}{OF}$

Since $OM = AB$, therefore

$$\frac{A'B'}{AB} = \frac{FB'}{OF} \quad \ldots(2)$$

From (1) and (2), we get

$$\frac{OB'}{OB} = \frac{FB'}{OF} = \frac{OF - OB'}{OF}$$

Using new Cartesian sign convention, we get

$$OB = -u\ ,\ OB' = -v\ ,\ OF = -f$$

$$\Rightarrow \quad \frac{-v}{-u} = \frac{-f + v}{-f}$$

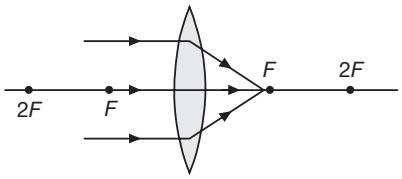
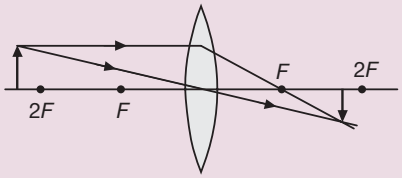
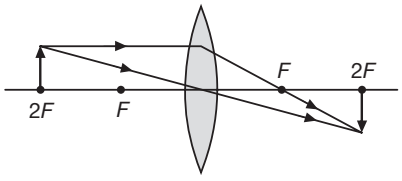
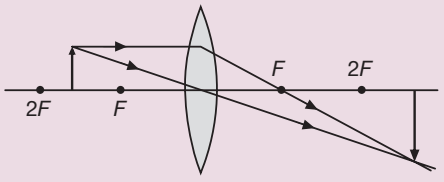
$$\Rightarrow \quad vf = uf - uv$$

$$\Rightarrow \quad uv = uf - vf$$

Dividing both sides by uvf, we again get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

IMAGE FORMATION BY CONVEX LENS

OBJECT POSITION	DIAGRAM	POSITION OF IMAGE	NATURE AND SIZE OF IMAGE
At infinity		At the principal Focus (F) or in the focal plane	Real, inverted and extremely diminished
Beyond $2F$		Between F and $2F$	Real, inverted and diminished
At $2F$		At $2F$	Real, inverted and of same size as the object
Between F and $2F$		Beyond $2F$	Real, inverted and magnified

(*Continued*)

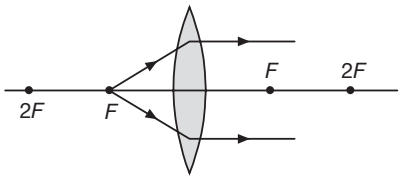
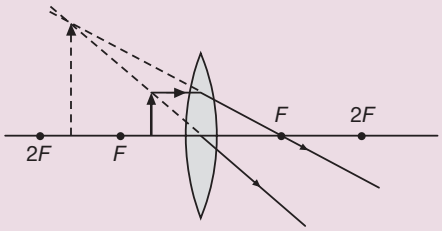
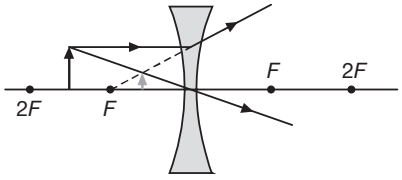
OBJECT POSITION	DIAGRAM	POSITION OF IMAGE	NATURE AND SIZE OF IMAGE
At F		At infinity	Real, inverted and highly magnified
Between F and optical centre		On the same side as the object	Virtual, erect and magnified

IMAGE FORMATION BY A CONCAVE LENS

OBJECT POSITION	DIAGRAM	POSITION OF IMAGE	NATURE AND SIZE OF IMAGE
For all positions of object		Images formed between the optical centre of the lens and the focus (F)	Always forms a Virtual, Erect and Diminished Image

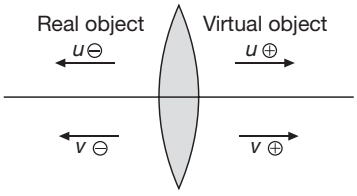
VARIATION CURVES OF IMAGE DISTANCE VS OBJECT DISTANCE FOR A THIN LENS

For Convex Lens

For a lens, we have $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

For convex lens of focal length f, we have $f=+f$.

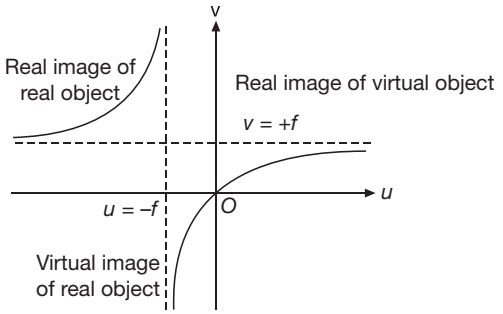
$$\Rightarrow \quad v=\frac{uf}{u+f} \qquad \ldots(1)$$



Substituting the following values of u in equation (1) to get the corresponding values of v for purpose of plotting the u-v graph.

u	$-\infty$	$-2f$	$-f$	$-\frac{f}{2}$	$-\frac{f}{4}$	0	$+\frac{f}{2}$	$+\frac{f}{4}$	$+f$	$+2f$	$+\infty$
v	$+f$	$+2f$	$+\infty$	$-f$	$-\frac{f}{3}$	0	$\frac{f}{3}$	$\frac{f}{5}$	$+\frac{f}{2}$	$+\frac{2f}{3}$	$+f$

The above function can be plotted as shown in figure for a convex lens.

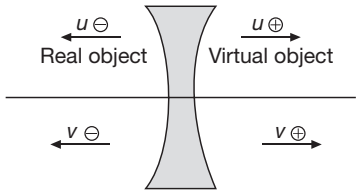


For Concave Lens

For a lens, we have $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

For concave lens of focal length f, we have $f=-f$.

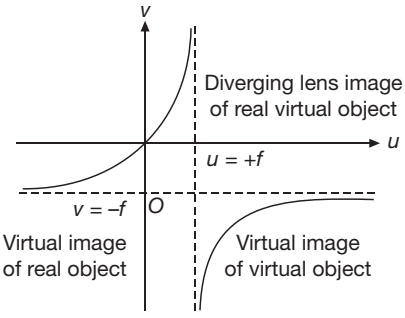
$$\Rightarrow \quad v=\frac{uf}{f-u} \qquad \ldots(1)$$



Substituting the following values of u in equation (1) to get the corresponding values of v for purpose of plotting the u-v graph.

u	$-\infty$	$-2f$	$-f$	$-\frac{f}{2}$	$-\frac{f}{4}$	0	$+\frac{f}{4}$	$+\frac{f}{2}$	$+f$	$+2f$	$+\infty$
v	$-f$	$-\frac{2f}{3}$	$-\frac{f}{2}$	$-\frac{f}{3}$	$-\frac{f}{5}$	0	$\frac{f}{3}$	$+f$	$+\infty$	$-2f$	$-f$

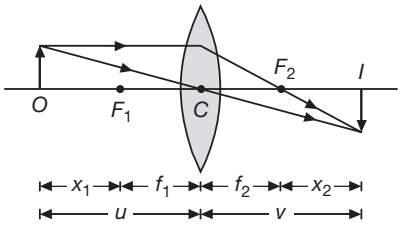
The above function can be plotted as shown in figure for a concave lens.



NEWTON'S FORMULA

If the distance of object and image are not measured from optical centre (C), but from first and second principal foci respectively, and if x_1 is the distance of the object from the first focus x_2 is the distance of the image from the second focus and if f is the focal length of the lens, then we have

$$u=-(f+x_1),\ v=f+x_2$$



According to the lens formula, we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

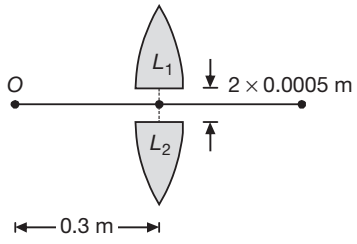
$$\Rightarrow \quad \frac{1}{f+x_2}-\frac{1}{-(f+x_1)}=\frac{1}{f}$$

$$\Rightarrow \quad x_1x_2=f^2$$

This is called Newton's formula.

ILLUSTRATION 79

A point object O is placed at a distance of 0.3 m from a convex lens of focal length 0.2 m. It then cut into two halves each of which is displaced by 0.0005 m as shown in figure. Find the position of the image. If more than one image is formed, find their number and the distance between them.



SOLUTION

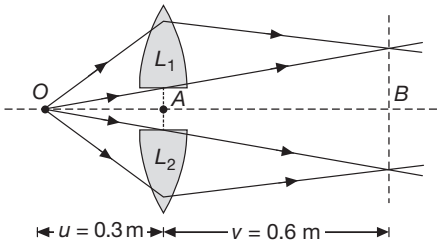
Each part will work as a separate lens and will form its own image. For any part, we have $u=-0.3$ m, $f=+0.2$ m.

Therefore, from lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we have

$$\frac{1}{v}-\frac{1}{0.3}=\frac{1}{0.2}$$

$$\Rightarrow \quad v=0.6 \text{ m}$$

So, each part forms a real image of the point object O at 0.6 m from the lens, as shown in figure.



Since the triangles OL_1L_2 and OI_1I_2 are similar. So, we have

$$\frac{I_1I_2}{L_1L_2}=\frac{OB}{OA}=\frac{u+v}{u}$$

$$\Rightarrow \quad \frac{I_1I_2}{L_1L_2}=\frac{0.3+0.6}{0.3}=\frac{0.9}{0.3}=3$$

$$\Rightarrow \quad I_1I_2=3\left(L_1L_2\right)=3\left(2\times 0.0005\right)=0.003 \text{ m}$$

ILLUSTRATION 80

An object is placed 45 cm from a converging lens of focal length 30 cm. A mirror of radius 40 cm is to be placed on the other side of lens so that the object coincides with its image. Find the position of the mirror if it is

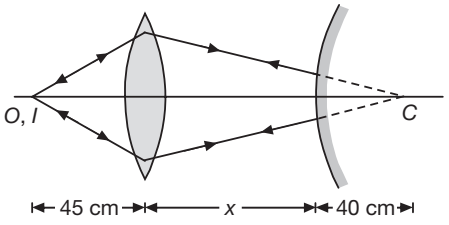
(a) convex?
(b) concave?

SOLUTION

The object and image will coincide only if the light ray retraces its path and it will happen only when the ray strikes the mirror normally. In other words the centre of the curvature of the mirror and the rays incident on the mirror are collinear.

(a) The rays after refraction from lens must be directed towards the centre of curvature of mirror at C. If x is the separation, then for the lens

$$u=-45 \text{ cm}, \; v=x+40, \; f=30 \text{ cm}$$

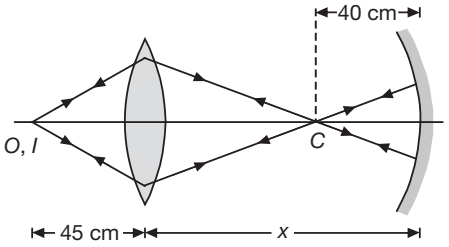


Using lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{x+40}-\frac{1}{-45}=\frac{1}{30}$$

$$\Rightarrow \quad x=\frac{45(30)}{45-30}-40=50 \text{ cm}$$

(b) In case of concave mirror, the refracted rays from lens meet at C, the centre of curvature (C) of the mirror.



Using lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ where $u=-45$ cm, $v=x-40$, $f=30$ cm, we get

$$\frac{1}{x-40}-\frac{1}{-45}=\frac{1}{30}$$

$$\Rightarrow \quad x-40=\frac{45\times 30}{45-30}$$

$$\Rightarrow \quad x=90+40=130 \text{ cm}$$

ILLUSTRATION 81

A lens with a focal length $f=30$ cm placed at a distance of $a=40$ cm from the object produces a sharp image of an object on the screen. A plane parallel plate with thickness of $d=9$ cm is placed between the lens and the object perpendicular to the optical axis of the lens. Through what distance should the screen be shifted for the image of the object to remain distinct? The refractive index of the glass of the plate is $\mu=1.8$.

SOLUTION

In the first case,

$$\frac{1}{v_1}+\frac{1}{40}=\frac{1}{30}$$

$$\Rightarrow \quad v_1 = 120 \text{ cm}$$

In the second case, shift due to the glass slab is given by

$$\Delta x = \left(1-\frac{1}{\mu}\right)t = \left(1-\frac{1}{1.8}\right)9 = 4 \text{ cm}$$

i.e., u will now become $(40-4)=36$ cm, so now we have

$$\frac{1}{v_2}+\frac{1}{36}=\frac{1}{30}$$

$$\Rightarrow \quad v_2 = 180 \text{ cm}$$

Therefore, the screen will have to be shifted 60 cm away from the lens.

ILLUSTRATION 82

Find the distance of an object from a convex lens of focal length 10 cm if the image formed is two times the size of object. Focal length of the lens is 10 cm.

SOLUTION

A convex lens forms both type of images, real as well as virtual. Since, nature of the image is not mentioned here, so we will have to consider both the cases.

CASE-1: When image is real

In this case v is positive and u is negative with $|v|=2|u|$, so when $u=-x$ then $v=2x$ and $f=10$ cm

Substituting in $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{2x}+\frac{1}{x}=\frac{1}{10}$$

$$\Rightarrow \quad \frac{3}{2x}=\frac{1}{10}$$

$$\Rightarrow \quad x = 15 \text{ cm}$$

$x = 15$ cm, means object lies between F and $2F$.

CASE-2: When image is virtual

In this case v and u both are negative. So when $u=-y$ then $v=-2y$ and $f=10$ cm

Substituting in, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get

$$\Rightarrow \quad \frac{1}{-2y}+\frac{1}{y}=\frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{2y}=\frac{1}{10}$$

$$\Rightarrow \quad y = 5 \text{ cm}$$

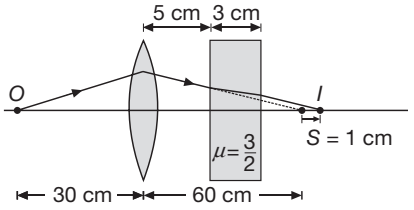
$y = 5$ cm, means object lies between F and P.

ILLUSTRATION 83

A convex lens of focal length 20 cm is placed at a distance 5 cm from a glass plate $\left(\mu=\frac{3}{2}\right)$ of thickness 3 cm. An object is placed at a distance 30 cm from lens on the other side of glass plate. Locate the final image produced by this optical setup.

SOLUTION

Figure shows the optical setup described in question and the ray diagram for image formation.



For lens formula to be used in refraction by lens, we use

$$u = -30 \text{ cm}$$

$$f = +20 \text{ cm}$$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}+\frac{1}{30}=\frac{1}{20}$$

$$\Rightarrow \quad v = \frac{20\times 30}{10} = 60 \text{ cm}$$

Shift of image due to refraction by the glass slab is given as

$$S = t\left(1-\frac{1}{\mu}\right)$$

$$\Rightarrow \quad S = 3\left(1-\frac{2}{3}\right) = 1 \text{ cm}$$

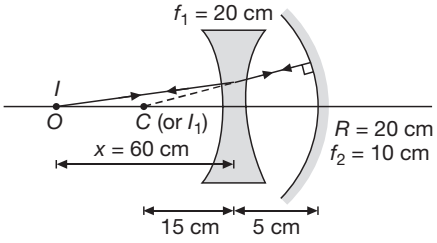
Thus position of final image $= 60+1 = 61$ cm .

ILLUSTRATION 84

A diverging lens of focal length 20 cm is placed coaxially 5 cm toward left of a converging mirror of focal length 10 cm . Where would an object be placed toward left of the lens so that a real image is formed on object itself.

SOLUTION

Due to reflection by a mirror, image of object is formed on itself when reflected rays falls normally on the mirror and retrace the path of incident rays. For this the image produced by the lens must be formed at the centre of curvature of the mirror as shown in ray diagram.



For lens formula, we use

$$u = -x$$
$$f = -20 \text{ cm}$$
$$v = -15 \text{ cm}$$
$$\Rightarrow \quad \frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$
$$\Rightarrow \quad -\frac{1}{15}+\frac{1}{x}=\frac{-1}{20}$$
$$\Rightarrow \quad \frac{1}{x}=\frac{1}{15}-\frac{1}{20}=\frac{4-3}{60}=\frac{1}{60}$$
$$\Rightarrow \quad x = 60 \text{ cm}$$

ILLUSTRATION 85

A thin converging lens of focal length $f = 25.0$ cm forms the image of an object, on the screen, at a distance 5 cm from the lens. The screen is then drawn closer by a distance 18 cm . By what distance should the object be shifted so that its image on the screen is sharp again?

SOLUTION

Since the image is formed on the screen, it is real

Now $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{u}=\frac{1}{f}-\frac{1}{v}=\frac{v-f}{fv}$$

$$\Rightarrow \quad u=\frac{fv}{v-f} \qquad \ldots(1)$$

In the second case, let the image is formed at $v-\Delta v$. Let the corresponding position of object be $u-\Delta u$. Now

$$u+\Delta u=\frac{f(v-\Delta v)}{(v-\Delta v)-f} \qquad \ldots(2)$$

The shift of the object $u+\Delta u-u=\Delta u$

Subtracting Equation (1) from Equation (2), we get

$$\Delta u=\frac{f(v-\Delta v)}{(v-\Delta v)-f}-\frac{fv}{v-f}$$

$$\Rightarrow \quad \Delta u=\frac{f(v-\Delta v)(v-f)-fv\big((v-\Delta v)f\big)}{\{(v-\Delta v)-f\}(v-f)}$$

$$\Rightarrow \quad \Delta u=\frac{f^2\Delta v}{(v-\Delta v-f)(v-f)}$$

$$\Rightarrow \quad \Delta u=\frac{f^2\Delta v}{(v-f)^2\left[1-\frac{\Delta v}{(v-f)}\right]}$$

$$\Rightarrow \quad \Delta u=\frac{f^2\Delta v}{(v-f)^2}\left[1-\frac{\Delta v}{(v-f)}\right]^{-1}$$

$$\Rightarrow \quad \Delta u\approx\frac{f^2\Delta v}{(v-f)^2} \qquad \text{(neglecting higher terms)}$$

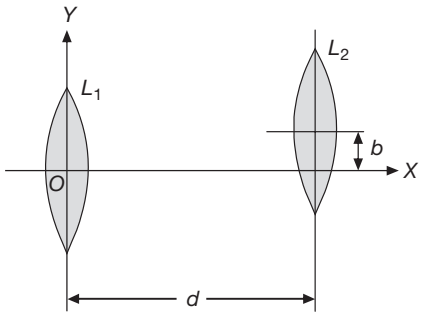
Substituting the given values, we have

$$\Delta u\approx\frac{(25)^2\times 18}{(500-25)^2}\approx\frac{(25)^2\times 18}{(475)^2}\approx 0.5 \text{ mm}$$

ILLUSTRATION 86

Two thin convex lenses of focal lengths f_1 and f_2 are separated by a horizontal distance d (where $d<f_1$, $d<f_2$) and their principal axes are separated by a vertical distance b as shown in the figure. Taking the

centre of the first lens (O) as the origin of co-ordinate system and considering a parallel beam of light coming from the left, find the x and y -coordinates of the focal point of this lens system.



SOLUTION

For the refraction through the first lens, we have $u \to \infty$, so

$$v_1 = f_1$$

Since, $d < f_2$, the first image (formed by L_1) lies to the right of second lens L_2, so

$$u_2 = +(f_1 - d)$$

Applying Lens Formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v_2} - \frac{1}{(f_1 - d)} = \frac{1}{f_2}$$

$$\Rightarrow \quad v_2 = \frac{f_2(f_1 - d)}{f_1 + f_2 - d}$$

$$\Rightarrow \quad x = v_2 + d = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$$

Magnification for second lens is given by

$$m = \frac{v_2}{u_2} = \frac{f_2}{f_1 + f_2 - d}$$

The image due to the second lens is formed below its principal axis and is of the size mb. So, the y coordinate of the focal point system is given by

$$y = b - mb$$

$$\Rightarrow \quad y = b - \frac{f_2 b}{f_1 + f_2 - d}$$

$$\Rightarrow \quad y = \frac{(f_1 - d)b}{f_1 + f_2 - d}$$

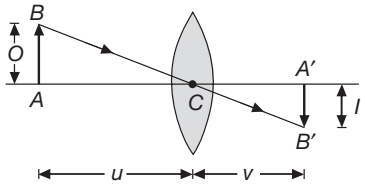
So, the coordinates of the focal point of this system are

$$(x,\ y) = \left[\frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d},\ \frac{(f_1 - d)b}{f_1 + f_2 - d}\right]$$

LINEAR OR LATERAL OR TRANSVERSE MAGNIFICATION (m)

The linear magnification (also called lateral or transverse magnification) m produced by a lens is defined as the ratio of the height of image to the height of the object. So,

$$m = \frac{\text{Height of Image}}{\text{Height of Object}} = \frac{I}{O}$$



Since triangles ABC and $A'B'C$ are similar, so

$$\frac{A'B'}{AB} = \frac{CA'}{CA}$$

Using Conventions, we get

$$A'B' = -I,\ AB = O,\ CA = -u,\ CA' = +v$$

$$\Rightarrow \quad -\frac{I}{O} = \frac{v}{-u}$$

$$\Rightarrow \quad m = \frac{I}{O} = \frac{v}{u}$$

Please note that for both the lens and mirror we have

m_{real} = NEGATIVE i.e. $m_{\text{real}} < 0$

m_{virtual} = POSITIVE i.e. $m_{\text{virtual}} > 0$

LONGITUDINAL OR AXIAL MAGNIFICATION BY A THIN LENS

Lateral magnification formula for thin lenses gives the image height above the principal axis of mirror and in this section we will discuss about the image width along the principal axis of a thin lens. The relation in

object and image width along the principal axis of mirror is called Longitudinal Magnification as given below.

Longitudinal magnification of image is given as

$$m_L = \frac{\text{Width of Image along Principal Axis}}{\text{Width of Object along Principal Axis}}$$

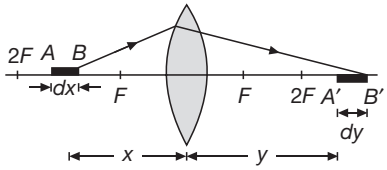


Figure shows image formation of an object located at a distance x from the convex lens of focal length f which produces an image of this object at a distance y which is real inverted and enlarged because object was placed between F and $2F$ points. Here we can see that object edge A was close to C so corresponding image edge A' is also closer to C. If we consider object is of very small width dx and image produced is having a width dy then from lens formula we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Here by coordinate sign convention we use $u = -x$, $f = +f$ and $v = +y$

$$\frac{1}{f} = \frac{1}{y} - \frac{1}{-x}$$

Differentiating this expression we get

$$0 = -\frac{1}{x^2}dx - \frac{1}{y^2}dy$$

From this relation we can get the Longitudinal Magnification as

$$m_L = \frac{dy}{dx} = -\frac{y^2}{x^2} = -m^2 \qquad \ldots(1)$$

For small width object if image is produced by a thin lens (converging or diverging) then image width can be calculated by using the Equation (1). But if object size is large then this relation cannot be used and in that case we need to calculate the image of both edges of the object along principal axis and take the difference of the image distances obtained.

Conceptual Note(s)

(a) Linear/Transverse/Lateral Magnification produced by a lens is

$$m = \frac{I}{O} = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}$$

where I is size of image perpendicular to Principal Axis and O is size of object perpendicular to principal Axis.

(b) Axial Magnification: Axial magnification is the ratio of the size of image along the principal axis to the size of the object along the principal axis. So

$$m_{axial} = \frac{\text{Size of Image along Principal Axis}}{\text{Size of Object along Principal Axis}} = \frac{dv}{du}$$

$$\Rightarrow \quad m_{axial} = \frac{dv}{du} = \frac{v^2}{u^2} = m^2$$

(c) Areal Magnification: Areal magnification is the ratio of the area of image to the area of object.

$$m_{areal} = \frac{\text{Area of Image}}{\text{Area of Object}} = \frac{A_I}{A_O}$$

$$m_{areal} = \frac{A_I}{A_o} = \frac{v^2}{u^2} = m^2$$

EFFECT OF MOTION OF OBJECT AND LENS ON IMAGE

When object or lens is in motion the distance between object and lens changes which affects the position and size of image. To find the image velocity and for analysis of image's motion we can differentiate the lens formula and find the rate at which distances between image and lens is changing. If we consider x and y as object and image distance from pole of mirror of focal length f then by lens formula we have

$$\frac{1}{f} = \frac{1}{y} - \frac{1}{x}$$

Differentiating the above relation with respect to time, we get

$$0 = -\frac{1}{x^2}\frac{dx}{dt} - \frac{1}{y^2}\frac{dy}{dt}$$

where $\frac{dx}{dt}$ is the relative velocity of object parallel to principal axis with respect to the lens and $\frac{dy}{dt}$ is the velocity of image parallel to principal axis with respect to lens.

$$\Rightarrow \quad \frac{dx}{dt} = (v_o)_{\parallel} \text{ and } \frac{dy}{dt} = (v_i)_{\parallel}$$

$$\Rightarrow \quad 0 = -\frac{1}{x^2}(v_o)_{\parallel} - \frac{1}{y^2}(v_i)_{\parallel}$$

$$\Rightarrow \quad (v_i)_{\parallel} = -\frac{y^2}{x^2}(v_o)_{\parallel} = -m^2(v_o)_{\parallel} \qquad \ldots(1)$$

where m is the linear magnification produced by the mirror. The expression of image speed as given in Equation (1) is valid only for the velocity component of the image and object along the principal axis of the lens.

If the object or mirror is in motion along the direction perpendicular to principal axis, then we can directly differentiate the height of object and image above principal axis which are related to each other as

$$h_i = mh_0 \qquad \ldots(2)$$

Differentiating (2) with respect to time we get

$$\frac{dh_i}{dt} = m\frac{dh_0}{dt}$$

where $\frac{dh_i}{dt} = (v_i)_{\perp}$ and $\frac{dh_0}{dt} = (v_o)_{\perp}$

$$\Rightarrow \quad (v_i)_{\perp} = m(v_o)_{\perp}$$

where $\frac{dh_i}{dt} = (v_i)_{\perp}$ and $\frac{dh_0}{dt} = (v_o)_{\perp}$ are the velocity components of image and object respectively in the direction perpendicular to the principal axis.

ILLUSTRATION 87

An object is approaching a thin convex lens of focal length 0.3 m with a speed of 0.01 ms^{-1}. Find the magnitudes of the rates of change of position and lateral magnification of image when the object is at a distance of 0.4 m from the lens.

SOLUTION

Differentiating the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ with respect to time, we get

$$-\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0 \qquad \{\because f = \text{constant}\}$$

$$\Rightarrow \quad \left(\frac{dv}{dt}\right) = \left(\frac{v^2}{u^2}\right)\frac{du}{dt} \qquad \ldots(1)$$

Further, substituting proper values in lens formula, we get

$$\frac{1}{v} + \frac{1}{0.4} = \frac{1}{0.3} \qquad \{\because u = -0.4 \text{ m}, f = 0.3 \text{ m}\}$$

$$\Rightarrow \quad v = 1.2 \text{ m}$$

Substituting the values in equation (1), we get
Magnitude of rate of change of position of image is

$$\frac{dv}{dt} = 0.09 \text{ ms}^{-1}$$

Lateral magnification, $m = \frac{v}{u}$

Rate of change of lateral magnification is given by

$$\frac{dm}{dt} = \frac{u\frac{dv}{dt} - v\frac{du}{dt}}{u^2} = \frac{(-0.4)(0.09) - (1.2)(0.01)}{(0.4)^2}$$

$$\Rightarrow \quad \frac{dm}{dt} = -0.3 \text{ per second}$$

$\Rightarrow$ Magnitude of rate of change of lateral magnification is

$$\frac{dm}{dt} = 0.3 \text{ per second}$$

ILLUSTRATION 88

A small pin of size 5 mm is placed along principal axis of a convex lens of focal length 6 cm at a distance 11 cm from the lens. Find the size of image of pin.

SOLUTION

For lens formula, we have

$$u = -11 \text{ cm}$$

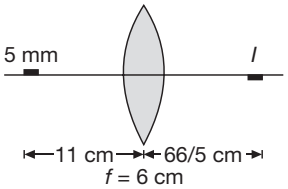
$$f = +6 \text{ cm}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} + \frac{1}{11} = \frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{6}-\frac{1}{11}=\frac{5}{66}$$

$$\Rightarrow \quad v=\frac{66}{5}\text{ cm}$$



Magnification by lens is given as

$$m=\frac{v}{u}=\frac{66}{5\times 11}=\frac{6}{5}$$

Longitudinal magnification is

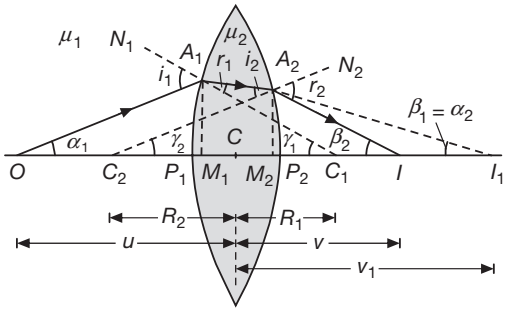
$$m_L=m^2=\frac{36}{25}$$

$$\text{Image size}=\frac{36}{25}\times 5\text{ mm}$$

$$\Rightarrow \quad \text{Image size}=\frac{36}{5}\text{ mm}=7.2\text{ mm}$$

LENS MAKER'S FORMULA FOR THIN LENS

Consider a thin lens having its optical centre at C and let O be the point object situated on its principal axis as shown in figure. Light starting from O strikes the first surface of the lens at A_1 and heads towards I_1, however, refraction takes place at the second surface, thereby giving a final real image at I.



Consider refraction at the first surface only. Let α_1, β_1 and γ_1 be the angles which the incident ray (OA_1), refracted ray (A_1I_1) and normal (A_1C_1) make with the principal axis.

According the general law of refraction, applied at A_1, we get

$$\mu_1\sin i_1=\mu_2\sin r_1$$

Since the angles are small, so $\sin i_1\cong i_1$ and $\sin r_1\cong r_1$

$$\Rightarrow \quad \mu_1 i_1=\mu_2 r_1 \qquad \ldots(1)$$

In ΔA_1C_1O, we have $i_1=\gamma_1+\alpha_1$

In $\Delta A_1C_1I_1$, we have $\gamma_1=r_1+\beta$

$$\Rightarrow \quad r_1=\gamma_1-\beta_1$$

Substituting for i_1 and r_1 in equation (1), we get

$$\mu_1(\gamma_1+\alpha_1)=\mu_2(\gamma_1-\beta_1)$$

$$\Rightarrow \quad \mu_2\beta_1+\mu_1\alpha_1=(\mu_2-\mu_1)\gamma_1$$

Since the angles are small, so they can be replaced by their tangents.

$$\Rightarrow \quad \mu_2\tan\beta_1+\mu_1\tan\alpha_1=(\mu_2-\mu_1)\tan\gamma_1$$

$$\Rightarrow \quad \mu_2\left(\frac{A_1M_1}{M_1I_1}\right)+\mu_1\left(\frac{A_1M_1}{M_1O}\right)=(\mu_2-\mu_1)\left(\frac{A_1M_1}{M_1C_1}\right)$$

For a thin lens, M_1 lies close to C. Therefore, all the distances measured from M_1 can be replaced by those measured from C. Hence we have

$$\frac{\mu_2}{CI_1}+\frac{\mu_1}{CO}=\frac{\mu_2-\mu_1}{CC_1} \qquad \ldots(2)$$

Now consider refraction at the second surface. The ray A_1A_2 which is the refracted ray for first surface becomes the incident ray for second surface. Applying general law of refraction at A_2 (light going from denser to rarer medium), we get

$$\mu_2\sin i_2=\mu_1\sin r_2$$

$$\Rightarrow \quad \mu_2 i_2=\mu_1 r_2 \qquad \ldots(3)$$

In $\Delta A_2C_2I_1$ we have $i_2=\gamma_2+\beta_1$

In ΔA_2C_2I, we have $r_2=\gamma_2+\beta_2$

Substituting for i_2 and r_2 in equation (2), we get

$$\mu_2(\gamma_2+\beta_1)=\mu_1(\gamma_2+\beta_2)$$

$$\Rightarrow \quad \mu_1\beta_2-\mu_2\beta_1=(\mu_2-\mu_1)\gamma_2$$

Since angles are small, so replacing the angles by their tangents, we get

$$\mu_1\tan\beta_2-\mu_2\tan\beta_1=(\mu_2-\mu_1)\tan\gamma_2$$

$$\Rightarrow \quad \mu_1\left(\frac{A_2M_2}{M_2I}\right)-\mu_2\left(\frac{A_2M_2}{M_2I_1}\right)=(\mu_2-\mu_1)\left(\frac{A_2M_2}{M_2C_2}\right)$$

Since the lens is thin, M_2 lies close to C, so we get

$$\frac{\mu_1}{CI}-\frac{\mu_2}{CI_1}=\frac{\mu_2-\mu_1}{CC_2} \quad \ldots(4)$$

Adding equations (2) and (4), we get

$$\frac{\mu_2}{CI_1}+\frac{\mu_1}{CO}+\frac{\mu_1}{CI}-\frac{\mu_2}{CI_1}=\frac{\mu_2-\mu_1}{CC_1}+\frac{\mu_2-\mu_1}{CC_2}$$

$$\Rightarrow \quad \mu_1\left(\frac{1}{CO}+\frac{1}{CI}\right)=(\mu_2-\mu_1)\left(\frac{1}{CC_1}+\frac{1}{CC_2}\right)$$

$$\Rightarrow \quad \frac{1}{CO}+\frac{1}{CI}=\left(\frac{\mu_2-\mu_1}{\mu_1}\right)\left(\frac{1}{CC_1}+\frac{1}{CC_2}\right)$$

$$\Rightarrow \quad \frac{1}{CO}+\frac{1}{CI}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{CC_1}+\frac{1}{CC_2}\right)$$

$$\Rightarrow \quad \frac{1}{CO}+\frac{1}{CI}=\left({}^1\mu_2-1\right)\left(\frac{1}{CC_1}+\frac{1}{CC_2}\right)$$

Applying sign convention, we have

$$CO=-u,\ CI=+v,\ CC_1=+R_1,\ CC_2=-R_2$$

$$\Rightarrow \quad \frac{1}{-u}+\frac{1}{v}=\left({}^1\mu_2-1\right)\left(\frac{1}{R_1}+\frac{1}{-R_2}\right)$$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{u}=\left({}^1\mu_2-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \ldots(5)$$

Since, focal length of a convex lens is defined as the distance of that point from the centre of lens where a beam coming parallel to principal axis comes to focus after refraction through the lens, so when

$$u\to\infty \text{ we have } v=f$$

Substituting in equation (5), we get

$$\frac{1}{f}=\left({}^1\mu_2-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \ldots(6)$$

If the first medium is air, then ${}^1\mu_2=\mu$, so we have

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

This formula is called Lens Maker's Formula.

Problem Solving Technique(s)

(a) A concave lens forms virtual, erect and diminished image.

(b) A convex lens may form real and virtual images. The real image is inverted, it may be diminished or magnified while virtual image formed by convex lens is erect and enlarged.

(c) Different Media on either side of Lens

If a lens of refractive index μ_2 has different media on either side, the medium of object space has refractive index μ_1 and that of image space has refractive index μ_3, then focal length f of lens is

$$\frac{\mu_3}{f}=\frac{\mu_2-\mu_1}{R_1}+\frac{\mu_3-\mu_2}{R_2}$$

As a special case if we put $\mu_3=\mu_1$, we get the Lens Maker's Formula.

ILLUSTRATION 89

A plano-convex lens has a thickness of **4 cm**. When placed on a horizontal table, with the curved surface in contact with it, the apparent depth of the bottom most point of the lens is found to be **3 cm**. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face is found to be $\frac{25}{8}$ cm. Find the focal length of the lens. Assume thickness to be negligible while finding its focal length.

SOLUTION

When placed on a horizontal table with curved surface in contact with it.

In this case refraction of the rays starting from O takes place from a plane surface as shown in Figure 1.1.

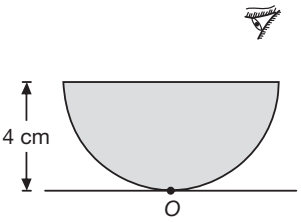


Figure 1.1

So, we can use

$$\text{Apparent Depth}=\frac{\text{Real Depth}}{\mu}$$

$$\Rightarrow \quad 3=\frac{4}{\mu}$$

$$\Rightarrow \quad \mu=\frac{4}{3}$$

When the plane surface is in contact with the horizontal table.

In this case refraction takes place from a spherical surface as shown in Figure 1.2.

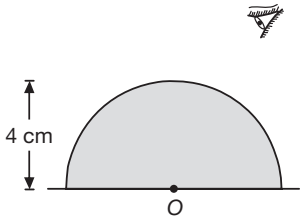


Figure 1.2

Hence, applying

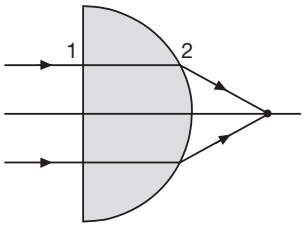
$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}\text{, we get}$$

$$\frac{1}{-\frac{25}{8}}-\frac{\frac{4}{3}}{-4}=\frac{1-\frac{4}{3}}{-R}$$

$$\Rightarrow \quad \frac{1}{3R}=\frac{1}{3}-\frac{8}{25}=\frac{1}{75}$$

$$\Rightarrow \quad R=25 \text{ cm}$$

To find the focal length, since we know that the parallel rays incident on the lens will converge at the focus of the lens. So using the lens maker formula, we get



$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f}=\left(\frac{4}{3}-1\right)\left(\frac{1}{\infty}-\frac{1}{-25}\right)=\frac{1}{75}$$

$$\Rightarrow \quad f=75 \text{ cm}$$

ILLUSTRATION 90

A point source of light is placed inside water and a thin converging lens of refractive index μ_2 is placed just outside the plane surface of water. The image of the source is formed at a distance x from the surface of water. If the lens is now placed just inside water and the image is now formed at a distance x' from the surface of water, show that

$$\frac{1}{x}-\frac{1}{x'}=\left(\frac{\mu_1-1}{\mu_2-1}\right)\frac{1}{f}$$

where f is the focal length of the lens and μ_1 is the refractive index of water.

SOLUTION

Let f and f_w be the focal lengths of the lens when it is outside and inside the water respectively, then

$$\frac{1}{f}=(\mu_2-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \ldots(1)$$

and $$\frac{1}{f_w}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \ldots(2)$$

When the lens is in air, let u be the distance of the object from the surface, then we use apparent depth of object to be $\frac{u}{\mu_1}$ from the lens. Using lens formula, we get

$$\frac{1}{x}-\frac{\mu_1}{u}=\frac{1}{f} \qquad \ldots(3)$$

When the lens is in water, then image is formed at a distance x' from lens, so due to refraction from water surface the final image is formed at a distance $\mu_1 x$ from the lens. Again using lens formula, we get

$$\frac{1}{\mu_1 x'}-\frac{1}{u}=\frac{1}{f_w} \qquad \ldots(4)$$

Multiplying equation (4) by μ_1, we get

$$\frac{1}{x'}-\frac{\mu_1}{u}=\frac{\mu_1}{f_W} \qquad \ldots(5)$$

Subtracting equations (5) and (3), we get

$$\frac{1}{x}-\frac{1}{x'}=\frac{1}{f}-\frac{\mu_1}{f_w} \qquad \ldots(6)$$

From equation (2), we have

$$\frac{1}{f_w}=\left(\frac{\mu_2-\mu_1}{\mu_1}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

From equation (1), we have

$$\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{f(\mu_2-1)}$$

$$\Rightarrow \quad \frac{1}{f_w}=\left(\frac{\mu_2-\mu_1}{\mu_1}\right)\frac{1}{f(\mu_2-1)}$$

Substituting this value in equation (6), we get

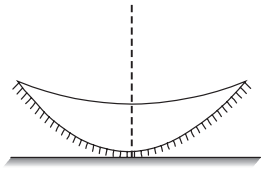
$$\frac{1}{x}-\frac{1}{x'}=\frac{1}{f}-\frac{1}{\mu_1 f}\left(\frac{\mu_2-\mu_1}{\mu_2-1}\right)$$

$$\Rightarrow \quad \frac{1}{x}-\frac{1}{x'}=\frac{1}{f}\left(1-\frac{\mu_2-\mu_1}{\mu_2-1}\right)$$

$$\Rightarrow \quad \frac{1}{x}-\frac{1}{x'}=\frac{1}{f}\left(\frac{\mu_1-1}{\mu_2-1}\right)$$

ILLUSTRATION 91

The convex surface of a thin concavo-convex lens of glass of the refractive index 1.5 has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface.



(a) Where should a pin be placed on the optic axis such that its image is formed at the same place?

(b) If the concave part is filled with water of refractive index $4/3$, find the distance through which the pin should be moved, so that the image of the pin again coincides with the pin.

SOLUTION

(a) Image of object will coincide with it, if the ray of light after refraction from the concave surface falls normally on concave mirror so formed by silvering the convex surface i.e., image after refraction from concave surface should be formed at centre of curvature of concave mirror or at a distance of 20 cm on same side of the combination. Let x be the distance of pin from the given optical system.

Applying, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$, we get for

$v=-20$ cm, $u=-x$, $R=-60$ cm

$$\frac{1.5}{-20}-\frac{1}{-x}=\frac{1.5-1}{-60}$$

$$\Rightarrow \quad \frac{1}{x}=\frac{3}{40}-\frac{1}{120}=\frac{8}{120}$$

$$\Rightarrow \quad x=\frac{120}{8}=15 \text{ cm}$$

(b) When the concave part is filled with water, then before striking with the concave surface, the ray is first refracted from a plane surface. So, let x be the distance of pin, then the plane surface will form its image at a distance of $h_{app}=\mu h$ i.e., $\frac{4x}{3}$ from it.

Now, using $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ with proper signs, we get

$$\frac{1.5}{-20}-\frac{\frac{4}{3}}{\left(\frac{4}{3}x\right)}=\frac{1.5-\frac{4}{3}}{-60}$$

$$\Rightarrow \quad \frac{1}{x}=\frac{-3}{40}+\frac{1}{360}=\frac{-26}{360}$$

$$\Rightarrow \quad x=\frac{360}{-26}=-13.84 \text{ cm}$$

LENS IMMERSED IN A LIQUID

If a lens (made of glass) of refractive index μ_g is immersed in a liquid of refractive index μ_l, then its focal length in liquid, f_l is given by

$$\frac{1}{f_l}=\left({}^{l}\mu_g-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

If f_a is the focal length of lens of air, then

$$\frac{1}{f_a}=\left({}^{a}\mu_g-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \left\{\because {}^{a}\mu_g=\mu_g\right\}$$

$$\Rightarrow \quad f_l=\left[\frac{\mu_g-1}{\frac{\mu_g}{\mu_l}-1}\right]f_a$$

Now three cases arise which are discussed here.

(a) If $\mu_g>\mu_l$, then f_l and f_a are of same sign and $f_l>f_a$.

That is the nature of lens remains unchanged, but its focal length increases and hence power of lens decreases. In other words the convergent lens becomes less convergent and divergent lens becomes less divergent.

(b) If $\mu_g = \mu_l$, then $f_l \to \infty$ and the lens behaves as a simple glass plate.

(c) If $\mu_g < \mu_l$, then f_l and f_a have opposite signs and the nature of lens changes i.e. a convergent lens becomes divergent and vice versa.

ILLUSTRATION 92

A lens has a power of $+5$ dioptre in air. Calculate its power if it is completely immersed in water? Given $\mu_g = \frac{3}{2}$ and $\mu_w = \frac{4}{3}$.

SOLUTION

Let f_a and f_w be the focal lengths of the lens in air and water respectively, then

$$P_a = \frac{1}{f_a} \quad \text{and} \quad P_w = \frac{\mu_w}{f_w}$$

Since lens has power $+5$ D in air, so

$$f_a = \frac{1}{P} = \frac{1}{5} = 0.2 \text{ m} = 20 \text{ cm}$$

Using Lens Maker's formula, we get

$$\frac{1}{f_a} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots(1)$$

Similarly, $\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \quad P_w = \frac{\mu_w}{f_w} = (\mu_g - \mu_w)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots(2)$$

Dividing equation (2) by equation (1), we get,

$$\frac{P_w}{P_a} = \frac{(\mu_g - \mu_w)}{(\mu_g - 1)} = \frac{1}{3}$$

$$\Rightarrow \quad P_w = \frac{P_a}{3} = +\frac{5}{3} \text{ D}$$

DISPLACEMENT METHOD

Consider an object and a screen fixed at a distance D apart. Let a lens of focal length f be placed between the object and the screen.

From figure we observe that

$$u + v = D \quad \Rightarrow \quad v = D - u$$

Also from Lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

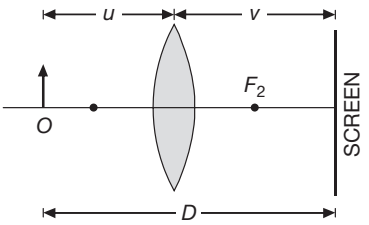
$$\Rightarrow \quad \frac{1}{D-u} - \frac{1}{-u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{D-u} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad Df = (D-u)u$$

$$\Rightarrow \quad u^2 - Du + Df = 0$$

$$\Rightarrow \quad u = \frac{D \pm \sqrt{D^2 - 4fD}}{2}$$



For u to be mathematically real,

$$D^2 - 4fD \geq 0$$

$$\Rightarrow \quad D \geq 4f$$

Problem Solving Technique(s)

So, if the object and the screen are placed at a distance less than $4f$, then a virtual image will be formed. Hence, for a real image to be formed $D \geq 4f$

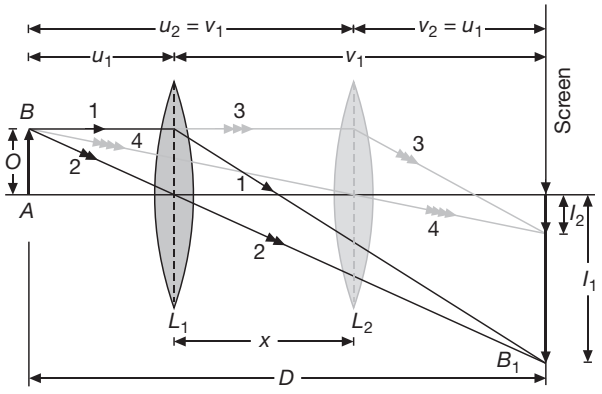
CASE-1: For $D = 4f$

$$u = v = \frac{D}{2}$$

i.e. the lens is placed exactly between the object and the screen.

CASE-2: For $D > 4f$

We get two different position of lens (L_1 and L_2) for which the image of object on the screen is distinct and clear.



L_1 First Position of Lens.
L_2 Second Position of the Same Lens (shown in grey).
Do not develop a misconception that there are two lenses, infact the same lens is displaced through x from position L_1 to L_2.

The object distances for these two positions are given by

$$u_1 = \frac{D - \sqrt{D^2 - 4fD}}{2} \quad \ldots(1)$$

$$u_2 = \frac{D + \sqrt{D^2 - 4fD}}{2} \quad \ldots(2)$$

Since $u + v = D$, so

$$v_1 = \frac{D + \sqrt{D^2 - 4fD}}{2} \quad \ldots(3)$$

$$v_2 = \frac{D - \sqrt{D^2 - 4fD}}{2} \quad \ldots(4)$$

We observe that

$$u_1 = v_2 = u(\text{say}) \quad \ldots(5)$$

$$v_1 = u_2 = v(\text{say}) \quad \ldots(6)$$

Let the lens be displaced through x, then we observe from figure that

$$x = v_1 - u_1 = \sqrt{D^2 - 4fD} \quad \ldots(7)$$

$$\Rightarrow \quad x^2 = D^2 - 4fD$$

$$\Rightarrow \quad \boxed{f = \frac{D^2 - x^2}{4D}} \quad \ldots(8)$$

Using (7) in (1), (2), (3) and (4), we get

$$u_1 = v_2 = \frac{D - x}{2} = u \quad \ldots(9)$$

$$v_1 = u_2 = \frac{D + x}{2} = v \quad \ldots(10)$$

If m_1 is the magnification for the first position of lens i.e. L_1, then

$$m_1 = \frac{I_1}{O} = \frac{v_1}{u_1} = \frac{v}{u} = \frac{D + x}{D - x} \quad \ldots(11)$$

If m_2 is the magnification for the second position of the Lens i.e. L_2, then

$$m_2 = \frac{I_2}{O} = \frac{v_2}{u_2} = \frac{u_1}{v_1} = \frac{u}{v} = \frac{D - x}{D + x} \quad \ldots(12)$$

From (11) and (12), we observe that

$$m_1 m_2 = 1 \quad \ldots(13)$$

So, if magnification for position L_1, is m, then magnification for position L_2 is $\frac{1}{m}$.

Also from (13), we get

$$\frac{I_1}{O}\frac{I_2}{O} = 1$$

$$\Rightarrow \quad O^2 = I_1 I_2$$

$$\Rightarrow \quad O = \sqrt{I_1 I_2}$$

i.e. size of the object (O) is the geometric mean of the sizes of the image for two position of lens L_1 and L_2.

Also,

$$m_1 - m_2 = \frac{D + x}{D - x} - \frac{D - x}{D + x}$$

$$\Rightarrow \quad m_1 - m_2 = \frac{4Dx}{D^2 - x^2}$$

$$\Rightarrow \quad m_1 - m_2 = \frac{x}{\left(\frac{D^2 - x^2}{4D}\right)}$$

$$\Rightarrow \quad m_1 - m_2 = \frac{x}{f}$$

$$\Rightarrow \quad f = \frac{x}{m_1 - m_2}$$

Further if $m_1 = m$, then $m_2 = \frac{1}{m}$

$$\Rightarrow \quad f = \frac{mx}{m^2 - 1}$$

Finally, we observe that

$$\frac{m_1}{m_2} = \left(\frac{D + x}{D - x}\right)^2$$

Problem Solving Technique(s)

Dear Students, you must keep in mind that actually "Displacement Method" is not in the syllabus, but the Examiner generally asks the problems not in its name but by its concept e.g. an examiner's mind may fabricate a problem not having the name Displacement method but then the problem must be having a clue which may state $D > 4f$ or the lens is displaced to get two real images on screen and stuff like that. So, you are advised not to overlook the topic as this is very important (not by name) but by the concept involved.

ILLUSTRATION 93

A thin converging lens of focal length f is moved between a candle and a screen. The distance between the candle and the screen is $D(>4f)$. Show that for two different positions of the lens, two different images can be obtained on the screen. If the ratio of dimensions of the image is β, find the value of $\left(\beta+\frac{1}{\beta}\right)$.

SOLUTION

Let x be the separation between two positions of the lens for which a real image is formed on the screen.

Then, $v+u=D$ …(1)

and $v-u=x$ …(2)

Solving we get $u=\frac{D-x}{2}$ and $v=\frac{D+x}{2}$

Now, $m_1=\frac{I_1}{O}=\frac{D+x}{D-x}$

and $m_2=\frac{I_2}{O}=\frac{D-x}{D+x}$

$\Rightarrow \quad \frac{I_1}{I_2}=\left(\frac{D+x}{D-x}\right)^2=\beta$

$\Rightarrow \quad \frac{D+x}{D-x}=\sqrt{\beta}$

$\Rightarrow \quad x=\left(\frac{\sqrt{\beta}-1}{\sqrt{\beta}+1}\right)D$

Now, $f=\frac{D^2-x^2}{4D}=\frac{D^2-\left(\frac{\sqrt{\beta}-1}{\sqrt{\beta}+1}\cdot D\right)^2}{4D}=\frac{D}{2+\sqrt{\beta}+\frac{1}{\sqrt{\beta}}}$

$\Rightarrow \quad \sqrt{\beta}+\frac{1}{\sqrt{\beta}}=\frac{D}{f}-2$

$\Rightarrow \quad \beta+\frac{1}{\beta}=\left(\frac{D}{f}-2\right)^2-2$

ILLUSTRATION 94

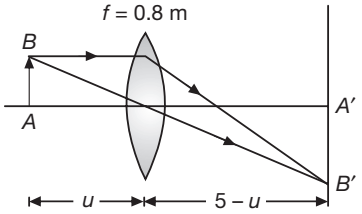
An object is 5 m to the left of a flat screen. A converging lens for which the focal length is $f=0.8$ m is placed between object and screen.

(a) Show that two lens positions exist that form images on the screen and determine how far are these positions from the object?

(b) How do the two images differ from each other?

SOLUTION

(a) Using the lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get



$$\frac{1}{5-u}-\frac{1}{-u}=\frac{1}{0.8}$$

$\Rightarrow \quad \frac{1}{5-u}+\frac{1}{u}=\frac{5}{4}$

$\Rightarrow \quad \frac{5-u+u}{(5-u)u}=\frac{5}{4}$

$\Rightarrow \quad 20=25u-5u^2$

$\Rightarrow \quad 5u^2-25u+20=0$

$\Rightarrow \quad 5u^2-20u-5u+20=0$

$\Rightarrow \quad 5u(u-4)-5(u-4)=0$

$\Rightarrow \quad (5u-5)(u-4)=0$

$\Rightarrow \quad u=1\text{ m}$ and $u=4\text{ m}$

Both the values are real, so this means that there exist two positions of lens that form images of object on the screen.

(b) $m=\frac{v}{u}$

$\Rightarrow \quad m_1=\frac{(5-4)}{(1-4)}=-0.25$ and $m_2=\frac{(5-1)}{(-1)}=-4$

Hence, both the images are real and inverted, the first has magnification -0.25 and the second -4.

Also, we observe that

$$m_1m_2=(-0.25)(-4)=1$$

ILLUSTRATION 95

For two positions of a converging lens between an object and a screen which are 96 cm apart, two real images are formed. The ratio of the lengths of the two images is 4. Calculate the focal length of the lens.

SOLUTION

Since, $\frac{m_1}{m_2} = 4$

$$\Rightarrow \quad \left(\frac{D+x}{D-x}\right)^2 = 4$$

$$\Rightarrow \quad \frac{D+x}{D-x} = 2$$

Substituting $D = 96$ cm, we get

$$\frac{96+x}{96-x} = 2$$

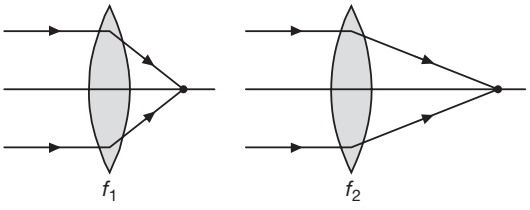
$$\Rightarrow \quad x = 32 \text{ cm}$$

Since, $f = \frac{D^2 - x^2}{4D}$

$$\Rightarrow \quad f = 21.33 \text{ cm}$$

POWER OF A LENS

The power of a lens P is actually the measure of its ability to deviate the incident rays towards axis. The greater the curvature of the two surfaces (i.e., the shorter the focal length f), the greater is the lens action. The shorter the focal length of a lens the more it converges or diverges the light, as shown in figure.



The power of a lens placed in air is actually the reciprocal of the focal length of the lens in metre and is given by

$$P = \frac{1}{f(\text{in metre})} = \frac{100}{f(\text{in cm})}$$

SI unit of power is dioptre (D).

Power of a lens placed in a medium is defined as

$$P_{\text{med}} = \frac{\mu}{f_{\text{med}}}$$

where μ is the refractive index of the medium and f_{med} is the focal length of the lens in that medium.

As a convention, the power of a converging lens (or **convex lens**) (with focal length positive) is taken to be **positive**. The power of a diverging lens (or **concave lens**) (with focal length negative) is taken to be **negative**.

Also we must note that for a mirror, power is defined as

$$P = -\frac{1}{f(\text{in metre})} = -\frac{100}{f(\text{in cm})}$$

Thus a convex lens and concave mirror have converging nature and hence they have positive power, whereas the concave lens and convex mirror have diverging nature and hence have negative power.

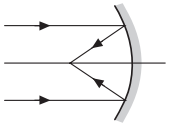
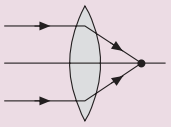
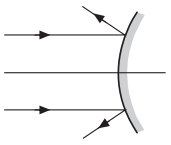
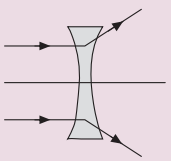
Nature of Lens Mirror	Focal Length (f)	Power $P_{\text{mirror}} = -\frac{1}{f}$, $P_{\text{lens}} = \frac{1}{f}$	Converging/ Diverging	Ray Diagram
Concave mirror	–ve	+ve	Converging	
Convex lens	+ve	+ve	Converging	
Convex mirror	+ve	–ve	Diverging	
Concave lens	–ve	–ve	Diverging	

ILLUSTRATION 96

A convergent lens of power 6 D is combined with a diverging lens of –2 D. Find the power and focal length of the combination.

SOLUTION

Here $P_1 = 6$ D, $P_2 = -2$ D

Power of the combination is given by

$$P = P_1 + P_2 = 6 - 2 = 4 \text{ D}$$

Since $f = \frac{1}{P}$

$$\Rightarrow \quad f = \frac{1}{4} = 0.25 \text{ m} = 25 \text{ cm}$$

ILLUSTRATION 97

An object 4 cm high is placed at a distance of 10 cm from a convex lens of focal length 20 cm. Find the position, nature and size of the image. Also find the power of the lens.

SOLUTION

Here, $u = -10$ cm (the object assumed to be kept to the left of optical centre)

$f = +20$ cm (positive for a convex lens)

$h_1 = +4$ cm (object kept above the principal axis)

Using the lens formula, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{-10} = \frac{1}{20}$$

$$\Rightarrow \quad v = -20 \text{ cm}$$

That is, the image is 20 cm from the lens, on the same side as the object. Hence, the image is **virtual**. The linear magnification,

$$m = \frac{h_2}{h_1} = \frac{v}{u}$$

So, size of the image is

$$h_2 = h_1\left(\frac{v}{u}\right) = 4 \times \frac{-20}{-10} = 8 \text{ cm}$$

The positive sign indicates that the image is **erect** (and **virtual**).

Since, the power of the lens is given by

$$P = \frac{1}{f(\text{in m})} = \frac{1}{+0.2} = +5 \text{ D}$$

ILLUSTRATION 98

A converging lens forms a five folds magnified image of an object. The screen is moved towards the object by a distance $d = 0.5$ m and the lens is shifted so that the image has the same size as the object. Find the lens power and the initial distance between the object and the screen.

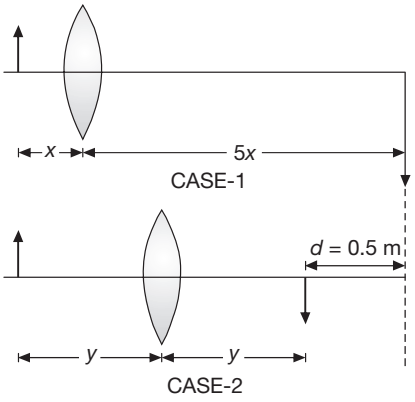
SOLUTION

In the first case image is five times magnified. Hence

$$|v| = 5|u|$$

In the second case image and object are of equal size. Hence

$$|v| = |u|$$



From the two figures, we get

$$6x = 2y + d$$

$$\Rightarrow \quad 6x - 2y = 0.5 \qquad \ldots(1)$$

Using the lens formula for both the cases, we get for

CASE-1,

$$\frac{1}{5x} - \frac{1}{-x} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{6}{5x} = \frac{1}{f} \qquad \ldots(2)$$

CASE-2,

$$\frac{1}{y}-\frac{1}{-y}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{2}{y}=\frac{1}{f} \qquad \ldots(3)$$

Solving these three equations, we get

$$x = 0.1875 \text{ m} \quad \text{and} \quad f = 0.15625 \text{ m}$$

Therefore, initial distance between the object and the screen is

$$6x = 1.125 \text{ m}$$

Power of the lens, $P=\frac{1}{f}=\frac{1}{0.15625}\ \text{D} = 6.4\ \text{D}$

LENSES IN CONTACT

If two or more lenses of focal lengths $f_1, f_2, \ldots$ are placed in contact, then their equivalent focal length f is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\ldots=\sum_{i=1}^{n}\frac{1}{f_i}$$

where f_1, f_2,.... are to be substituted with proper signs attached.

The power of combination

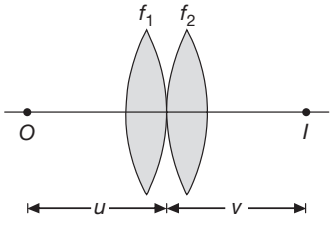
$$P = P_1 + P_2 + \ldots = \sum_{i=1}^{n} P_i$$

Here too, P_1, P_2,..... are to be substituted with proper signs attached.

The magnification of the combination is

$$M = m_1 \times m_2 \times \ldots = \prod_{i=1}^{n} m$$

In many optical instruments, the combination of lenses in contact are used so as to improve the performance of the instrument.



Consider two lenses of focal lengths f_1 and f_2 kept in contact. Let a point object O be placed at a distance u from the combination. The first image (say I_1) after refraction from the first lens is formed at a distance v_1 (whatever may be the sign of v_1) from the combination. This image I_1 acts as an object for the second lens and let v be the distance of the final image from the combination. Applying the lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get

For the first lens, $\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f_1}$ …(1)

and for the second lens, $\frac{1}{v}-\frac{1}{v_1}=\frac{1}{f_2}$ …(2)

Adding equations, (1) and (2), we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f}\ \text{(say)}$$

where, f is the equivalent focal length of the combination. Thus,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

TWO THIN LENSES SEPARATED BY A DISTANCE

If two thin lenses of focal lengths f_1, f_2 are placed at a distance x apart, then equivalent focal length of combination is

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1 f_2}$$

or Power for the combination is

$$P = P_1 + P_2 - xP_1P_2$$

The net magnification of the combination is still remains

$$m = m_1 \times m_2$$

ILLUSTRATION 99

Consider a co-axial system of two thin convex lenses of focal length f each separated by a distance d. Draw ray diagrams for image formation corresponding to an object at infinity placed on the principal axis in the following cases: (a) $d < f$ (b) $d = f$ (c) $f < d < 2f$ (d) $d = 2f$ and (e) $d > 2f$. Indicate the

nature of the combination (concave, convex or plane) in each case.

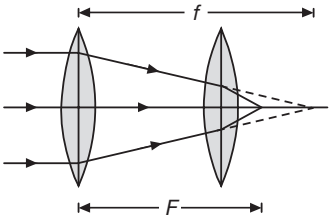
SOLUTION

The formula

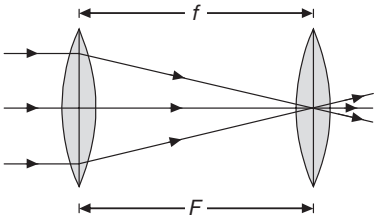
$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

is valid only for small values of d compared to f_1 and f_2. Therefore, we cannot use this formula in the given cases. However, we can draw the ray diagram to decide the nature of the combination.

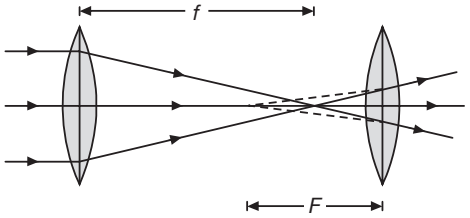
(a) When $d < f$: The ray diagram is shown in figure. The out-coming rays are convergent. Obviously, the combination is a convex lens with $F < f$.



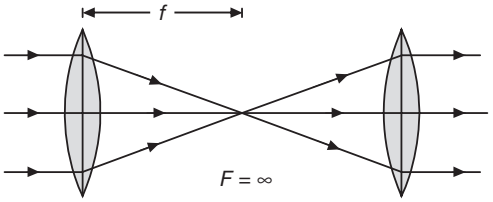
(b) When $d = f$: The incident parallel beam converges to a point and then passes without any more deviation. The combination behaves like a convex lens of $F = f$.



(c) When $f < d < 2f$: The incident parallel beam emerges out as a divergent beam. the combination behaves as a divergent or concave lens.



(d) When $d = 2f$: The incident parallel beam emerges out as a parallel beam but inverted. The combination behaves as a plane glass slab, which inverts the beam.



(e) When $d > 2f$: The incident parallel beam emerges out as a convergent beam. The combination behaves as a convergent or convex lens.

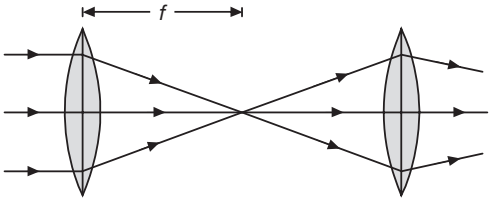


ILLUSTRATION 100

Two equi-convex lenses of focal lengths 30 cm and 70 cm, made of material of refractive index = 1.5, are held in contact coaxially by a rubber band round their edges. A liquid of refractive index 1.3 is introduced in the space between the lenses filling it completely. Find the position of the image of a luminous point object placed on the axis of the combination lens at a distance of 90 cm from it.

SOLUTION

According to Lens Maker's Formula, we have

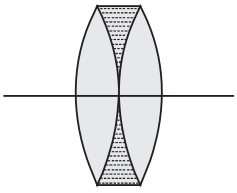
$$\frac{1}{30}=(1.5-1)\left(\frac{1}{R_1}-\frac{1}{-R_1}\right)$$

$$\Rightarrow \quad R_1 = 30 \text{ cm}$$

Similarly, radius of curvature of the second lens is 70 cm. Since

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3} \qquad \ldots(1)$$

Here, $f_1 = 30$ cm, $f_2 = 70$ cm



Now f_3 is calculated again using the Lens Maker's Formula, so we get

$$\frac{1}{f_3} = (1.3-1)\left(\frac{1}{-30} - \frac{1}{70}\right)$$

$\Rightarrow \quad f_3 = -70$ cm

$\Rightarrow \quad F = 30$ cm {from equation (1)}

According to Lens formula, applied on the combination of lenses, we have

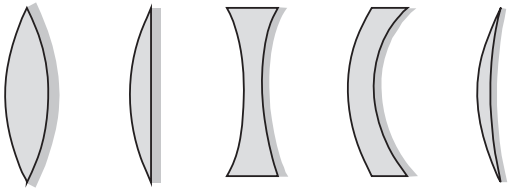
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{(-90)} = \frac{1}{30}$$

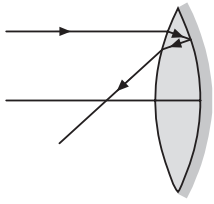
$\Rightarrow \quad v = 45$ cm

LENSES WITH ONE SILVERED SURFACE

When one face of a lens is silvered as shown in figure it acts like a lens-mirror combination.



It is obvious from the ray diagram as shown in figure that the incident ray of light is refracted through the lens twice (i.e., once when light is incident on the lens and second time when reflected by the mirror) and reflected from the mirror once.



The combination acts like a mirror whose effective power is given by

$$P_{net} = 2P_l + P_m$$

where P_l is the power of the lens and P_m is the power of the mirror.

Since for a mirror we have

$$P_m = -\frac{1}{f_m}$$

and for a lens, we have

$$P_l = \frac{1}{f_l} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

So, the combination acts like a mirror having net focal length given by

$$F_{net} = -\frac{1}{P_{net}}$$

$$\Rightarrow \quad P = -\frac{1}{F} = \frac{2}{f_l} - \frac{1}{f_m}$$

where f_l is focal length of lens and f_m is focal length of spherical mirror formed due to silvering of surface.

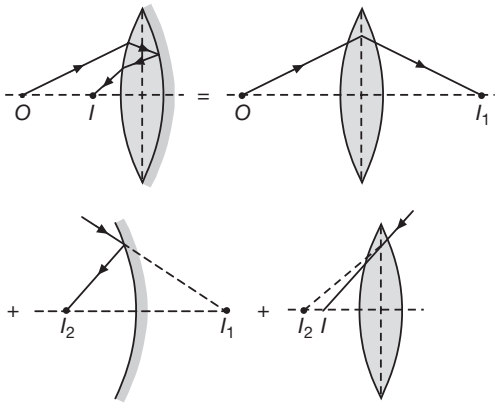
To have a fundamental understanding of this we can understand the silvering of lenses using the following arguments.

A ray incident on a lens with its backside silvered will be refracted through the lens twice and will be reflected from the mirror once, as shown.

(a) Light from object O passes through lens to form image I_1.

(b) The image I_1 acts as an object (virtual) for the curved mirror to form image I_2.

(c) The image I_2 acts as an object (virtual) for the lens to form the final image I.



The silvered lens acts like a mirror with equivalent focal length F, given by

$$-\frac{1}{F}=\frac{1}{f_l}-\frac{1}{f_m}+\frac{1}{f_l}=\frac{2}{f_l}-\frac{1}{f_m}$$

where f_l is focal length of lens and f_m is focal length of spherical mirror formed due to silvering of surface.

SIGN CONVENTION

While using the above formula, we make use of the following sign conventions.

(a) f is positive for converging (convex) lens and concave mirror.

(b) f is negative for diverging (concave) lens and convex mirror.

For example, for a plano-convex lens, from Lens Maker's Formula we get

$$\frac{1}{f_l}=(\mu-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)=\frac{\mu-1}{R}$$

$$\Rightarrow \quad f_l=\frac{R}{\mu-1}$$

(a) when plane surface is silvered, $f_m \to \infty$

Since we know that

$$-\frac{1}{F}=\frac{2}{f_l}-\frac{1}{\infty}=\frac{2(\mu-1)}{R}$$

$$\Rightarrow \quad F=-\frac{R}{2(\mu-1)}$$

(b) when convex surface is silvered, then in general we know the relation between radius of curvature and the focal length is given by

$$f_m=\frac{R}{2}$$

Since we know that

$$-\frac{1}{F}=\frac{2}{f_l}+\frac{2}{R}=\frac{2(\mu-1)}{R}+\frac{2}{R}=\frac{2\mu}{R}$$

$$\Rightarrow \quad F=-\frac{R}{2\mu}$$

ILLUSTRATION 101

The plane surface of a plano-convex lens of focal length 60 cm is silver plated. A point object is placed at a distance 20 cm from the convex face of lens. Find the position and nature of the final image formed.

SOLUTION

Since, $P=2P_l+P_m$

$$\Rightarrow \quad -\frac{1}{F}=\frac{2}{f_l}-\frac{1}{f_m}$$

where, $f_l=+60$ cm and $f_m \to \infty$

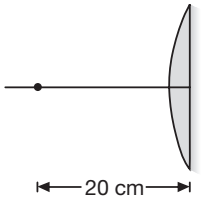
$$\Rightarrow \quad -\frac{1}{F}=\frac{2}{60}-\frac{1}{\infty}=\frac{1}{30}$$

$$\Rightarrow \quad F=-30 \text{ cm}$$

The problem is reduced to a simple case where a point object is placed in front of a concave (converging) mirror of focal length 30 cm.

Using mirror formula i.e.,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$



where $u=-20$ cm and $f=-30$ cm

$$\Rightarrow \quad \frac{1}{v}+\frac{1}{-20}=\frac{1}{-30}$$

$$\Rightarrow \quad v=60 \text{ cm}$$

The image is virtual and erect

ILLUSTRATION 102

A concave mirror has the form of a hemisphere with a radius of $R=60$ cm. A thin layer of an unknown transparent liquid is poured into the mirror. The mirror-liquid system forms one real image and another real image is formed by mirror alone of the source in a certain position.

(a) Image produced by combination coincides with the source and that produced by mirror alone is located at a distance of $l=30$ cm from the source away from mirror. Find the refractive index μ of the liquid in this case.

(b) In another case, if the image formed by mirror coincides with the source and that produced by the combination is produced at a distance 30 cm from the source away from mirror, then find the refractive index of the liquid in this case also.

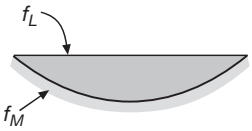
SOLUTION

(a) For concave mirror with unknown liquid, equivalent focal length of the combined mirror is given as

$$\frac{1}{f_{eq}}=\frac{2}{f_L}+\frac{1}{f_M}$$

Where $\frac{1}{f_L}=(\mu-1)\left(\frac{1}{\infty}-\frac{1}{-60}\right)$

$\Rightarrow \frac{1}{f_L}=\left(\frac{\mu-1}{60}\right)$



and focal length of mirror is

$$f_M=\frac{60}{2}=30 \text{ cm}$$

Thus equivalent focal length of the combination mirror is given as

$$\frac{1}{f_{eq}}=2\left(\frac{\mu-1}{60}\right)+\frac{2}{60}=\frac{2\mu}{60}$$

$\Rightarrow f_e=\frac{30}{\mu}$

As image formed by the mirror liquid system coincides with the source, the location of object is at $2f_e$

$\Rightarrow u=2f_e=\frac{60}{\mu}$

According to the given condition, $\frac{60}{\mu}+30$ is the distance of the image formed by the mirror itself, thus using mirror formula we have

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$\Rightarrow \frac{1}{v}+\frac{1\times\mu}{-60}=-\frac{1}{30}$

$\Rightarrow v=\frac{60}{\mu-2}=-\left(\frac{60}{2-\mu}\right)$

Image distance from the mirror is $\frac{60}{2-\mu}$, thus form the already obtained condition we use

$$\frac{60}{\mu}+30=\frac{60}{2-\mu}$$

$\Rightarrow \mu^2+2\mu-4=0$

$\Rightarrow \mu=-1\pm\sqrt{5}$

Since μ cannot be negative, so

$\mu=-1+\sqrt{5}$

$\Rightarrow \mu=2.236-1=1.236$

(b) Mirror produces its image on source when the source is located at the centre of curvature thus source position must be at 60 cm from the pole of mirror.

Now we use mirror formula for calculation of image distance for mirror liquid combination

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f_e}$$

$\Rightarrow \frac{1}{v}+\frac{1}{-60}=-\frac{\mu}{30}$

$\Rightarrow v=\frac{60}{1-2\mu}=-\left(\frac{60}{2\mu-1}\right)$

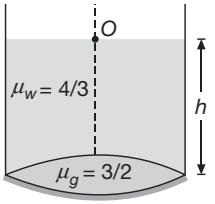
According to the given condition we use

$$\frac{60}{2\mu-1}+30=60$$

$\Rightarrow \mu=1.5$

ILLUSTRATION 103

Bottom of a glass beaker is made of a thin equi-convex lens having bottom side silver polished as shown in the figure.



Water is filled in the beaker upto a height 4 m. The image of point object, floating at middle point of beaker at the surface of water coincides with it. Find out the radius of curvature of the lens. Given that refractive index of glass is $\frac{3}{2}$ and that of water is $\frac{4}{3}$.

SOLUTION

The silvered lens placed at the bottom of tank behaves like an equivalent mirror and if object is placed at the centre of curvature of the mirror then its image is produced on itself. Here the focal length of the glass lens with respect to water in surrounding is gives as

$$\frac{1}{f_L}=\left(\frac{\frac{3}{2}}{\frac{4}{3}}-1\right)\left(\frac{1}{R}+\frac{1}{R}\right)$$

$$\Rightarrow \quad \frac{1}{f_L}=\frac{1}{8}\times\frac{2}{R}=\frac{1}{4R}$$

$$\Rightarrow \quad f_2=4R$$

Focal length of mirror is $\frac{R}{2}$, so the equivalent focal length of combination is given as

$$\frac{1}{f_{eq}}=\frac{2}{f_L}+\frac{1}{f_M}=\frac{2}{4R}+\frac{2}{R}=\frac{5}{2R}$$

$$\Rightarrow \quad f_{eq}=-\frac{2}{5}R,$$

Thus object is to be placed at $2f_{eq}$ so that its image is produced on itself, thus we have object height given as

$$h=2\times\frac{2}{5}R=4 \text{ m}$$

$$\Rightarrow \quad R=5 \text{ m}$$

ILLUSTRATION 104

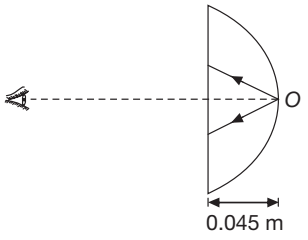
The greatest thickness of a planoconvex lens when viewed normally through the plane surface appears to be 0.03 m and when viewed normally through the curved surface it appears to be 0.036 m. If the actual thickness is 0.045 m, find the

(a) refractive index of the material of the lens.
(b) radius of curvature of lens.
(c) focal length if its plane surface is silvered.
(d) focal length when the curved surface is silvered.

SOLUTION

(a) Since, $\mu=\dfrac{\text{Real Depth}}{\text{Apparent Depth}}$

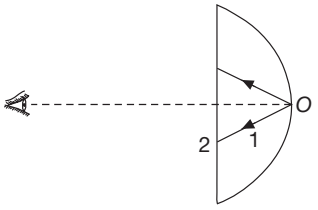
$$\Rightarrow \quad \mu=\frac{d_{\text{actual}}}{d_{\text{app}}}=\frac{0.045}{0.03}=1.5$$



(b) Using, $\dfrac{\mu_2}{v}-\dfrac{\mu_1}{u}=\dfrac{\mu_2-\mu_1}{R}$, we get

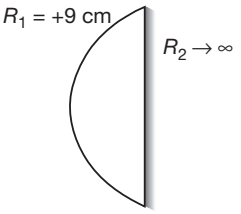
$$\frac{1}{(-0.036)}-\frac{1.5}{(-0.045)}=\frac{1-1.5}{(-R)}$$

$$\Rightarrow \quad R=0.09 \text{ m}=9 \text{ cm}$$



(c) If the plane surface is silvered, then

$$\frac{1}{F}=\frac{2}{f_l}+\frac{1}{f_m}$$



But $f_m \to \infty$

$$\Rightarrow \quad \frac{1}{F}=\frac{2}{f_l}$$

where, $\dfrac{1}{f_l}=(\mu-1)\left(\dfrac{1}{R_1}-\dfrac{1}{\infty}\right)$

$$\Rightarrow \quad \frac{1}{F}=\frac{2(\mu-1)}{R_1}$$

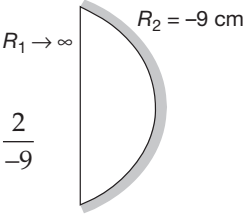
$$\Rightarrow \quad \frac{1}{F}=\frac{2(1.5-1)}{+9}$$

$$\Rightarrow \quad F=9 \text{ cm}$$

The nature is given by applying negative sign to the final result. So, this will behave as a concave mirror.

(d) When curved surface is silvered then $R_1 \to \infty$, $R_2=-9$ cm

$$\frac{1}{F}=\frac{2}{f_l}-\frac{1}{f_m}$$



$$\Rightarrow \quad \frac{1}{F}=2(\mu-1)\left(\frac{1}{\infty}-\frac{1}{-9}\right)-\frac{2}{-9}$$

$$\Rightarrow \quad \frac{1}{F}=2\left(\frac{\mu-1}{9}\right)+\frac{2}{9}$$

$$\Rightarrow \quad \frac{1}{F}=\frac{2\mu}{9}$$

$$\Rightarrow \quad \frac{1}{F}=\frac{1.5\times2}{+9}$$

$$\Rightarrow \quad f=3 \text{ cm}$$

Test Your Concepts-VII

Based on Lens Formula

(Solutions on page H.21)

1. The distance between two point sources of light is 24 cm. Find out where would you place a converging lens of focal length 9 cm, so that the images of both the sources are formed at the same point.

2. An object is moved along the principal axis of a convex lens. An image three times the size of the object is obtained when the object is at a distance of 16 cm from the lens and at a distance of 8 cm from the lens. Find the focal length of the lens.

3. The radius of curvature of the convex surface of a plano-convex lens is 10 cm and its focal length is 30 cm. What should be the refractive index of its material?

4. One face of an equi-convex lens ($\mu = 1.5$) of focal length 60 cm is silvered. Does it behave like a concave mirror or convex mirror? Also determine the equivalent focal length of the mirror.

5. A biconvex lens made of glass with a refractive index of $\mu = 1.6$ has a focal length of $f = 10$ cm in air. Calculate the focal length of this lens if it is placed into a transparent medium
(a) with a refractive index of $\mu_1 = 1.5$
(b) with a refractive index of $\mu_2 = 1.7$

6. A biconvex thin lens is prepared from glass of refractive index $\frac{3}{2}$. The two bounding surfaces have equal radii of 25 cm each. One of the surfaces is silvered from outside to make it reflecting. Where should an object be placed before this lens so that the image coincides with the object.

7. A converging lens of focal length 5 cm is placed in contact with a diverging lens of focal length 10 cm. Find the combined focal length of the system.

8. A biconvex lens of refractive index 1.5 has a focal length of $f_1 = 10$ cm. One of the lens surfaces having a radius of curvature of $R = 10$ cm is coated with silver. Determine the position of the image if the object is at a distance of $u = 15$ cm from the lens.

9. A convex lens is held 45 cm above the bottom of an empty tank. The image of a point at the bottom of a tank is formed 36 cm above the lens. Now a liquid is poured into the tank to a depth of 40 cm. It is found that the distance of the image of the same point on the bottom of the tank is 48 cm above the lens. Find the refractive index of the liquid.

10. A concave spherical mirror with a radius of curvature of 0.2 m is filled with water. Calculate the focal length of this system? Given that refractive index of water is $\frac{4}{3}$.

11. A convex lens of focal length f_1 is placed in front of a luminous point P so that the distance of the point P from lens is greater than focal length and the image formed is at the shortest possible distance. If now a concave lens of very large focal length f_2 be placed in contact with first, find the shift in the position of the image.

12. At what distance from a biconvex lens of focal length $f = 1$ m should a concave spherical mirror with a radius of curvature of $R = 1$ m be placed for a beam incident on the lens parallel to the major optical axis of the system to leave the lens, remaining parallel to the optical axis, after being reflected from the mirror? Find the image of the object produced by the given optical system.

13. A convex lens of focal length f_1 is placed in front of a luminous point object. The separation between the object and the lens is $3f_1$. A glass slab of thickness t is placed between object and the lens. A real image of the object is formed at the shortest possible distance from the object.
(a) Find the refractive index of the slab.
(b) If a concave lens of very large focal length f_2 is placed in contact with the convex lens, find the shifting of the image.

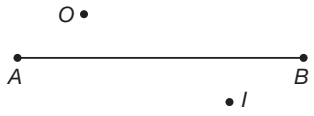
14. An optical system consists of two convergent lenses with focal lengths $f_1 = 20$ cm and $f_2 = 10$ cm. The distance between the lenses is $d = 30$ cm. An object is placed at a distance of $u_1 = 30$ cm from the first lens. At what distance from the second lens will the image be obtained?

15. If r be the radius of curvature of each face of thin converging lens whose one face is silvered and μ is the refractive index of lens material, prove that the lens is equivalent to a concave mirror of focal length $\frac{r}{4\mu - 2}$.

16. Three convergent thin lenses of focal lengths $4a$, a and $4a$ respectively are placed in order along the axis so that the distance between consecutive lenses is $4a$. Prove that this combination simply inverts every small object on the axis without change of magnitude or position.

17. The distance between an object and a divergent lens is m times greater than the focal length of the lens. How many times will the image be smaller than the object?

18. An image I is formed of point object O by a lens whose optic axis is AB as shown in figure.



(a) State whether it is a convex lens or concave?
(b) Draw a ray diagram to locate the lens and its focus.

19. Two thin lenses having focal lengths $f_1 = 7$ cm and $f_2 = 6$ cm are placed at a distance $d = 3$ cm apart. What is the distance of the focus of the system from the second lens? Assume the system to be a centred one.

20. Two glasses with refractive indices of $\mu_1 = 1.5$ and $\mu_2 = 1.7$ are used to make two identical double convex lenses.
(a) Find the ratio between their focal lengths.
(b) How will each of these lenses act on a ray parallel to its optical axis if the lenses are submerged into a transparent liquid with a refractive index of 1.6?

21. A parallel beam of light is incident on a system consisting of three thin lenses with a common optical axis. The focal lengths of the lenses are equal to $f_1 = +10$ cm, $f_2 = -20$ cm and $f_3 = +9$ cm, respectively. The distance between the first and the second lenses is 15 cm and between the second and the third 5 cm. Find the position of the point at which the beam converges when it leaves the system of lenses.

22. Consider a plano-concave lens with one of the radii of curvature r made up of a transparent material whose refractive index varies with intensity (I) of incident light as $\mu = \mu_0 + aI$, where $a > 0$ and $0 < \mu_0 < \frac{3}{2}$. Calculate the intensity when the focal length is equal to two times the radius of curvature r.

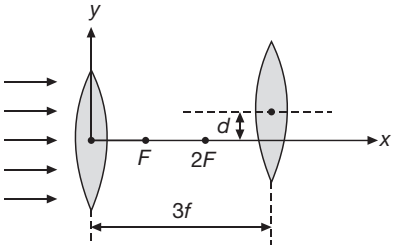
23. Paraxial rays are incident on surfaces of a thin equiconvex glass lens of refractive index μ and having radius of curvature R. If the final image is formed after n internal reflections, calculate distance of this image from pole of the lens.

24. When the plane surface of a plano-convex lens is silvered it is found that the image of the object pin is formed at the position of the object pin placed at a distance of x_1 from the silvered lens. When the same lens is silvered on the curved surface the image of the object pin is formed at the position of the object pin placed at a distance of x_2 from the silvered lens. Find, in terms of x_1 and x_2, the
(a) focal length of lens
(b) radius of curvature of the curved surface and
(c) index of refraction of the medium of lens.

25. A small fish, 0.4 m below the surface of a lake, is viewed through a simple converging lens of focal length 3 m. The lens is kept at 0.2 m above the water surface such that the fish lies on the optical axis of the lens. Find the position of image of the fish seen by the observer. The refractive index of water is $\frac{4}{3}$.

26. Two symmetric double convex lenses A and B have same focal length but the radii of curvature differ so that $R_A = 0.9\, R_B$. If refractive index of A is 1.63, find the refractive index of B.

27. In the figure it is shown, the focal length f of the two thin convex lenses is the same. They are separated by a horizontal distance $3f$ and their optical axes are displaced by a vertical separation $d\,(d \ll f)$, as shown. Taking the origin of coordinates O at the centre of the first lens, find the x and y coordinates of the point where a parallel beam of rays coming from the left finally gets focussed?



DEFECTS OF IMAGES: ABERRATIONS

The simple theory of image formation developed for mirrors and lenses suffers from various approximations. As a result, the actual images formed contain several defects. These defects can be broadly divided in two categories.

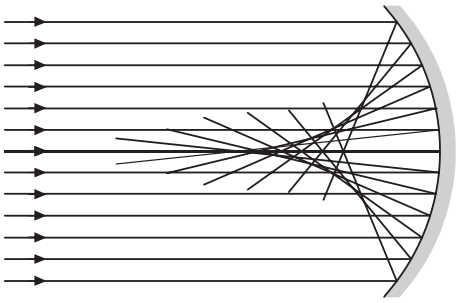
(a) Monochromatic Aberration: The defects, which arise when light of a single colour is used, are called **monochromatic aberrations**.

(b) Chromatic Aberration: The index of refraction of a transparent medium differs for different wavelengths of the light used. The defects arising from such a variation of the refractive index are termed as **chromatic aberrations**.

MONOCHROMATIC ABERRATIONS

Spherical Aberration

Throughout the discussion of lenses and mirrors with spherical surfaces, it has been assumed that the aperture of the lens or the mirror is small and the light rays of interest make small angles with the principal axis. Only then, it is possible to have a point image of a point object.

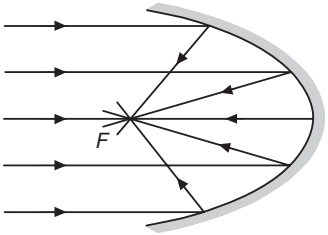


The rays reflect or refract from points at different distances from the principal axis. In general, they meet each other at different points. Thus, the image of a point object is a blurred surface. Such a defect is called **Spherical Aberration**. Figure shows spherical aberration for a concave mirror for an object at infinity. The rays parallel to the principal axis are incident on the spherical surface of the concave mirror. The rays close to the principal axis **(Paraxial Rays)** are focused at the geometrical focus F of the mirror. The rays farthest from the principal axis **(Marginal Rays)** are focused at a point F' somewhat closer to the mirror. The intermediate rays focus at different points between F and F'. Also, the rays reflected from a small portion away from the pole meet at a point off the axis. Thus, a three- dimensional blurred image is formed.

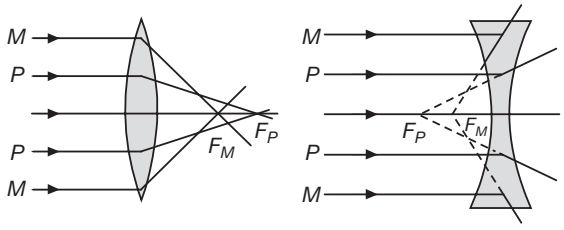
The intersection of this image with the plane of figure is called the **Caustic Curve**.

If a screen is placed perpendicular to the principal axis, a disc image is formed on the screen. As the screen is moved parallel to itself, the disc becomes smallest at one position. This disc is closest to the ideal image and its periphery is called the **Circle of Least Confusion**. The magnitude of spherical aberration may be measured from the distance FF' between the point where the paraxial rays converge and the point where the marginal rays converge.

The parallel rays may be brought to focus at one point if a parabolic mirror is used. Also, if a point source is placed at the focus of a parabolic mirror, the reflected rays will be very nearly parallel. The reflectors used in automobile headlights are made parabolic and the bulb is placed at the focus. The light beam is then nearly parallel and goes up to large distance.



A lens too produces a blurred disc type image of a point object (due to finite aperture of lens). Figure shows the situation for a convex and a concave lens for the rays coming parallel to the principal axis.



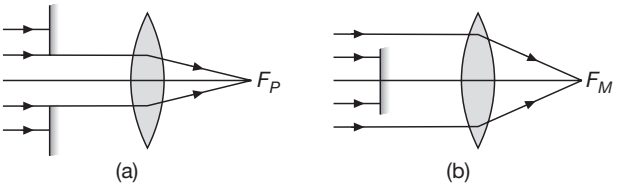
We see from the figure that the marginal rays deviate a bit strongly and hence, they meet at a point different from that given by geometrical optics formulae. Also, in the situation shown, the spherical aberration is opposite for convex and concave lens. The point F_M, where the marginal rays meet, is to the left of the focus for convex lens and is to the right of the focus for the concave lens.

The magnitude of spherical aberration for a lens depends on the radii of curvature and the object distance. For minimum spherical aberration the ratio of radii of curvature of lens is

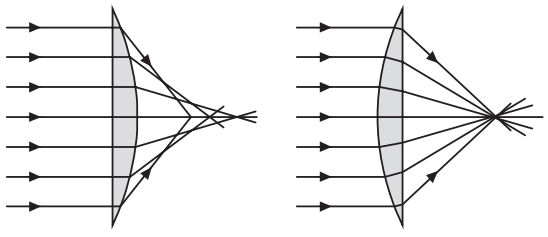
$$\frac{R_1}{R_2} = \frac{2\mu^2 - \mu - 4}{\mu(2\mu + 1)}$$

However, it cannot be reduced to zero for a single lens which forms a real image of a real object.

A simple method to reduce spherical aberration is to use a **stop** before and in front of the lens. A stop is an opaque sheet with a small circular opening in it. It only allows a narrow pencil of rays to go through the lens hence reducing the aberration. However, this method reduces the intensity of the image as most of the light is cut off.



Otherwise, the spherical aberration is less if the total deviation of the rays is distributed over the two surfaces of the lens. Example for this is a planoconvex lens forming the image of a distant object. If the plane surface faces the incident rays, the spherical aberration is much larger than that in the case when the curved surface faces the incident rays. In the former case, the total deviation occurs at a single surface whereas it is distributed at both the surfaces in the latter case.

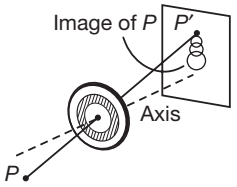


The spherical aberration can also be reduced by using a combination of convex and concave lenses. A suitable combination can reduce the spherical aberration by compensation of positive and negative aberrations. If two thin lenses are separated by a distance d, then condition for minimum spherical aberration is

$$d = f_1 - f_2$$

Coma

It has been observed that if a point object is placed on the principal axis of a lens and the image is received on a screen perpendicular to the principal axis, the image has a shape of a disc because of spherical aberration. The basic reason is that the rays passing through different regions of the lens meet the principal axis at different points. If the point object is placed away from the principal axis and the image is received on a screen perpendicular to the axis, the shape of the image is like a comet. This defect is called **Coma**. the lens fails to converge all the rays passing at different distances from the axis at a single point. The paraxial rays form an image of P at P'. The rays passing through the shaded zone forms a circular image on the screen above P'. The rays through outer zones of the lens form bigger circles placed further above P'. The image seen on the screen thus have a comet-like appearance.



Coma can be reduced by properly designing the radii of curvature of lens surfaces. It can also be reduced by using appropriate stops placed at appropriate distance from the lens.

Astigmatism

Spherical aberration and coma refer to the spreading of the image of a point object in a plane perpendicular to the principal axis. The image is also spread along the principal axis. Consider a point object placed at a point off the axis of a converging lens. A screen is placed perpendicular to the axis and is moved along the axis. At a certain distance, an approximate line

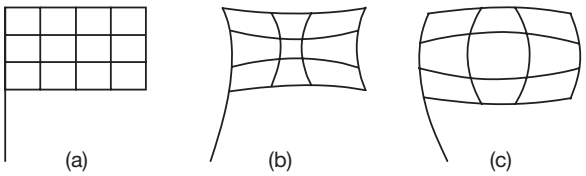
image is focused. If the screen is moved further away, the shape of the image changes but it remains on the screen for quite a distance moved by the screen. The spreading of image along the principal axis is known as **Astigmatism** (you must not confuse this with a defect of vision having the same name).

Curvature

So far we have considered the image formed by a lens on a plane. However, it must be kept in mind that the best image may not be formed along a plane. For a point object placed off the axis, the image is spread both along and perpendicular to the principal axis. The best image is, in general, obtained not on a plane but on a curved surface. This defect is known as **curvature**. It is intrinsically related to astigmatism. The astigmatism or the curvature may be reduced by using proper stops placed at proper locations along the axis.

Distortion

It is the defect arising when extended objects are imaged. Different portions of the object are, in general, at different distances from the axis. The relation between the object distance and the image distance is not linear and hence, the magnification is not the same for all portions of the extended object. Hence a line object is not imaged into a line but into a curve and shown.



Object (a) and its distorted images (b) & (c)

CHROMATIC ABERRATION

The inability of a lens to form the white image of a white object is called **chromatic aberration**. In this case the lens forms coloured images of a white object.

The chromatic aberration arises due to the fact that the focal length of a lens depends upon the refractive index of material of the lens. The lens has different refractive indices for different colours or wavelengths in accordance with Cauchy's formula given by

$$\mu = A + \frac{B}{\lambda^2}$$

Accordingly, the refractive index is maximum for violet $(\lambda = 4000\text{Å})$ and minimum for red $(\lambda = 7800\text{Å})$. Since

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

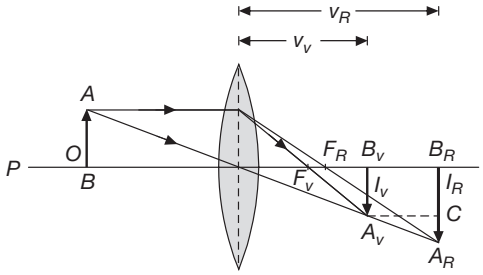
$$\Rightarrow \quad f \propto \frac{1}{\mu - 1}$$

Hence focal length of a lens is maximum for red and minimum for violet

$$\Rightarrow \quad f_{\text{red}} > f_{\text{violet}}.$$

Figure represents the chromatic aberration caused by a lens in the image of an object AB of size O.

F_R and F_V are second principal foci for red and violet colours respectively. The images of object AB are of different sizes and of different colours between $A_V B_V$ and $A_R B_R$. The chromatic aberration is of two types.



Chromatic aberration

Axial or Longitudinal Chromatic Aberration

This is the spread of images along the principal axis and is given by $dv = (v_R - v_V)$ as this spread is very small.

$$v_R - v_V = \frac{\omega v^2}{f}$$

where ω is dispersive power, v is distance of image from lens for mean (yellow) colour and f is mean focal length of lens such that $f = \sqrt{f_V f_R}$

If object is at infinity, then axial chromatic aberration,

$$f_R - f_V = \omega f$$

Lateral Chromatic Aberration

This is the spread of images perpendicular to principal axis and is given by

$$I_R - I_V = \frac{v_R O}{u} - \frac{v_V O}{u} = (v_R - v_V)\frac{O}{u} = \left(\frac{\omega v^2}{f}\right)\frac{O}{u}$$

ACHROMATISM AND ACHROMATIC DOUBLET

The lens system free from chromatic aberration is called **achromatic combination**. This is obtained by using two lenses of different materials and different focal lengths and process is called, **to Achromatise** which satisfies the relation

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

$$\Rightarrow \quad \frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2}$$

where ω_1 and ω_2 are dispersive powers of materials of lenses for focal length f_1 and f_2 respectively.

(a) As ω_1 and ω_2 are always positive, therefore f_1/f_2 must be negative. This means the combination must have *one lens convergent and other divergent.*

(b) For the achromatic combination (also called **Achromatic Doublet**) to be convergent, the power of convex lens must be greater or the focal length of convex lens must be smaller than that of concave lens. As dispersive power for crown glass is less than that for flint glass, therefore the convex lens must be made of crown glass while concave lens must be made of flint glass.

Condition for minimum chromatic aberration obtained by two thin lenses of same medium separated by a distance d is

$$d = \frac{f_1 + f_2}{2}$$

ILLUSTRATION 105

An achromatic convergent lens of focal length 150 cm is made by combining flint and crown glass lenses. Calculate the focal lengths of both the lenses and point out which one is divergent, if the ratio of the dispersive powers of flint and crown glasses are 3 : 2.

SOLUTION

For the given combination, we have

$$\frac{1}{150} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \ldots(1)$$

Condition of achromatism is

$$\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2} = -\frac{2}{3} \qquad \ldots(2)$$

Solving the Equations (1) and (2) we get

$$f_1 = +50 \text{ cm}$$

and $\quad f_2 = -75 \text{ cm}$

ILLUSTRATION 106

A thin biconvex lens is placed with its principal axis first along a beam of parallel red light and then along a beam of parallel blue light. If the refractive indices of the lens for red and blue light are respectively 1.514 and 1.524 and if the radius of curvature of the faces are 30 cm and 20 cm, calculate the separation of foci for red and blue light. If the focal length for the mean colour (yellow) is 23.1 cm, find the dispersive power of the material of the lens.

SOLUTION

By lens maker's formula, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here for red light, we use

$$\frac{1}{f_r} = (1.514 - 1)\left(\frac{1}{20} + \frac{1}{30}\right)$$

$$\Rightarrow \quad \frac{1}{f_r} = 0.514 \times \left(\frac{1}{12}\right)$$

$$\Rightarrow \quad f_r = 23.33$$

For blue light, we use

$$\frac{1}{f_b} = (1.524 - 1)\left(\frac{1}{20} + \frac{1}{30}\right)$$

$$\Rightarrow \quad f_b = \frac{12}{0.524} = 22.9 \text{ cm}$$

Separation between the focal points is

$$\Delta f = f_r - f_b = 23.33 - 22.9 = 0.43 \text{ cm}$$

We use $\frac{1}{f_b} - \frac{1}{f_r} = \left(\frac{\mu_b - \mu_r}{\mu - 1}\right)\frac{1}{f} = \frac{\omega}{f}$

where dispersive power of the lens material is given as

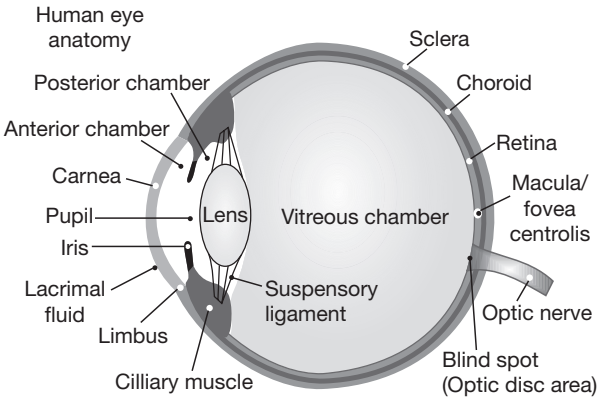
$$\omega = \left(\frac{\mu_b - \mu_r}{\mu - 1}\right)$$

$$\Rightarrow \quad f_r - f_b = \frac{\omega}{f} \times (f_b \times f_r) = \frac{\omega f^2}{f} = \omega f$$

$$\Rightarrow \quad \omega = \frac{\text{separation}}{\text{mean focal length}} = \frac{0.43}{23.1}$$

$$\Rightarrow \quad \omega = 0.019$$

HUMAN EYE



In a number of ways, the human eye works much like a digital camera as discussed.

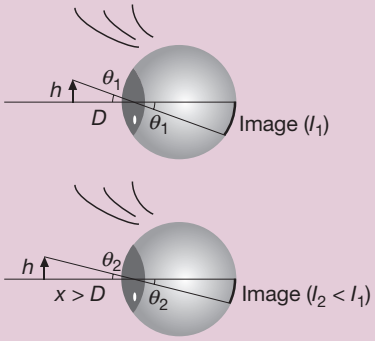
1. Light is focussed primarily by the cornea, the clear front surface of the eye, which acts like a camera lens.
2. The iris of the eye functions like the diaphragm of a camera, controlling the amount of light reaching the back of the eye by automatically adjusting the size of the pupil (aperture).
3. The eye's crystalline lens is located directly behind the pupil and further focusses light. Through a process called accommodation, this lens helps the eye automatically focus on near and approaching objects, like an autofocus camera lens.
4. Light focused by the cornea and crystalline lens (and limited by the iris and pupil) then reaches the retina (a light-sensitive inner lining of the back of the eye). The retina acts like an electronic image sensor of a digital camera, converting optical images into electronic signals. The optic nerve then transmits these signals to the visual cortex. (Cortex is the part of the brain that controls our sense of sight).

Conceptual Note(s)

(a) Human eye lens has a power to adjust its focal length to see the near and far objects. Normally an eye can see objects lying in front of it at distances ranging from 25 cm to infinity (∞). That is a normal eye can see very distant objects clearly but near objects can be seen clearly if they are at a distance greater than equal to 25 cm from it. This ability of the eye to see objects from ∞ to 25 cm by adjusting its focal length is called **the power of accommodation**.

So when the object is brought too closer to the eye, then the focal length cannot be adjusted to form the image on the retina. So, we conclude that there should be a minimum separation between the eye and the object for a clear vision of the object and this separation is called **Least Distance of Distinct Vision** (+*D*). For a normal eye, *D* is generally taken to be 25 cm.

(b) If the object is at infinity the eye is least strained i.e. relaxed, however when the object lies at *D*, then the eye is maximum strained and the visual angle subtended at the eye is maximum.



(c) Persistence of vision is the time interval between two light pulses arriving at the eye which the eye can see distinctly. Persistence of vision of human eye is $\frac{1}{10}$ sec. This simply means that if two light pulses arrive at the eye in a time interval less than $\frac{1}{10}$ sec then these two pulses will be seen by the eye as one single pulse.

(d) Resolving limit or limit of resolution of eye is the minimum angular separation between two objects so that they are just resolved (i.e. can be seen distinctly by eye). For eye, resolving limit (R.L.) is 1 minute i.e. 1′. So,

$$(\text{R.L.}) = 1' = \left(\frac{1}{60}\right)^{o} = \left(\frac{1}{60} \times \frac{\pi}{180}\right) \text{ radian}$$

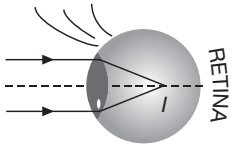
So two objects will not be visible distinctly (as two), when the angle subtended by these two objects at the eye is less than 1′.

DEFECTS OF EYE

A normal eye has nearer point at $D(25\text{ cm})$ called distance of distinct vision and far point at ∞.

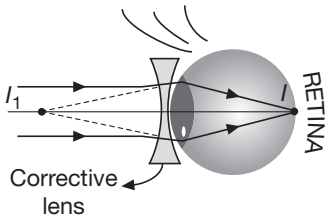
Short-sightedness or Myopia

1. A short-sighted eye can see only nearer objects.
2. It is due to elongation of eye-ball due to which radius of curvature of the lens decreases and hence power P of the lens increases.
3. The image is formed before the retina and it appears as if the separation between the eye lens and retina has increased.



DEFECTIVE EYE
Image is not created on the retina

4. In this case, the far point comes closer to the eye. The far point of a normal eye is generally at infinity but in case of myopia, the person is able to see up to a certain distance and not beyond that. This maximum distance upto which the person can see clearly is called as the **defected far point**.
5. Myopia is corrected by using a concave lens of focal length equal to the far point of defective eye also called as the defected far point.
   It simply means that the concave lens would make the image of an object lying at infinity at the defected far point and then this image will be seen as object by the eye so that the final image is formed at the retina.



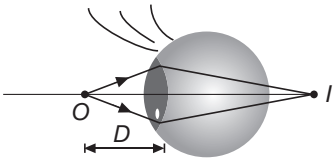
DEFECT CORRECTED

6. If a person can see upto a distance x (defected far point) but wants to see an object placed at distance $y\ (>x)$, then the focal length of the concave lens to be used is

$$f = \frac{xy}{x-y}$$

Long-sightedness or Hypermetropia

1. A long sighted eye can see only farther objects.
2. It is due to contraction of eye-ball due to which radius of curvature of the lens increases and hence power P of the lens decreases.
3. The image is formed behind the retina and it appears as if the separation between the eye lens and retina has decreased.

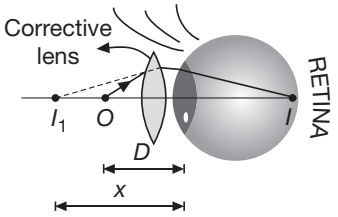


DEFECTIVE EYE
Image is created beyond the retina

4. In this case, the near point moves away from the eye. The near point of a normal eye is generally at $D = 25$ cm but in case of hypermetropia it shifts to a distance > 25 cm and is also called as the **defected near point**.
5. Hypermetropia is corrected by using convex lens. This lens brings the defected nearer point

i.e. the nearer point of defective eye at a distance which equals to the distance of distance of distinct vision $D(=25\text{ cm})$.

It simply means that for an object lying at the defected near point, the convex lens would make the image of this object at $D=25$ cm and then this image made by the convex lens will act as object for the eye so that the final image is formed at the retina.



DEFECT CORRECTED

6. If a person cannot see before a distance $d(>D)$ but wants to see an object placed at $D=25$ cm, then the focal length of the convex lens to be used is

$$f=\frac{Dd}{d-D}$$

Presbyopia

A presbyopic eye can see objects only within a definite range. This defect is corrected by using bifocal lenses.

Astigmatism

It arises due to distortion in spherical shape in cornea. This defect is corrected by using cylindrical lenses.

ILLUSTRATION 107

The accommodation of eye of a short-sighted man lies between 12 cm and 60 cm. He wears spectacles through which he can see remote objects distinctly. Calculate the minimum distance at which the man can read a book through his spectacles.

SOLUTION

As per the given situation, a man can manage to see objects clearly if placed between 12 cm and 60 cm (accommodation of eye). If v is the distance between eye lens and retina then the focal length of eye lens when an object is placed at 60 cm distance is obtained by using lens formula.

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{(-60)}=\frac{1}{f_e} \qquad \ldots(1)$$

If he uses spectacles having lens of focal length f, then he can see far objects clearly. This means, that for far objects, the combination of eye lens and spectacles lens produces the image at retina at distance v from the eye lens. For this combination of the lenses, we use lens formula as

$$\frac{1}{v}-\frac{1}{\infty}=\frac{1}{f_e}+\frac{1}{f} \qquad \ldots(2)$$

From equation (1) and (2), we get

$$f=-60\text{ cm}$$

For the near point of the eye at 12 cm, if the focal length of the eye lens is f_e' then by lens formula, we get

$$\frac{1}{v}-\frac{1}{(-12)}=\frac{1}{f_e'} \qquad \ldots(3)$$

The minimum distance D at which the man can read a book through his spectacles (when he places the book at distance D from the eye with spectacles on) is obtained by using the lens formula

$$\frac{1}{v}-\frac{1}{(-D)}=\frac{1}{f_e'}+\frac{1}{-60} \qquad \ldots(4)$$

From equation (3) and (4), we get

$$D=15\text{ cm}$$

OPTICAL INSTRUMENTS

An optical instrument is a device which is constructed by a suitable combination of mirrors, prisms and lenses so that it assists the eye in viewing an object. The principle of working of an optical instrument in based on the laws of reflection and refraction of light. The common types of optical instrument are

(a) Projection instruments: These are used to project on the screen a real, inverted and magnified image of an opaque or transparent object so as to be viewed by a large audience. The object is, however, so fitted that its image is seen in erect form.

An eye, a photographic camera, a projection lantern, an episcope, an epidiascope, an over-head projector, a film projector, etc., are examples of projection instruments.

(b) Microscopes: These are used to see very small objects in magnified form which otherwise cannot be seen distinctly when placed close to the naked eye.

EXAMPLE

A simple microscope and a compound microscope.

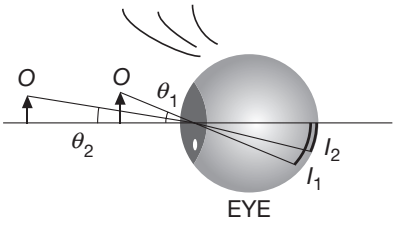
(c) Telescopes: These are used to see astronomical and distant objects in magnified form which, otherwise cannot be seen clearly with the naked eye.

EXAMPLE

An astronomical telescope, a Galilean telescope, a terrestrial telescope, a reflecting telescope, etc.

VISUAL ANGLE

The size of the object as perceived by the eye depends upon the size of image formed at the retina. The size of image formed at the retina is roughly proportional to the angle subtended by the object at the eye called as the visual angle. When the object lies close to the eye its visual angle is large and hence image I_1 formed at the retina is large. When the object is taken far away from the eye its visual angle becomes small and hence the image I_2 formed at the retina is also small.



θ_1 is the visual angle subtended at the eye for near position of object and θ_2 is the visual angle subtended at the eye for the far position of the object. Since $\theta_1 > \theta_2$, so $I_1 > I_2$.

Optical instruments are used for increasing the visual angle artificially so as to increase the size of the image formed at the retina.

MAGNIFYING POWER OR ANGULAR MAGNIFICATION (*M*)

Magnifying power or angular magnification M is the factor by which an image formed on the retina can be enlarged by using a microscope or a telescope.

For Microscope, the magnifying power is the ratio of the visual angle subtended (or formed) by the final image at the eye to the visual angle subtended by the object at the eye (when kept at the distance of distinct vision).

$$M_{\text{microscope}} = \frac{\left(\begin{array}{c}\text{Visual angle subtended by final}\\ \text{image at eye}\end{array}\right)}{\left(\begin{array}{c}\text{Visual angle subtended by the}\\ \text{object at eye when kept at } D\end{array}\right)}$$

For Telescope, the magnifying power is the ratio of the visual angle subtended by the final image at the eye to the visual angle subtended by the object at the eye (when seen from the naked eye).

$$M_{\text{telescope}} = \frac{\left(\begin{array}{c}\text{Visual angle subtended by final}\\ \text{image at eye}\end{array}\right)}{\left(\begin{array}{c}\text{Visual angle subtended by the object}\\ \text{at eye when seen by the naked eye}\end{array}\right)}$$

Conceptual Note(s)

(a) The term linear magnification (m) is different from the term magnifying power (M).

(b) Linear magnification $m = \frac{h_i}{h_o} = \pm\frac{v}{u}$ is the ratio of the height of the image to the height of the object.

(c) Magnifying power (as discussed above) is the ratio of the apparent increase in size of the image seen by the eye.

(d) While m is unitless, the unit of M is X. So if magnifying power of a microscope is 11, then it is written as 11X.

SIMPLE MICROSCOPE (MAGNIFYING GLASS)

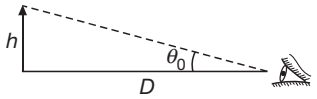
A convex lens of short focal length can be used to see magnified image of a small object and is called **a magnifying glass** or a **simple microscope**.

When a small object is placed between optical centre and focus of a convex lens, its virtual, erect and magnified image is formed on the same side of the lens. The lens is held close to eye and the distance of the object is adjusted, till the image is formed at the least distance of distinct vision from the eye.

When we view an object with naked eyes, the object must be placed somewhere between infinity and the near point. The maximum angle is subtended on the eye when the object is placed at the near point. This angle

$$\theta_0 = \frac{h}{D} \qquad \ldots(1)$$

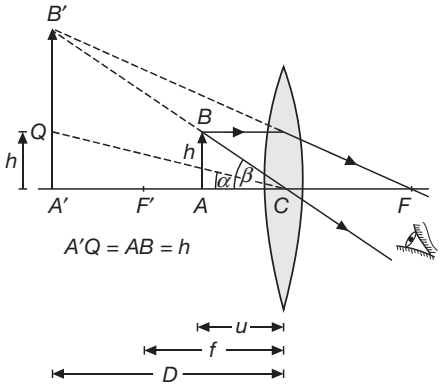
where h is the size of the object and D is the least distance for clear vision.



Magnifying Power (M)

CASE-1: When image is formed at D i.e. near point adjustment or strained viewing

In this case $M = M_D$ is defined as the ratio of the angle subtended by the image at the eye to the angle subtended by the object (at the eye) seen directly, when both lie at the least distance of distinct vision.



By definition, magnifying power of the simple microscope is given by

$$M_D = \frac{\beta}{\alpha}$$

Since angles α and β are small, therefore, angles α and β can be replaced by their tangents i.e.

$$M_D = \frac{\tan\beta}{\tan\alpha} = \frac{h/u}{h/D} = \frac{D}{u} \qquad \ldots(1)$$

If f is focal length of the lens acting as simple microscope, then

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Since, image is formed at distance of distinct vision, so according to new Cartesian sign convention.

$$u = -u\,,\ v = -D \text{ and } f = +f$$

$$\Rightarrow \quad \frac{1}{(-D)} - \frac{1}{(-u)} = \frac{1}{f}$$

$$\Rightarrow \quad -\frac{1}{D} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{D}{u} = 1 + \frac{D}{f} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$M_D = \frac{D}{u} = 1 + \frac{D}{f}$$

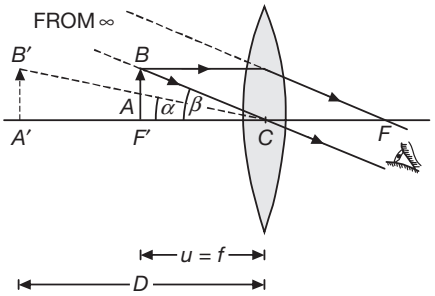
Conceptual Note(s)

(a) From above it follows that lesser is the focal length of the convex lens used as simple microscope, greater is the value of the magnifying power obtained.

(b) Further, the positive value of magnifying power of a simple microscope tells that image formed is erect and hence virtual.

CASE-2: When image is formed at infinity i.e. far point adjustment or relaxed viewing

In this case $M = M_\infty$ is defined as the ratio of the visual angle β subtended (at the eye) by the final image to the visual angle α subtended (at the eye) by the object when kept at D.



Draw a line $A'B' = AB$ and perpendicular to principal axis at a distance $CA' = D$ (least distance of distinct vision) join $B'C$. Then $\angle B'CA' = \alpha \approx \frac{h}{D}$ is the angle formed by object at the eye, when situated at distance D.

The angle formed by the image situated at infinity at the eye is same as the angle formed by the object AB at the eye. Thus, $\angle BCA = \beta \approx \frac{h}{f}$ is the angle formed by the image at the eye.
By definition,

$$M_\infty = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{h/f}{h/D} = \frac{D}{f}$$

Conceptual Note(s)

It follows that magnifying power of the simple microscope is one less, when the image is formed at infinity. However, the viewing of the image is more comfortable.

USES

(a) Jewellers and watch makers make use of convex lens of short focal length to obtain a magnified view of the fine jewellery work and the small components of the watches.
(b) In science laboratories, a magnifying glass is used to see slides and to read the Vernier scales attached to the instruments.
(c) The use of magnifying glass enables us to place the object close to eye, making it appear bright and yet clearly visible. In position AB, object lies close to the eye. In absence of lens, the object will not be clearly visible.
(d) It is also used by astrologers to read the fate lines of the hand.
(e) Used by Biology students to see slides.
(f) Used by detective department to match finger prints.

ILLUSTRATION 108

A man with normal near point (25 cm) reads a book with small print using a magnifying glass (a thin convex lens) of focal length 5 cm. Find the

(a) closest and farthest distance at which he can read the book when viewing through the magnifying glass.
(b) maximum and minimum magnifying power possible using the above simple microscope.

SOLUTION

(a) For a normal eye, far and near points are ∞ and 25 cm, respectively. So, we have

$$v_{max} \to -\infty \text{ and } v_{min} = -25 \text{ cm}$$

Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad u = \frac{f}{\left(\frac{f}{v}\right) - 1}$$

So, u will be minimum, when v is minimum i.e., $v_{min} = -25$ cm

$$\Rightarrow \quad (u)_{min} = \frac{5}{-\left(\frac{5}{25}\right) - 1} = -\frac{25}{6} = -4.17 \text{ cm}$$

And u will be maximum, when v is maximum i.e., $v_{max} \to \infty$

$$\Rightarrow \quad (u)_{max} = \frac{5}{\left(\frac{5}{\infty}\right) - 1} = -5 \text{ cm}$$

(b) Since magnifying power for a lens is

$$m = \frac{v}{u}$$

Magnifying power will be minimum, when u is maximum i.e., $u_{max} = -5$ cm

$$\Rightarrow \quad (m)_{min} = \frac{D}{f} = \frac{-25}{-5} = 5$$

m will be maximum, when u is minimum i.e.,

$$u_{min} = -\frac{25}{6} = -4.17 \text{ cm}$$

$$\Rightarrow \ (m)_{max} = \frac{-25}{-\frac{25}{6}} = 6 \qquad \left\{ = 1 + \frac{D}{f} \right\}$$

COMPOUND MICROSCOPE

A compound microscope is used to see extremely small objects. It consists of two lenses. A lens of **short aperture and short focal length** facing the object is called **object lens (or objective lens)** and another lens of **large focal length** and **large aperture** is called **eye lens (or eye piece or ocular)**. The two lenses are placed coaxially at the two ends of a tube. To focus over an object, the distance of the object lens from the object is adjusted with the help of rack and pinion arrangement.

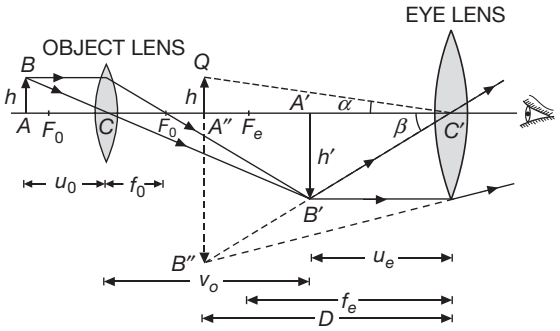
When a small object is placed just outside the focus of the object lens, its real, inverted and magnified image is produced on the other side of the lens between F and $2F$. The image produced by object lens acts as object for the eye lens. The distance of object from the object lens is so adjusted that the final image is formed at the least distance of distinct vision from the eye.

Let AB be an object placed just outside the focus F_0 of the object lens. Its virtual image $A'B'$ is formed on the other side of the lens. The image $A'B'$ lies between focus F_e and optical centre C' of the eye lens and it acts as object for the eye lens. Using the rack and pinion arrangement, the distance between object lens and the object AB is adjusted, till it virtual and magnified image $A''B''$ is formed on the same side at the least distance of distinct vision.

MAGNIFYING POWER (M)

CASE-1: When image is formed at D

In this case $M = M_D$ is defined as the ratio of the visual angle β subtended by the final image at the eye to the visual angle α subtended by the object seen directly, when both are placed at the least distance of distinct vision.



Let $\angle A''C'B'' = \angle A'C'B' = \beta$ be the angle subtended by the final image at the eye. Let us cut $A''Q$ equal to AB and join QC'. Then, $\angle A''C'Q = \alpha$, the angle subtended by the object at the eye, when situated at the least distance of distinct vision. By definition, magnifying power of the compound microscope,

$$M_D = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha}$$

Since the angles α and β are small, so they can be replaced by their tangents.

$$M_D = \frac{\tan\beta}{\tan\alpha} = \frac{h'/u_e}{h/D} = \left(\frac{h'}{h}\right)\left(\frac{D}{u_e}\right) \qquad \ldots(1)$$

Since, for the objective lens, we have

$$\frac{h_i}{h_o} = \frac{h'}{h} = \frac{v_o}{u_o} \qquad \ldots(2)$$

For the eye piece, we have $u = -u_e$, $v = v_e = -D$ and $f = +f_e$

Since, $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$

$$\Rightarrow \quad \frac{1}{-D} + \frac{1}{u_e} = \frac{1}{f_e}$$

$$\Rightarrow \quad \frac{D}{u_e} = 1 + \frac{D}{f_e} \qquad \ldots(3)$$

Substituting (2) and (3) in (1), we get

$$M_D = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$$

In general, the focal length if the objective is very small so that $\frac{v_o}{f_o} \gg 1$ and also the first image is formed close to the eye piece so that $v_o \approx L$ (where L is the length of the microscope tube i.e. the separation between the objective and the eye piece).

Since, $\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$

$$\Rightarrow \quad 1 - \frac{v_o}{u_o} = \frac{v_o}{f_o}$$

$$\Rightarrow \quad \frac{v_o}{u_o} = 1 - \frac{v_o}{f_o} \approx -\frac{v_o}{f_o} \approx -\frac{L}{f_o}$$

$$\Rightarrow \quad M_D = -\frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$$

If we do not take into account the approximation, then the length of the compound microscope tube in this case is denoted by L_D and is given by

$$L_D = v_o + u_e = \frac{u_o f_o}{u_o - f_o} + \frac{f_e D}{f_e + D}$$

Conceptual Note(s)

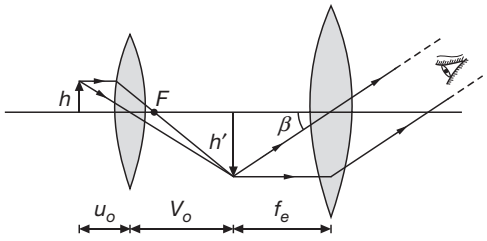
(a) From the above expression, it follows that a compound microscope will have large magnifying power, if both the object lens and the eye lens are of small focal length. In practice, focal length of object lens is smaller than that of eye lens i.e. $f_0 < f_e$. Further the **negative** value of magnifying power of compound microscope tells that final image formed is **inverted**.

(b) In practice, to eliminate chromatic aberration, a combination of two lenses in contact is used. It is called **objective**.

(c) In place of an eye lens, a combination of two lenses at certain distance apart satisfying certain conditions (to minimize chromatic and spherical aberrations) is used. It is called **eye piece**.

CASE-2: When image is formed at infinity

In this case $M = M_\infty$ is defined as the ratio of the visual angle β subtended (at the eye) by the final image to the visual angle α subtended (at the eye) by the object when kept at D.



So, $M_\infty = \frac{\beta}{\alpha} \approx \frac{\tan\beta}{\tan\alpha} = \frac{h'/f_e}{h/D} = \left(\frac{h'}{h}\right)\left(\frac{D}{f_e}\right) = \left(\frac{v_o}{u_o}\right)\left(\frac{D}{f_e}\right)$

and $L_\infty = v_o + f_e$

$$\Rightarrow \quad v_o = L_\infty - f_e$$

Since, we know that $\frac{1}{v_o} - \frac{1}{(-u_o)} = \frac{1}{f_o}$

$$\Rightarrow \quad \frac{v_o}{u_o} = \frac{v_o - f_o}{f_o}$$

Since $M_\infty = \left(\frac{v_o}{u_o}\right)\left(\frac{D}{f_e}\right)$

$$\Rightarrow \quad M_\infty = \left(\frac{v_o}{u_o}\right)\left(\frac{D}{f_e}\right) = \left(\frac{v_o - f_o}{f_o}\right)\left(\frac{D}{f_e}\right)$$

$$\Rightarrow \quad M_\infty = \left(\frac{L_\infty - f_e - f_o}{f_o f_e}\right)D$$

where $L_\infty = v_o + f_e = \frac{u_o f_o}{u_o - f_o} + f_e$

ILLUSTRATION 109

A compound microscope has a magnifying power 30. The focal length of its eye-piece is 5 cm. Assuming the final image to be at the least distance of distinct vision (25 cm), find the magnification produced by the objective.

SOLUTION

For a compound microscope, we have

$$M = m_0 m_e \qquad \ldots(1)$$

Since the final image is formed at least distance of distinct vision, the magnification of eye-piece is

$$m_e = \left(1 + \frac{D}{f_e}\right) = 1 + \frac{25}{5} = 6$$

From equation (1), we get

$$-30 = m_0 \times 6$$

$$\Rightarrow \quad m_0 = -\frac{30}{6} = -5$$

Negative sign implies that image formed by the objective is inverted.

ILLUSTRATION 110

In a compound microscope the objective and eyepiece have focal lengths of 0.95 cm and 5 cm respectively, and are kept at a distance of 20 cm. The final image is formed at a distance of 25 cm from eyepiece. Calculate the position of the object and the total magnification.

SOLUTION

From the lens formula for eyepiece, we use

$$v_e = -25 \text{ cm} \quad \text{and} \quad f_e = +5 \text{ cm}$$

$$\Rightarrow \quad \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{5} = -\frac{6}{25}$$

$$\Rightarrow \quad u_e = -\left(\frac{25}{6}\right) \text{ cm}$$

For the objective, we use

$$f_0 = 0.95 \text{ cm} \quad \text{and} \quad v_0 = 20 - \left(\frac{25}{6}\right) = \frac{95}{6} \text{ cm}$$

Using lens formula, we have

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$$

$$\Rightarrow \quad \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{u_0}$$

$$\Rightarrow \quad \frac{1}{u_0} = \frac{6}{5} - \frac{1}{0.96} = \frac{6-100}{95}$$

$$\Rightarrow \quad \frac{1}{u_0} = -\frac{94}{95}$$

$$\Rightarrow \quad u_0 = \frac{95}{94} \text{ cm}$$

Total Magnification $M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$

$$\Rightarrow \quad M = \frac{\left(\frac{95}{6}\right)}{-\left(\frac{95}{94}\right)}\left(1 + \frac{25}{5}\right) = -94$$

ILLUSTRATION 111

The focal lengths of the objective and eye-lens of a microscope are 1 cm and 5 cm respectively. If the magnifying power for the relaxed eye is 45, then calculate the length of the tube.

SOLUTION

Given that $f_o = 1 \text{ cm}$, $f_e = 5 \text{ cm}$, $M_\infty = 45$

Since, we have $M_\infty = \frac{(L_\infty - f_o - f_e)}{f_o f_e}$

$$\Rightarrow \quad 45 = \frac{(L_\infty - 1 - 5) \times 25}{1 \times 5}$$

$$\Rightarrow \quad L_\infty = 15 \text{ cm}$$

ILLUSTRATION 112

The focal length of the objective of a microscope is $f_o = 3$ mm and of the eyepiece $f_e = 5$ cm. An object is placed at a distance of 3.1 mm from the objective. Find the magnification of the microscope for a normal eye, if the final image is produced at a distance 25 cm from the eye (or eyepiece). Also final the separation between the lenses of microscope.

SOLUTION

For the objective, we use

$$u_0 = -0.31 \text{ cm} \quad \text{and} \quad f_0 = 0.3 \text{ cm}$$

Using lens formula, we have

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$$

$$\Rightarrow \quad \frac{1}{v_0} - \frac{1}{-0.31} = \frac{1}{0.3}$$

$$\Rightarrow \quad v_0 = +9.3 \text{ cm}$$

For the eyepiece, we use

$$v_e = -25 \text{ cm} \quad \text{and} \quad f_e = +5 \text{ cm}$$

Using lens formula, we have

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$\Rightarrow \quad \frac{1}{-25} - \frac{1}{u_e} = \frac{1}{5}$$

$$\Rightarrow \quad u_0 = -\frac{25}{6} \text{ cm} = -4.166 \text{ cm}$$

Magnifying power of telescope is given as

$$M = \frac{v_0}{u_0}\left(1+\frac{D}{f_e}\right) = \frac{9.3}{0.31}\left(1+\frac{25}{5}\right) = 180$$

Separation between lenses is

$$d = v_0 + u_e = 9.3 + 4.166 = 13.466 \text{ cm}$$

ASTRONOMICAL TELESCOPE (REFRACTING TYPE)

An astronomical telescope is used to see heavenly objects. It produces a virtual and inverted image. As such bodies are round, the inverted image does not affect the observation.

An astronomical refracting telescope consists of two lens systems. The lens system facing the object is called **objective**. It has large aperture and is of **large focal** length (f_0). The other lens system is called **eye-piece**. It has **small aperture** and is of **short focal length** (f_e). The objective and the eye-piece are mounted coaxially in two metallic tubes. The tube holding the eye-piece can be made to slide into the tube holding the objective with the help of rack and pinion arrangement.

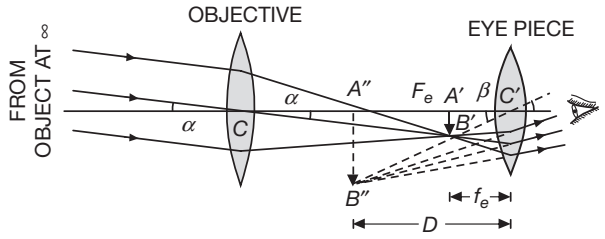
The objective forms the real and inverted image of the distant object in its focal plane. The position of the eye-piece is adjusted, till the final image is formed **at least distance of distinct vision**.

In case, position of the eye-piece is adjusted such that final image is formed at infinity, the telescope is said to be in **normal adjustment**.

Magnifying Power (*M*)

CASE-1: When final image is formed at least distance of distinct vision

When a parallel beam of light rays from the distant object falls on the objective, its real and inverted image $A'B'$ is formed on the other side of the objective. The position of eye-piece is adjusted so that the final image $A''B''$ is formed at least distance of distinct vision. Under such a situation the magnifying power of a telescope is defined as the ratio of the angle subtended at the eye by the image formed at the least distance of distinct vision to angle subtended at the eye by the object lying at infinity, when seen directly.



Again, as the object is at a very large distance, the angle α subtended by it at the objective is practically the same as that subtended by it at the eye. Therefore, if $\angle A''C'B'' = \beta$, then

$$M_D = \frac{\beta}{\alpha}$$

Again, as angle α and β are small, they can be replaced by their tangents,

$$\Rightarrow \quad M_D = \frac{\tan\beta}{\tan\alpha} = \frac{CA'}{C'A'}$$

Since, $CA' = f_0$ and $C'A' = u_e$

$$\Rightarrow \quad M_D = \frac{f_0}{u_e}$$

For eye lens, we have

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$\Rightarrow \quad \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$\Rightarrow \quad \frac{1}{u_e} = -\frac{1}{f_e}\left(1-\frac{f_e}{v_e}\right)$$

$$\Rightarrow \quad M_D = -\frac{f_0}{f_e}\left(1-\frac{f_e}{v_e}\right)$$

Applying new Cartesian sign conventions we get

$$f_0 = +f_0,\ v_e = -D,\ f_e = +f_e$$

$$\Rightarrow \quad M_D = -\frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$$

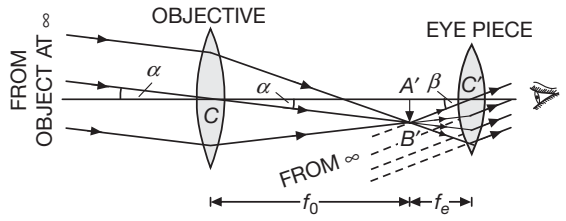
Therefore, a refracting telescope will have large magnifying power, if the object lens is of large focal length and eye lens is of short focal length. Further, the **negative** value of magnifying power of the telescope tells that the final image formed is **inverted**

and **real**. Out of the two adjustments discussed, this adjustment gives a higher magnification, since the factor $\left(1+\frac{f_e}{D}\right)$ is greater than one.

Also, a telescope does not increase the size of object, but it forms an image nearer to the eye, so that the angle of vision is increased and hence it appears to us as if the bigger image of object is formed.

CASE-2: When final image is formed at infinity (Normal adjustment)

When a parallel beam of light rays from the distant object falls on the objective, its real and inverted image $A'B'$ is formed on the other side of the objective. If the position of eye-piece is adjusted, so that the image $A'B'$ lies at its focus, then the final highly magnified image will be formed at infinity. Under such a situation i.e. in normal adjustment the magnifying power of a telescope is defined as the ratio of the angle subtended by the image at the eye as seen through the telescope to the angle subtended by the object seen directly, when both the object and the image lie at infinity. It is also called **angular magnification** of the telescope and is denoted by $M = M_\infty$.



As the object is at a very large distance, the angle subtended by it at the eye is practically the same as that subtended by it at the objective.

Thus, $\angle A'CB' = \alpha$ may be considered as the angle subtended by object at the eye. Let $\angle A'C'B' = \beta$.

Then

$$M_\infty = \frac{\beta}{\alpha}$$

Since the angles α and β are small, $\alpha \approx \tan\alpha$ and $\beta \approx \tan\beta$. Therefore,

$$M_\infty = \frac{\tan\beta}{\tan\alpha} = \frac{CA'}{C'A'}$$

Using new Cartesian sign conventions we get

$$CA' = +f_0$$

{$\because$ distance of $A'B'$ from object lens is along the incident light}

$$C'A' = -f_e$$

{$\because$ distance of $A'B'$ from eye lens is against incident light}

$$\Rightarrow \quad M_\infty = \frac{f_0}{f_e}$$

Conceptual Note(s)

(a) It follows that the magnifying power of a telescope in normal adjustment will be large, if objective is of large focal length and the eye-piece is of short focal length.

(b) Further, when telescope is in normal adjustment, the distance between the two lenses is equal to sum of their focal lengths $(f_0 + f_e)$.

(c) Further, the **negative** value of the magnifying power of the telescope tells that final image formed is inverted and real.

ILLUSTRATION 113

The objective of a telescope is a convex lens of focal length 100 cm. Its eye-piece is also a convex lens of focal length 5 cm. Determine the magnifying power of the telescope for normal adjustment.

SOLUTION

For normal adjustment, the magnifying power of a telescope is given by

$$M_\infty = \frac{f_o}{f_e}$$

Here, $f_o = 100$ cm, $f_e = 5$ cm

$$\Rightarrow \quad M_\infty = \frac{100}{5} = 20$$

ILLUSTRATION 114

An astronomical telescope in normal adjustment has a tube length of 93 cm and magnification (angular) of 30. If the eye-piece is to be drawn out by 3 cm to focus a near object, with the final image at infinity,

find how far away is the object and the magnification (angular) is this case.

SOLUTION

In the normal adjustment, magnification (angular) produced by telescope is

$$M_\infty = \frac{f_o}{f_e} = 30$$

$$\Rightarrow \quad f_o = 30 f_e$$

and tube length $L_\infty = f_o + f_e = 93$

$$\Rightarrow \quad f_e = \frac{93}{31} = 31 \text{ cm} \quad \text{and} \quad f_o = 90 \text{ cm}$$

Since $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$

$$\Rightarrow \quad \frac{1}{\infty} - \frac{1}{-u_e} = \frac{1}{+3}$$

$$\Rightarrow \quad u_e = 3 \text{ cm}$$

Since $v_o = f_o + u_e = 90 + 3 = 93$ cm and $\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$

$$\Rightarrow \quad \frac{1}{+93} - \frac{1}{-u_o} = \frac{1}{+90}$$

$$\Rightarrow \quad \frac{1}{u_o} = \frac{1}{90} - \frac{1}{93} = \frac{3}{90 \times 93}$$

$$\Rightarrow \quad u_e = 30 \times 93 = 2790 \text{ cm} = 27.9 \text{ m}$$

$$\text{Magnification} = \frac{v_o}{u_e} = \frac{93}{3} = 31$$

TERRESTRIAL TELESCOPE

A terrestrial telescope is used to observe objects on earth.

An astronomical telescope is used to view heavenly objects since the inversion of their images does not produce any complication. While viewing earthly objects we would prefer to have their images erect and hence, astronomical telescope is not suitable in such cases. By using an additional convex lens O (of focal length f) in between O_1 and O_2 of an astronomical telescope, we can have the final erect image. The lens O is called **erecting lens**, while the improved version of the telescope is called **Terrestrial Telescope**.

Rays from the distant object get refracted through the objective O_1, giving a real inverted image A_1B_1. The erecting lens O is so adjusted that its distance from A_1B_1 is equal to twice its (erecting lens) focal length. An image A_2B_2 having same size as that of A_1B_1, inverted w.r.t. A_1B_1 and hence erect w.r.t. the object is obtained at a distance $2f$ on other side of O. A_2B_2 acts as an object for lens at O_2 and finally an erect and magnified image is obtained after refraction through O_2. If the distance O_2B_2 is equal to focal length f_e of the eye lens O_2, final image is formed at infinity and the telescope is said to be in **normal adjustment** as in Figure 1.3.

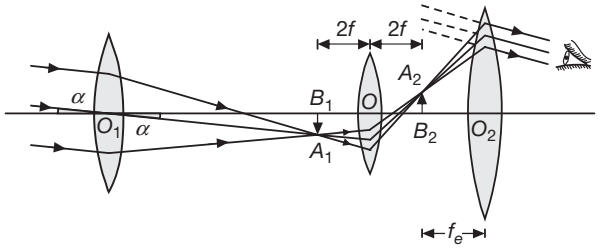


Figure 1.3

If the distance O_2B_2 is less than f_e then corresponding to a certain value of this distance, a virtual and magnified image is obtained at the distance of distinct vision as shown in Figure 1.4.

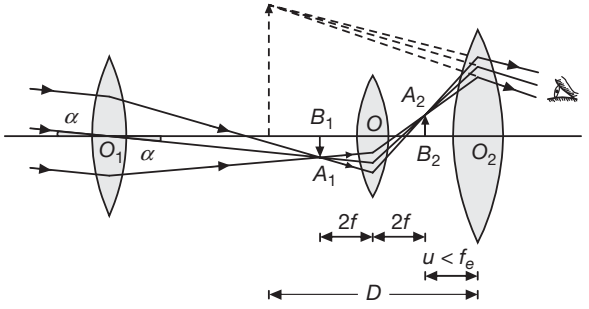
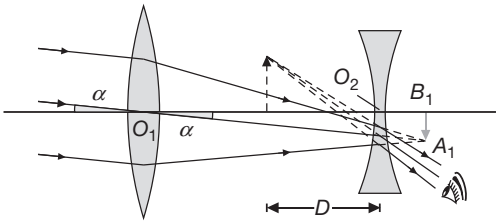


Figure 1.4

Since the sizes of A_2B_2 and A_1B_1 are same, introduction of the erecting lens O has nor produced any change in its magnifying power but, has helped in getting the final image erect only. It may also be noted that the use of erecting lens O results in an increase (equal to four times the focal length of erecting lens) in the length of the tube of telescope.

GALILEO'S TELESCOPE

Instead of using a combination of two, lens O_1 and O_2 for getting an erect image, Galileo used only one concave lenses to get the final erect image.



Parallel beam of incident rays from infinity are focussed by the objective O_1. An inverted image A_1B_1 (shown in grey) would have been formed after refraction through O_1. Before the rays meet at A_1, a concave lens $(\text{at } O_2)$ intercepts them. The beam diverges and the final erect image A_2B_2 is obtained. The distance O_2B_1 is so adjusted that final image is formed at the distance of distinct vision. If O_2B_1 is equal to the focal length f_e of eye lens at O_2 final image is formed at infinity and the telescope is said to be set in normal **adjustment**. In such a case the length of the tube is equal to the difference between the focal lengths of two lenses. The field of view of this telescope is small because of the use of concave lens.

When set in normal adjustment, its magnifying power M is given by

$$M = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{\dfrac{A_1B_1}{B_1O_2}}{\dfrac{A_1B_1}{B_1O_1}} = \frac{B_1O_1}{B_1O_2}$$

$$\Rightarrow \quad M = \frac{F}{f} = \frac{\text{Focal length of objective}}{\text{Focal length of eye lens}}$$

LIMIT OF RESOLUTION AND RESOLVING POWER

The *Resolving Power* (RP) of an optical instrument is defined as the reciprocal of smallest angular separation between two neighbouring objects whose images are just distinctly formed by the instrument. The smallest angular separation is called the *Resolving Limit* denoted by RL (also called as **Limit of Resolution**).

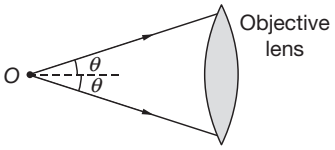
$$\text{Resolving Power} = \frac{1}{\text{Limit of Resolution}}$$

For Microscope

For a microscope, the resolving limit is given by

$$RL = \frac{\lambda}{2\mu\sin\theta}$$

where μ is refractive index of medium between object and objective lens, $\mu\sin\theta$ is called numerical aperture and λ is wavelength of light used to illuminate the object.



The resolving power for the microscope is given by

$$RP = \frac{1}{RL} = \frac{2\mu\sin\theta}{\lambda}$$

Since $RP \propto \dfrac{1}{\lambda}$, therefore for high resolution of microscope a beam of electrons is used which has wavelength of the order of $1\ \text{Å}$.

For Telescope

If a is aperture or diameter of telescope and λ the wavelength of light, then resolving limit (can also be denoted by $d\theta$ in this case) is

$$d\theta \propto \frac{\lambda}{a}$$

For spherical aperture $d\theta = \dfrac{1.22\lambda}{a}$

Resolving power $\propto \dfrac{a}{\lambda}$

Resolving power of telescope or microscope has no concern with focal lengths of lenses.

PHOTOMETRY

Radiant Flux

It is the radiant energy emitted by a body per second in all directions including all wavelengths. Its unit is watt.

Luminous Flux

The radiant energy emitted by a body per second in the visible region (i.e., between wavelengths from 4000Å to 7800Å) is called luminous flux. Its unit is lumen.

Luminous Intensity (I) of a Light Source

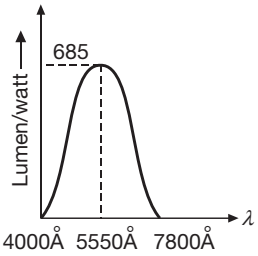
The luminous intensity (I) of a light source in any direction may be defined as the luminous flux (F) per unit solid angle (Ω) in that direction. Its unit is lumen/steradian or candela.

$$\Rightarrow \quad I = \frac{\Delta F}{\Delta \Omega} \qquad \text{(in lumen/steradian or candela)}$$

If the light source is isotropic, then luminous flux is uniform in all directions, so that total luminous flux is given by

$$F = 4\pi I \qquad \text{(since total solid angle for all directions is } 4\pi\text{)}$$

If we plot the ratio luminous flux/radiant flux against wavelength of radiation, we get a graph as shown in figure.



The graph indicates that the ratio luminous flux/radiant flux is maximum for 5550Å i.e. for yellow colour. This indicates that our eye is most sensitive for 5550Å i.e. yellow colour. The maximum value of ratio luminous flux/radiant flux is 685 lumen/watt. Thus when the ratio luminous flux/radiant flux is 685 lumen/watt, the luminous efficiency is said to be 100%.

The tungsten filament bulb converts 2-3% electrical energy into visible light energy, while fluorescent tube converts 8-9% electrical energy into visible light energy.

Illuminance (E) of a Surface

It may be defined as the luminous flux falling per unit area on the surface. Its unit is lumenm^{-2} or lux.

$$E = \frac{\Delta F}{\Delta A}$$

Inverse Square Law Obeyed by Illuminance

When radiant energy falls normally on a surface, the illuminance E of the surface is inversely proportional to the square of distance of surface point from source i.e.

$$E \propto \frac{1}{r^2} \qquad \ldots(1)$$

Lambert's Cosine Law Obeyed by Illuminance

When radiant energy falls obliquely on a surface, the illuminance E of surface is directly proportional to the cosine of angle made by normal to the surface with the direction of incident radiation. So,

$$E \propto \cos\theta \qquad \ldots(2)$$

Combining (1) and (2), we get

$$E = \frac{I\cos\theta}{r^2} \qquad \ldots(3)$$

Total Luminous Energy falling on a surface is given by

$$Q = EAt$$

where E = illuminance of the surface
A = area and
t = exposure time.

The total luminous energy required to be incident on a given type of camera film is constant.

For a box type camera, the time of exposure $\propto \left(\frac{f}{d}\right)^2$, where, d is the diameter of camera lens and f is the focal length of camera lens.

The Luminance (L) or Brightness

Luminance of a surface is the luminous flux reflected by unit area of the surface normally.

$\Rightarrow$ Luminance = Illuminance × Reflection Coefficient.

Principle of Photometry

If two sources of light of illuminating power I_1 and I_2 are placed at distances r_1 and r_2 from the screen, then the screen will be equally illuminated due to two sources when

$$\frac{I_1}{r_1^2} = \frac{I_2}{r_2^2}$$

Test Your Concepts-VIII

Based on Aberrations, Human Eye and Optical Instruments

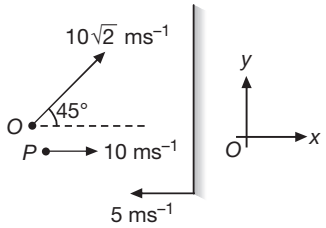
(Solutions on page H.26)

1. For a normal eye, the far point is at infinity and the near point of distinct vision is about 25 cm in front of the eye. The cornea of the eye provides a converging power of about 40 dipotres, and the least converging power of the eye-lens behind cornea is about 20 dioptres. Find the range of accommodation (converging power of eye lens) of the normal eye
2. The focal lengths of a thin convex lens are 1 m and 0.968 m for red and blue colour of light rays respectively. Calculate the chromatic aberration and dispersive power of material of the lens.
3. A short-sighted person cannot see objects situated beyond 2 m from him distinctly. What should be the power of the lens which he should use for seeing distant objects clearly?
4. A projector lens has a focal length 10 cm. It throws an image of a 2 cm × 2 cm slide on a screen 5 metre from the lens. Find
   (i) the size of the picture on the screen and
   (ii) the ratio of illumination of the slide and of the picture on the screen.
5. The eyepiece and objective of a microscope, of focal lengths 0.3 m and 0.4 m respectively, are separated by a distance of 0.2 m. The eyepiece and the objective are to be interchanged such that the angular magnification of the instrument remains same. What is the new separation between the lenses?
6. A compound microscope is used to enlarge an object kept at a distance of 0.03 m from its objective which consists of several convex lenses in contact and has focal length 0.02 m. If a lens of focal length 0.1 m is removed from the objective, find the distance by which the eyepiece of the microscope must be moved to refocus the image.
7. The focal lengths of the objective and eyepiece of a microscope are 4 mm and 25 mm respectively, and the length of the tube is 16 cm. If the final image is formed at infinity and the least distance of distinct vision is 25 cm, then calculate the magnifying power of the microscope.
8. The focal lengths of the objective and the eyepiece of an astronomical telescope are 0.25 m and 0.02 m, respectively. The telescope is adjusted to view an object at a distance of 1.5 m from the objective, the final image being 0.25 m from the eye of the observer. Calculate the tube length of the telescope and the magnification produced by it.
9. An astronomical telescope consisting of two convex lenses of focal length 50 cm and 5 cm is focussed on the moon. What is the distance between the two lenses in this position? If the telescope is then turned towards an object 10 m away, how much would the eye-piece have to be moved to focus on the object without altering the accommodation of the eye? Calculate the magnification (angular) produced by the telescope in the two adjustments.

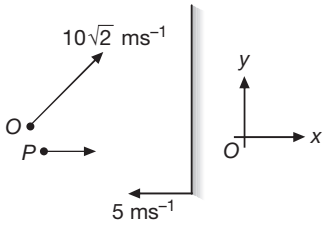
SOLVED PROBLEMS

PROBLEM 1

A plane mirror is moving with a uniform speed of $5\ \text{ms}^{-1}$ along negative x-direction and an observer P is moving with a velocity of $10\ \text{ms}^{-1}$ along $+x$ direction. Calculate the velocity of image of an object O, moving with a velocity of $10\sqrt{2}\ \text{ms}^{-1}$ as shown in the figure, as observed by the observer. Also find its magnitude and direction.



SOLUTION



Let $\vec{v}_O$ be the velocity of the object O, $\vec{v}_P$ be the velocity of the observer P, $\vec{v}_M$ be the velocity of the mirror and $\vec{v}_I$ be the velocity of image (Assume all these velocities w.r.t. ground), then

$$\vec{v}_O = \frac{10\sqrt{2}}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$$

$$\vec{v}_O = 10\left(\hat{i}+\hat{j}\right)$$

$$\vec{v}_P = 10\hat{i}$$

$$\vec{v}_M = -5\hat{i}$$

$(\vec{v}_{IM})_\perp = -(\vec{v}_{OM})_\perp$, where the axis perpendicular to the mirror is the x-axis.

$$\Rightarrow \quad (\vec{v}_I)_x - (\vec{v}_M)_x = -(\vec{v}_O)_x + (\vec{v}_M)_x$$

$$\Rightarrow \quad (\vec{v}_I)_x = 2(\vec{v}_M)_x - (\vec{v}_O)_x$$

$$\Rightarrow \quad (\vec{v}_I)_x = 2(-5\hat{i}) - 10\hat{i}$$

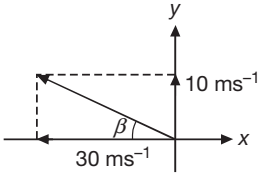
$$\Rightarrow \quad (\vec{v}_I)_x = -20\hat{i}$$

Further, parallel to the mirror, i.e., along y-axis, we have

$$(\vec{v}_I)_y = (\vec{v}_O)_y = 10\hat{j}$$

Since $\vec{v}_I = (\vec{v}_I)_x + (\vec{v}_I)_y$

So, absolute velocity of the image is



$$\vec{v}_I = -20\hat{i} + 10\hat{j}$$

Now $\vec{v}_{IP} = \vec{v}_I - \vec{v}_P$

$$\Rightarrow \quad \vec{v}_{IP} = -20\hat{i} + 10\hat{j} - 10\hat{i}$$

$$\Rightarrow \quad \vec{v}_{IP} = -30\hat{i} + 10\hat{j}$$

$$\Rightarrow \quad |\vec{v}_{IP}| = \sqrt{900+100} = 10\sqrt{10}\ \text{ms}^{-1}$$

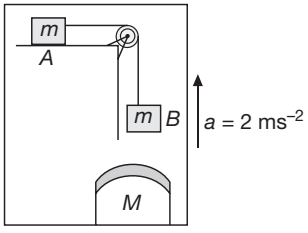
If β is the angle made by $\vec{v}_{IP}$ with $-x$ axis, then

$$\tan\beta = \frac{10}{30}$$

$$\Rightarrow \quad \beta = \tan^{-1}\left(\frac{1}{3}\right), \text{ with } -x \text{ axis}$$

PROBLEM 2

Consider the situation shown in figure. The elevator is going up with an acceleration of $2\ \text{ms}^{-2}$ and the focal length of the mirror is $12\ \text{cm}$. All the surfaces are smooth and the pulley is light. The mass pulley system is released from rest (w.r.t. the elevator) at $t = 0$ when the distance of B from the mirror is $42\ \text{cm}$. Find the distance between the image of the block B and the mirror at $t = 0.2\ \text{s}$. Take $g = 10\ \text{ms}^{-2}$.

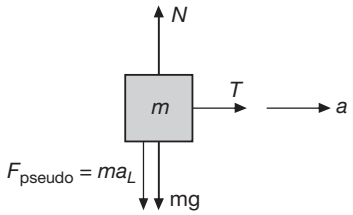


SOLUTION

Let us assume that the acceleration of blocks A and B to be a w.r.t. lift and a_L be the acceleration of lift. Consider block A, as seen from the reference frame attached to the lift (a non-inertial frame), we get

$$N = mg + ma_L \quad \ldots(1)$$

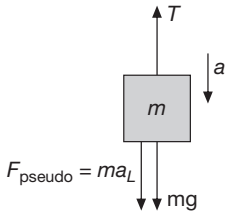
$$T = ma \quad \ldots(2)$$



Free body diagram of A

Now, consider block B, as seen from the reference frame attached to the lift, we have

$$mg + ma_L - T = ma \quad \ldots(3)$$



Free body diagram of B

On adding equations (2) and (3), we get

$$a = \frac{g + a_L}{2} = \frac{10+2}{2} = 6 \text{ ms}^{-2}$$

So, distance fallen by block (B) is $x = \frac{1}{2}at^2$

$$\Rightarrow \quad x = \frac{1}{2} \times 6 \times (0.2)^2$$

$$\Rightarrow \quad x = 0.12 \text{ m} = 12 \text{ cm}$$

Now, consider reflection at convex mirror, we have

$$u = -(42-12) = -30 \text{ cm}$$

$$f = +12 \text{ cm}$$

Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v} + \frac{1}{(-30)} = \frac{1}{12}$$

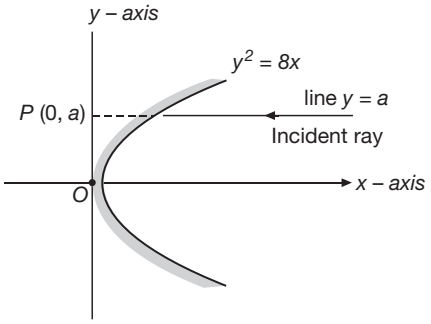
$$\Rightarrow \quad v = \frac{(30)(12)}{40+12} = \frac{360}{42}$$

$$\Rightarrow \quad v = 8.57 \text{ cm}$$

Therefore, the distance between the image of block (B) and mirror is 8.57 cm

PROBLEM 3

Figure shows a parabolic reflector in x-y plane given by $y^2 = 8x$. A ray of light travelling along the line $y = a$ is incident on the reflector. Find where the ray intersects the x-axis after reflection.



SOLUTION

The point at which light ray is incident satisfies $y^2 = 8x$

Since $y = a$

$$\Rightarrow \quad x = \frac{a^2}{8} \text{ i.e., the point at which ray is incident is } P\left(\frac{a^2}{8}, a\right)$$

After reflection ray passes through Q.

In ΔPQC, $PQ = QC$ (because sides opposite to equal angles are equal)

In right triangle PNQ

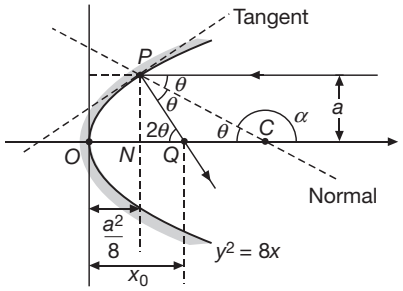
$$\tan(2\theta) = \frac{a}{\left(x_0 - \frac{a^2}{8}\right)} = \frac{8a}{8x_0 - a^2} \quad \ldots(1)$$

Slope of tangent to a curve is $\frac{dy}{dx}$.

Since $y^2 = 8x$

$$\Rightarrow \quad 2y\frac{dy}{dx} = 8$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{4}{y}$$



If m_1 is slope of tangent and m_2 is slope of normal, then $m_1 m_2 = -1$

$$\Rightarrow \quad \text{Slope of normal is } m_2 = -\frac{1}{\left(\frac{dy}{dx}\right)} = -\frac{y}{4}$$

$$\Rightarrow \quad \tan\alpha = -\frac{a}{4} \qquad \{\because\ y = a\}$$

$$\Rightarrow \quad \tan\theta = \frac{a}{4} \qquad \{\because\ \theta = 180 - \alpha\} \qquad \ldots(2)$$

From (1), we get

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{8a}{8x_0 - a^2}$$

$$\Rightarrow \quad \frac{2\times\frac{a}{4}}{1-\frac{a^2}{16}} = \frac{8a}{8x_0 - a^2}$$

$$\Rightarrow \quad x_0 = 2$$

Please note that x_0 is independent of a and the rays intersect at the focus of the parabola.

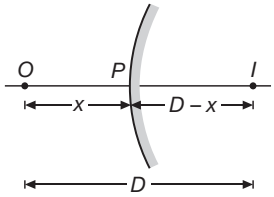
PROBLEM 4

An observer whose least distance of distinct vision is D, views his own face in a convex mirror of radius of curvature R. Prove that the magnification produced cannot exceed $\frac{R}{D+\sqrt{D^2+R^2}}$.

SOLUTION

For clear vision, the distance between object and image OI must be more than D. Let object be placed at a distance x from mirror and hence image is at distance $(D-x)$ from mirror.

Since, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$



$$v = +(D-x),\ u = -x \text{ and } f = +\frac{R}{2}(\text{for } x > 0)$$

$$\frac{1}{D-x} - \frac{1}{x} = \frac{2}{R}$$

$$\Rightarrow \quad \frac{x - D + x}{(D-x)(x)} = \frac{2}{R}$$

$$\Rightarrow \quad -2x^2 + 2xD = 2Rx - RD$$

Forming a quadratic in x, we get

$$x = \frac{D - R \pm \sqrt{R^2+D^2}}{2}$$

Discarding the negative value, we get

$$x = \frac{D - R + \sqrt{R^2+D^2}}{2},$$

Magnification produced by the mirror is

$$m = -\frac{v}{u} = -\frac{D-x}{x} = \frac{D+R-\sqrt{R^2+D^2}}{D-R+\sqrt{R^2+D^2}}$$

Rationalising the above equation, we get

$$m = \frac{(D+R)-\left(\sqrt{R^2+D^2}\right)^2}{\left(D-R+\sqrt{R^2+D^2}\right)\left(D+R+\sqrt{R^2+D^2}\right)}$$

$$\Rightarrow \quad m = \frac{2RD}{2D^2+2D\left(\sqrt{R^2+D^2}\right)} = \frac{r}{D+\sqrt{R^2+D^2}}$$

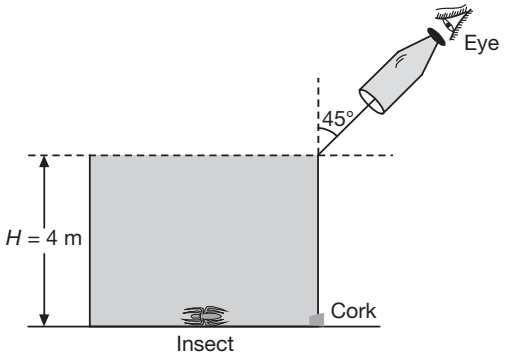
Thus the maximum magnification produced by the mirror can be given as

$$m_{\max} = \frac{R}{D+\sqrt{R^2+D^2}}$$

PROBLEM 5

A fixed cylindrical tank of height $H = 4$ m and radius $R = 3$ m is filled up with a liquid. An observer observes through a telescope fitted at the top of the

wall of the tank and inclined at $\theta = 45°$ with the vertical. When the tank is completely filled with liquid, he notices an insect, which is at the center of the bottom of the tank. At $t = 0$, he opens a cork of radius $r = 3$ cm at the bottom of tank. The insect moves in such a way that it is visible for a certain time. Determine



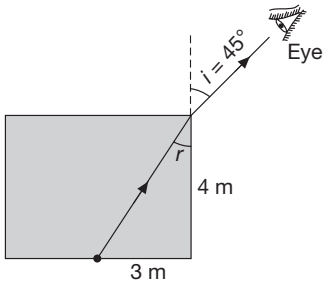
(a) the refractive index of the liquid
(b) the velocity of insect as a function of time.

SOLUTION

(a) At $t = 0$

$$\sin r = \frac{3}{5}$$

$$\mu = \frac{\sin i}{\sin r} = \frac{5}{3\sqrt{2}}$$



(b) Let at time t, insect be at a distance x from centre of the tank. Since,

$$\frac{x_1}{h} = \tan r = \frac{3}{4}$$

$$\Rightarrow \quad x_1 = \frac{3}{4}h$$

So, $x = (H - h) + x_1 - 3$

$$\Rightarrow \quad x = 4 - h + \frac{3}{4}h - 3$$

$$\Rightarrow \quad x = 1 - \frac{h}{4}$$

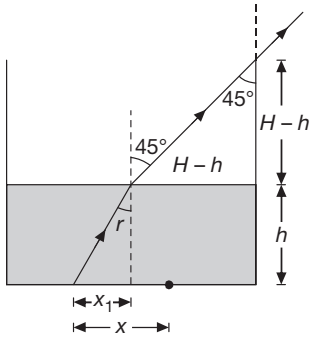
$$\Rightarrow \quad \left(\frac{dx}{dt}\right) = \frac{1}{4}\left(-\frac{dh}{dt}\right) \qquad \ldots(1)$$

From Equation of Continuity, we have

$$-A_1\left(\frac{dh}{dt}\right) = A_2\sqrt{2gh}$$

$$\Rightarrow \quad \left(-\frac{dh}{\sqrt{h}}\right) = \frac{A_2\sqrt{2g}}{A_1}dt$$

$$\Rightarrow \quad -\int_H^h \frac{dh}{\sqrt{h}} = \frac{\pi(3\times10^{-2})^2}{\pi(3)^2}\sqrt{2\times9.8}\int_0^t dt$$



Substituting $H = 4$ m, we get

$$h = (2 - 2.21\times10^{-4}t)^2$$

$$\Rightarrow \quad -\frac{dh}{dt} = 4.42\times10^{-4}(2 - 2.21\times10^{-4}t)$$

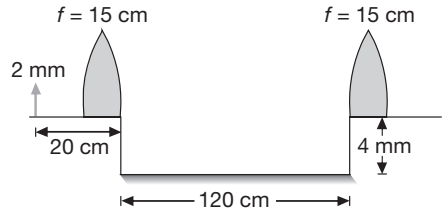
So, speed of insect is

$$v = \frac{dx}{dt} = \frac{1}{4}\left(-\frac{dh}{dt}\right)$$

$$\Rightarrow \quad v = 1.1\times10^{-4}(2 - 2.21\times10^{-4}t)\ \text{ms}^{-1}$$

PROBLEM 6

A convex lens of focal length 15 cm is split into two halves and the two halves are placed at a separation of 120 cm. Between these two halves of the convex lens, a plane mirror is placed horizontally and at a distance of 4 mm below the principal axis of the lens halves. An object of length 2 mm is placed at a distance of 20 cm from one half lens as shown in figure.



(a) Find the position and size of the final image.
(b) Trace the path of rays forming the image.

SOLUTION

(a) For refraction at first half lens, using lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{v}-\frac{1}{-20}=\frac{1}{15}$$

$$\Rightarrow \quad v=60 \text{ cm}$$

Magnification, $m=\frac{v}{u}=\frac{60}{-20}=-3$

The image formed by first half lens is shown in Figure 1

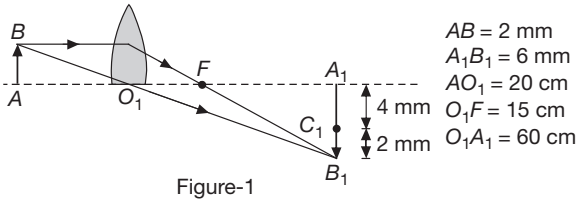


Figure-1

Now, the point B_1 is 6 mm below the principal axis of the lenses. Plane mirror is 4 mm below it. Hence, 4 mm length of A_1B_1 (i.e., A_1C_1) acts as real object for mirror. Mirror forms its virtual image A_2C_2. So, 2 mm length of A_1B_1 (i.e., C_1B_1) acts as virtual object for mirror. Real image C_2B_2 is formed of this part. Image formed by plane mirror is shown in Figure 2.

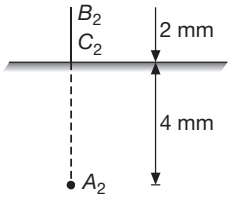


Figure-2

For the second half of the lens, using lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{v}-\frac{1}{-60}=\frac{1}{15}$$

$$\Rightarrow \quad v=+20$$

$$m=\frac{v}{u}=\frac{20}{-60}=-\frac{1}{3}$$

So, length of final image $A_3B_3=\frac{1}{3}A_2B_2=2$ mm.

However, point B_2 is 2 mm below the optic axis of second half lens. Hence, its image B_3 is formed $\frac{2}{3}$ mm above the principal axis.

Similarly, point A_2 is 8 mm below the principal axis. Hence, its image is $\frac{8}{3}$ mm above it.

Therefore, image is at a distance of 20 cm behind the second half lens and at a distance of $\frac{2}{3}$ mm above the principal axis.
The size of image is 2 mm and is inverted as compared to the given object. Image formed by second half lens is shown in Figure 3.

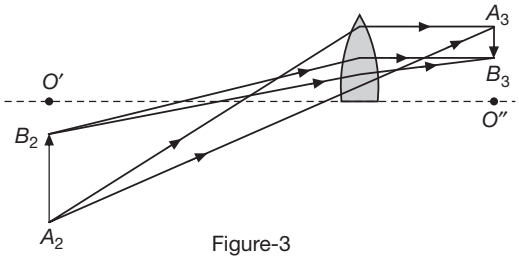


Figure-3

(b) The ray diagram for the final image is shown in Figure 4.

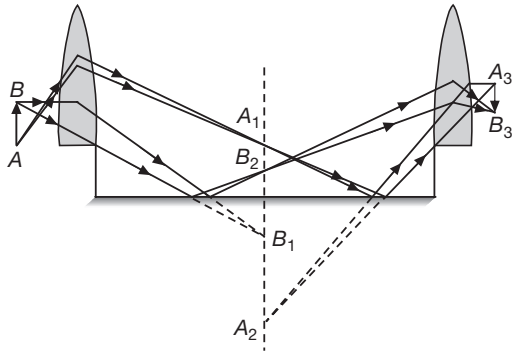
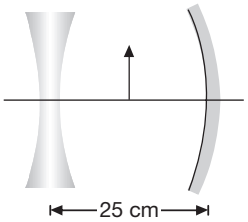


Figure-4

PROBLEM 7

An object lies midway between the lens and a mirror. The mirror's radius of curvature is 20 cm and the lens has a focal length of –16.7 cm . Considering that the rays that leave the object travel first towards the mirror, locate the final image formed by this system. Is this image real or virtual. Is it upright or inverted? What is the overall magnification?



SOLUTION

STEP-1: Image formed by mirror

Using mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{R}$, we get

$$\frac{1}{v_1}+\frac{1}{-12.5}=\frac{2}{-20}$$

$$\Rightarrow \quad v_1=-50 \text{ cm}$$

$$m_1=-\frac{v}{u}=-\frac{(-50)}{(-12.5)}=-4$$

So, the image formed by the mirror is at a distance of 50 cm from the mirror to the left of it. It is inverted and four times larger.

STEP-2: Image formed by lens

The image formed by mirror acts as an object for the lens and the image formed by the mirror is at a distance of 25 cm to the left of lens. Using the lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we get

$$\frac{1}{v_2}-\frac{1}{25}=\frac{1}{-16.7}$$

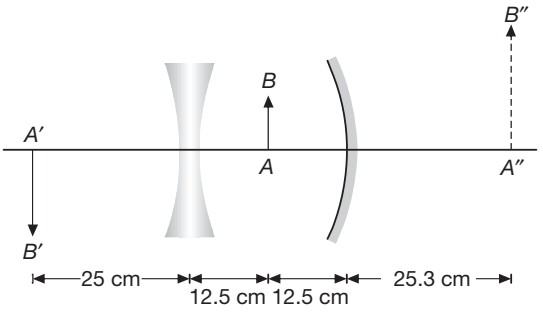
$$\Rightarrow \quad v_2=-50.3 \text{ cm}$$

and $m_2=\frac{v}{u}=\frac{-50.3}{25}=-2.012$

Net magnification is given by

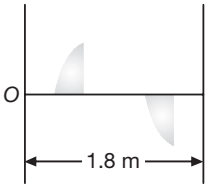
$$m=m_1\times m_2\approx 8$$

Hence, the final image is at a distance 25.3 cm to the right of the mirror, virtual, upright enlarged and approximately 8 times. Positions of the two images are shown in figure.



PROBLEM 8

A thin plano-convex lens of focal length f is split into two halves. One of the halves is shifted along the optical axis as shown in figure. The separation between object and image planes is 1.8 m . The magnification of the image, formed by one of the half lens is 2. Find the focal length of the lens and separation between the two halves. Draw the ray diagram for image formation.



SOLUTION

For both the halves, position of object and image is same, however the only difference is of magnification. Magnification for one of the halves is given as $2(>1)$. This can be for the first one, because for this, $|v|>|u|$. Therefore, magnification, $|m|=\left|\frac{v}{u}\right|>1$. So, for the first half, we have

$$\left|\frac{v}{u}\right|=2$$

$$\Rightarrow \quad |v|=2|u|$$

Let $u=-x$, then $v=+2x$

and $|u|+|v|=1.8$ m

$$\Rightarrow \quad 3x=1.8 \text{ m}$$

$$\Rightarrow \quad x=0.6 \text{ m}$$

Hence, $u = -0.6$ m and $v = +1.2$ m

Using $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{1.2} - \frac{1}{-0.6} = \frac{1}{0.4}$

$\Rightarrow \quad f = 0.4$ m

For the second half, we have

$$\frac{1}{f} = \frac{1}{1.2-d} + \frac{1}{-(0.6+d)}$$

$$\Rightarrow \quad \frac{1}{0.4} = \frac{1}{1.2-d} + \frac{1}{(0.6+d)}$$

Solving this, we get $d = 0.6$ m

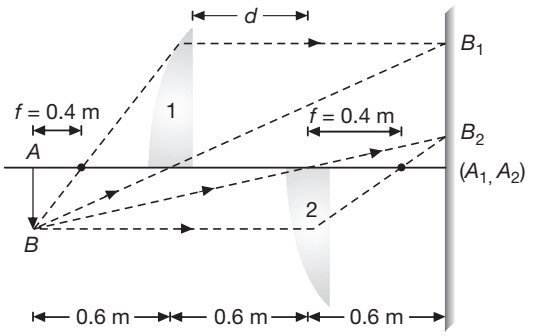
Magnification for the second half will be

$$m_2 = \frac{v}{u} = \frac{0.6}{-(1.2)} = -\frac{1}{2}$$

and magnification for the first half is

$$m_1 = \frac{v}{u} = \frac{1.2}{-(0.6)} = -2$$

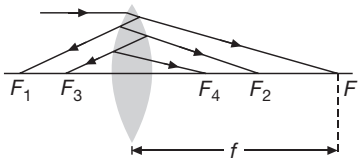
The ray diagram is as follows:



PROBLEM 9

A strong source of light when used with a convex lens produces a number of images of the source owing to feeble internal reflections and refraction called flare spots as shown in figure. These extra images are F_1, F_2,.... If F_n is the position of n^{th} flare spot, then show that

$$\frac{1}{F_n} = \frac{(n+1)\mu - 1}{f(\mu - 1)}$$



SOLUTION

Light converges at F_1 after two refractions and one reflection from the lens. So we use

$$\frac{1}{F_1} = \frac{2}{f_e} + \frac{1}{f_m}$$

Where focal length of equivalent independent lens is given do

$$\frac{1}{f_e} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f} = (\mu - 1)\left(\frac{1}{+R} - \frac{1}{-R}\right) = (\mu - 1)\frac{2}{R}$$

$$\Rightarrow \quad R = 2(\mu - 1)f$$

$$\Rightarrow \quad \frac{1}{F_1} = \frac{2}{f} + \frac{2}{2(\mu - 1)f}$$

$$\Rightarrow \quad \frac{1}{F_1} = \frac{2\mu - 1}{(\mu - 1)f}$$

For F_2, there are three refractions and two reflections

$$\frac{1}{F_2} = \frac{3}{f_1} + \frac{2}{f_m}$$

$$\Rightarrow \quad \frac{1}{F_2} = \frac{3}{f} + \frac{2}{\frac{R}{2}} = \frac{3}{f} + \frac{4}{R}$$

$$\Rightarrow \quad = \frac{3}{f} + \frac{4}{2(\mu - 1)f}$$

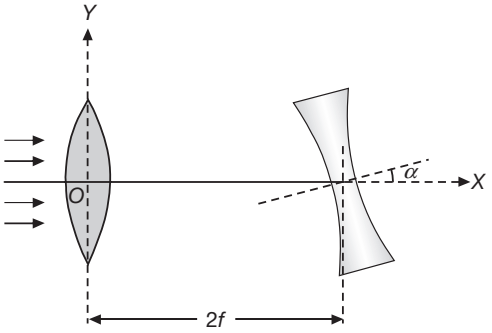
$$\Rightarrow \quad = \frac{3}{f} + \frac{2}{(\mu - 1)f}$$

$$\Rightarrow \quad = \frac{3(\mu - 1) + 2}{(\mu - 1)f} = \frac{3\mu - 1}{(\mu - 1)f}$$

$$\Rightarrow \quad \frac{1}{F_n} = \frac{(n+1)\mu - 1}{(\mu - 1)f}$$

PROBLEM 10

Two thin lenses of same focal length f are arranged with their principal axes inclined at an angle α as shown in figure. The separation between the optical centers of the lenses is $2f$. A point object lies on the principal axis of the convex lens at a large distance to the left of convex lens.



(a) Find the co-ordinates of the final image formed by the system of lenses taking O as the origin of the co-ordinate axes.

(b) Draw the ray diagram.

SOLUTION

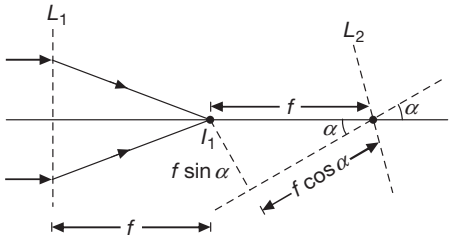
(a) For concave lens (L_2)

$$\frac{1}{v}-\frac{1}{-f\cos\alpha}=\frac{-1}{f}$$

$$\Rightarrow\quad v=-\left(\frac{f\cos\alpha}{1+\cos\alpha}\right) \qquad \ldots(1)$$

The magnification is given by

$$m=\frac{v}{u}=\frac{1}{1+\cos\alpha}$$



So, height of I_2 from the principal axis of L_2 is

$$h=(f\sin\alpha)m=\frac{f\sin\alpha}{1+\cos\alpha} \qquad \ldots(2)$$

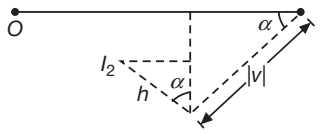
Hence the x-co-ordinate of image I_2 is given by

$$x=2f-|v|\cos\alpha-h\sin\alpha$$

$$\Rightarrow\quad x=2f-\frac{f\cos^2\alpha}{1+\cos\alpha}-\frac{f\sin^2\alpha}{1+\cos\alpha}$$

$$\Rightarrow\quad x=2f-\frac{f}{1+\cos\alpha}$$

$$\Rightarrow\quad x=f\left(\frac{2\cos\alpha+1}{\cos\alpha+1}\right)$$



Similarly, y co-ordinate of image I_2

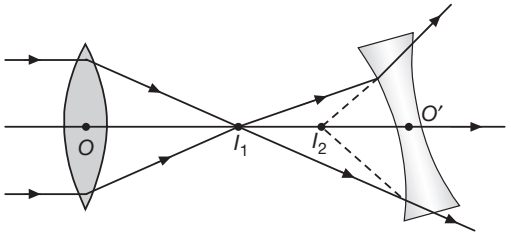
$$y=-(|v|\sin\alpha-h\cos\alpha)$$

On substituting the values of $|v|$ and h from (1) and (2), we get $y=0$.

So, the coordinates of the final image are

$$\left[f\left(\frac{2\cos\alpha+1}{\cos\alpha+1}\right),\ 0\right]$$

(b) Ray diagram is shown in figure

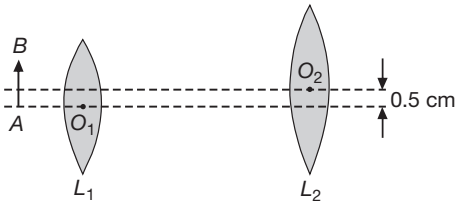


Conceptual Note(s)

The Y-co-ordinate of I_2 is zero is very obvious because a ray of light starting from I_2 and passing through O′ will suffer no deviation. Hence, I_2 must be formed on this line itself i.e., $y=0$.

PROBLEM 11

Two thin lenses $f_1=10$ cm and $f_2=20$ cm are separated by a distance $d=5$ cm. Their optical centres are displaced a distance $\Delta=0.5$ cm. A linear object of size 3 cm placed at 30 cm from the optical centre of left lens. Find the nature position and size of final image.



SOLUTION

STEP-1: Refraction from the first lens L_1

Using the lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{v_1}-\frac{1}{-30}=\frac{1}{10}$$

$$\Rightarrow \quad v_1=15 \text{ cm}$$

and $$m_1=\frac{v}{u}=\frac{15}{-30}=-\frac{1}{2}$$

STEP-2: Refraction from the second lens L_2

Again using the lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{v_2}-\frac{1}{(15-5)}=\frac{1}{20}$$

$$\Rightarrow \quad v_2=\frac{20}{3} \text{ cm} \approx 6.67 \text{ cm}$$

and $$m_2=\frac{v}{u}=\frac{20/3}{10}=\frac{2}{3}$$

About Final Image

Net magnification is given by

$$m=m_1m_2=-\frac{1}{3}$$

i.e., height of the image is $3\times\frac{1}{3}=1$ cm

Since, the net magnification is negative, so the final image is inverted.

Further y-coordinate of a point of the image will be,

$$y_1=my_0-m_2\Delta \qquad \ldots(1)$$

with respect to the principal axis of L_1

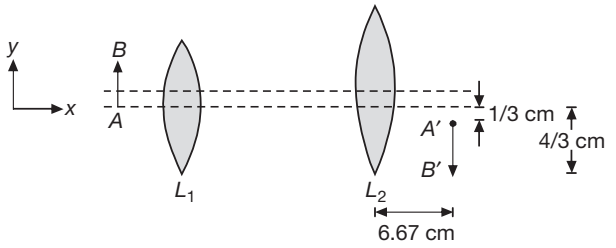
So, y-coordinate of image of A is

$$y_{A'}=\left(-\frac{1}{3}\right)(0)-\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)=-\frac{1}{3} \text{ cm and}$$

y-coordinate of image of B is

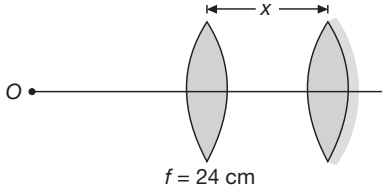
$$y_{B'}=\left(-\frac{1}{3}\right)(3)-\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)=-\frac{4}{3} \text{ cm}$$

Thus final image is as shown in figure.



PROBLEM 12

The radius of curvature of the curved surfaces of an equiconvex lens is 32 cm and its refractive index is $\mu=1.5$. One of its side is silvered and placed 14 cm away from an object as shown in figure. At what distance x should a second convex lens of focal length 24 cm be placed so that the image coincides with the object.



SOLUTION

For the convex lens, we use

$$f=+24 \text{ cm},$$

and $$u=-(14-x)$$

By refraction formula, we use

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{u}+\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{24}-\frac{1}{(14-x)}=\frac{14-x-24}{24(14-x)}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{-(x+10)}{24(14-x)}$$

$$\Rightarrow \quad v=-\left[\frac{(336-24x)}{(x+10)}\right]$$

The image will coincide the object if light rays after refraction from un-silvered face fall normally upon silvered face so that these rays will retrace the path

of incident rays. This is possible when first surface forms the image at 32 cm from it. Now for the unsilvered surface of the silvered lens, we use

$$\mu_2 = 1.5\,,\ \mu_1 = 1\,,\ v_1 = -32\text{ cm}\,,$$

$$u_1 = -(x - v) \text{ and } R = +32\text{ cm}$$

By using refraction formula, we have

$$\frac{\mu}{v_1} - \frac{1}{u_1} = \frac{\mu - 1}{R}$$

$$\Rightarrow \quad \frac{1.5}{-32} + \frac{1}{\left[x - \left\{-\frac{336 - 24x}{(x+10)}\right\}\right]} = \frac{1.5 - 1}{32}$$

$$\Rightarrow \quad \frac{(x+10)}{x(x+10) + (336 - 24x)} = \frac{0.5}{32} + \frac{1.5}{32} = \frac{1}{16}$$

$$\Rightarrow \quad x^2 + 10x + 336 - 24x = 16x + 160$$

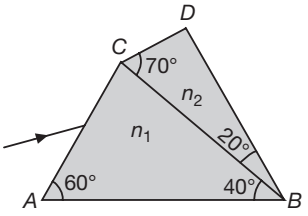
$$\Rightarrow \quad x^2 - 30x + 176 = 0$$

$$\Rightarrow \quad x = 8 \text{ OR } x = 22\text{ cm}$$

Hence the lens should be placed 8 cm from silvered surface.

PROBLEM 13

A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together with a gap as shown in the figure. The angles of the prism are as shown, n_1 and n_2 depend on λ, the wavelength of light according to the relations given by $n_1 = 1.20 + \dfrac{10.8 \times 10^4}{\lambda^2}$ and $n_2 = 1.45 + \dfrac{1.80 \times 10^4}{\lambda^2}$ where λ is in nm.



(a) Calculate the wavelength λ_0 for which rays incident at any angle on the interface BC pass through without bending at that interface.

(b) For light of wavelength λ_0, find the angle of incidence i on the face AC such that the deviation produced by the combination of prisms is minimum.

SOLUTION

Since, $n_1 = 1.20 + \dfrac{10.8 \times 10^4}{\lambda^2}$ and $n_2 = 1.45 + \dfrac{1.80 \times 10^4}{\lambda^2}$

where, λ is in nm.

(a) The incident ray will not deviate at BC only if $n_1 = n_2$

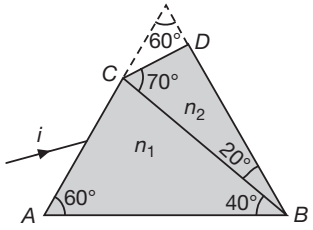
$$\Rightarrow \quad 1.2 + \frac{10.8 \times 10^4}{\lambda_o^2} = 1.45 + \frac{1.80 \times 10^4}{\lambda_o^2} \left(\lambda = \lambda_0\right)$$

$$\Rightarrow \quad \frac{9 \times 10^4}{\lambda_o^2} = 0.25$$

$$\Rightarrow \quad \lambda_0 = \frac{3 \times 10^2}{0.5}$$

$$\Rightarrow \quad \lambda_0 = 600\text{ nm}$$

(b) The given system happens to be a part of an equilateral prism of prism angle 60° as shown in figure.



At minimum deviation, we have

$$r_1 = r_2 = \frac{60°}{2} = 30° = r \qquad \{\text{say}\}$$

Since according to Snell's Law, we have

$$n_1 = \frac{\sin i}{\sin r}$$

$$\Rightarrow \quad \sin i = n_1 \sin(30°)$$

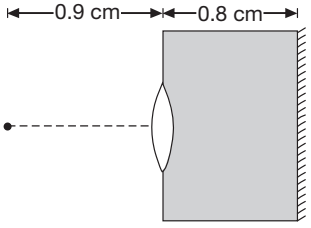
Since, $n_1 = 1.2 + \dfrac{10.8 \times 10^4}{\lambda_0^2}$, where $\lambda_0 = 600$ nm

$$\Rightarrow \quad \sin i = \left\{1.2 + \frac{10.8 \times 10^4}{(600)^2}\right\}\left(\frac{1}{2}\right) = \frac{1.5}{2} = \frac{3}{4}$$

$$\Rightarrow \quad i = \sin^{-1}\left(\frac{3}{4}\right)$$

PROBLEM 14

A thin equiconvex lens of glass of refractive index $\mu = \frac{3}{2}$ and of focal length 0.3 m in air is sealed into an opening at one end of a tank filled with water $\mu = \frac{4}{3}$. On the opposite side of the lens, a mirror is placed inside the tank on the tank wall perpendicular to the lens axis, as shown in figure. The separation between the lens and the mirror is 0.8 m. A small object is placed outside the tank in front of. Find the position (relative to the lens) of the image of the object formed by the system.



SOLUTION

Applying Lens Maker's Formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{0.3} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$$\{\because \ R_1 = R \text{ and } R_2 = -R\}$$

$$\Rightarrow \quad R = 0.3$$

Now applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ at air glass surface, we get

$$\frac{\frac{3}{2}}{v_1} - \frac{1}{-(0.9)} = \frac{\left(\frac{3}{2}\right) - 1}{0.3}$$

$$\Rightarrow \quad v_1 = 2.7 \text{ m}$$

So, the first image I_1 will be formed at 2.7 m from the lens.

This image I_1 will act as the virtual object for glass water surface.

Therefore, applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ at glass water surface, we get

$$\frac{\frac{4}{3}}{v_2} - \frac{\frac{3}{2}}{2.7} = \frac{\left(\frac{4}{3}\right) - \left(\frac{3}{2}\right)}{-0.3}$$

$$\Rightarrow \quad v_2 = 1.2 \text{ m}$$

So, the second image I_2 is formed at 1.2 m from the lens or 0.4 m from the plane mirror.

This image I_2 will act as a virtual object for mirror. Therefore, third real image I_3 will be formed at a distance of 0.4 m in front of the mirror after reflection from it. Now this image acts as a real object for water-glass interface. Hence applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, we get

$$\frac{\frac{3}{2}}{v_4} - \frac{\frac{4}{3}}{-(0.8 - 0.4)} = \frac{\left(\frac{3}{2}\right) - \left(\frac{4}{3}\right)}{0.3}$$

$$\Rightarrow \quad v_4 = -0.54 \text{ m}$$

So, the fourth image is formed to the right of the lens at a distance of 0.54 m from it.

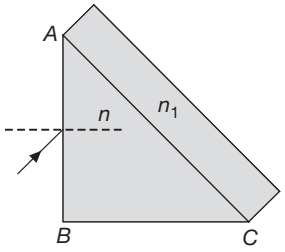
Now finally applying the same formula for glass-air surface, we get

$$\frac{1}{v_5} - \frac{\frac{3}{2}}{-0.54} = \frac{1 - \left(\frac{3}{2}\right)}{-0.3} = -0.9 \text{ m}$$

Hence, the position of final image is 0.9 m relative to the lens (rightwards) i.e., the image is formed 0.1 m behind the mirror.

PROBLEM 15

A right angle prism $(45° - 90° - 45°)$ of refractive index n has a plane of refractive index $n_1 (n_1 < n)$ cemented to its diagonal face. The assembly is in air. The ray is incident on AB.



(i) Calculate the angle of incidence at AB for which the ray strikes the diagonal face at the critical angle.

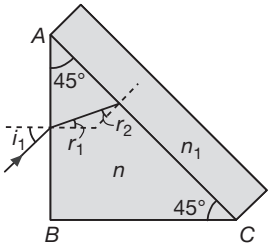
(ii) Assuming $n = 1.352$, calculate the angle of incidence at AB for which the refracted ray passes through the diagonal face undeviated.

SOLUTION

(i) Critical angle C at face AC will be given by

$$C = \sin^{-1}\left(\frac{n_1}{n}\right)$$

$$\Rightarrow \sin C = \frac{n_1}{n}$$



Now, it is given that $r_2 = C$

$$\Rightarrow r_1 = A - r_2 = (45° - C)$$

Applying Snell's Law at face AB, we get

$$n = \frac{\sin i_1}{\sin r_1}$$

$$\Rightarrow \sin i_1 = n \sin r_1$$

$$\Rightarrow i_1 = \sin^{-1}(n \sin r_1)$$

Substituting value of r_1, we get

$$i_1 = \sin^{-1}\{n \sin(45° - C)\}$$

$$\Rightarrow i_1 = \sin^{-1}[n(\sin 45° \cos C - \cos 45° \sin C)]$$

Since $\sin C = \frac{n_1}{n}$

$$\Rightarrow i_1 = \sin^{-1}\left(\frac{n}{\sqrt{2}}\sqrt{1 - \sin^2 C} - \sin C\right)$$

$$\Rightarrow i_1 = \sin^{-1}\left[\frac{n}{\sqrt{2}}\left(\sqrt{1 - \frac{n_1^2}{n^2}} - \frac{n_1}{n}\right)\right]$$

$$\Rightarrow i_1 = \sin^{-1}\left[\frac{1}{\sqrt{2}}\left(\sqrt{n^2 - n_1^2} - n_1\right)\right]$$

Therefore, required angles of incidence (i_1) at face AB for which the ray strikes at AC at critical angle is given by

$$i_1 = \sin^{-1}\left[\frac{1}{\sqrt{2}}\left(\sqrt{n^2 - n_1^2} - n_1\right)\right]$$

(ii) The ray will pass undeviated through face AC when

$$n_1 = n \text{ or } r_2 = 0°$$

i.e., ray falls normally on face AC

Since it is given that $n_1 < n$, so the option $n_1 = n$ is ruled out, hence

$$r_2 = 0°$$

$$\Rightarrow r_1 = A - r_2 = 45° - 0° = 45°$$

Now applying Snell's Law at face AB, we get

$$n = \frac{\sin i_1}{\sin r_1}$$

$$\Rightarrow 1.352 = \frac{\sin i_1}{\sin(45°)}$$

$$\Rightarrow \sin i_1 = (1.352)\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \sin i_1 = 0.956$$

$$\Rightarrow i_1 = \sin^{-1}(0.956) \approx 73°$$

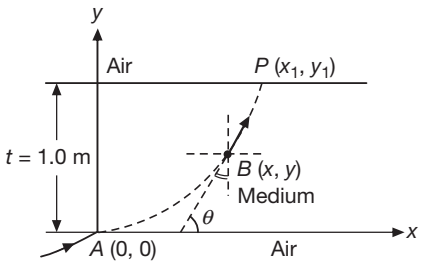
Therefore, required angle of incidence is $i_1 = 73°$.

PROBLEM 16

A ray of light travelling in air is incident at grazing angle (incident angle $= 90°$) on a long rectangular slab of a transparent medium of thickness $t = 1.0$ m. The point of incidence is the origin $A(0, 0)$. The medium has a variable index of refraction $n(y)$ given by

$$n(y) = \left[ky^{3/2} + 1\right]^{1/2} \text{ where } k = 1.0(\text{meter})^{-3/2}$$

The refractive index of air is 1.0



(a) Obtain a relation between the slope of the trajectory of the ray at a point $B(x, y)$ in the medium and the incident angle at the point.

(b) Obtain an equation for the trajectory $y(x)$ of the ray in the medium.

(c) Determine the co-ordinates (x_1, y_1) of the point P, where the ray intersects three upper surface of the slab-air boundary.

(d) Indicate the path of the ray subsequently.

SOLUTION

(a) $i+\theta=90°$, $\theta=90°-i$,

Slope of tangent $=\tan\theta=\tan(90°-i)=\cot i$

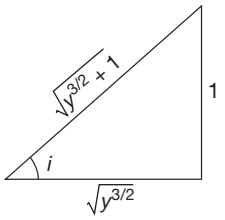
(b) $\tan\theta=\dfrac{dy}{dx}$

$\therefore \quad \dfrac{dy}{dx}=\cot i \qquad \ldots(1)$

Applying Snell's Law at A and B

$n_A \sin i_A = n_B \sin i_B$

$n_A = 1$ because $y=0$



$\sin i_A = 1$ because $i_A = 90°$ Grazing incidence

$n_B=\sqrt{Ky^{3/2}+1}=\sqrt{y^{3/2}+1}$

because $K=1.0(m)^{-3/2}$

$\therefore \quad (1)(1)=\sqrt{\left(y^{3/2}+1\right)}\sin i$

$\Rightarrow \quad \sin i=\dfrac{1}{\sqrt{y^{3/2}+1}}$

$\Rightarrow \quad \cot i=\sqrt{y^{3/2}} \text{ or } y^{3/4} \qquad \ldots(2)$

Equating equations (1) and (2), we get

$\dfrac{dy}{dx}=y^{3/4}$ or $y^{-3/4}dy=dx$

$\Rightarrow \quad \displaystyle\int_0^y y^{-3/4}dy=\int_0^x dx$ or $4y^{1/4}=x \qquad \ldots(3)$

The required equation of trajectory is $4y^{1/4}=x$

$\Rightarrow \quad y=\dfrac{x^4}{4^4}=\dfrac{x^4}{256}$

(c) At the point of intersection on the upper surface,

$y=1$ m

$\Rightarrow \quad x=(256)^{1/4}=4$ m

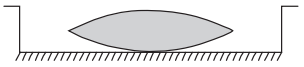
So the co-ordinates are $(4\text{ m}, 1\text{ m})$

(d) As $n_A \sin i_A = n_P \sin i_P$ and as $n_A = n_P = 1$

Therefore, $i_P = i_A = 90°$ i.e., the ray will emerge parallel to the boundary at P i.e., at grazing emergence.

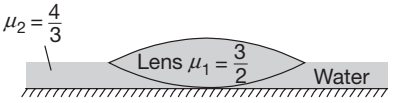
PROBLEM 17

A thin biconvex lens of refractive index $\dfrac{3}{2}$ is placed on a horizontal plane mirror as shown in the figure. The space between the lens and the mirror is then filled with water of refractive index $\dfrac{4}{3}$. It is found that when a point object is placed 15 cm above the lens on its principal axis, the object coincides with its own image. On repeating with another liquid, the object and the image again coincide at a distance 25 cm from the lens. Calculate the refractive index of the liquid.



SOLUTION

Let R be the radius of curvature of both the surfaces of the equi-convex lens, then in the first case, the situation is shown in figure.



Let f_1 be the focal length of equi-convex lens of refractive index μ_1 and f_2 be the focal length of plano-concave lens (made of water) of refractive index μ_2. The focal length of the combined lens system is given by

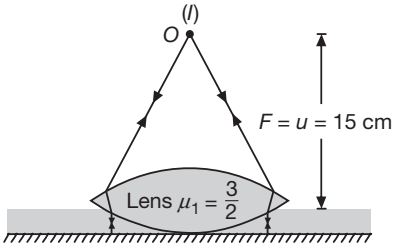
$\dfrac{1}{F}=\dfrac{1}{f_1}+\dfrac{1}{f_2}$

$\Rightarrow \quad \dfrac{1}{F}=(\mu_1-1)\left(\dfrac{1}{R}-\dfrac{1}{-R}\right)+(\mu_2-1)\left(\dfrac{1}{-R}-\dfrac{1}{\infty}\right)$

$\Rightarrow \quad \dfrac{1}{F}=\left(\dfrac{3}{2}-1\right)\left(\dfrac{2}{R}\right)+\left(\dfrac{4}{3}-1\right)\left(-\dfrac{1}{R}\right)=\dfrac{1}{R}-\dfrac{1}{3R}=\dfrac{2}{3R}$

$$\Rightarrow \quad F = \frac{3R}{2}$$

Now, image coincides with the object when ray of light retraces its path i.e., it is falls normally on the plane mirror. This is possible only when object lies at centre of curvature of the lens system.



$$\Rightarrow \quad F = 15 \text{ cm} \qquad \{\because \text{ Distance of object is 15 cm}\}$$

$$\Rightarrow \quad \frac{3R}{2} = 15 \text{ cm}$$

$$\Rightarrow \quad R = 10 \text{ cm}$$

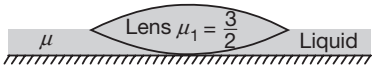
In the second case, let μ be the refractive index of the liquid filled between lens and mirror and let F' be the focal length of new lens system. Then,

$$\frac{1}{F'} = (\mu_1 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) + (\mu - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

$$\Rightarrow \quad \frac{1}{F'} = \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right) - \frac{(\mu - 1)}{R} = \frac{1}{R} - \frac{\mu - 1}{R} = \frac{(2 - \mu)}{R}$$

$$\Rightarrow \quad F' = \frac{R}{2 - \mu} = \frac{10}{2 - \mu} \qquad \{\because R = 10 \text{ cm}\}$$

Now, the image coincides with the object when it is placed at 25 cm distance.



$$\Rightarrow \quad F' = 25$$

$$\Rightarrow \quad \frac{10}{2 - \mu} = 25$$

$$\Rightarrow \quad 50 - 25\mu = 10$$

$$\Rightarrow \quad 25\mu = 40$$

$$\Rightarrow \quad \mu = \frac{40}{25} = 1.6$$

$$\Rightarrow \quad \mu = 1.6$$

PROBLEM 18

A telescope has an objective of focal length 50 cm and eyepiece of focal length 5 cm. The distance of distinct vision is 25 cm. The telescope is focussed for distinct vision on a scale 200 cm away from the objective. Calculate

(i) The separation between the objective and eye-piece,

(ii) The magnification produced.

SOLUTION

The situation is shown in figure with ray diagram.

(i) If the separation between the two lenses be x then for lens formula for refraction at objective lens we use

$$u_0 = -200 \text{ cm} \quad \text{and} \quad f_0 = +50 \text{ cm}$$

From lens formula, we have

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$$

$$\Rightarrow \quad \frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{50} - \frac{1}{200}$$

$$\Rightarrow \quad \frac{1}{v_0} = \frac{4 - 1}{200} = \frac{3}{200}$$

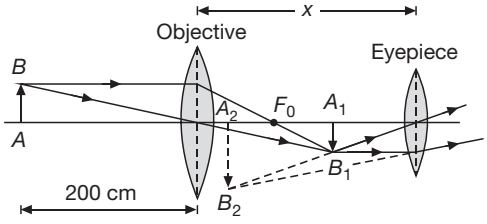
$$\Rightarrow \quad v_0 = +\frac{200}{3} \text{ cm}$$

Thus a real image is formed at a distance of $\frac{200}{3}$ from the objective. This image acts as object for the eyepiece. For refraction through eyepiece, we use

$$u_e = -\left(x - \frac{200}{3}\right)$$

$$v_e = -25 \text{ cm} \quad \text{and} \quad f_e = +5 \text{ cm}$$

$$\Rightarrow \quad -\frac{1}{25} + \frac{1}{\left(x - \frac{200}{3}\right)} = \frac{1}{5}$$



$$\Rightarrow \quad \frac{1}{\left(x-\frac{200}{2}\right)}=\frac{1}{5}+\frac{1}{25}=\frac{6}{25}$$

$$\Rightarrow \quad 6x-400=25$$

$$\Rightarrow \quad 6x=425$$

$$\Rightarrow \quad x=\frac{425}{6}=70.80 \text{ cm}$$

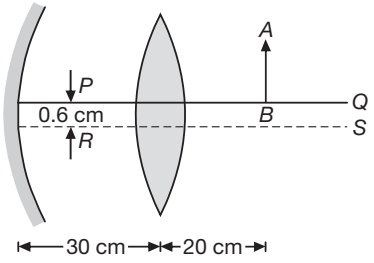
(ii) Magnification of Objective $=\frac{v_0}{u_0}=\frac{200}{3\times 200}=\frac{1}{3}$

Magnification of Eyepiece $=\frac{v_e}{u_e}=\frac{25\times 6}{25}=6$

Total magnification $=\frac{1}{3}\times 6=2$.

PROBLEM 19

A convex lens of focal length 15 cm and a concave mirror of focal length 30 cm are kept with their optic axis PQ and RS parallel but separated in vertical direction by 0.6 cm as shown. The distance between the lens and mirror is 30 cm. An upright object AB of height 1.2 cm is placed on the optic axis PQ of the lens at a distance of 20 cm from the lens. If $A'B'$ is the image after refraction from the lens and the reflection from the mirror, find the distance of $A'B'$ from the pole of the mirror and obtain its magnification. Also locate positions of A' and B' with respect to the optic axis RS.

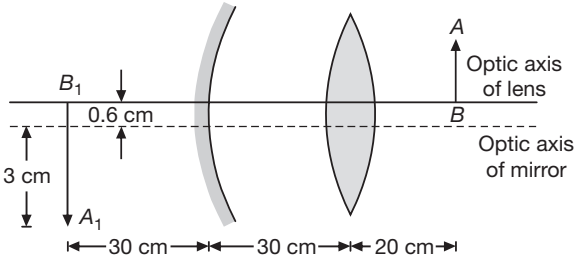


SOLUTION

Rays coming from object AB first refract from the lens and then reflect from the mirror.

For refraction from the lens, we have

$u=-20$ cm, $f=+15$ cm



Applying lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{v}-\frac{1}{(-20)}=\frac{1}{15}$$

$$\Rightarrow \quad v=+60 \text{ cm}$$

and linear magnification is given by

$$m_1=\frac{v}{u}=\frac{+60}{(-20)}=-3$$

So, the first image formed by the lens will be 60 cm from it (or 30 cm from the mirror) towards left and 3 times magnified but inverted. Length of first image A_1B_1 would be $A_1B_1=1.2\times 3=3.6$ cm (inverted).

For reflection from mirror, we have

Image formed by lens (A_1B_1) will behave like a virtual object for the mirror at a distance of 30 cm from it as shown. Therefore $u=+30$ cm, $f=-30$ cm.

Applying mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{v}+\frac{1}{30}=-\frac{1}{30}$$

$$\Rightarrow \quad v=-15 \text{ cm}$$

and linear magnification is given by

$$m_2=-\frac{v}{u}=-\frac{(-15)}{+30}=+\frac{1}{2}$$

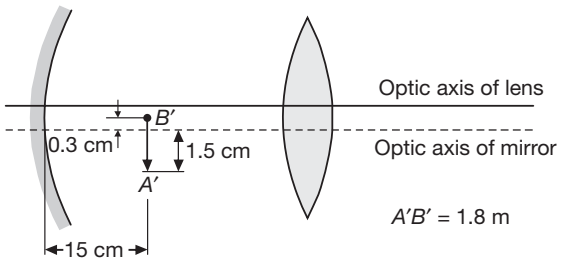
So, the final image $A'B'$ will be located at a distance of 15 cm from the mirror (towards right) and since magnification is $+\frac{1}{2}$, length of final image would be

$$A'B'=3.6\times\frac{1}{2}=1.8 \text{ cm}$$

$$\Rightarrow \quad A'B'=1.8 \text{ cm}$$

Since the point B_1 is 0.6 cm above the optic axis of mirror, therefore, its image B′ would be $(0.6)\left(\frac{1}{2}\right)=0.3$ cm above optic axis.

Similarly, point A_1 is 3 cm below the optic axis, therefore, its image A' will be $3\times\frac{1}{2}=1.5$ cm below the optic axis as shown.



Net magnification of the image is given by

$$m=m_1\times m_2=(-3)\left(+\frac{1}{2}\right)=-\frac{3}{2}$$

$$\Rightarrow \quad A'B'=(m)(AB)=\left(-\frac{3}{2}\right)(1.2)=-1.8 \text{ cm}$$

Conceptual Note(s)

If the co-ordinates of the object (X_0, Y_0) are generally known to us with reference to the pole of an optical instrument (whether it is a lens or a mirror), the corresponding co-ordinates of image (X_i, Y_i) are found as follows.

X_i is obtained using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ (for a mirror)

OR $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ (for a lens)

Here, v is actually X_i and u is X_0 i.e., the above formula can be written as $\frac{1}{X_i}\pm\frac{1}{X_0}=\frac{1}{f}$

Similarly, Y_i is obtained from $m=\frac{I}{O}$

Here, I is Y_i and O is Y_0 i.e., the above formula can be written as $m=\frac{Y_i}{Y_0}$ or $Y_i=mY_0$.

PROBLEM 20

The optical powers of the objective and eyepiece of a microscope are equal to 100 D and 20 D respectively. The microscope magnification is equal to 50 when image is produced at near point of eye. What will be magnification of the microscope be when the distance between the objective and eyepiece is increased by 2 cm ?

SOLUTION

For the eyepiece, we use

$$v_e=-25 \text{ cm} \quad \text{and} \quad f_e=5 \text{ cm}$$

Using lens formula for eyepiece, we have

$$\frac{1}{v_e}-\frac{1}{u_e}=\frac{1}{f_e}$$

$$\Rightarrow \quad \frac{1}{u_e}=\frac{1}{-25}-\frac{1}{5}=-\frac{6}{25} \text{ cm}$$

$$\Rightarrow \quad u_e=-\frac{25}{6} \text{ cm}$$

Magnification of eyepiece is

$$M_e=\frac{v_e}{u_e}=\frac{25}{\left(\frac{25}{6}\right)}=6$$

The magnification of microscope is given as

$$50=M_e\times M_0$$

$$\Rightarrow \quad 50=6\times M_0$$

$$\Rightarrow \quad M_0=\frac{50}{6}=\text{Magnification of objective}$$

The magnifying power of objective is given by

$$M_0=\frac{v_0}{u_0}=\frac{v_0-f_0}{f_0}$$

$$\Rightarrow \quad M_0f_0=v_0-f_0$$

$$\Rightarrow \quad v_0=M_0f_0+f_0=f_0(M_0+1)$$

$$\Rightarrow \quad v_0=f_0\left(\frac{50}{6}+1\right)=\frac{56}{6}f_0=\frac{56}{6}$$

When the distance is increased by 2 cm , then new value of v_0 will become

$$v_0'=\frac{56}{6}+2=\frac{68}{6}$$

now magnification by objective will be

$$M_0' = \frac{\left(\frac{68}{6}\right)}{1} - 1 = \frac{62}{6}$$

As magnification by eyepiece will remain same, total magnification will now be

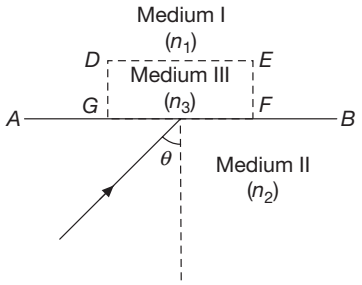
$$M_T = M_0' \times M_e = \frac{62}{6} \times 6 = 62$$

PROBLEM 21

Monochromatic light is incident on a plane interface AB between two media of refractive indices n_1 and n_2 $(n_2 > n_1)$ at an angle of incidence θ as shown in the figure. The angle θ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now if a transparent slab $DEFG$ of uniform thickness and of refractive index n_3 is introduced on the interface (as shown in the figure), show that for any value of n_3 all light will ultimately be reflected back again into medium II. Consider separately the cases

(a) $n_3 < n_1$ and

(b) $n_3 > n_1$



SOLUTION

At interface AB, θ is infinitesimally greater (slightly greater) than the critical angle for interface, so

$$\theta > \sin^{-1}\left(\frac{n_1}{n_2}\right)$$

(a) When $n_3 < n_1$

$$\Rightarrow \quad n_3 < n_1 < n_2$$

$$\Rightarrow \quad \frac{n_3}{n_2} < \frac{n_1}{n_2}$$

$$\Rightarrow \quad \sin^{-1}\left(\frac{n_3}{n_2}\right) < \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Hence, critical angle for Medium III and Medium II will be less than the critical angle for Medium II and Medium I. So, if TIR is taking place between Medium I and Medium II, then TIR will definitely take place between Medium I and Medium III.

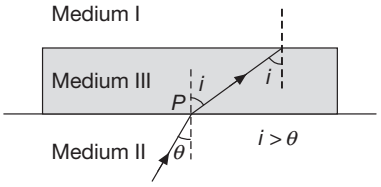
(b) When $n_3 > n_1$, then two further cases may arise.

CASE-1: $n_1 < n_3 < n_2$

In this case there will be no TIR between Medium I and Medium III but TIR will take place between Medium III and Medium II. This is because :

Ray of light first enters from Medium II to Medium III i.e., from denser to rarer. So,

$i > \theta$



Applying Snell's Law at P, we get

$$n_2 \sin\theta = n_3 \sin i$$

$$\Rightarrow \quad \sin i = \left(\frac{n_2}{n_3}\right)\sin\theta$$

Since, $\sin\theta$ is slightly greater than $\frac{n_1}{n_2}$, so

$\sin i$ is slightly greater than $\frac{n_2}{n_3} \times \frac{n_1}{n_2} = \frac{n_1}{n_3}$

However, $\frac{n_1}{n_3}$ is nothing but $\sin C$ for Medium I, Medium III interface, so

$\sin i$ is slightly greater than $\sin C$ for Medium I, Medium III interface.

$$\Rightarrow \quad i > (C)_{I,\ III}$$

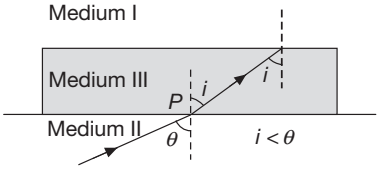
Hence, TIR will now take place on Medium I and Medium III interface and the ray will be reflected back to Medium III.

CASE-2: $n_1 < n_2 < n_3$

This time while moving from Medium II to Medium III, ray of light will bend towards normal. Again applying Snell's Law at P, we get

$$n_2 \sin\theta = n_3 \sin i$$

$$\Rightarrow \quad \sin i = \frac{n_2}{n_3}\sin\theta$$



Since, $\sin\theta$ slightly greater than $\frac{n_1}{n_2}$

So, $\sin i$ will be slightly greater than $\frac{n_2}{n_3}\times\frac{n_1}{n_2}=\frac{n_1}{n_3}$

However $\frac{n_1}{n_3}$ is $\sin C$ for Medium I and Medium III interface, so

$\sin i > \sin C$ for Medium I and Medium III interface.

$\Rightarrow \quad i > (C)_{I,III}$

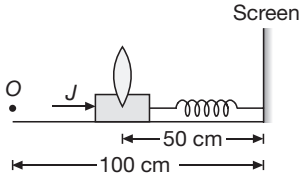
Therefore, TIR will again take place between Medium I and Medium III and the ray will be reflected back.

Conceptual Note(s)

The Cases 1 and 2 for $n_3 > n_1$ can be explained by single equation only. But two cases are deliberately formed for better understanding of refraction, Snell's Law and total internal reflection (TIR).

PROBLEM 22

A point object O is located at a distance of 100 cm from a screen. A lens of focal length 23 cm mounted on a movable frictionless stand is kept between the source and the screen. The stand is attached to a spring of natural length 50 cm and spring constant $800\ \text{Nm}^{-1}$ as shown in figure.



Mass of the stand with lens is 2 kg. How much impulse J should be imparted to the stand so that a real image of the object is formed on the screen after a fixed time gap. Also find this time gap. (Neglect the width of the stand)

SOLUTION

Let the distance of the lens from the object be L when a real image is formed on the screen. Then, we have

$$u=-L,\ v=+(100-L),\ f=+23\ \text{cm}$$

Using lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{100-L}-\frac{1}{-L}=\frac{1}{23}$$

$$\Rightarrow \quad L^2-100L+2300=0$$

Solving, we get

$$L=\left(50\pm10\sqrt{2}\right)\ \text{cm}$$

Since the lens is executing SHM and a real image is formed after a fixed time gap, then this time gap must be such that real image is obtained when the lens passes through two positions at same distance from the mean position and hence separated by a time gap equal to one fourth of the time period of SHM i.e. $\Delta t=\frac{T}{4}$.

So phase difference between the two positions of real image formation must be $\frac{\pi}{2}$, because the two positions are symmetrically located about the mean position and phase difference of any of these positions from origin must be $\frac{\pi}{4}$.

If A is the amplitude of SHM and x be the displacement of the lens executing SHM, then we have

$$10\sqrt{2}\ \text{cm}=A\sin\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \quad A=20\ \text{cm}$$

Velocity of lens, at mean position, in this case is

$$v_0=A\omega=A\sqrt{\frac{K}{m}}$$

Since impulse is equal to the change in momentum of the body, so impulse required to attain this speed is given by

$$J=mv_0=A\sqrt{Km}=8\ \text{kgms}^{-1}$$

PROBLEM 23

A small block of mass m and a concave mirror of radius R fitted with a stand, lie on a smooth horizontal table with a separation d between them. The

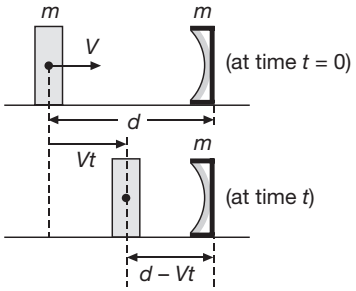
mirror together with its stand has a mass m. The block is pushed at $t = 0$ towards the mirror so that it starts moving towards the mirror at a constant speed V and collides with it. The collision is perfectly elastic. Find the velocity of the image

(a) at a time $t < \frac{d}{V}$

(b) at a time $t > \frac{d}{V}$

SOLUTION

(a) $t < \frac{d}{V}$



$$u = -(d - Vt)$$

$$f = -\frac{R}{2}$$

We know that $\vec{V}_{I/m} = -m^2\vec{V}_{O/m}$

Here, $m = \frac{f}{f-u}$

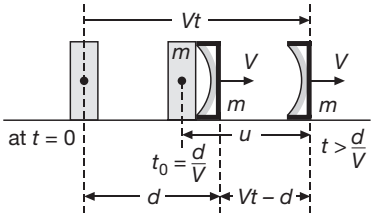
$$\Rightarrow\ m = \frac{-\frac{R}{2}}{-\frac{R}{2} + (d - Vt)} = \frac{-R}{2(d-Vt) - R}$$

So, velocity of image is $v_1 = -\left[\frac{-R}{2(d-Vt)-R}\right]^2 v$

$$\Rightarrow\ v_1 = \frac{R^2 V}{[2(d-Vt)-R]^2}$$

(b) $t > \frac{d}{V}$

Block will collide with mirror assembly after time $t_0 = \frac{d}{V}$. Applying Conservation of Linear Momentum, block and mirror assembly will exchange their momentum i.e., block will stop and mirror starts moving with velocity V. So, now



$$u = -(Vt - d)$$

$$\Rightarrow\ u = -V\left(t - \frac{d}{V}\right)$$

Since $m = \frac{f}{f+u}$

$$\Rightarrow\ m = \frac{-\frac{R}{2}}{-\frac{R}{2} + V\left(t - \frac{d}{V}\right)}$$

Also, we know that, $\vec{V}_{I/m} = -m^2\vec{V}_{O/m}$

$$\Rightarrow\ \vec{V}_I - \vec{V}_m = -m^2\left(\vec{V}_{O/m}\right)$$

Let us assume rightward direction as positive, then

$$v_1 - V = -m^2(-V)$$

$$\Rightarrow\ v_1 = (1 + m^2)V$$

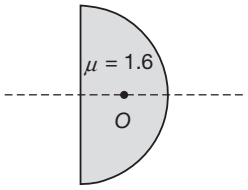
$$\Rightarrow\ v_1 = V\left[1 + \frac{R^2}{[2(Vt-d) - R^2]}\right]$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. A transparent hemisphere has a radius of curvature 8 cm and an index of refraction of 1.6. A small object O is placed on the axis halfway between the plane surface and the spherical surface i.e., 4 cm from each. The distance between the two images when viewed along the axis from the two sides of the hemisphere is approximately



(A) 7.5 cm (B) 8.5 cm
(C) 2.5 cm (D) 13.5 cm

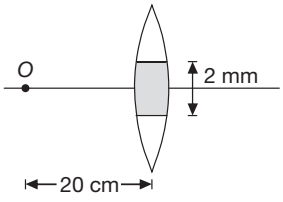
2. A square wire of side 3.0 cm is placed 25 cm in front of a concave mirror of focal length 10 cm with its centre on the axis of the mirror and its plane normal to the axis. The area enclosed by the image of the wire is

(A) 7.5 cm^2 (B) 6.0 cm^2
(C) 4.0 cm^2 (D) 3.0 cm^2

3. An object is placed at a distance $2f$ from the pole of a convex mirror of focal length f. The linear magnification is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$
(C) $\frac{3}{4}$ (D) 1

4. A convex lens of focal length 10 cm is painted black at the middle portion as shown in figure. An object is placed at a distance of 20 cm from the lens. Then



(A) the distance between the images is 2 mm
(B) the distance between the images is 4 mm
(C) the distance between the two images formed by such a lens is 6 mm
(D) only one image will be formed by the lens

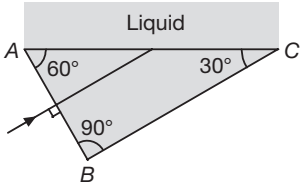
5. An object is placed at 20 cm from a convex mirror of focal length 20 cm. The distance of the image from the pole of the mirror is

(A) infinity (B) 10 cm
(C) 15 cm (D) 40 cm

6. A point object is placed at a distance of 25 cm from a convex lens of focal length 20 cm. When a glass slab of thickness t and refractive index 1.5 is inserted between the lens and the object, the image is formed at infinity. The thickness t of the slab is

(A) 5 cm (B) 10 cm
(C) 15 cm (D) 20 cm

7. Light is incident normally on face AB of a prism as shown in figure. A liquid of refractive index μ is placed on face AC of the prism. The prism is made of glass of refractive index $\frac{3}{2}$. The limits of μ for which total internal reflection takes place at the face AC is



(A) $\mu < \frac{\sqrt{3}}{2}$ (B) $\mu > \sqrt{3}$
(C) $\mu < \frac{3\sqrt{3}}{4}$ (D) $\mu > \frac{\sqrt{3}}{2}$

8. An object is placed in front of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cm, there is no parallax between the images formed by the two mirrors. The radius of curvature of the convex mirror is

(A) 60 cm (B) 50 cm
(C) 30 cm (D) 25 cm

9. A sharp image of an extended object which is placed perpendicular to the principal axis of a lens is η times that of the object for a particular position of object on a screen. Without disturbing the position of object and screen, by shifting the lens, a position can be obtained where the sharp image is $\frac{1}{\eta}$ times that of object. Ratio of difference between the two positions of lens to the focal length of lens is

(A) $\frac{\eta^2-1}{\eta}$ if $\eta>1$

(B) $\frac{\eta^2-1}{\eta}$ if $\eta<1$

(C) $\frac{\eta^2-1}{\eta}$ for all values of η

(D) η

10. A concave lens forms the image of an object such that the distance between the object and image is 10 cm and the magnification produced is $\frac{1}{4}$. The focal length of the lens will be

(A) 10 cm (B) 8.6 cm

(C) 6.2 cm (D) 4.4 cm

11. For a concave mirror, the magnification of a real image was found to be twice as great when the object was 15 cm from the mirror as it was when the object was 20 cm from the mirror. The focal length of the mirror is

(A) 5.0 cm (B) 7.5 cm

(C) 10 cm (D) 12.5 cm

12. The image formed by a convex mirror of focal length 20 cm is half the size of the object. The distance of the object from the mirror is

(A) 10 cm (B) 20 cm

(C) 30 cm (D) 40 cm

13. A concave mirror of focal length f in vacuum is placed in a medium of refractive index 2. Its focal length in the medium is

(A) $\frac{f}{2}$ (B) f

(C) $2f$ (D) $4f$

14. A spherical mirror forms an erect image three times the size of the object. If the distance between the object and the image is 80 cm, the nature and the focal length of the mirror are

(A) concave, 30 cm

(B) convex, 30 cm

(C) concave, 15 cm

(D) convex, 15 cm

15. A boy of height 1.5 m with his eye level at 1.4 m stands before a plane mirror of length 0.75 m fixed on the well. The height of the lower edge of the mirror above the floor is 0.8 m. Then

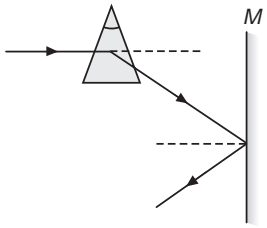
(A) the boy will see his full image.

(B) the boy cannot see his hair.

(C) the boy cannot see his feet.

(D) the boy cannot see both his hair and feet.

16. A horizontal ray of light passes through a prism of $\mu=1.5$ whose apex angle is $4°$ and then strikes a vertical mirror M as shown. For the ray, after reflection to become horizontal, the mirror must be rotated through an angle of



(A) $1°$ (B) $2°$

(C) $3°$ (D) $4°$

17. A man of height 1.6 m wishes to see his full image in a plane mirror placed at a distance of 2 m. The minimum length of the mirror should be

(A) 0.4 m (B) 0.8 m

(C) 1.6 m (D) 2.4 m

18. A ray of light falls on a plane mirror. When the mirror is turned, about an axis which is at right angle to the plane of the mirror through $30°$, the angle between the incident ray and new reflected ray is $45°$. The angle between the incident ray and original reflected ray was

(A) $60°$ (B) $30°$

(C) $60°$ or $30°$ (D) $45°$

19. A plane mirror reflects a beam of light to form a real image. The incident beam is

(A) parallel (B) convergent

(C) divergent (D) any one of the above

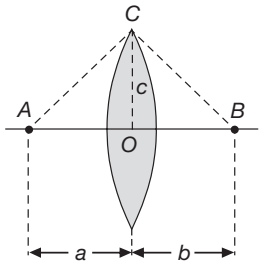
20. A plane mirror is approaching you at 10 cms^{-1}. You can see your image in it. The image will approach you with a speed

(A) 5 cms^{-1} (B) 10 cms^{-1}

(C) 15 cms^{-1} (D) 20 cms^{-1}

21. An object is placed at $A(OA>f)$, where, f is the focal length of the lens. The image is formed at B. A perpendicular is erected at O and C is chosen such

that $\angle BCA = 90°$. Then the value of f (in terms of a, b and c) is



(A) $\frac{(a+b)^3}{c^2}$ (B) $\frac{(a+b)c}{(a+c)}$

(C) $\frac{c^2}{a+b}$ (D) $\frac{a^2}{a+b+c}$

22. An observer moves towards a plane mirror with a speed of 2 ms^{-1}. The speed of the image with respect to the observer is

(A) 1 ms^{-1} (B) 2 ms^{-1}

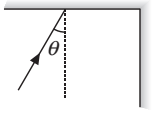
(C) 4 ms^{-1} (D) 8 ms^{-1}

23. A concave mirror of focal length f produces a real image n times the size of the object. The distance of the objet from the mirror is

(A) $(n-1)f$ (B) $(n+1)f$

(C) $\left(\frac{n+1}{n}\right)f$ (D) $\left(\frac{n-1}{n}\right)f$

24. Two plane mirrors are arranged at right angles to each other as shown in figure. A ray of light is incident on the horizontal mirror at an angle θ. The value of θ for which the ray emerges parallel to the incoming ray after reflection from the vertical mirror is



(A) 30° (B) 45°

(C) 60° (D) all of the above

25. A convex mirror of focal length f produces an image $\left(\frac{1}{n}\right)^{th}$ of the size of the object. The distance of the object from the mirror is

(A) nf (B) $\frac{f}{n}$

(C) $(n+1)f$ (D) $(n-1)f$

26. A real image formed by a concave mirror is 4.5 times the size of the object. If the mirror is 20 cm from the object, its focal length is

(A) $\frac{90}{11}$ cm (B) $\frac{120}{11}$ cm

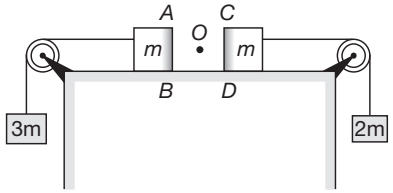
(C) $\frac{150}{11}$ cm (D) $\frac{180}{11}$ cm

27. An object is placed 10 cm in front of a convex mirror of focal length 20 cm. The distance of the image from the mirror is

(A) $\frac{10}{3}$ cm (B) $\frac{20}{3}$ cm

(C) 10 cm (D) $\frac{40}{3}$ cm

28. Two blocks each of mass m lie on a smooth table. They are attached to two other masses as shown in the figure. The pulleys and strings are light. An object O is kept at rest on the table. The sides AB and CD of the two blocks are made reflecting. The acceleration of two images formed in those two reflecting surfaces w.r.t. each other is



(A) $\frac{5g}{6}$ (B) $\frac{5g}{3}$

(C) $\frac{g}{3}$ (D) $\frac{17g}{6}$

29. A concave mirror forms the image of an object on a screen. If the lower half of the mirror is covered with an opaque card, the effect would be

(A) to make the image less bright.

(B) to make the lower half of the image disappear.

(C) to make the upper half of the image disappear.

(D) to make the image blurred.

30. Two plane mirrors are inclined at 70°. A ray incident on one mirror at angle θ, after reflection falls on the second mirror and is reflected from there parallel to the first mirror. θ is

(A) 45° (B) 50°

(C) 55° (D) 60°

31. A man stands in a room with his eyes at the centre of the room. The height of the ceiling is H. The length of the shortest plane mirror, fixed on the wall in front of the man, so that the man can see the full image of the wall behind him is

(A) $\frac{2H}{3}$ (B) $\frac{H}{2}$

(C) $\frac{H}{3}$ (D) $\frac{H}{4}$

32. An object is placed between two parallel mirrors. The number of images formed is
(A) 2 (B) 4
(C) 8 (D) infinite

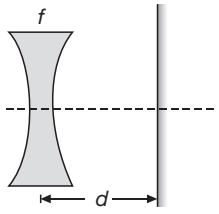
33. A bulb is placed between two plane mirrors inclined at an angle of 60°. The number of images formed is
(A) 5 (B) 6
(C) 4 (D) 3

34. Two plane mirrors are placed perpendicular to each other. A ray strikes one mirror and after reflection falls on the second mirror. The ray after reflection from the second mirror will be
(A) perpendicular to the original ray.
(B) parallel to the original ray.
(C) at 45° to the original ray.
(D) can be at any angle to the original ray.

35. A real image is formed by a convex lens, then it is brought in contact with a concave lens such that again a real image is formed. This image will
(A) remain in its original position
(B) shift towards the lens system
(C) shift away from the lens system
(D) shift to infinity

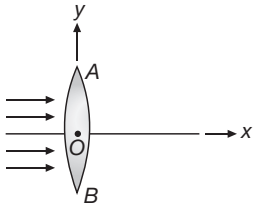
36. Plane mirrors A and B are kept at an angle θ with respect to each other. Light falls on A, is reflected, then falls on B and is reflected. The emergent ray is opposite to the incident direction. Then the angle θ is equal to
(A) 30° (B) 45°
(C) 60° (D) 90°

37. A diverging lens of focal length 10 cm is placed 10 cm in front of a plane mirror as shown in the figure. Light from a very far away source falls on the lens. The final image is at a distance



(A) 20 cm behind the mirror
(B) 7.5 cm in front of the mirror
(C) 7.5 cm behind the mirror
(D) 2.5 cm in front of the mirror

38. Monochromatic light rays parallel to x-axis strike a convex lens AB of refractive index 0.5. If the lens oscillates such that AB tilts upto a small angle θ (in radian) on either side of y-axis, then find the distance between extreme positions of oscillating image



(A) $f\sec\theta$
(B) $f\sec^2\theta$
(C) $f(\sec\theta-1)$
(D) The image will not move

39. A thin rod of length $\frac{f}{3}$ lies along the axis of a concave mirror of focal length f. One end of its image touches an end of the rod. The length of the image is
(A) f (B) $\frac{f}{2}$
(C) $2f$ (D) $\frac{f}{4}$

40. How many images will be formed if two mirrors are fitted on adjacent walls and one mirror on ceiling?
(A) 5 (B) 7
(C) 11 (D) 2

41. The wavefront that represents the light waves travelling in vacuum along the y-axis is
(A) $x+y+z=$ constant (B) $x=$ constant
(C) $y=$ constant (D) $z=$ constant

42. A boy stands straight in front of a mirror at a distance of 30 cm from it. He sees his erect image whose height is $\frac{1}{5}$ of his real height. The mirror he is using is
(A) plane (B) convex
(C) concave (D) plano-concave

43. The image of an object placed in front of a concave mirror of focal length 12 cm is formed at a point which is 10 cm more distant from the mirror that the object. The magnification of the image is
(A) 1.5 (B) 2
(C) 2.5 (D) 3

44. The minimum value of the refractive index for a $90°-45°-45°$ prism which is used to deviate a beam through 90° by total internal reflection is

(A) $\frac{5}{\sqrt{3}}$ (B) $\sqrt{2}$

(C) $\frac{3}{2}$ (D) $\sqrt{3}$

45. An object is moving towards a concave mirror of focal length 24 cm. When it is at a distance of 60 cm from the mirror its speed is $9\ \text{cms}^{-1}$. The speed of its image at that instant, is

(A) $4\ \text{cms}^{-1}$ towards the mirror

(B) $9\ \text{cms}^{-1}$ towards the mirror

(C) $4\ \text{cms}^{-1}$ away from the mirror

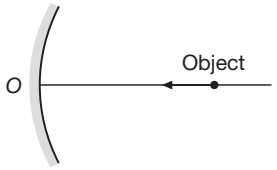
(D) $9\ \text{cms}^{-1}$ away from the mirror

46. A ray of light passes through an equilateral prism such that the angle of emergence is equal to the angle of incidence and each is equal to $\left(\frac{3}{4}\right)^{\text{th}}$ of the angle of prism. The angle of deviation is

(A) 45° (B) 39°

(C) 20° (D) 30°

47. A point object is moving along principal axis of a concave mirror with uniform velocity towards pole. Initially the object is at infinite distance from pole on right side of the mirror as shown in the figure. Before the object collides with mirror, the number of times at which the distance between object and its image is 40 cm are



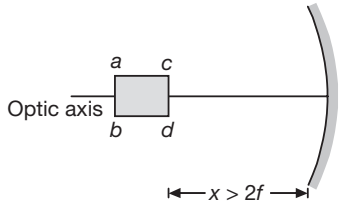
(A) One time

(B) Two times

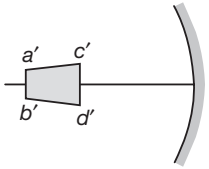
(C) Three times

(D) Data insufficient

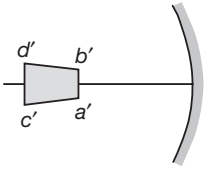
48. An object is placed in front of a concave mirror of focal length f as shown in figure. The correct shape of the image is represented by



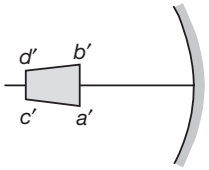
(A)



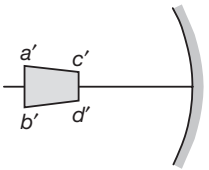
(B)



(C)



(D)



49. The index of refraction of diamond is 2.0. Velocity of light in diamond in cms^{-1} is approximately

(A) 6×10^{10} (B) 3×10^{10}

(C) 2×10^{10} (D) 1.5×10^{10}

50. A plane mirror is placed at origin parallel of y-axis, facing the positive x-axis. An object starts from $(2, 0, 0)$ m with a velocity of $\left(2\hat{i} + 2\hat{j}\right)\text{ms}^{-1}$. The relative velocity of image with respect to object is along

(A) positive x-axis (B) positive y-axis

(C) negative x-axis (D) negative y-axis

51. A ray of light passes from vacuum into a medium of refractive index n. If the angle of incidence is twice the angle of refraction, then the angle of incidence is

(A) $\cos^{-1}\left(\frac{n}{2}\right)$ (B) $\sin^{-1}\left(\frac{n}{2}\right)$

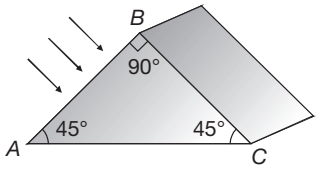
(C) $2\cos^{-1}\left(\frac{n}{2}\right)$ (D) $2\sin^{-1}\left(\frac{n}{2}\right)$

52. A point of source of light is placed at the bottom of a vessel containing a liquid of refractive index $\frac{5}{3}$. A person is viewing the source from above the surface. There is an opaque disc of radius 1 cm floating on the surface. The centre of the disc lies vertically above the source. The liquid from the vessel is gradually drained out through a tap. The maximum height of the liquid for which the source cannot be seen at all from above is

(A) $\frac{3}{2}$ cm (B) $\frac{4}{3}$ cm

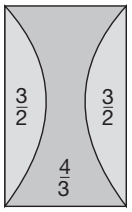
(C) $\frac{2}{3}$ cm (D) $\frac{3}{4}$ cm

53. A beam of light consisting of red, green and blue colours is incident on a right-angled prism as shown. The refractive index of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will



(A) separate part of the red colour from the green and blue colours.
(B) separate part of the blue colour from the red and green colours.
(C) separate all the three colours from one another.
(D) not separate even partially any colour from the other two colours.

54. Two plano-convex lenses each of focal length 10 cm and refractive index $\frac{3}{2}$ are placed as shown. Water $\left(\mu = \frac{4}{3}\right)$ is filled in the space between the two lenses. The whole arrangement is in air. The optical power of the system in diopters is



(A) 6.67 (B) –6.67 (C) 33.3 (D) 20

55. Total internal reflection of a ray of light is possible when the ray goes from
(A) denser to rarer medium and the angle of incidence is greater than the critical angle.
(B) denser to rarer medium and the angle of incidence is less than the critical angle.
(C) rarer to denser medium and the angle of incidence is greater than the critical angle.
(D) rarer to denser medium and the angle of incidence is less than the critical angle.

56. The critical angle of light going from medium A into medium B is θ. The speed of light in medium A is v. The speed of light in medium B is

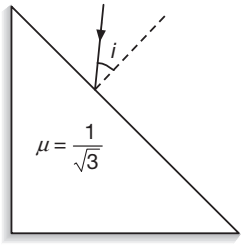
(A) $\frac{v}{\sin\theta}$ (B) $v\sin\theta$

(C) $\frac{v}{\tan\theta}$ (D) $v\tan\theta$

57. Glass has refractive index $\frac{3}{2}$ and water has refractive index $\frac{4}{3}$. If the speed of light in glass is 2.00×10^8 ms^{-1}, the speed of light in water in ms^{-1} is

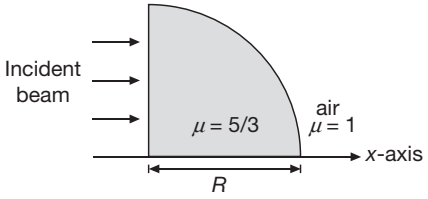
(A) 1.50×10^8 (B) 1.78×10^8
(C) 2.25×10^8 (D) 2.67×10^8

58. Two sides of an isosceles right prism are coated with a reflecting coating. A ray of light falls on the hypotenuse at an arbitrary angle i. The value of i for which the ray leaving the prism is parallel to the incident ray is



(A) 30°
(B) 60°
(C) 45°
(D) any arbitrary angle from $0 < i < \frac{\pi}{2}$

59. A uniform, horizontal parallel beam of light is incident on a quarter cylinder, of radius 5 cm having refractive index $\frac{5}{3}$. The width of the region at which the incident rays after normal incidence on plane surface and subsequent refraction at curved surface intersect the x axis is (Neglect the ray which travels along x-axis)



(A) 4 cm (B) $\frac{5}{4}$ cm
(C) $\frac{9}{4}$ cm (D) $\frac{25}{4}$ cm

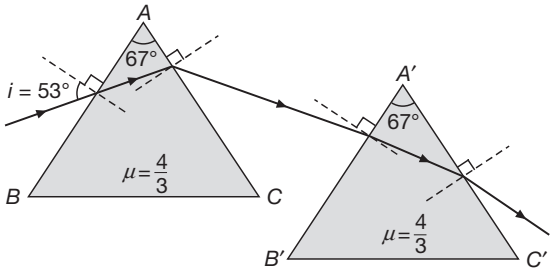
60. A diver in a lake wants to signal his distress to a person sitting on the edge of the lake flashing his water proof torch. He should direct the beam
(A) vertically upwards.
(B) horizontally.
(C) at an angle to the vertical which is slightly less than the critical angle.
(D) at an angle to the vertical which is slightly more than the critical angle.

61. Critical angle of light passing from a glass to water is minimum for

(A) red colour (B) green colour
(C) yellow colour (D) violet colour

62. Mirage is observed in a desert due to the phenomenon of
(A) interference
(B) total internal reflection
(C) scattering
(D) double refraction

63. A ray is incident on the first prism at an angle of incidence $53°$ as shown in the figure. The angle between side CA and $B'A'$ for the net deviation by both the prisms to be double of the deviation produced by the first prism, will be

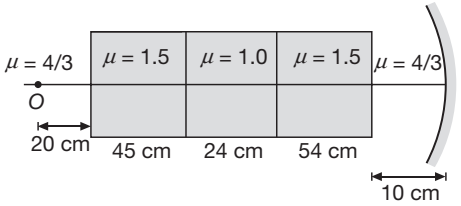


(A) $\sin^{-1}\left(\frac{2}{3}\right)+53°$ (B) $\sin^{-1}\left(\frac{2}{3}\right)+37°$
(C) $\cos^{-1}\left(\frac{2}{3}\right)+53°$ (D) $2\sin^{-1}\left(\frac{2}{3}\right)$

64. The distances of an object and its virtual image from the focus of a convex lens of focal length f are 1 cm each, then f is
(A) $(2+\sqrt{2})$ cm (B) $(\sqrt{2}+1)$ cm
(C) $2\sqrt{2}$ cm (D) 4 cm

65. Total internal reflection can occur when light tends to pass from
(A) a denser to a rarer medium.
(B) a rarer to a denser medium.
(C) one medium to another of different refractive index irrespective of which medium has greater refractive index.
(D) one medium to another of equal refractive index.

66. A composite slab consisting of different media is placed in front of a concave mirror of radius of curvature 150 cm. The whole arrangement is placed in water. An object O is placed at a distance 20 cm from the slab. The refractive indices of different media are given in the diagram shown in figure. The final image formed by the system lies



(A) to the left of object
(B) at the object
(C) To the right of object
(D) Data insufficient to arrive at a conclusion

67. A ray incident at an angle of incidence $60°$ enters a glass sphere of refractive index $\mu=\sqrt{3}$. This ray is reflected and refracted at the farther surface of the sphere. The angle between reflected and refracted rays at this surface is
(A) $40°$ (B) $60°$
(C) $70°$ (D) $90°$

68. A water film is formed on a glass block. A light ray is incident on water film from air at an angle 60°. What is the angle of incidence on glass block?
(Refractive Index of Glass = 1.5, Refractive Index of Water $= 4/3$)
(A) $\sin^{-1}\left(\frac{3\sqrt{3}}{8}\right)$ (B) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(C) $\sin^{-1}\left(\frac{4\sqrt{3}}{9}\right)$ (D) $\sin^{-1}\left(\frac{9\sqrt{3}}{16}\right)$

69. A stone lies at the bottom of a stream. A boy wants to hit it with a stick. Taking aim the boy holds the stick in the air at an angle of 45°. At what distance from the stone will the stick hit the bottom, if the depth is 32 cm (given ${}^{a}\mu_{w}=4/3$)
(A) 8 cm (B) 12 cm
(C) 16 cm (D) $12\sqrt{2}$ cm

70. When the surface of the lake is calm, a fish submerged in water will see the entire out-side world within inverted cone whose apex is situated at the eye of the fish and the cone subtends an angle of
(A) $10°$ (B) $60°$
(C) $98°$ (D) $30°$

71. A ray of light strikes a glass slab of thickness t. It emerges on the opposite face, parallel to the incident ray but laterally displaced. The lateral displacement is Δx.
(A) $\Delta x=0$ (B) $\Delta x=t\sin(i-r)\cos r$
(C) $\Delta x=\frac{t\sin i}{\cos r}$ (D) $\Delta x=\frac{t\sin(i-r)}{\cos r}$

72. In cold countries the phenomenon of looming (i.e. ship appears in the sky) takes place because
(A) refractive index of air decreases with height.
(B) refractive index of air increases with height.
(C) refractive index does not change with height.
(D) refractive index becomes infinity at the surface.

73. If D is the deviation of a normally falling light beam on a thin prism of angle A and δ is the dispersive power of the same prism then
(A) D is independent of A.
(B) D is independent of refractive index.
(C) δ is independent of refractive index.
(D) δ is independent of A.

74. For an equilateral prism, it is observed that when a ray strikes grazingly at one face it emerges grazingly at the other. Its refractive index will be
(A) $\frac{\sqrt{3}}{2}$ (B) $\sqrt{\frac{3}{2}}$
(C) 2 (D) $\sqrt{2}$

75. A rectangular block of glass (refractive index $3/2$) is kept in water (refractive index $4/3$). The critical angle for total internal reflection is
(A) $\sin^{-1}\left(\frac{8}{9}\right)$ for a ray of light passing from glass to water.
(B) $\sin^{-1}\left(\frac{8}{9}\right)$ for a ray of light passing from water to glass.
(C) $\sin^{-1}\left(\frac{2}{3}\right)$ for a ray of light passing from water to glass.
(D) $\sin^{-1}\left(\frac{8}{9}\right)$ for a ray of light passing from glass to air.

76. The refractive index of a given piece of transparent quartz is greatest for
(A) red light (B) violet light
(C) green light (D) yellow light

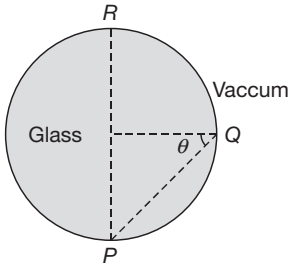
77. A well cut diamond appears bright because
(A) it emits light
(B) it is radioactive
(C) of total internal reflection
(D) of dispersion

78. The maximum refracting angle of a prism of refractive index 2 is
(A) 30^o (B) 45^o
(C) 60^o (D) 90^o

79. When light passes from one medium to another, the physical quantity that remains unchanged is
(A) velocity (B) wavelength
(C) frequency (D) None of these

80. A monochromatic beam of light passes from a denser to a rarer medium. As a result its
(A) velocity increases (B) velocity decreases
(C) frequency decreases (D) frequency increases

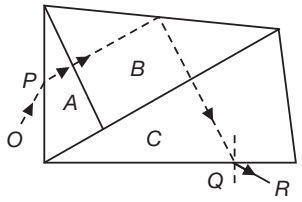
81. It is found that all electromagnetic signals sent from P towards Q reach point R. The speed of electromagnetic signals in glass can not be



(A) 1.0×10^8 ms^{-1} (B) 2.4×10^8 ms^{-1}
(C) 2×10^7 ms^{-1} (D) 4×10^7 ms^{-1}

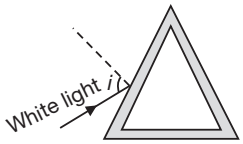
82. A number of images of a candle flame are seen in a thick mirror
(A) the first image is the brightest.
(B) the second image is the brightest.
(C) the last image is the brightest.
(D) all images are equally bright.

83. Three glass prisms A, B and C of same refractive index are placed in contact with each other as shown in figure with no air gap between the prisms. Monochromatic ray of light OP passes through the prism assembly and emerges as QR. The condition of minimum deviation is satisfied in the prisms



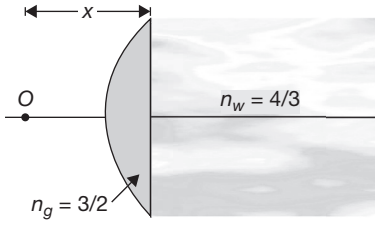
(A) A and C
(B) B and C
(C) A and B
(D) in all prisms A, B and C

84. A beam of white light is incident on a hollow prism of glass. Then



(A) the light emerging from prism gives no spectrum.
(B) the light emerging from prism gives spectrum but the bending of all colours is away from base.
(C) the light emerging from prism gives spectrum, all the colours bend towards base, the violet most and red the least.
(D) the light emerging from prism gives spectrum, all the colours bend towards base, the violet the least and red the most.

85. An object O is kept in air in front of a thin plano-convex lens of radius of curvature 10 cm. It's refractive index is $\frac{3}{2}$ and the medium towards right of plane surface is water of refractive index $\frac{4}{3}$. What should be the distance x of the object so that the rays become parallel finally.



(A) 5 cm (B) 10 cm
(C) 20 cm (D) 40 cm

86. If the critical angle for the medium of a prism is C and the angle of prism is A, then there will be no emergent ray when
(A) $A < 2C$ (B) $A = 2C$
(C) $A > 2C$ (D) $A \le 2C$

87. The angle of a prism is 60°. What is the angle of incidence for minimum deviation? The refractive index of the material of the prism is $\sqrt{2}$.
(A) 45° (B) 60°
(C) 30° (D) $\sin^{-1}\left(\frac{2}{3}\right)$

88. A ray of light is incident at angle i on one surface of a prism of small angle A and emerges normally from the opposite surface. If the refractive index of the material of the prism is μ, the angle of incidence i is nearly equal to
(A) $\frac{A}{\mu}$ (B) $\frac{A}{2\mu}$
(C) μA (D) $\frac{\mu A}{2}$

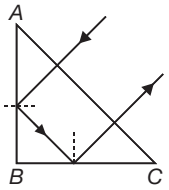
89. If ${}^{i}\mu_{j}$ represents the refractive index when a ray of light goes from medium i to medium j, then the product ${}^{2}\mu_{1} \times {}^{3}\mu_{2} \times {}^{4}\mu_{3}$ is equal to
(A) ${}^{3}\mu_{1}$ (B) ${}^{3}\mu_{2}$
(C) $\frac{1}{{}^{1}\mu_{4}}$ (D) ${}^{4}\mu_{2}$

90. An air bubble inside a glass slab $\left(\mu = \frac{3}{2}\right)$ appears to be 6 cm deep when viewed from one side and 4 cm deep when viewed from the opposite side. The thickness of the slab is
(A) 10 cm (B) 6.67 cm
(C) 15 cm (D) None of the above

91. The refracting angle of a prism is A and the refractive index of the material of the prism is $\cot\left(\frac{A}{2}\right)$. The angle of minimum deviation is
(A) $180° - 3A$ (B) $180° + 2A$
(C) $90° - A$ (D) $180° - 2A$

92. The angle of a prism is 30°. The rays incident at 60° at one refracting face suffer a deviation of 30°. The angle of emergence is
(A) 0° (B) 30°
(C) 60° (D) 90°

93. A ray falls on a prism $ABC(AB = BC)$ and travels as shown in the figure. The minimum refractive index of the prism material should be



(A) $\frac{4}{3}$ (B) $\sqrt{2}$
(C) $\frac{3}{2}$ (D) $\sqrt{3}$

94. Critical angle is minimum when a light ray passes from
(A) air to glass (B) glass to air
(C) glass to water (D) water to glass

95. A point source of light is placed 4 m below the surface of a liquid of refractive index $\frac{5}{3}$. The minimum diameter of a disc, which should be placed over the source, on the surface of the liquid to cut off all light coming out of water, is

(A) ∞ (B) 6 m
(C) 4 m (D) 3 m

96. A man standing in a swimming pool looks at a stone lying at the bottom. The depth of the swimming pool is h. At what distance from the surface of water is the image of the stone formed? Line of vision is normal. Refractive index of water is n.

(A) $\frac{h}{n}$ (B) $\frac{n}{h}$
(C) h (D) hn

97. The path of a refracted ray of light in a prism is parallel to the base of the prism only when the
(A) light is of a particular wavelength.
(B) ray is incident normally at one face.
(C) ray undergoes minimum deviation.
(D) prism is made of a particular type of glass.

98. A convex lens forms a real image three times larger than the object on a screen. The object and screen are moved until the image becomes twice the size of the object. If the shift of the object is 6 cm then the screen has to be shifted by

(A) 9 cm (B) 18 cm
(C) 36 cm (D) 72 cm

99. A vessel of depth d is half filled with a liquid of refractive index μ_1 and the other half is filled with a liquid of refractive index μ_2. The apparent depth of the vessel, when looked at normally, is

(A) $d(\mu_1+\mu_2)$ (B) $d\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)$
(C) $\frac{d}{2}(\mu_1+\mu_2)$ (D) $\frac{d}{2}\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)$

100. Two point sources S_1 and S_2 are 24 cm apart. Where should a convex lens of focal length 9 cm be placed in between them so that the images of both sources are formed at the same place?

(A) 6 cm from S_1 (B) 10 cm from S_1
(C) 12 cm from S_1 (D) 15 cm from S_1

101. Light travels through a glass plate of thickness t and having refractive index n. If c is the velocity of light in vacuum, the time taken by light to travel this thickness of glass is

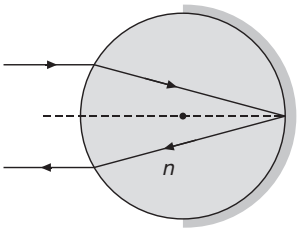
(A) $\frac{t}{nc}$ (B) $\frac{t}{n^2c}$
(C) $\frac{nt}{c}$ (D) $\frac{n^2t}{c}$

102. A fish looking up through the water sees the outside world contained in a circular horizon. If the refractive index of water is $\frac{4}{3}$ and the fish is 12 cm below the surface of water, the radius of the circle in cm is

(A) $36\sqrt{5}$ (B) $36\sqrt{7}$
(C) $\frac{36}{\sqrt{7}}$ (D) $4\sqrt{5}$

103. A diver inside water sees the setting sun at
(A) 41° to the horizon (B) 49° to the horizon
(C) 0° to the horizon (D) 45° to the horizon

104. A transparent cylinder has its right half polished so as to act as a mirror. A paraxial light ray is incident from left, that is parallel to principal axis, exits parallel to the incident ray as shown. The refractive index n of the material of the cylinder is



(A) 1.2 (B) 1.5
(C) 1.8 (D) 2.0

105. If the refractive index of water is $\frac{4}{3}$ and that of glass is $\frac{5}{3}$, then the critical angle of incidence for light tending to go from glass to water is

(A) $\sin^{-1}\left(\frac{3}{4}\right)$ (B) $\sin^{-1}\left(\frac{3}{5}\right)$
(C) $\sin^{-1}\left(\frac{4}{5}\right)$ (D) $\sin^{-1}\left(\frac{2}{3}\right)$

106. Two media A and B of refractive indices $\mu_1 = 1.5$ and $\mu_2 = 2$ are separated by x-z plane. A ray of light travels from A to B. The incident ray and the reflected ray are represented by unit vectors $\vec{u}_1 = a\hat{i} + b\hat{j}$ and $\vec{u}_2 = c\hat{i} + d\hat{j}$. Then

(A) $\frac{a}{c}=\frac{3}{4}$ (B) $\frac{a}{c}=\frac{4}{3}$

(C) $\frac{b}{d}=\frac{3}{4}$ (D) $\frac{b}{d}=\frac{4}{3}$

107. The speed of light in medium A is 2.0×10^8 ms^{-1} and that in medium B is 2.4×10^8 ms^{-1}. The critical angle of incidence for light tending to go from medium A to medium B is

(A) $\sin^{-1}\left(\frac{5}{12}\right)$ (B) $\sin^{-1}\left(\frac{5}{6}\right)$

(C) $\sin^{-1}\left(\frac{2}{3}\right)$ (D) $\sin^{-1}\left(\frac{3}{4}\right)$

108. The speed of light in glass of refractive index 1.5 is 2×10^8 ms^{-1}. In a certain liquid the speed of light is 2.5×10^8 ms^{-1}. The refractive index of the liquid is

(A) 0.64 (B) 0.80

(C) 1.20 (D) 1.44

109. A ray of light travelling inside a rectangular glass block of refractive index $\sqrt{2}$ is incident on the glass-air surface at an angle of incidence of 45°. The refractive index of air is 1. The ray will

(A) emerge into air without any deviation.

(B) be reflected back into glass.

(C) be absorbed.

(D) emerge into air with an angle of refraction equal to 90°.

110. A fish in water sees an object which is 24 cm above the surface of water. The height of the object above the surface of water that will appear to the fish is

(A) 24 cm (B) 32 cm

(C) 18 cm (D) 48 cm

111. The angle of minimum deviation equals the angle of prism A of an equilateral glass prism. The angle of incidence at which minimum deviation will be obtained is

(A) $\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)$ (B) 30°

(C) 60° (D) 45°

112. Light is incident at an angle α on one planar end of a transparent cylindrical rod of refractive index n. The least value of n for which the light entering the rod will not emerge from the curved surface of rod, irrespective of value of α is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$

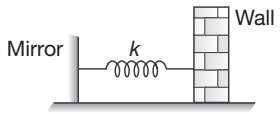
(C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{3}$

113. For a prism the refractive index (μ) is related to wavelength (λ) as $\mu=A+\frac{B}{\lambda^2}$. The dispersive power is large if

(A) A is large (B) B is large

(C) μ is large (D) A and μ are large

114. A plane mirror having a mass m is tied to the free end of a massless spring of spring constant k. The other end of the spring is attached to a wall. The spring with the mirror held vertically to the floor on which it can slide smoothly. When the spring is at its natural length, the mirror is found to be moving at a speed of v cms^{-1}. The separation between the images of a man standing before the mirror, when the mirror is in its extreme positions



(A) $v\sqrt{\frac{m}{k}}$ (B) $\frac{v}{2}\sqrt{\frac{m}{k}}$

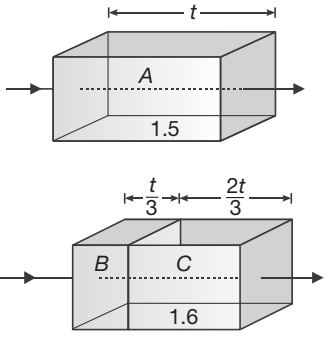
(C) $2v\sqrt{\frac{m}{k}}$ (D) $4v\sqrt{\frac{m}{k}}$

115. An infinitely long rod lies along the axis of a concave mirror of focal length f. The near end of the rod is at a distance $u>f$ from the mirror. The length of the image of the rod is

(A) $\frac{uf}{u+f}$ (B) $\frac{f^2}{u+f}$

(C) $\frac{f^2}{u-f}$ (D) $\frac{uf}{u-f}$

116. Two transparent slabs have the same thickness as shown in figure. One is made of material X of refractive index 1.5. The other is made of two materials Y and Z having thicknesses in the ratio 1 : 2. The refractive index of Z is 1.6. If a monochoromatic parallel beam passing through the slabs has the same number of wavelengths inside both, the refractive index of Y is



(A) 1.1 (B) 1.2
(C) 1.3 (D) 1.4

117. A curved mirror of focal length f (in vacuum) is placed in a medium of refractive index 2. Its new focal length in the medium is f'.

(A) $f' < f$ (B) $f' > f$
(C) $f' = f$ (D) $f' \cong f$

118. If ε_0 is the absolute permittivity of free space, μ_0 is absolute permeability of free space, ε is the permittivity of medium, μ is permeability of medium and n is the refractive index of medium then,

(A) $n = \sqrt{\frac{\mu_0\mu}{\varepsilon_0\varepsilon}}$ (B) $n = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$

(C) $n = \sqrt{\frac{\mu_0\varepsilon_0}{\mu\varepsilon}}$ (D) $n = \frac{\mu\varepsilon}{\mu_0\varepsilon_0}$

119. The critical angle of glass $\left(\mu_g = \frac{3}{2}\right)$ is θ_1 and that of water $\left(\mu_w = \frac{4}{3}\right)$ is θ_2. The critical angle for water-glass interface is

(A) less than θ_1 (B) less than θ_2
(C) between θ_1 and θ_2 (D) greater than θ_2

120. Two plane mirrors M_1 and M_2 are inclined to each other at $70°$. A ray incident on the mirror M_1 at an angle θ falls on M_2 and is then reflected parallel to M_1 for

(A) $\theta = 45°$ (B) $\theta = 50°$
(C) $\theta = 55°$ (D) $\theta = 60°$

121. An object is placed at 20 cm from a convex mirror of focal length 20 cm. The distance of the image from the pole of the mirror is

(A) infinite (B) 10 cm
(C) 15 cm (D) 40 cm

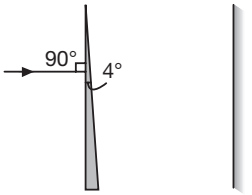
122. The sun (diameter D) subtends an angle θ radian at the pole of a concave mirror of focal length f. The diameter of the image of the sun formed by the mirror is

(A) $f\theta$ (B) $2f\theta$
(C) $\frac{2f\theta}{D}$ (D) $D\theta$

123. Inside a solid glass sphere of radius R, a point source of light is embedded at a distance $x(<R)$ from centre of the sphere. The solid sphere is surrounded by air of refractive index 1. The maximum angle of incidence for rays incident on the spherical glass-air interface directly from the point source is

(A) $\cos^{-1}\left(\frac{x}{R}\right)$ (B) $\sin^{-1}\left(\frac{x}{R}\right)$
(C) $\cos^{-1}\left(\sqrt{\frac{x}{R}}\right)$ (D) $\sin^{-1}\left(\sqrt{\frac{x}{R}}\right)$

124. A prism having an apex angle $4°$ and refractive index 1.5 is located in front of a vertical plane mirror as shown in figure. The total angle through which the ray is deviated after reflection from the mirror is given by

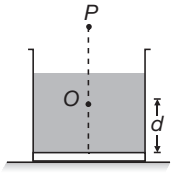


(A) $176°$ (B) $4°$
(C) $178°$ (D) $2°$

125. A slab of glass of thickness 3 cm and refractive index $\frac{3}{2}$ is placed with its face perpendicular to the principal axis of the concave mirror. If the radius of mirror is 10 cm, the distance at which an object must be placed from the mirror so that the image coincides with the object is

(A) 9 cm (B) 10 cm
(C) 11 cm (D) 12 cm

126. A tank contains a transparent liquid of refractive index n the bottom of which is made of a mirror as shown. An object O lies at a height d above the mirror. A person P vertically above the object sees O and its image in the mirror and finds the apparent separation to be



(A) $2nd$ (B) $\dfrac{2d}{n-1}$

(C) $\dfrac{2d}{n}$ (D) $\dfrac{d}{n}(1+n)$

127. A ray of light enters an anisotropic medium from vacuum at grazing incidence. If θ is the angle made by the reflected ray inside the medium with the interface and $n(\theta)$ is the refractive index of the medium then,

(A) $n(\theta)\sin\theta = 1$ (B) $n(\theta)\cos\theta = 1$

(C) $\dfrac{n(\theta)}{\sin\theta} = 1$ (D) $\dfrac{n(\theta)}{\cos\theta} = 1$

128. A person runs with a speed u towards a bicycle moving away from him with speed v. The person approaches his image in the mirror fixed at the rear of bicycle with a speed of

(A) $u - v$ (B) $u - 2v$

(C) $2u - v$ (D) $2(u - v)$

129. Light travels through a glass plate of thickness t having refractive index μ. If c is the velocity of light in vacuum, the time taken by the light to travel this thickness of glass is

(A) $\dfrac{t}{\mu c}$ (B) $\dfrac{t}{\mu^2 c}$

(C) $\dfrac{\mu t}{c}$ (D) $\dfrac{\mu^2 t}{c}$

130. A real image I is formed by a converging lens L on its optic axis. On introduction of a rectangular glass slab of thickness d and refractive index μ between the image and lens the image displaces it by

(A) $d(\mu - 1)$ away from L

(B) $d(\mu - 1)$ towards L

(C) $d\left(1 - \dfrac{1}{\mu}\right)$ away from L

(D) $d\left(1 - \dfrac{1}{\mu}\right)$ towards L

131. When a ray is refracted from one medium to another, the wavelength changes from 6000 Å to 4000 Å. The critical angle for the interface will be

(A) $\cos^{-1}\left(\dfrac{2}{3}\right)$ (B) $\sin^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$

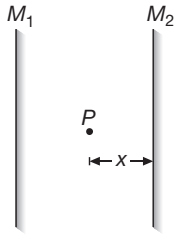
(C) $\sin^{-1}\left(\dfrac{2}{3}\right)$ (D) $\cos^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$

132. A boy stands straight in front of a mirror at a distance of 30 cm away from it. He sees his erect image whose height is one fifth of the original height. The mirror used by him is

(A) plane (B) convex

(C) concave (D) plano concave

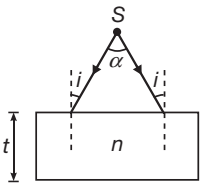
133. Two plane mirrors M_1 and M_2 are parallel to each other and 3 m part. A person P standing x metre from the right mirror M_2 looks into this mirror and sees a series of images. The distance between the first and second image is 4 m. Then the value of x is



(A) 4 m (B) 3 m

(C) 1 m (D) 2 m

134. A diverging beam of light from a point source S having divergence angle α, falls symmetrically on a glass slab as shown. The angles of incidence of two extreme rays are equal. If the thickness of the glass slab is t and the refractive index n, then the divergence angle of the emergent beam is



(A) Zero (B) α

(C) $\sin^{-1}\left(\dfrac{1}{n}\right)$ (D) $2\sin^{-1}\left(\dfrac{1}{n}\right)$

135. The light on reflection from a plane mirror can give a real image when

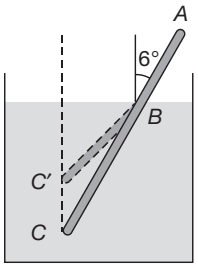
(A) the convergent rays are incident on the mirror.

(B) the divergent rays are incident on the mirror.

(C) an object is placed very close to the mirror.

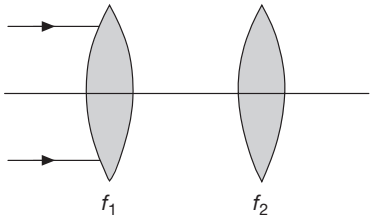
(D) an object is placed very far away from the mirror.

136. A small rod ABC is put in water making an angle $6°$ with vertical. If it is viewed paraxially from above, it will look like bent shaped ABC'. The angle of bending $(\angle CBC')$ will be in degree is $\left(n_w = \frac{4}{3}\right)$.



(A) $2°$ (B) $3°$
(C) $4°$ (D) $4.5°$

137. Parallel beam of light is incident on the system of two convex lenses of focal length $f_1 = 20$ cm and $f_2 = 10$ cm. The distance between the two lenses, so that rays after refraction from both the lenses pass undeviated is



(A) 30 cm (B) 40 cm
(C) 60 cm (D) 90 cm

138. The plane faces of two identical plano convex lenses, each with focal length f are pressed against each other using an optical glue to form a usual convex lens. The distance from the optical centre at which an object must be placed to obtain the image same as the size of object is

(A) $\frac{f}{4}$ (B) $\frac{f}{2}$
(C) f (D) $2f$

139. A parallel beam of light incident on a concave lens of focal length 10 cm emerges as a parallel beam from a convex lens placed coaxially, the separation between the lenses being 10 cm. The focal length of the convex lens in cm is

(A) 10 (B) 20
(C) 15 (D) 30

140. A ray of light is incident on a glass sphere of refractive index $\frac{3}{2}$. The angle of incidence for which a ray that enters the sphere does not come out of the sphere is

(A) $\tan^{-1}\left(\frac{2}{3}\right)$ (B) $\sin^{-1}\left(\frac{2}{3}\right)$
(C) $45°$ (D) $90°$

141. A thin prism P_1 of angle $4°$ and made from glass of refractive index 1.54, is combined with another thin prism P_2 made from a glass with refractive index 1.72, to produce dispersion without deviation. The angle of P_2 is

(A) $5.33°$ (B) $4°$
(C) $3°$ (D) $2.6°$

142. A transparent sphere of radius R made of material of refractive index $\frac{3}{2}$ is kept in air. The distance from the centre of the sphere must a point object be placed so as to form a real image at the same distance from the sphere is

(A) R (B) $2R$
(C) $3R$ (D) $4R$

143. An air bubble in water is to be placed in a way such that a real image is obtained at the same distance from bubble. Taking $\mu_{\text{water}} = \frac{4}{3}$ we have the distance of object from the air bubble as

(A) R
(B) $2R$
(C) $3R$
(D) An air bubble is incapable to form a real image.

144. An object is placed at a distance of 12 cm from a convex lens on its principal axis and a virtual image of certain size is formed. On moving the object 8 cm away from the lens, a real image of the same size as that of virtual image is formed. The focal length of the lens in cm is

(A) 15 (B) 16
(C) 17 (D) 18

145. An air bubble inside a glass slab appears to be 6 cm deep when viewed from one side and 4 cm deep when viewed from the other side. Assuming $\mu_{\text{glass}} = \frac{3}{2}$, the thickness of slab is

(A) 10 cm (B) $\frac{20}{3}$ cm

(C) 15 cm (D) 20 cm

146. On two sides of an oily paper screen, two bulbs A and B are placed at a distance of 20 cm and 30 cm, so that equal intensity is obtained on both sides of screen. If P_A and P_B be the powers of the bulbs A and B respectively then $\frac{P_A}{P_B}$ is

(A) 0.44 (B) 2.25

(C) 1.5 (D) 0.67

147. An achromatic combination pair of a telescope objective will be

(A) lenses of $f = -50$ cm and power +2 D

(B) lenses of powers 3 D and –5 D respectively

(C) lenses of $f = +20$ cm and power –4.5 D

(D) lenses of $f = +40$ cm and power +2 D

148. A ray of light enters the face of a glass prism of refracting angle A, refractive index μ at an angle of incidence i. It is observed that no ray emerges from the other face. For this the minimum value of i should be

(A) $\mu \sin A - \cos A$

(B) $\sin^{-1}(\sin A - \mu \cos A)$

(C) $\sin^{-1}\left[\sqrt{\mu^2 - 1}\sin A - \cos A\right]$

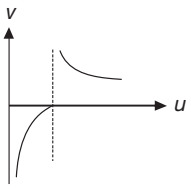
(D) $\sqrt{\mu^2 - 1}\sin A - \cos A$

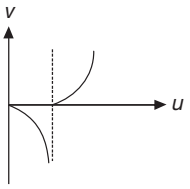
149. A beaker containing liquid is placed on the table underneath a microscope which can be moved along a vertical scale. The microscope is focussed, through the liquid onto a mark on the table when the reading on the scale is a. It is next focussed on the upper surface of liquid and the reading is b. More liquid is added and the observations are repeated. The corresponding readings are c and d. The refractive index of liquid is

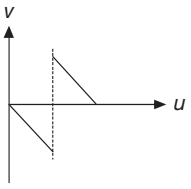
(A) $\frac{d-b}{d-c-b+a}$ (B) $\frac{d-c-b+a}{d-b}$

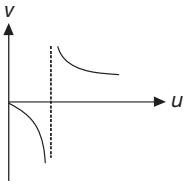
(C) $\frac{b-d}{d-c-b+a}$ (D) $\frac{d-c-b+a}{b-d}$

150. As the position of an object (u) reflected from a concave mirror is varied, the position of the image (v) also varies. By allowing the u to change from 0 to $+\infty$, the graph between v versus u will be

(A) 

(B) 

(C) 

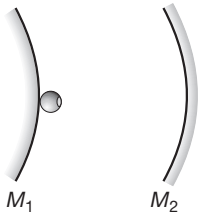
(D) 

151. A parallel beam of light emerges from the opposite surface of the sphere when a point source of light lies at the surface of the sphere. The refractive index of the sphere is

(A) $\frac{3}{2}$ (B) $\frac{5}{3}$

(C) 2 (D) $\frac{5}{2}$

152. Two spherical mirrors M_1 and M_2, one convex and other concave having same radius of curvature R are arranged coaxially at a distance $2R$ (consider their pole separation to be $2R$). A bead of radius a is placed at the pole of the convex mirror as shown. The ratio of the sizes of the first three images of the bead is



(A) 1 : 2: 3 (B) $1 : \frac{1}{2} : \frac{1}{3}$

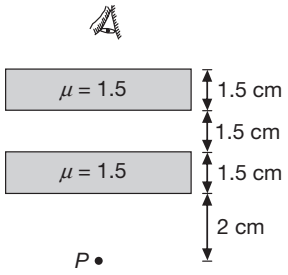
(C) $\frac{1}{3} : \frac{1}{11} : \frac{1}{41}$ (D) 3 : 11 : 41

153. A ray of light is incident on one face of prism with refracting angle $A\,(< 90°)$. The incident ray is normal to the other face of the prism. If C is the critical angle for prism-air interface, then the ray will emerge from this face only if

(A) $\cot C < \cot A + 1$ (B) $\cot C > \cot A + 1$

(C) $\cot A < \cot C + 1$ (D) $\cot A > \cot C + 1$

154. The image of point P when viewed from top of the glass slabs is



(A) 2 cm above P (B) 0.5 cm above P

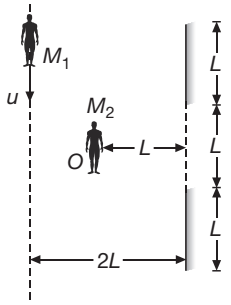
(C) 0.5 cm below P (D) 1 cm above P

155. An isosceles prism has refracting angle A. Its one face is silvered (other than the base). A ray of light falling normally on the face not silvered emerges through the base of the prism normal to it.

(A) $A = 45°$ (B) $A = 90°$

(C) $A = 36°$ (D) $A = 72°$

156. Two plane mirrors of length L are separated by distance L and a man M_2 is standing at distance L from the connecting line of mirrors as shown in figure.

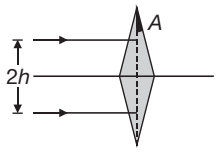


A man M_1 is walking in a straight line at distance $2L$ parallel to mirrors at speed u, then man M_2 at O will be able to see image of M_1 for total time

(A) $\frac{4L}{u}$ (B) $\frac{3L}{u}$

(C) $\frac{6L}{u}$ (D) $\frac{9L}{u}$

157. Two identical thin isosceles prisms of refracting angle A and refractive index μ are placed with their bases touching each other and this system can collectively act as a crude converging lens. A parallel beam of light is incident on this system as shown. The focal length of this so called converging lens is



(A) $f = \frac{h}{\mu A}$ (B) $f = \frac{h}{A}$

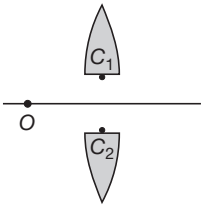
(C) $f = \frac{h}{(\mu - 1)A}$ (D) $f = \frac{\mu h}{\mu - 1}$

158. Rays of light from a luminous object are brought to focus at a point A. The rays are intercepted, before meeting at A by a convex lens of focal length 20 cm placed at 24 cm from A and are forced to meet at B. Then AB equals (in cm)

(A) 12 (B) 24

(C) 6 (D) 48

159. A point object is placed at a distance of 0.3 m from a convex lens of focal length 0.2 m cut into two equal halves, each of which is displaced by 0.0005 m, as shown in figure. If C_1 and C_2 be their optical centres then,



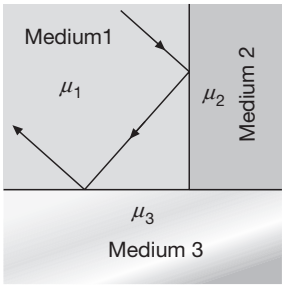
(A) an image is formed at a distance of 0.6 m from C_1 or C_2 along principal axis.

(B) two images are formed, one at a distance of 0.6 m and other at a distance of 1.2 m from C_1 or C_2 along principal axis.

(C) an image is formed at a distance of 0.12 m from C_1 or C_2 along principal axis.

(D) two images are formed at a distance of 0.6 m from C_1 or C_2 along principal axis at a separation of 0.003 m.

160. In the figure shown, light is incident on the interface between medium 1 (refractive index μ_1) and 2 (refractive index μ_2) at angle slightly greater than the critical angle, and is totally reflected. The light is then also totally reflected at the interface between medium 1 and 3 (refractive index μ_3), after which it travels in a direction opposite to its initial direction. The medium must have a refractive indices such that



(A) $\mu_1 < \mu_2 < \mu_3$ (B) $\mu_1^2 - \mu_3^2 > \mu_2^2$

(C) $\mu_1^2 - \mu_2^2 < \mu_3^2$ (D) $\mu_1^2 + \mu_2^2 > \mu_3^2$

161. All of the following statements are correct except that
(A) the magnification produced by a convex mirror is always less than one.
(B) a virtual, erect, same sized image can be obtained by using the plane mirror.
(C) a virtual, erect, magnified image can be formed by using the concave mirror.
(D) a real, inverted, same sized image can be formed by using a convex mirror.

162. A ray of light undergoes deviation of $30°$ when incident on an equilateral prism of refractive index $\sqrt{2}$. The angle made by the ray inside the prism with the base of the prism is

(A) $0°$ (B) $15°$

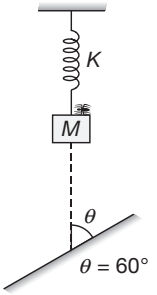
(C) $30°$ (D) $45°$

163. A convex lens of focal length f forms an image of a heavenly body. The area of the image formed is proportional to

(A) f^0 (B) f^1

(C) f^2 (D) f^3

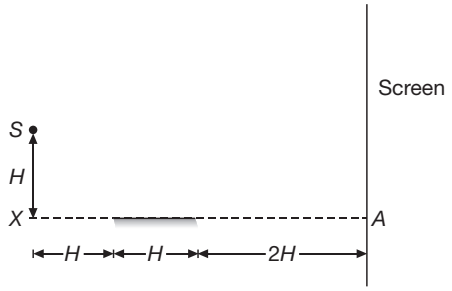
164. An insect of negligible mass is sitting on a block of mass M, tied with a spring of force constant k. The block performs simple harmonic motion with amplitude A in front of a plane mirror placed as shown in figure. The maximum speed of insect relative to its image will be



(A) $A\sqrt{\frac{k}{M}}$ (B) $\frac{A\sqrt{3}}{2}\sqrt{\frac{k}{M}}$

(C) $A\sqrt{3}\sqrt{\frac{k}{M}}$ (D) $2A\sqrt{\frac{M}{k}}$

165. A point source has been placed as shown in the figure. The length on the screen that will receive reflected light from the mirror is



(A) $2H$ (B) $3H$

(C) $4H$ (D) H

166. A plano convex lens has a thickness of 4 cm. When placed on a horizontal table with curved surface in contact with it, the apparent depth of the bottom most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of plane face is found to be $\frac{25}{8}$ cm. The focal length of the lens is

(A) 50 cm (B) 75 cm

(C) 100 cm (D) 150 cm

167. If an object is placed between two parallel mirrors, an infinite number of images are formed. If the mirrors are at a distance $2b$ and an object is placed at the middle of the two mirrors, the distance of the nth image from the object is

(A) nb (B) $\frac{1}{2}nb$

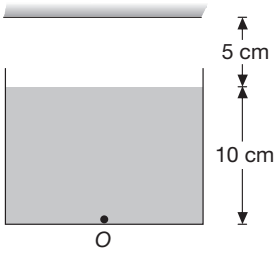
(C) $2nb$ (D) $\frac{1}{4}nb$

168. A ray of light is incident on the plane mirror at rest. The mirror starts turning at a uniform acceleration of 2π rads^{-2}. The reflected ray, at the end of $\frac{1}{4}$ s must have turned through

(A) $90°$ (B) $45°$

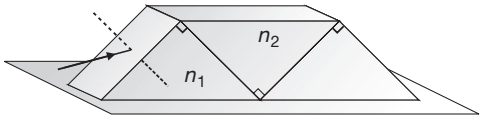
(C) $22.5°$ (D) $11.25°$

169. In the situation shown in figure, water $\left(\mu_w = \frac{4}{3}\right)$ is filled in a beaker upto a height of 10 cm. A plane mirror is fixed at a height of 5 cm from the surface of water. Distance of image from the mirror after reflection from it of an object O at the bottom of the beaker is



(A) 7.5 cm (B) 10 cm
(C) 12.5 cm (D) 15 cm

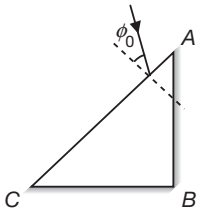
170. Three right angled prisms of refractive indices n_1, n_2 and n_3 are fixed together using an optical glue as shown in figure. If a ray passes through the prisms without suffering any deviation, then



(A) $n_1 = n_2 = n_3$ (B) $n_1 = n_2 \neq n_3$
(C) $1 + n_1 = n_2 + n_3$ (D) $1 + n_2^2 = n_1^2 + n_3^2$

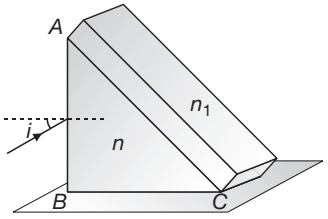
171. Four lenses are made from same type of glass. The radius of curvature of each face is given . Out of these, the lens having the greatest positive power is
(A) 10 cm convex and 15 cm convex.
(B) 20 cm convex and 30 cm concave.
(C) 15 cm convex and plane.
(D) 5 cm convex and 10 cm concave.

172. The sides of an isosceles right angled prism are silvered. A ray of light falls on the hypotenuse of the prism at an angle ϕ_0 as shown. The ray leaving the prism will



(A) graze the face AC.
(B) emerge normally to the face AC.
(C) be parallel to the incident ray.
(D) make an angle of 30° with incident ray.

173. A right angled prism (45°-90°-45°) of refractive index n has a plate of refractive index $n_1 (n_1 < n)$ cemented to its diagonal face. The assembly is in air. A ray is incident on AB as shown. If the ray strikes the diagonal face AC at critical angle then



(A) $\sin i = \left(\frac{n_1}{n}\right)$

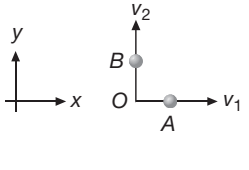
(B) $\sin i = \left(\frac{n}{n_1}\right)$

(C) $\sin i = \sqrt{\frac{n^2 - n_1^2}{2}}$

(D) $\sin i = \frac{\sqrt{n^2 - n_1^2} - n_1}{\sqrt{2}}$

174. A fish in near the centre of a spherical fish bowl filled with water of refractive index $\frac{4}{3}$. A child stands in air at a distance $2R$ (R is the radius of curvature of the sphere) from the centre of the bowl. At what distance from the centre would the child's nose appear to the fish situated at the centre
(A) R (B) $2R$
(C) $3R$ (D) $4R$

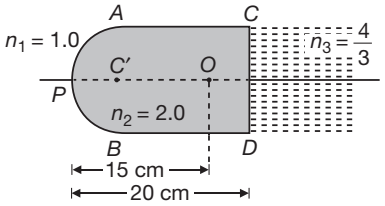
175. Two particles A and B of mass m_1 and m_2 respectively start moving from O with speeds v_1 and v_2. A moves towards the plane mirror and B moves parallel to mirror horizontally. The mirror is in y-z plane. The absolute-speed of image of centre of mass of the system (image of A + image of B) is



(A) Zero (B) $\frac{m_1 v_1}{m_2}$

(C) $\frac{m_2 v_2}{m_1}$ (D) $\frac{\sqrt{m_1^2 v_1^2 + m_2^2 v_2^2}}{m_1 + m_2}$

176. The slab of a material of refractive index 2 shown in figure has a curved surface APB of radius of curvature 10 cm and a plane surface CD. On the left of APB is air and on the right of CD is water with refractive indices as given in figure. An object O is placed at a distance of 15 cm from pole P as shown. The distance of the final image of O from P, as viewed from the left is



(A) 20 cm (B) 30 cm
(C) 40 cm (D) 50 cm

177. A thin rod of length $\frac{1}{3}f$ is placed along the optic axis of a concave mirror of focal length f such that its image which is real and elongated just touches the rod. The magnification is

(A) $\frac{4}{3}$ (B) $\frac{5}{3}$

(C) $\frac{3}{2}$ (D) None of above

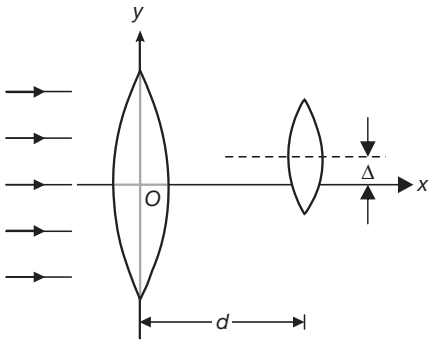
178. A 2 cm diameter coin lies flat at the bottom of a bowl in which the water $\left(\mu_w = \frac{4}{3}\right)$, is 20 cm deep. If the coin is viewed directly from above, the apparent diameter of the coin is

(A) 1.67 cm (B) 1.5 cm
(C) 2.67 cm (D) 2 cm

179. A ray of light undergoes deviation of 30° when incident on an equilateral prism of refractive index $\sqrt{2}$. The angle made by the ray inside the prism with the base of the prism is

(A) 30° (B) 45°
(C) 60° (D) 0°

180. Two thin convex lenses of focal lengths f_1 and f_2 are separated by a horizontal distance d (where $d < f_1$ and $d < f_2$) and their centres are displaced by a vertical separation Δ as shown. Taking the origin of coordinates O, at the centre of the first lens, the x and y coordinates of the focal point of this lens system, for a parallel beam of rays coming from the left, are given by



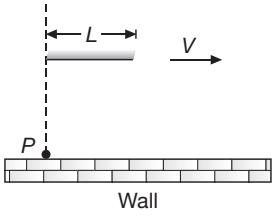
(A) $x = \frac{f_1 f_2}{f_1 + f_2}$, $y = \Delta$

(B) $x = \frac{f_1(f_2 + d)}{f_1 + f_2 - d}$, $y = \frac{\Delta^2}{f_1 + f_2}$

(C) $x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$, $y = \frac{\Delta(f_1 - d)}{f_1 + f_2 - d}$

(D) $x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$, $y = 0$

181. The mirror of length L moves horizontally as shown in the figure with a velocity v. The mirror is illuminated by a point source of light P placed on the ground. The rate at which the length of the light spot on the ground increases is



(A) v (B) zero
(C) $2v$ (D) $3v$

182. A prism has a refractive index $\sqrt{\frac{3}{2}}$ and refracting angle 90°. Find the minimum deviation produced by the prism.

(A) 40° (B) 45°
(C) 30° (D) 49°

183. A circular beam of light of diameter $d = 2$ cm falls on a plane surface of a glass slab. The angle of incidence is $60°$ and refractive index of glass is $\mu = \frac{3}{2}$. The diameter of the refracted beam is

(A) 2.52 cm (B) 3 cm
(C) 3.26 cm (D) 4 cm

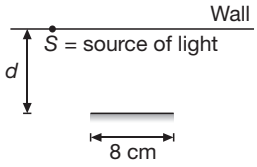
184. Two thin lenses, when in contact, produce a combination of power $+10$ D. When they are 0.25 m apart, the power reduces to $+6$ D. The focal lengths of the lenses (in m) are

(A) 0.125 and 0.5 (B) 0.125 and 0.125
(C) 0.5 and 0.75 (D) 0.125 and 0.75

185. A luminous point object is moving along the principal axis of a concave mirror of focal length 12 cm towards it. When its distance from the mirror is 20 cm its velocity is $4\ \text{cms}^{-1}$. The velocity of the image (in cms^{-1}) at that instant is

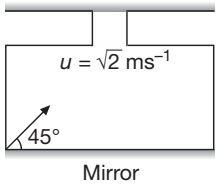
(A) 6, towards the mirror
(B) 6, away from the mirror
(C) 9, away from the mirror
(D) 9, towards the mirror

186. A plane mirror of length 8 cm is present near a wall in situation as shown in figure. The length of spot formed on the wall is



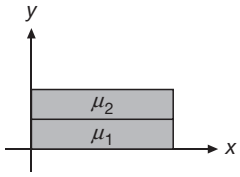
(A) 4 cm (B) 8 cm
(C) 16 cm (D) 32 cm

187. An elevator at rest which is at tenth floor of a building is having a plane mirror fixed to its floor. A particle is projected with a speed $\sqrt{2}\ \text{ms}^{-1}$ and at $45°$ with the horizontal as shown in the figure. At the very instant of projection, the cable of the elevator breaks and the elevator starts falling freely. The separation between the particle and its image, 0.5 s after the instant of projection is



(A) 0.5 m (B) 1 m
(C) 1.5 m (D) 2 m

188. Two thin slabs of refractive indices μ_1 and μ_2 are placed parallel to each other in the x-z plane. If the direction of propagation of a ray in the two media are along the vectors $\vec{r}_1 = a\hat{i} + b\hat{j}$ and $\vec{r}_2 = c\hat{i} + d\hat{j}$ then we have



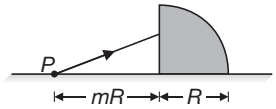
(A) $\mu_1 a = \mu_2 b$

(B) $\frac{\mu_1 a}{\sqrt{a^2+b^2}} = \frac{\mu_2 a}{\sqrt{c^2+d^2}}$

(C) $\mu_1(a^2+b^2) = \mu_2(c^2+d^2)$

(D) None of these

189. A quarter cylinder of radius R and refractive index 1.5 is placed on a table. A point object P is kept at a distance of mR from it. The value of m for which a ray from P will emerge parallel to the table is

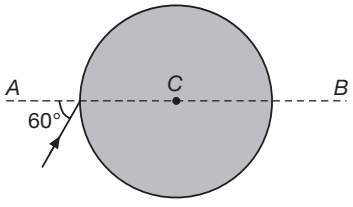


(A) $\frac{1}{3}$ (B) $\frac{2}{3}$
(C) 1 (D) $\frac{4}{3}$

190. A light ray is incident on a prism of angle $A = 60°$ and refractive index $\mu = \sqrt{2}$. The angle of incidence at which the emergent ray grazes the surface is given by

(A) $\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$ (B) $\sin^{-1}\left(\frac{1-\sqrt{3}}{2}\right)$
(C) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (D) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$

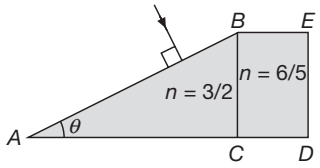
191. A ray of light falls on a transparent sphere of refractive index μ, having centre at C as shown in figure. The ray emerges from the sphere parallel to line AB, then



(A) $\mu = \sqrt{2}$ (B) $\mu = \frac{3}{2}$

(C) $\mu = \sqrt{3}$ (D) $\mu = 2$

192. In the figure ABC is the cross section of a right angled prism and $BCDE$ is the cross section of a glass slab. The value of θ so that light incident normally on the face AB does not cross the face BC is (given $\sin^{-1}\left(\frac{3}{5}\right) = 37°$)



(A) $\theta \le 37°$ (B) $\theta < 37°$

(C) $\theta \le 53°$ (D) $\theta < 53°$

193. The curvature radii of a concavo-convex glass lens are 20 cm and 60 cm. The convex surface of the lens is silvered. With the lens horizontal, the concave surface is filled with water. The focal length of the effective mirror is (μ of glass $= 1.5$, μ of water $= \frac{4}{3}$)

(A) $\frac{90}{13}$ cm (B) $\frac{80}{13}$ cm

(C) $\frac{20}{3}$ cm (D) $\frac{45}{8}$ cm

194. A beaker is filled with water as shown in Figure I. The bottom surface of the beaker is a concave mirror of large radius of curvature and small aperture. The height of water is $h = 40$ cm. It is found that when an object is placed 4 cm above the water surface, the image coincides with the object. Now the water level h is reduced to zero but there will still be some water left in the concave part of the mirror as shown in Figure II. The new height of the object h' above the new water surface so that the image again coincides with the object, will be (Refractive index of water $= \frac{4}{3}$)

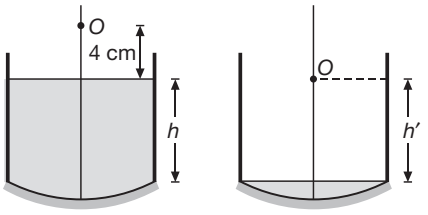


Figure I Figure II

(A) 34 cm (B) 10 cm

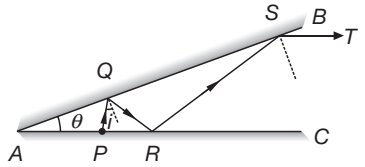
(C) 74 cm (D) Zero

195. Two identical equiconvex lenses of focal length f, made of glass $\left(\mu_g = \frac{3}{2}\right)$ are kept in contact. The space between the two lenses is filled with water $\left(\mu_w = \frac{4}{3}\right)$. The focal length of the combination is

(A) $\frac{f}{2}$ (B) $\frac{3f}{4}$

(C) f (D) $\frac{4f}{3}$

196. Two plane mirrors AB and AC are inclined at an angle $\theta = 20°$. A ray of light starting from point P is incident at point Q on the mirror AB, then at R on mirror AC and again on S on AB. Finally the ray ST goes parallel to mirror AC. The angle which the ray makes with the normal at point Q on mirror AB is



(A) 20° (B) 30°

(C) 40° (D) 60°

197. A thin lens of focal length f and its aperture has a diameter d. It forms an image of intensity I. Now the central part of the aperture upto diameter $\left(\frac{d}{2}\right)$ is blocked by an opaque paper. The focal length and image intensity would change to

(A) $\frac{f}{2}, \frac{I}{2}$ (B) $f, \frac{I}{4}$

(C) $\frac{3f}{4}, \frac{I}{2}$ (D) $f, \frac{3I}{4}$

198. A certain prism is found to produce a minimum deviation of $38°$. It produces a deviation of $44°$ when the angle of incidence is either $42°$ or $62°$. What will be the angle of incidence when it undergoes minimum deviation?

(A) 45° (B) 49°

(C) 40° (D) 55°

199. Critical angle for a prism is $36°$. The maximum angle of prism for which the emergent ray is possible is

(A) 18° (B) 36°

(C) 72° (D) 144°

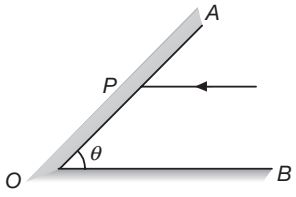
200. A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different positions separated from each other by 20 cm . The lens has a focal length of

(A) 32.1 cm (B) 15.8 cm
(C) 21.4 cm (D) 10.7 cm

201. A prism, having refractive index $\sqrt{2}$ and refracting angle 30°, has one of the refracting surfaces polished. A beam of light incident on the other refracting surface will retrace its path if the angle of incidence is

(A) 0° (B) 30°
(C) 45° (D) 60°

202. Two plane mirrors are inclined at angle θ as shown in figure. If a ray parallel to OB strikes the other mirror at P and finally emerges parallel to OA after two reflections, then θ equals



(A) 30° (B) 45°
(C) 60° (D) 90°

203. An object is placed 20 cm in front of a block of glass 10 cm thick having its farther side silvered. The image is formed 23.2 cm behind the silvered face. The refractive index of glass is

(A) 1.41 (B) 1.46
(C) 1.51 (D) 1.61

204. Two parallel rays are travelling in a medium of refractive index $\mu_1 = \frac{4}{3}$. However, one of the rays passes through a parallel glass slab of thickness t and refractive index $\mu_2 = \frac{3}{2}$. The path difference between the two rays due to the glass slab is

(A) $\frac{t}{8}$ (B) $\frac{t}{6}$
(C) $\frac{4t}{3}$ (D) $\frac{3t}{2}$

205. A ray of light entering from air to glass (refractive index 1.5) is partly reflected and partly refracted. If the incident and the reflected rays are at right angles to each other, the angle of refraction is

(A) $\sin^{-1}\left(\sqrt{\frac{2}{3}}\right)$ (B) $\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)$
(C) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (D) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

206. A beam of light is converging towards a point on a screen. A plane parallel sided plate of glass of thickness t and refractive index μ is introduced in the path of the beam. The convergence point is shifted by

(A) $t\left(1-\frac{1}{\mu}\right)$ away (B) $t\left(1+\frac{1}{\mu}\right)$ away
(C) $t\left(1-\frac{1}{\mu}\right)$ nearer (D) $t\left(1+\frac{1}{\mu}\right)$ nearer

207. The distance of an object from the first focus of an equiconvex lens is 10 cm and the distance of its real image from the second focus is 40 cm . The focal length of the lens is

(A) 10 cm (B) 20 cm
(C) 25 cm (D) 40 cm

208. A beam of monochromatic light is incident on one face of an equilateral prism, the angle of incidence being 55°. If the angle of emergence is 46° then the angle of minimum deviation is

(A) 41° (B) $< 41°$
(C) $> 41°$ (D) $\geq 41°$

209. When a ray of light is refracted by a prism such that the angle of deviation is minimum, then

(A) the angle of emergence is equal to the angle of incidence.
(B) the angle of emergence is greater than the angle of incidence.
(C) the angle of emergence is smaller than the angle of incidence.
(D) the sum of the angle of incidence and the angle of emergence is equal to 90°.

210. The image of a square hole in a screen illuminated by light is obtained on another screen with the help of a converging lens. The distance of the hole from the lens is 40 cm . If the area of the image is nine times that of the hole, the focal length of the lens is

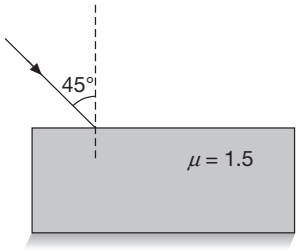
(A) 30 cm (B) 50 cm
(C) 60 cm (D) 75 cm

211. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror. The size of the image is approximately equal to

(A) $b\sqrt{\frac{u-f}{f}}$ (B) $b\left(\frac{u-f}{f}\right)^2$

(C) $b\sqrt{\frac{f}{u-f}}$ (D) $b\left(\frac{f}{\mu-f}\right)^2$

212. One side of a glass slab of refractive index 1.5 is silvered as shown. A ray of light is incident on the other side at angle of incidence $i = 45°$. The deviation of the ray of light from its initial path when it comes out of the slab is



(A) 45° (B) 90°

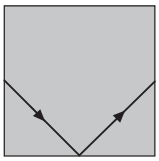
(C) 120° (D) 180°

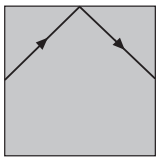
213. If f_B and f_R are the focal lengths of a convex lens for blue and red lights respectively and F_B and F_R are the respective values for a concave lens, then

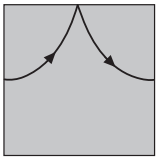
(A) $f_B > f_R$ and $F_B > F_R$ (B) $f_B < f_R$ and $F_B > F_R$

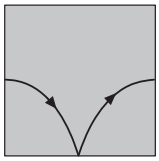
(C) $f_B > f_R$ and $F_B > F_R$ (D) $f_B < f_R$ and $F_B < F_R$

214. A cubic container is filled with a liquid whose refractive index increases linearly from top to bottom. The path of a ray of light inside the liquid is best represented by

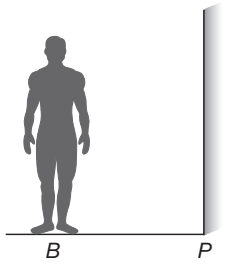
(A) 

(B) 

(C) 

(D) 

215. A person AB of height 170 cm is standing in front of a plane mirror. His eyes are at height 164 cm. At what distance from P should a hole be made in the mirror so that he cannot see the top of his head



(A) 167 cm (B) 161 cm

(C) 163 cm (D) None of these

216. The focal length of a convex lens is f and the distance of an object from the principal focus is x. The ratio of the size of the real image to the size of the object is

(A) $\frac{f}{x}$ (B) $\frac{x}{f}$

(C) $\frac{f+x}{f}$ (D) $\frac{f}{f+x}$

217. Focal length of a convex mirror is 10 cm

(A) image of an object placed at 20 cm is also at 20 cm

(B) image of an object placed at 10 cm is at infinity

(C) both (A) and (B) are correct

(D) both (A) and (B) are incorrect

218. An object is placed at a distance x_1 from the principal focus of a lens and its real image is formed at a distance x_2 from the principal focus. The focal length of the lens is

(A) x_1x_2 (B) $\frac{x_1x_2}{2}$

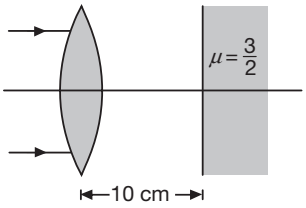
(C) $\frac{x_1+x_2}{2}$ (D) $\sqrt{x_1x_2}$

219. The plane faces of two identical planoconvex lenses, each having focal length of 40 cm, are pressed against each other to form a usual convex lens. The distance in cm from this lens, at which an object must be placed to obtain a real image with magnification unity is

(A) 10 (B) 20

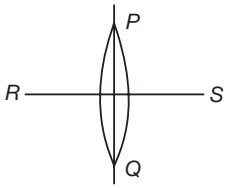
(C) 40 (D) 80

220. A plane refracting surface of refractive index $\frac{3}{2}$ is placed at a distance of 10 cm from a thin convex lens of focal length 30 cm. The parallel rays incident on lens will converge at a distance of



(A) 30 cm from the lens. (B) 25 cm from the lens.
(C) 20 cm from the lens. (D) 40 cm from the lens.

221. The figure shows an equiconvex lens of focal length f. If the lens is cut along PQ, the focal length of each half will be

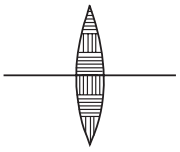


(A) $\frac{f}{2}$ (B) f
(C) $2f$ (D) $4f$

222. In PROBLEM 221, if the lens is cut along PQ and RS simultaneously, the focal length of each part will be

(A) $\frac{f}{2}$ (B) f
(C) $2f$ (D) $4f$

223. The layered lens as shown is made of two types of transparent materials-one indicated by horizontal lines and the other by vertical lines. The number of images formed of an object will be



(A) 1 (B) 2
(C) 3 (D) 6

224. The distance between an object and its real image formed by a convex lens cannot be

(A) greater than $2f$ (B) less than $2f$
(C) greater than $4f$ (D) less than $4f$

225. Two thin symmetrical lenses of different nature and of different material have equal radii of curvature $R = 15$ cm are placed close together and immersed in water $\left(\mu_w = \frac{4}{3}\right)$. The focal length of the system in water is 30 cm. The difference between refractive indices of the two lenses is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$
(C) $\frac{1}{4}$ (D) $\frac{3}{4}$

226. A needle of length 5 cm, placed 45 cm from a lens forms an image on a screen placed 90 cm on the other side of the lens. The type of lens and its focal length are

(A) convex, 30 cm (B) concave, 30 cm
(C) convex, 60 cm (D) concave, 60 cm

227. In PROBLEM 226, the nature and size of the image are

(A) real, 20 cm (B) real, 10 cm
(C) virtual, 20 cm (D) virtual, 10 cm

228. An object is placed 50 cm in front of a convex surface of radius 20 cm. If the surface separates air from glass of refractive index 1.5, the distance of the image from the lens and its nature are

(A) 30 cm, real (B) 30 cm, virtual
(C) 300 cm, real (D) 300 cm, virtual

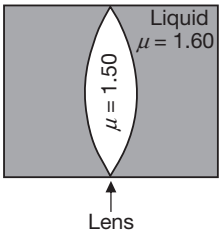
229. One of the refracting surfaces of a prism of angle 30° is silvered. A ray of light incident at an angle of 60° retraces its path. The refractive index of the material of prism is

(A) $\sqrt{2}$ (B) $\frac{3}{2}$
(C) $\sqrt{3}$ (D) 2

230. A slab of glass of refractive index 1.5 and thickness 3 cm is placed with the faces perpendicular to the principal axis of a concave mirror. If the radius of curvature of the mirror is 10 cm, the distance at which an object must be placed from the mirror so that the image coincides with the object is

(A) 9 cm (B) 10 cm
(C) 11 cm (D) 12 cm

231. Figure represents a convergent lens placed inside a cell filled with a liquid. The lens has a focal length +20 cm when in air and its material has refractive index 1.50. If the liquid has a refractive index 1.60, the focal length of the lens in the new system is



(A) –80 cm (B) +80 cm

(C) –160 cm (D) –24 cm

232. A point object O is placed on the principal axis of a convex lens of focal length 20 cm at a distance of 40 cm to the left of it. The diameter of the lens is 10 cm. If the eye is placed 60 cm to the right of the lens at a distance h below the principal axis, then the maximum value of h to see the image will be

(A) 0 cm (B) 5 cm

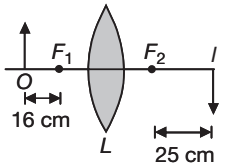
(C) 2.5 cm (D) 10 cm

233. For two positions of a lens, the images are obtained on a fixed screen. If the size of object is 2 cm and the size of diminished image is 0.5 cm, the size of the other image will be

(A) 1 cm (B) 4 cm

(C) 8 cm (D) 16 cm

234. The medium on both sides of lens is air. The distances of object O, image I from first and second foci F_1 and F_2 are shown in figure. The focal length of lens is



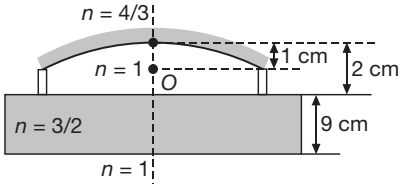
(A) 16 cm

(B) 25 cm

(C) 20 cm

(D) cannot be estimated with given data

235. A concave mirror of focal length 2 cm is placed on a glass slab as shown in the figure. Then the image of object O formed due to reflection at mirror and then refraction by the slab is



(A) virtual and will be at 2 cm from the pole of the concave mirror

(B) virtual and formed on the pole of the mirror

(C) real and on the object itself

(D) None of these

236. A plane mirror is moving with velocity $4\hat{i}+4\hat{j}+8\hat{k}$. A point object in front of the mirror moves with a velocity $3\hat{i}+4\hat{j}+5\hat{k}$. Here $\hat{k}$ is along the normal to the plane mirror and facing towards the object. The velocity of the image is

(A) $-3\hat{i}-4\hat{j}+5\hat{k}$ (B) $3\hat{i}+4\hat{j}+11\hat{k}$

(C) $-4\hat{i}+5\hat{j}+11\hat{k}$ (D) $7\hat{i}+9\hat{j}+3\hat{k}$

237. A mango tree is at the bank of a river and one of the branch of tree extends over the river. A tortoise lives in river. A mango falls just above the tortoise. The acceleration of the mango falling from tree appearing to the tortoise is (Refractive index of water is $\frac{4}{3}$ and the tortoise is stationary)

(A) g (B) $\frac{3g}{4}$

(C) $\frac{4g}{3}$ (D) $\frac{7g}{3}$

238. An equiconvex lens, having radius of curvature 33 cm, is placed on a horizontal plane mirror and a pin held 20 cm above the lens coincides with its image. Now the space between the lens and the mirror is filled with a liquid. In order to coincide with the image the pin has to be raised by 5 cm. The refractive index of the liquid is

(A) 1.33 (B) 1.53

(C) 2.33 (D) 2.66

239. A real image is formed by a convex lens. If we put a concave lens in contact with it, the combination again forms a real image. The new image

(A) is closer to the lens system.

(B) is farther from the lens system.

(C) is at the original position.

(D) may be anywhere depending on the focal length of the concave lens.

240. A concave mirror has a focal length 20 cm. The distance between the two positions of the object for which the image size is double of the object size is

(A) 60 cm (B) 40 cm

(C) 30 cm (D) 20 cm

241. A light ray gets reflected from a pair of mutually perpendicular mirrors, not necessarily along axes. The intersection point of mirrors is at origin. The incident light ray is along $y=x+2$. If the light ray strikes both mirrors in succession, then it may get reflected finally along the line

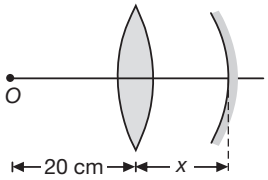
(A) $y=2x-2$ (B) $y=-x+2$

(C) $y=-x-2$ (D) $y=x-4$

242. Two lenses of powers +12 D and –2 D are in contact. The focal length of the combination is

(A) 10 cm (B) 12.5 cm
(C) 16.6 cm (D) 8.33 cm

243. A point object O is placed at a distance of 20 cm from a convex lens of focal length 10 cm as shown in figure. The distance x from the lens where a concave mirror of focal length 60 cm has to be placed so that final image coincides with the object is

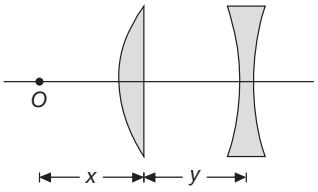


(A) 10 cm
(B) 20 cm
(C) 40 cm
(D) final image can never coincide with the object under the conditions provided.

244. A lens forms a sharp image on a screen. On inserting a parallel sided glass slab between the lens and the screen, it is found necessary to move the screen a distance d away from the lens in order for the image to be sharp again. If the refractive index of the material of the slab is n, the thickness of the slab is

(A) nd (B) $\frac{d}{n}$
(C) $\frac{(n-1)d}{n}$ (D) $\frac{nd}{n-1}$

245. A plano convex glass lens $\left(\mu_g = \frac{3}{2}\right)$ of radius of curvature $R = 10$ cm is placed at a distance of y from a concave lens of focal length 20 cm. The distance x of a point object O from the plano convex lens so that the position of final image is independent of y is



(A) 20 cm (B) 30 cm
(C) 40 cm (D) 60 cm

246. A thin lens has focal length f, and its aperture has diameter D. It forms an image of intensity I. If the central part of the aperture, of diameter $\frac{D}{2}$, is blocked by an opaque paper, the focal length of the lens and the intensity of image will become

(A) $\frac{f}{2}, \frac{I}{2}$ (B) $f, \frac{I}{4}$
(C) $\frac{3f}{4}, \frac{I}{2}$ (D) $f, \frac{3I}{4}$

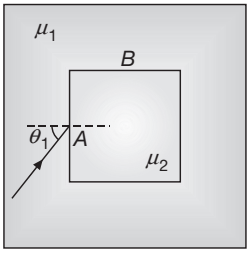
247. When a ray of light goes from air to a glass slab, then
(A) its wavelength increases
(B) its wavelength decreases
(C) its frequency increases
(D) neither its wavelength nor its frequency changes

248. In the displacement method, a convex lens is placed in between an object and a screen. If the magnification in the two positions are m_1 and m_2 $(m_1 > m_2)$, and the distance between the two positions of the lens is x, the focal length of the lens is

(A) $\frac{x}{m_1 + m_2}$ (B) $\frac{x}{m_1 - m_2}$
(C) $\frac{x}{(m_1 + m_2)^2}$ (D) $\frac{x}{(m_1 - m_2)^2}$

249. A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. The focal length of the lens is
(A) 10.7 cm (B) 21.4 cm
(C) 15.8 cm (D) 32.1 cm

250. A ray of light is incident on the left vertical face of a glass cube of refractive index μ_2, as shown in figure.



The plane of incidence is the plane of the page and the cube is surrounded by liquid of refractive index μ_1. What is the largest angle of incidence θ_1 for which total internal reflection occurs at the top surface is

(A) $\sin\theta_1 = \sqrt{\left(\frac{\mu_2}{\mu_1}\right)^2 - 1}$ (B) $\sin\theta_1 = \sqrt{\left(\frac{\mu_2}{\mu_1}\right)^2 + 1}$
(C) $\sin\theta_1 = \sqrt{\left(\frac{\mu_1}{\mu_2}\right)^2 + 1}$ (D) $\sin\theta_1 = \sqrt{\left(\frac{\mu_1}{\mu_2}\right)^2 - 1}$

251. A plano-convex lens, when silvered at its plane surface is equivalent to a concave mirror of focal length 28 cm . When its curved surface is silvered and the plane surface not silvered, it is equivalent to a concave mirror of focal length 10 cm , then the refractive index of the material of the lens is

(A) $\frac{9}{14}$ (B) $\frac{14}{9}$

(C) $\frac{17}{9}$ (D) None of these

252. A lens is placed between the source of light and a wall. It forms images of area A_1 and A_2 on the wall for its two different positions. The area of the source of light is

(A) $\sqrt{A_1A_2}$ (B) $\frac{A_1+A_2}{2}$

(C) $\sqrt{\left[\frac{\sqrt{A_1}+\sqrt{A_2}}{2}\right]^2}$ (D) $\left(\frac{1}{A_1}+\frac{1}{A_2}\right)^{-1}$

253. The plane face of a plano-convex lens is silvered. If μ be the refractive index and r the radius of curvature of the curved surface, then the system behaves like a concave mirror of radius

(A) $\frac{r}{\mu}$ (B) $\frac{r}{\mu-1}$

(C) $r\mu$ (D) $r(\mu-1)$

254. A ray of light falls on the surface of a spherical paper weight making an angle β with the normal and is refracted in the medium at an angle β. The angle of deviation of the emergent ray from the direction of the incident ray is

(A) $(\alpha-\beta)$ (B) $2(\alpha-\beta)$

(C) $\frac{(\alpha-\beta)}{2}$ (D) $\beta-\alpha$

255. The magnification of an object placed in front of a convex lens of focal length 20 cm is +2 . To obtain a magnification of −2 , the object has to be moved by a distance equal to

(A) 40 cm (B) 30 cm

(C) 20 cm (D) 10 cm

256. A concave mirror is placed on a horizontal table with its axis directed vertically upward. Let (O) be the pole of the mirror and C its centre of curvature. A point object is placed at C . It has a real image also located at C . If the mirror is now filled with water, the image will be

(A) real and will remain at C.
(B) real and located at a point between C and ∞.
(C) virtual and located at a point between C and O.
(D) real and located at a point between C and O.

257. All of the following statements are correct except

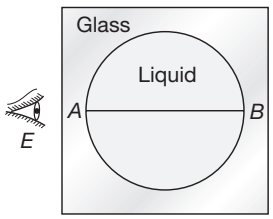
(A) the magnification produced by a convex mirror is always less than one.
(B) a virtual, erect, same-sized image can be obtained using a plane mirror.
(C) a virtual, erect, magnified image can be formed using a concave mirror.
(D) a real, inverted, same-sized image can be formed using a convex mirror.

258. When an object is at distances x and y from a lens, a real image and a virtual image is formed respectively having same magnification. The focal length of the lens is given by

(A) $x-y$ (B) $x+y$

(C) $\sqrt{xy}$ (D) $\frac{x+y}{2}$

259. Figure shows a spherical cavity in a solid glass block. The cavity is filled with a liquid and from outside an observer sees the distance AB which is the diameter of the cavity and it appear as infinitely large to the observer. If refractive index of liquid is μ_1 and that of glass is μ_2 , then $\frac{\mu_1}{\mu_2}$ is



(A) 2 (B) $\frac{1}{2}$

(C) 4 (D) None of these

260. If the central portion of a convex lens is wrapped in black paper as shown in the figure,

(A) no image will be formed by the remaining portion of the lens.
(B) full image will be formed, but it will be less bright.

(C) the central portion of the image will be missing.
(D) there will be two images, each produced by one of the exposed portions of the lens.

261. A plane mirror made of glass slab $(\mu_g = 1.5)$ is 2.5 cm thick and silvered at the back. A point object is placed 5 cm in front of the unsilvered face of the mirror. The position of the final image is

(A) $\frac{16}{3}$ cm from unsilvered face

(B) $\frac{25}{3}$ cm from unsilvered face

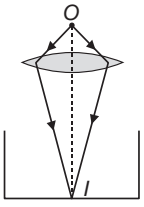
(C) 12 cm from unsilvered face

(D) 14 cm from unsilvered face

262. The distance between an object and the screen is 100 cm. A lens produces an image on the screen when placed at either of two positions 40 cm apart. The power of the lens is approximately

(A) 4.25 D (B) 4.50 D
(C) 4.75 D (D) 5.0 D

263. A real image of an object is formed by a convex lens at the bottom of an empty beaker. The beaker is now filled with a liquid of refractive index 1.4 to a depth of 7 cm. In order to get the image again at the bottom, the beaker should be moved



(A) downward by 2 cm
(B) upward by 2 cm
(C) downward by 3 cm
(D) upward by 3 cm

264. The convex surface of a thin concavo-convex lens (refractive index 1.5) has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface. At what distance from the lens should a pin be placed on the optic axis such that its image is formed at the same place?

(A) 15 cm (B) 7.5 cm
(C) 22.5 cm (D) 30 cm

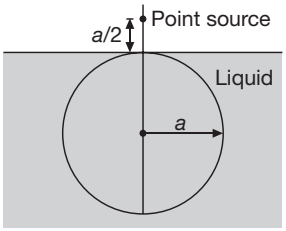
265. In PROBLEM 264, if the concave part is filled with water (refractive index $\frac{4}{3}$), the distance from the lens at which the pin should be placed to form the image at the same place is

(A) $\frac{90}{13}$ cm (B) $\frac{45}{13}$ cm
(C) $\frac{135}{13}$ cm (D) $\frac{180}{13}$ cm

266. An equiconvex lens of glass $(\mu_g = 1.5)$ of focal length 10 cm, silvered on one side behaves like a

(A) convex mirror of focal length 5 cm
(B) convex mirror of focal length 20 cm
(C) concave mirror of focal length 2.5 cm
(D) concave mirror of focal length 10 cm

267. An opaque sphere of radius a is just immersed in a transparent liquid as shown in figure. A point source is placed on the vertical diameter of the sphere at a distance $\frac{a}{2}$ from the top of the sphere. One ray originating from the point source after refraction from the air liquid interface forms tangent to the sphere. The angle of refraction for that particular ray is $30°$. The refractive index of the liquid is



(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{3}{\sqrt{5}}$
(C) $\frac{4}{\sqrt{5}}$ (D) $\frac{4}{\sqrt{7}}$

268. A plano-convex lens of focal length 30 cm has its plane surface silvered. An object is placed 40 cm from the lens on the convex side. The distance of the image from the lens is

(A) 18 cm (B) 24 cm
(C) 30 cm (D) 40 cm

269. Refraction takes place at a concave spherical boundary separating glass air medium. If $\mu_g = \frac{3}{2}$, then for the image to be real, the object distance

(A) is independent of the radius of curvature of the refracting surface
(B) should be greater than the radius of curvature of the refracting surface
(C) should be greater than two times the radius of curvature of the refracting surface
(D) should be greater than three times the radius of curvature of the refracting surface

270. A parallel beam of light incident on a concave lens of focal length 10 cm emerges as a parallel beam from a convex lens placed coaxially, the distance between the lenses being 10 cm. The focal length of the convex lens in cm is

(A) 10 (B) 20
(C) 30 (D) 40

271. An object is placed at a distance of 12 cm from a convex lens on its principal axis and a virtual image of certain size is formed. If the object is moved 8 cm away from the lens, a real image of the same size as that of the virtual image is formed. The focal length of the lens in cm is

(A) 15 (B) 16
(C) 18 (D) 19

272. A thin converging lens of refractive index 1.5 has a power of $+0.5\,D$. When this lens is immersed in a liquid, it acts as a diverging lens of focal length 100 cm. The refractive index of the liquid is

(A) $\frac{4}{3}$ (B) $\frac{3}{2}$
(C) $\frac{5}{3}$ (D) 2

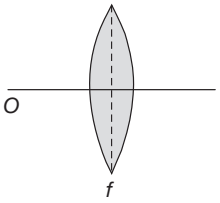
273. The distance between an object and its real image formed by a lens is D. If the magnification is m, the focal length of the lens is

(A) $\frac{(m-1)D}{m}$ (B) $\frac{mD}{m+1}$
(C) $\frac{(m-1)D}{m^2}$ (D) $\frac{mD}{(m+1)^2}$

274. A plano convex lens of focal length 16 cm, is to be made of glass of refractive index 1.5. The radius of curvature of the curved surface should be

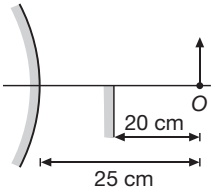
(A) 8 cm (B) 12 cm
(C) 16 cm (D) 24 cm

275. A real image of a point object O was formed by an equi-convex lens of focal length f and the magnification was found to be unity. Now the lens is cut into two symmetrical pieces as shown by the dotted line and the right part is removed. The position of the image formed by the remaining part is at



(A) f (B) $2f$
(C) $\frac{f}{2}$ (D) Infinity

276. In the figure, an object is placed 25 cm from the surface of a convex mirror, and a plane mirror is set so that the image formed by the two mirrors lie adjacent to each other in the same plane. The plane mirror is placed at 20 cm from the object. The radius of curvature of the convex mirror is

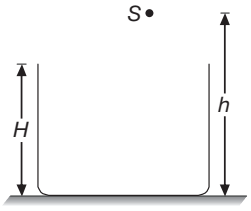


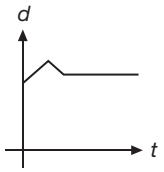
(A) $R = 25$ cm (B) $R = 50$ cm
(C) $R = 75$ cm (D) $R = 100$ cm

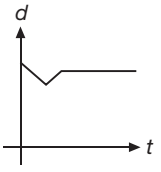
277. A convex lens, made of a material of refractive index 1.5 and having a focal length of 10 cm is immersed in a liquid of refractive index 3.0. The lens will behave as a

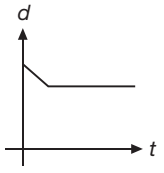
(A) converging lens of focal length 10 cm.
(B) diverging lens of focal length 10 cm.
(C) converging lens of focal length $\frac{10}{3}$ cm.
(D) diverging lens of focal length 30 cm.

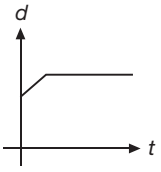
278. A point source S is placed at a height h from the bottom of a vessel of height $H(<h)$. The vessel is polished at the base. If the water is gradually filled in the vessel at a constant rate α m^3s^{-1}, the distance d of image of the source from the bottom of the vessel varies with time t as



(A) 

(B) 

(C) 

(D) 

279. A convex lens of glass has power P in air. If it is immersed in water its power will be
(A) more than P
(B) less than P
(C) P
(D) more than P for some colours and less than P for others

280. A biconvex lens, made of a material of refractive index 1.5, has radius of curvature of each side equal to 0.5 m. The power of the lens is
(A) 0.5 D (B) 1.0 D
(C) 1.5 D (D) 2.0 D

281. A convex lens forms a real image 4 cm long on a screen. When the lens is shifted to a new position without disturbing the object or the screen, again real image is formed on the screen which is 16 cm long. The length of the object is
(A) 8 cm (B) 10 cm
(C) 12 cm (D) 6 cm

282. The maximum and minimum distances between a convex lens and an object, for the magnification of a real image to be greater than one are
(A) $2f$ and f (B) f and zero
(C) ∞ and $2f$ (D) $4f$ and $2f$

283. A point object is placed on the optic axis of a convex lens of focal length f at a distance of $2f$ to the left of it. The diameter of the lens is d. An observer has his eye at a distance of $3f$ to the right of the lens and a distance h below the optic axis. The maximum value of h to see the image is
(A) $\frac{d}{4}$ (B) $\frac{d}{3}$
(C) $\frac{d}{2}$ (D) d

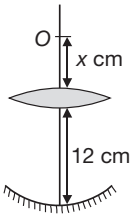
284. A convex lens is immersed in a liquid of refractive index greater than that of glass. It will behave as a
(A) convergent lens
(B) divergent lens
(C) plane glass
(D) homogeneous liquid

285. If the top half of a convex lens is covered with black paper,
(A) the bottom half of the image will disappear.
(B) the top half of the image will disappear.
(C) the magnification will be reduced to half.
(D) the intensity will be reduced to half.

286. In displacement method, the lengths of images in the two positions of the lens between the object and the screen are 9 cm and 4 cm respectively. The length of the object must be
(A) 6.25 cm (B) 1.5 cm
(C) 6 cm (D) 36 cm

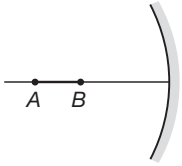
287. A convex lens of focal length 15 cm is placed on a plane mirror. An object is placed 20 cm from the lens. The image is formed
(A) 12 cm in front of the mirror
(B) 60 cm behind the mirror
(C) 60 cm in front of the mirror
(D) 30 cm in front of the mirror

288. A convex lens of focal length 40 cm is held coaxially 12 cm above a concave mirror of focal length 18 cm. An object held x cm above the lens gives rise to an image coincident with it. The x is equal to



(A) 12 cm (B) 15 cm
(C) 18 cm (D) 30 cm

289. A linear object AB is placed along the axis of a concave mirror. The object is moving towards the mirror with speed V. The speed of the image of the point A is $4V$ and the speed of the image of B is also 4 V. If centre of the line AB is at a distance L from the mirror then length of the object AB will be



(A) $\frac{3L}{2}$ (B) $\frac{5L}{3}$
(C) L (D) $\frac{4L}{3}$

290. Two thin lenses of powers 2 D and 3 D are placed in contact. An object is placed at a distance of 30 cm from the combination. The distance in cm of the image from the combination is
(A) 30 (B) 40
(C) 50 (D) 60

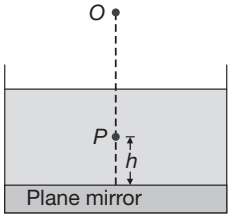
291. Two convex lenses of focal lengths f_1 and f_2 are mounted coaxially separated by a distance. If the power of the combination is zero, the distance between the lenses is

(A) $|f_1 - f_2|$ (B) $f_1 + f_2$

(C) $\dfrac{f_1 f_2}{|f_1 - f_2|}$ (D) $\dfrac{f_1 f_2}{f_1 + f_2}$

292. Chromatic aberration in a lens is caused by
(A) reflection (B) interference
(C) diffraction (D) dispersion

293. A plane mirror is placed at the bottom of a tank containing a liquid of refractive index μ. A small object P lies at a height h above the mirror. An observer O, vertically above P, outside the liquid, observe P and its image in the mirror. The apparent distance between these two will be

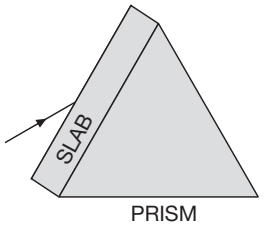


(A) $h\left(1+\dfrac{1}{\mu}\right)$ (B) $\dfrac{2h}{\mu-1}$

(C) $2\mu h$ (D) $\dfrac{2h}{\mu}$

294. A person can see clearly between 1 m and 2 m. His corrective lenses should be
(A) bifocals with power –0.5 D and additional +3.5 D
(B) bifocals with power –1.0 D and additional +3.0 D
(C) concave with power 1.0 D
(D) convex with power 0.5 D

295. A parallel glass slab of refractive index $\sqrt{3}$ is placed in contact with an equilateral prism of refractive index $\sqrt{2}$. A ray is incident on left surface of slab as shown. The slab and prism combination is surrounded by air. The magnitude of minimum possible deviation of this ray by slab-prism combination is



(A) 30° (B) 45°

(C) 60° (D) $60° - \sin^{-1}\sqrt{\dfrac{2}{3}}$

296. Spherical aberration in a thin lens can be reduced by
(A) using a monochromatic light.
(B) using a doublet combination.
(C) using a circular annular mask over the lens.
(D) increasing the size of the lens.

297. A virtual image of an object is formed with a magnification of 2, when the object is placed infront of a concave mirror of focal length f. To obtain a real image of same magnification, the object has to moved by a distance

(A) $\dfrac{f}{2}$ (B) $\dfrac{2f}{3}$

(C) f (D) $\dfrac{3f}{2}$

298. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eyepiece is 36 cm and the final image is formed at infinity. The focal length f_0 of the objective and f_e of the eyepiece are

(A) $f_0 = 45$ cm and $f_e = -9$ cm
(B) $f_0 = 50$ cm and $f_e = 10$ cm
(C) $f_0 = 7.2$ cm and $f_e = 5$ cm
(D) $f_0 = 30$ cm and $f_e = 6$ cm

299. A plano convex lens of radius of curvature R fits exactly into a plano concave lens such that their plane surfaces are parallel to each other. If the lenses are made of different materials of refractive indices μ_1 and μ_2, then focal length of the combination is given by

(A) $\dfrac{R}{2-(\mu_1+\mu_2)}$ (B) $\dfrac{R}{2(\mu_1-\mu_2)}$

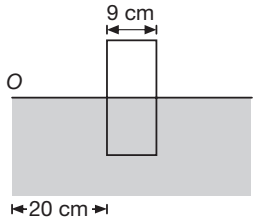
(C) $\dfrac{2R}{\mu_2-\mu_1}$ (D) $\dfrac{R}{\mu_1-\mu_2}$

300. A compound microscope has an objective of focal length 2.0 cm and an eye piece of focal length 6.25 cm separated by 15 cm. If the final image is formed at the least distance of distinct vision (25 cm), the distance of the object from the objective is
(A) 1.5 cm (B) 2.5 cm
(C) 3.0 cm (D) 4.0 cm

301. In PROBLEM 300, the magnifying power of the microscope is

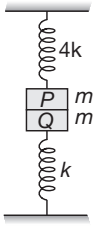
(A) 10 (B) 15
(C) 20 (D) 30

302. A point object is placed at a distance of 20 cm from a glass slab $\left(\mu_g = \frac{3}{2}\right)$ half immersed in water $\left(\mu_w = \frac{4}{3}\right)$ as shown in figure. The distance between two images when seen from the other side of the slab is



(A) 1 cm (B) 2 cm
(C) 4 cm (D) 6 cm

303. In the figure shown, blocks P and Q are in contact but do not stick to each other. The lower face of P behaves as a plane mirror. The springs are in their natural lengths.



The system is released from rest. The distance between Q and its image, when Q is at the lowest point for the first time is

(A) $\frac{2mg}{K}$ (B) $\frac{4mg}{K}$
(C) $\frac{3mg}{K}$ (D) 0

304. A compound microscope has a magnification of 30. The focal length of the eye-piece is 5 cm. If the final image is formed at the least distance of distinct vision (25 cm), the magnification produced by the objective is

(A) 5 (B) 7.5
(C) 10 (D) 15

305. The least distance of distinct vision is 25 cm. The focal length of a convex lens is 5 cm . It can act as a simple microscope of magnifying power

(A) 4 (B) 5
(C) 6 (D) None of these

306. An astronomical telescope has an eye piece of focal length 5 cm. If the angular magnification of normal adjustment is 10, the distance between the objective and the eye piece is

(A) 45 cm (B) 50 cm
(C) 55 cm (D) 110 cm

307. The focal lengths of the objective and the eyepiece of an astronomical telescope are 100 cm and 20 cm respectively. Its magnifying power in normal adjustment is

(A) 5 (B) 2
(C) 25 (D) 4

308. Two convex lenses of focal lengths 0.3 m and 5 cm are used to make a telescope. The distance kept between them is equal to

(A) 0.35 m (B) 5.3 cm
(C) 5.3 m (D) 0.15 m

309. To have larger magnification by a telescope

(A) the objective should be of large focal length and the eyepiece should be of small focal length
(B) both the objective and the eyepiece should be of large focal lengths
(C) both the objective and the eyepiece should be of small focal lengths
(D) the objective should be of small focal length and the eyepiece should be of large focal length

310. The angle of incidence for an equilateral prism is 60°. The refractive index of prism so that the ray inside the prism is parallel to the base of the prism is

(A) $\frac{9}{8}$ (B) $\sqrt{2}$
(C) $\frac{4}{3}$ (D) $\sqrt{3}$

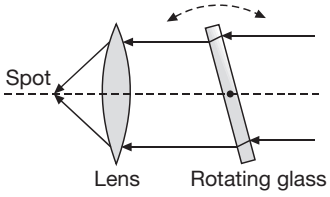
311. Four convergent lenses have focal lengths 100 cm, 10 cm, 4 cm and 0.3 cm. For a telescope with maximum possible magnification, we choose the lenses of focal lengths

(A) 100 cm, 0.3 cm (B) 10 cm, 0.3 cm
(C) 10 cm, 4 cm (D) 100 cm, 4 cm

312. The angular magnification of a telescope which contains an objective of focal length f_1 and eyepiece of focal length f_2 is

(A) $\frac{f_2}{f_1}$ (B) $\frac{f_1 + f_2}{f_2}$
(C) $\frac{f_1}{f_2}$ (D) $\frac{f_1 f_2}{f_1 + f_2}$

313. A slab of high quality flat glass, with parallel faces, is placed in the path of a parallel light beam before it is focussed to a spot by a lens. The glass is rotated slightly back and forth from the dotted centre about an axis coming out of the page, as shown in the diagram. According to ray optics the effect on the focussed spot is



(A) There is no movement of the spot.
(B) The spot moves towards then away from the lens.
(C) The spot moves up and down parallel to the lens.
(D) The spot moves along a line making an angle (neither zero nor $90°$) with axis of lens.

314. An achromatic combination is to be made using a convex and a concave lens. The two lenses should have
(A) their power equal.
(B) their refractive indices equal.
(C) their dispersive powers equal.
(D) the product of their powers and dispersive powers equal.

315. For a thin equiconvex lens, the optics axis coincides with the x-axis and the optical centre coincides with the origin. The co-ordinates of a point object and its image are $(-40, 1)$ cm and $(50, -2)$ cm respectively. Lens is located at

(A) $x = 0$ (B) $x = -10$ cm
(C) $x = +20$ cm (D) $x = -30$ cm

316. The near point of a person is 50 cm and the far point is 1.5 m. The spectacles required for reading purpose and for seeing distant objects are respectively

(A) $+2$ D, $-\left(\frac{2}{3}\right)$ D (B) $+\left(\frac{2}{3}\right)$ D, -2 D
(C) -2 D, $+\left(\frac{2}{3}\right)$ D (D) $-\left(\frac{2}{3}\right)$ D, $+2$ D

317. Astigmatism for a human eye can be removed by using
(A) concave lens (B) convex lens
(C) cylindrical lens (D) prismatic lens

318. A hollow convex lens of glass behaves like a
(A) plane mirror (B) concave lens
(C) convex lens (D) glass plate

319. The far point of a myopic eye is 250 cm. The correcting lens should be a
(A) diverging lens of focal length 250 cm.
(B) converging lens of focal length 250 cm.
(C) diverging lens of focal length 125 cm.
(D) converging lens of focal length 125 cm.

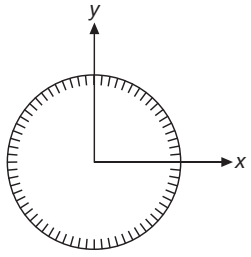
320. A parallel beam of light passes parallel to the principal axis and falls on one face of a thin convex lens of focal length f and after two internal reflections from the second face forms a real image. The distance of image from lens if the refractive index of material of lens is 1.5

(A) $\frac{f}{7}$ (B) $\frac{f}{2}$
(C) $7f$ (D) $\frac{f}{6}$

321. A person cannot see clearly beyond 50 cm. The power of the lens required to correct his vision is

(A) -0.5 D (B) $+0.5$ D
(C) -2 D (D) $+2$ D

322. A ray travelling in negative x-direction is directed towards positive y-direction after being reflected from a surface at point P. The reflecting surface is represented by the equation $x^2 + y^2 = a^2$. Then co-ordinates of point P are



(A) $(a, 0)$ (B) $(0.6a, 0.8a)$
(C) $(0.8a, 0.6a)$ (D) $\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$

323. A person cannot see clearly objects at a distance less than 100 cm,. The power of the spectacles required to see clearly objects at 25 cm is

(A) $+1$ D (B) $+3$ D
(C) $+4$ D (D) $+2$ D

324. An object is kept at a distance of 16 cm from a thin lens and the image formed is real. If the object is kept at a distance of 6 cm from the same lens the image formed is virtual. If the size of the images formed are equal, the focal length of the lens will be

(A) 8 cm (B) 5 cm

(C) 11 cm (D) $\sqrt{96}$ cm

325. A person can see clearly objects lying between 25 cm and 2 m from his eye. His vision can be corrected by using spectacles of power

(A) +0.25 D (B) +0.5 D

(C) –0.25 D (D) –0.5 D

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

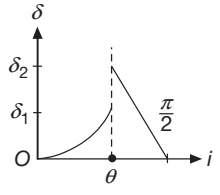
1. The x-y plane is the boundary between two transparent media. Medium 1 with $z \geq 0$ has a refractive index $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. The refracted ray makes angle r with $+z$ axis and incident ray makes an angle i with $-z$ axis. Then,

(A) $i = 120°$ (B) $i = 60°$

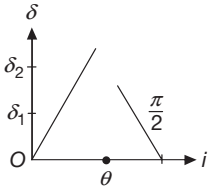
(C) $r = 45°$ (D) $r = 135°$

2. A ray of light travels from a medium of refractive index μ to air. Its angle of incidence in the medium is i, measured from the normal to the boundary, and its angle of deviation is δ. The curve that best represents the plot of deviation δ (along y-axis) with angle of incidence i (along x-axis) is

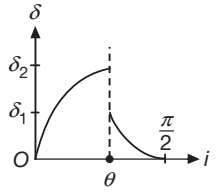
(A)



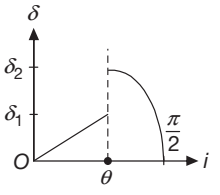
(B)



(C)



(D)



3. In PROBLEM 2,

(A) $\theta = \sin^{-1}\left(\frac{1}{\mu}\right)$ (B) $\theta = \frac{\pi}{2} - \sin^{-1}\left(\frac{1}{\mu}\right)$

(C) $\frac{\delta_2}{\delta_1} = \mu$ (D) $\frac{\delta_2}{\delta_1} = 2$

4. A ray of light from a denser medium strikes a rarer medium at angle of incidence i. The reflected and the refracted rays make an angle of 90° with each other. The angles of reflection and refraction are r and r' respectively. The critical angle is

(A) $\sin^{-1}(\tan r)$ (B) $\sin^{-1}(\tan i)$

(C) $\sin^{-1}(\tan r')$ (D) $\tan^{-1}(\sin i)$

5. A single converging lens is used as a simple microscope. In the position of maximum magnification. Select the correct statement(s).

(A) the object is placed at the focus of the lens.

(B) the object is placed between the lens and its focus.

(C) the image is formed at infinity.

(D) the object and the image subtend the same angle at the eye.

6. A light of wavelength 6000 Å in air enters a medium of refractive index 1.5. Inside the medium, its frequency is ν and its wavelength is λ.

(A) $\nu = 5 \times 10^{14}$ Hz (B) $\nu = 7.5 \times 10^{14}$ Hz

(C) $\lambda = 4000$ Å (D) $\lambda = 9000$ Å

7. If a converging beam of light is incident on a concave mirror, the reflected light

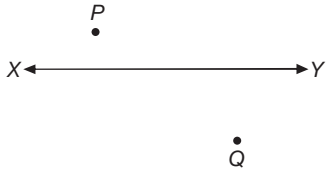
(A) may form a real image

(B) must form a real image

(C) may form a virtual image

(D) may be a parallel beam

8. Two points P and Q lie on either side of an axis XY as shown. It is desired to produce an image of P at Q using a spherical mirror, with XY as the optic axis. The mirror must be



(A) converging
(B) diverging
(C) positioned to the left of P
(D) positioned to the right of Q

9. An object and a screen are fixed at a distance d apart. When a lens of focal length f is moved between the object and the screen, sharp images of the object are formed on the screen for two positions of the lens. The magnifications produced at these two positions are M_1 and M_2.

(A) $d > 2f$ (B) $d > 4f$

(C) $M_1 M_2 = 1$ (D) $|M_1| - |M_2| = \dfrac{d}{f}$

10. Resolving power of an electron microscope is R_e and that of optical microscope is R_0.

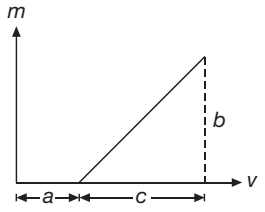
(A) $R_e > R_0$ (B) $R_e < R_0$

(C) $R_e = R_0$ (D) Data Insufficient

11. In PROBLEM 10, the correct argument for the correct selected option is that
(A) electrons have greater wavelength than visible light.
(B) electrons have lesser wavelength than visible light.
(C) resolving power is inversely proportional to the wavelength of the wave used for detecting an object by the microscope.
(D) resolving power is inversely proportional to the square of the wavelength of the wave used for detecting an object by the microscope.

12. The distance between two point objects P and Q is 32 cm. A convex lens of focal length 15 cm is placed between them so that the images of both the objects are formed at the same place. The distance of P from the lens could be
(A) 20 cm (B) 18 cm
(C) 16 cm (D) 12 cm

13. The graph shows the variation of magnification m produced by a convex lens with the image distance v. The focal length of the lens is



(A) $\dfrac{b}{c}$ (B) $\dfrac{c}{b}$

(C) a (D) $\dfrac{ab}{c}$

14. A lens of focal length f is placed in between an object and screen fixed at a distance D. The lens forms two real images of object on the screen for two of its different positions, a distance x apart. The two real images have magnifications m_1 and m_2 respectively $(m_1 > m_2)$.

(A) $f = \dfrac{x}{m_1 - m_2}$ (B) $m_1 m_2 = 1$

(C) $f = \dfrac{D^2 - x^2}{4D}$ (D) $D \geq 4f$

15. A parallel beam of white light falls on a combination of a concave and a convex lens, both of same material. Their focal lengths are 15 cm and 30 cm respectively for the mean wavelength in white light. On the other side of the lens system, one sees
(A) a coloured pattern with violet at the outer edge.
(B) a coloured pattern with red at the outer edge.
(C) white light again.
(D) that it is unable for the lens to converge the rays at a point.

16. Consider a ray of light going from A to B. Let the ray traverse, in going from A to B, distances $s_1, s_2, s_3, \ldots\ldots\ldots\ldots s_m$ in media of indices $n_1, n_2, n_3, \ldots\ldots\ldots\ldots n_m$ respectively.

(A) Total time of flight $t = \dfrac{1}{c}\sum_{i=1}^{m} n_i s_i$

(B) Total time of flight $t = \dfrac{1}{c}\sum_{i=1}^{m} s_i$

(C) Optical path length is (O.P.L.) $= \sum_{i=1}^{m} n_i s_i$

(D) For inhomogeneous media the O.P.L. $= \int_{A}^{B} n(s)\,ds$ and the ray travels along 'Stationary Pathways'.

17. A point object is placed at 30 cm from a convex glass lens $\left(\mu_g = \dfrac{3}{2}\right)$ of focal length 20 cm. For the final image of object to be formed at infinity, which of the following is/are correct?

(A) A concave lens of focal length 60 cm is placed in contact with the convex lens
(B) A convex lens of focal length 60 cm is placed at a distance of 30 cm from the convex lens.
(C) The entire convex lens system is immersed in a liquid of refractive index $\frac{4}{3}$
(D) The entire convex lens system is immersed in a liquid of refractive index $\frac{9}{8}$

18. For a mirror, the linear magnification is +2. The conclusion(s) that can be drawn from this information is/are
(A) The mirror is concave
(B) The mirror can be convex or concave but it cannot be plane
(C) The object lies between pole and focus
(D) The object lies beyond focus

19. A ray of light has speed v_0 frequency f_0 and wavelength λ_0 in vacuum. When this ray of light enters in a medium of refractive index μ, corresponding values are v, f and λ. Then

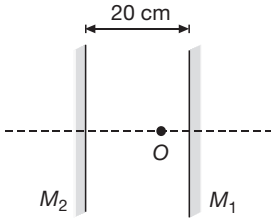
(A) $f = \frac{f_0}{\mu}$ (B) $\lambda = \frac{\lambda_0}{\mu}$
(C) $v = \frac{v_0}{\mu}$ (D) $f = f_0$

20. For which of the pairs of u and f for curved mirror(s), the image formed is smaller in size.
(A) $u = -45$ cm, $f = -10$ cm
(B) $u = -10$ cm, $f = 20$ cm
(C) $u = -60$ cm, $f = 30$ cm
(D) $u = -20$ cm, $f = -30$ cm

21. A diverging lens of focal length f_1 is placed in front of and coaxially with a concave mirror of focal length f_2. Their separation is d. A parallel beam of light incident on the lens returns as a parallel beam from the arrangement. Select the correct statement(s).
(A) The beam diameters of the incident and reflected beams must be the same.
(B) $d = 2|f_2| - |f_1|$
(C) $d = |f_2| - |f_1|$
(D) If the entire arrangement is immersed in water, the conditions will remain unaltered.

22. An astronomical telescope and a Galilean telescope use identical objective lenses. They have the same magnification, when both are in normal adjustment. The eyepiece of the astronomical telescope has a focal length f. Select the correct statement(s).
(A) The tube lengths of the two telescopes differ by f.
(B) The tube lengths of the two telescopes differ by $2f$.
(C) The Galilean telescope has shorter tube length.
(D) The Galilean telescope has longer tube length.

23. Two plane mirrors M_1 and M_2 are placed parallel to each other 20 cm apart. A luminous point object 'O' is placed between them at 5 cm from M_1 as shown.



(A) The distances (in cm) of three nearest images from mirror M_1 are 5, 35 and 45 respectively.
(B) The distances (in cm) of three nearest images from mirror M_2 are 5, 35 and 45 respectively.
(C) The distances (in cm) of three nearest images from mirror M_1 are 15, 25 and 55 respectively.
(D) The distances (in cm) of three nearest images from mirror M_2 are 15, 25 and 55 respectively.

24. In the case of hypermetropia
(A) the image of a near object is formed behind the retina.
(B) the image of a distant object is formed in front of the retina.
(C) a concave lens should be used for correction.
(D) a convex lens should be used for correction.

25. Which of the following produce a virtual image longer in size than the object?
(A) Concave lens
(B) Convex lens
(C) Concave mirror
(D) Convex mirror

26. A concave mirror has focal length 15 cm. Where should an object be placed in front of the mirror so that the image formed is three times the size of the object?
(A) 7.5 cm (B) 10 cm
(C) 17.5 cm (D) 20 cm

27. A concave mirror of focal length f forms an image 2 times the size of object. The object distance from the mirror is

(A) $\frac{f}{4}$ (B) $\frac{4f}{3}$

(C) $\frac{3f}{2}$ (D) $\frac{f}{2}$

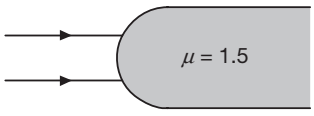
28. A point object P moves towards a convex mirror with a constant speed V, along its optic axis. The speed of the image
(A) is always less than V.
(B) may be less than, equal to or greater than V, depending on the position of P.
(C) increases as P comes closer to the mirror.
(D) decrease as P comes closer to the mirror.

29. A bird flies down vertically towards a water surface. To a fish inside the water, vertically below the bird, the bird will appear to
(A) be closer than its actual distance.
(B) be farther away than its actual distance.
(C) move slower than its actual speed.
(D) move faster than its actual speed.

30. There are three optical media 1, 2 and 3 with their refractive indices $\mu_1 > \mu_2 > \mu_3$. Select the correct statement(s)
(A) When a ray of light travels from 3 to 1 no TIR will take place.
(B) Critical angle between 1 and 2 is less than the critical angle between 1 and 3.
(C) Critical angle between 1 and 2 is more than the critical angle between 1 and 3.
(D) Chances of TIR are more when ray of light travels from 1 to 3 as compare to the case when it travel from 1 to 2.

31. An equilateral prism has a refractive index $\sqrt{2}$. Select the correct alternative(s).
(A) Minimum deviation from this prism can be $30°$
(B) Minimum deviation from this prism can be $45°$
(C) At angle of incidence $45°$, deviation is minimum
(D) At angle of incidence $60°$, deviation is minimum

32. Parallel rays of light are falling on a convex spherical surface of radius of curvature $R = 20$ cm and refractive index $\mu = 1.5$ as shown. After refraction from the spherical surface, the parallel rays



(A) appear to meet after extending the refracted rays backwards.
(B) actually meet at some point.
(C) meet (or appear to meet) at a distance of 60 cm from the spherical surface.
(D) meet (or appear to meet) at a distance of 30 cm from the spherical surface.

33. The focal length of a lens in air and refractive index are f and μ respectively. The focal length changes to f_1 when the lens is immersed in a liquid of refractive index $\frac{\mu}{2}$ and it becomes f_2 when the lens is immersed in a liquid of refractive index 2μ. Then

(A) $f_1 = -\frac{2(\mu-1)}{f}$ (B) $f_2 = -\frac{2(\mu-1)}{f}$

(C) $f_1 = \frac{\mu-1}{f}$ (D) $f_2 = \frac{\mu-1}{f}$

34. Two thin lenses, when in contact, produce a combination of power +10 dioptre. When they are 0.25 m apart, the power is reduced to +6 dioptre. The respective powers of the lenses in dioptre, are
(A) 1 and 9 (B) 2 and 8
(C) 4 and 6 (D) 5 each

35. A solid, transparent sphere has a small, opaque dot at its centre. When observed from outside, the apparent position of the dot will be
(A) independent of the refractive index of the sphere.
(B) closer to the eye than its actual position.
(C) farther away from the eye than its actual position.
(D) the same as its actual position.

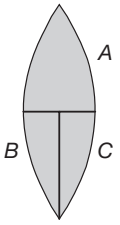
36. For a concave mirror
(A) virtual image is always larger in size
(B) real image is always smaller in size
(C) real image is always larger in size
(D) real image may be smaller or larger in size

37. During refraction, ray of light passes undeviated, then
(A) medium on both sides is same
(B) angle of incidence is $90°$
(C) angle of incidence is $0°$
(D) medium on other side is rarer

38. A convex lens made of glass $\left(\mu_g = \frac{3}{2}\right)$ has focal length f in air. The image of an object placed in front of it is real, inverted and magnified. Now the whole arrangement is immersed in water $\left(\mu_w = \frac{4}{3}\right)$ without changing the distance between object and lens, then
(A) the new focal length becomes $4f$
(B) the new focal length becomes $\frac{f}{4}$

(C) the new image formed will be virtual and magnified.
(D) the new image formed will be real and diminished.

39. A thin, symmetric double-convex lens of power P is cut into three parts A, B and C as shown. The power of



(A) A is P (B) A is $2P$
(C) B is $\frac{P}{2}$ (D) B is $\frac{P}{4}$

40. A watch glass having uniform thickness and having average radius of curvature of its two surfaces much larger than its thickness is placed in the path of a beam of parallel light. The beam will
(A) be completely unaffected.
(B) converge slightly.
(C) diverge slightly.
(D) converge or diverge slightly depending on whether the beam is incident from the concave or the convex side.

41. A converging lens of focal length f_1 is placed in front of and coaxially with a convex mirror of focal length f_2. Their separation is d. A parallel beam of light incident on the lens returns as a parallel beam from the arrangement. Select the correct statement(s).
(A) The beam diameters of the incident and reflected beams must be the same.
(B) $d = f_1 - 2|f_2|$
(C) $d = f_1 - |f_2|$
(D) If the entire arrangement is immersed in water, the conditions will remain unaltered.

42. Check the wrong statement(s)
(A) A concave mirror can give a virtual image.
(B) A concave mirror can give a diminished virtual image.
(C) A convex mirror can give a real image.
(D) A convex mirror can give a diminished virtual image.

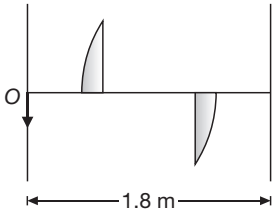
43. When lights of different colours move through water, they must have different
(A) wavelengths (B) frequencies
(C) velocities (D) amplitudes

44. A thin concavo-convex lens has two surfaces of radii of curvature R and $2R$. The material of the lens has a refractive index μ. When kept in air, the focal length of the lens
(A) will depend on the direction from which light is incident on it.
(B) will be the same, irrespective of the direction from which light is incident on it.
(C) will be equal to $\frac{2R}{\mu - 1}$.
(D) will be equal to $\frac{R}{\mu - 1}$.

45. A convex mirror is used to form an image of a real object. The image
(A) always lies between the pole and the focus.
(B) is diminished in size.
(C) is erect.
(D) is real.

46. A ray of light is incident on a prism of refracting angle A. C is the critical angle for the material of the prism with respect to the surrounding material (say air/vacuum).
(A) An emergent ray will be there for all values of C.
(B) An emergent ray will be there only for $A < 2C$.
(C) A ray incident at an angle i can pass through the prism if $\sin i > \frac{\sin(A - C)}{\sin C}$ for $C < A < 2C$.
(D) None of above is correct.

47. A thin plane-convex lens of focal length f is split into two equal halves. One of the halves is shifted along the optical axis as shown. The separation between object and image planes is 1.8 m and the magnification of image formed by one of the half lens is 2. The separation between two halves is d.



(A) $f = 0.4$ m (B) $f = 0.6$ m
(C) $d = 0.6$ m (D) $d = 0.4$ m

48. A point source of light is placed at a distance h below the surface of a large and deep lake. If f is the fraction of light energy that escapes directly from water surface and μ is refractive index of water then,

(A) f varies as a function of h

(B) f is independent of value of h

(C) $f = \dfrac{1}{2\sqrt{\mu^2 - 1}}$

(D) $f = \dfrac{1}{2}\left[1 - \sqrt{1 - \dfrac{1}{\mu^2}}\right]$

49. An object is placed at a distance $2f$ from the pole of a curved mirror of focal length f.

(A) The linear magnification is 1 for both types of curved mirror.

(B) The linear magnification is 1 for a concave mirror.

(C) The linear magnification is $\dfrac{1}{3}$ for a convex mirror.

(D) Data Insufficient.

50. If a convergent beam of light passes through a diverging lens, the result

(A) may be a convergent beam.

(B) may be a divergent beam.

(C) may be a parallel beam.

(D) must be a parallel beam.

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.

Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.

Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.

Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** A parallel beam of light traveling in air can be displaced laterally by a parallel transparent slab by distance more than the thickness of the plate.
Statement-2: The lateral displacement of light traveling in air increases with rise in value of refractive index of slab.

2. **Statement-1:** Even in absolutely clear water, a diver cannot see very clearly.
Statement-2: Velocity of light is reduced in water.

3. **Statement-1:** Spherical aberration of a lens can be reduced by blocking the central portion or peripheral portion of the lens.
Statement-2: Spherical aberration arises on account of inability of the lens to focus central and peripheral rays at the same point.

4. **Statement-1:** For total internal reflection, angle of incident in denser medium must be greater than critical angle for the pair of media in contact.
Statement-2: $\mu = \dfrac{1}{\sin C}$, where the symbols have their standard meaning.

5. **Statement-1:** The images formed due to total internal reflections are much brighter than those formed by mirrors or lenses.
Statement-2: There is no loss of intensity in total internal reflection.

6. **Statement-1:** A bird in air is diving vertically with speed v_0 over a tank filled with water and having flat silvered bottom serving as plane mirror, it observes velocity of its image in silvered bottom of tank as $2v_0$ upward relative to itself .
Statement-2: Bird and its image in bottom mirror are always equidistant from bottom mirror.

7. **Statement-1:** We cannot produce a real image by plane or convex mirrors under any circumstances.
Statement-2: The focal length of a convex mirror is always taken as positive.

8. **Statement-1:** If a light ray is incident on any one of the two mirrors inclined at $90°$ with each other, then finally the emergent ray is antiparallel with incident ray.
Statement-2: Finally, the reflected and initially incident rays are in same phase when successively reflected from two perpendicularly inclined mirrors.

9. **Statement-1:** The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature.
Statement-2: Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.

10. **Statement-1:** The focal length of the mirror is f and distance of the object from the focus is u, the magnification of the mirror is $\frac{f}{u}$.

Statement-2: Magnification $= \frac{\text{size of image}}{\text{size of object}}$.

11. **Statement-1:** Although the surfaces of the lens used in goggles are curved, it does not have any power.
Statement-2: In case of goggles, the lenses are concavo-convex and both the surface of lens have equal radii of curvature.

12. **Statement-1:** A convex lens behaves as a concave lens when placed in a medium of refractive index greater than the refractive index of its material.
Statement-2: Light in that case will travel through the convex lens from denser to rarer medium. It will bend away from normal, i.e., the convex lens would diverge the rays and behave as concave.

13. **Statement-1:** The minimum distance between an object and its real image formed by a convex lens is $2f$.

Statement-2: The distance between an object and its real image is minimum when its magnification is one.

14. **Statement-1:** The lens formula $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ indicates that focal length of a lens depends on distances of object and image from the lens.
Statement-2: The formula does indicate but when u is changed v also changes, so that f of a particle lens remains constant.

15. **Statement-1:** For observing traffic at our back, we prefer to use a convex mirror.
Statement-2: A convex mirror has a much larger field of view than a plane mirror or a concave mirror.

16. **Statement-1:** When a ray of light enters glass from air, its frequency decreases.
Statement-2: The velocity of light in glass is less than that in air.

17. **Statement-1:** A ray incident along normal to the mirror retraces its path.
Statement-2: In reflection, angle of incidence is always equal to angle of reflection.

18. **Statement-1:** A concave mirror of focal length in air is used in a medium of refractive index 2. Then the focal length of mirror in medium becomes double.
Statement-2: The radius of curvature of a mirror is double of the focal length.

19. **Statement-1:** Light from an object falls on a concave mirror forming a real image of the object. If both the object and mirror are immersed in water, there is no change in position of the image.
Statement-2: The formation of image by reflection does not depend on surrounding medium, so there is no change in position of image.

20. **Statement-1:** A convex lens can be convergent in one medium and divergent in other medium.
Statement-2: In denser medium, convex lens is convergent and in rarer medium, convex lens is divergent.

21. **Statement-1:** For a prism of refracting angle $60°$ and refractive index $\sqrt{2}$ minimum deviation is $30°$.

Statement-2: At minimum deviation, $r_1 = r_2 = \frac{A}{2} = 30°$.

22. **Statement-1:** There exist two angles of incidence for the same magnitude of deviation (except minimum deviation) by a prism kept in air.
Statement-2: For a prism kept in air, a ray is incident on first surface and emerges out of second surface (of prism) along the previous emergent ray, then this ray emerges out of first surface along the previous incident ray. This principle is called the Principle of Reversibility of Light.

23. **Statement-1:** A plane convex lens is silvered from plane surface. It can act as a diverging mirror.
Statement-2: Focal length of concave mirror is independent of medium.

24. **Statement-1:** Maximum distance of image formed by convex mirror from pole of mirror equals $'f'$ for all the objects (real/virtual).
Statement-2: Convex mirrors forms virtual images for objects placed in front of mirror.

25. **Statement-1:** We cannot produce a real image by plane or convex mirror under any circumstances.
Statement-2: Reflection Law is valid for plane mirror as well as convex mirror.

26. **Statement-1:** There is no dispersion of light refracted through a rectangular glass slab.
Statement-2: Dispersion of light is the phenomenon of splitting of a beam of white light into its constituent colours.

27. **Statement-1:** Convex mirror always form a virtual image.
Statement-2: Focal length of a mirror is half of the radius of curvature.

28. **Statement-1:** A fish inside a pond will see a person standing outside taller than he is actually.
Statement-2: Light rays from person converges into eyes of fish on entering water from air.

29. **Statement-1:** Optical fibre has thin glass core coated by glass of small refractive index and is used to send light signals.
Statement-2: All the rays of light entering the fibre are totally reflected even at very small angles of incidence.

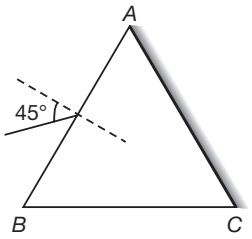
30. **Statement-1:** The mirror used in search light are parabolic and not concave spherical.
Statement-2: In a concave spherical mirror the image formed is always virtual.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

A ray of light is incident at $45°$ on the face AB of an equilateral prism ABC which has the face AC silvered. Based on the information provided answer the following questions.



1. The refractive index μ of the material of the prism so that when the ray falls on face BC (after reflecting from AC) it makes an angle $60°$ with it is
(A) $\sqrt{3}$ (B) $\sqrt{2}$
(C) 2 (D) 1.5

2. The total deviation, when the ray of light finally emerges from BC is
(A) $120°$ (B) $180°$
(C) $150°$ (D) $90°$

Comprehension 2

A convex lens of focal length 20 cm and a concave lens of focal length 10 cm are placed 20 cm apart. In between them an object is placed at distance x from the convex lens.
Based on the information provided answer the following questions.

3. The value of x (in cm) so that images formed by both the lenses coincide is
(A) $20(\sqrt{3}-1)$ (B) $\dfrac{20\sqrt{3}-1}{\sqrt{3}}$
(C) $\dfrac{20}{\sqrt{3}}$ (D) $10(\sqrt{3}-1)$

4. The linear magnification produced by convex lens and concave lens individually is
(A) $(\sqrt{3}+1)$ and $\dfrac{1}{\sqrt{3}}$
(B) $\sqrt{3}$ and $\dfrac{1}{\sqrt{3}}$
(C) $(\sqrt{3}+1)$ and $(\sqrt{3}-1)$
(D) $\sqrt{3}$ and $(2\sqrt{3}-3)$

Comprehension 3

A telescope is an optical instrument used to increase the visual angle of distant objects such as stars, planets etc. An astronomical telescope consists of two converging lenses. The one facing the object is called objective and the lens close to the eye is called an eyepiece. It can be adjusted by displacing relative to the objective. The angular magnification is defined as the ratio of focal length of objective and eyepiece. One can see the image with unstrained eye if it forms at infinity. An astronomical telescope has an objective of focal length 50 cm and a magnification of 20. Based on above information, answer the following questions.

5. Focal length of the eyepiece is
(A) 2.5 cm
(B) 5 cm
(C) 7.5 cm
(D) None of these

6. To view remote object by an unstrained normal eye, separation between two lenses will be
   (A) 55 cm (B) 57.5 cm
   (C) 60 cm (D) 52.5 cm

7. If object is at a distance 600 m from the telescope to see the image with unstrained eye separation between two lenses should be (in cm)
   (A) 46.65 (B) 47.65
   (C) 49.96 (D) 49.65

Comprehension 4

Speed of light in a medium of refractive index n is given by $\frac{c}{n}$ where c is speed of light in vacuum refractive index of a medium depends on wavelength (λ). As wavelength increases refractive index decreases. It is also given

$$\lambda_{red} > \lambda_{orange} > \lambda_{yellow}$$

Based on above information, answer the following questions.

8. In glass
   (A) orange light travels faster than yellow light
   (B) yellow light travels faster than orange light
   (C) yellow light travels faster than red light
   (D) orange light travels faster than red light

9. The quantity that remains unchanged if light enters from water to glass is
   (A) Wavelength and colour
   (B) Refractive index and frequency
   (C) Frequency and velocity
   (D) Colour and frequency

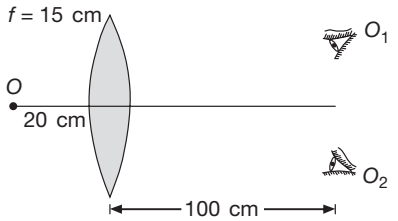
10. The phenomenon that happens because of variation of wavelength is
    (A) Aberration
    (B) Dispersion
    (C) Total internal reflection
    (D) Bending of light

11. Which of the following statement is true?
    (A) Time taken (t) by yellow light to travel distance x_0 in refractive index n can be $t \le \frac{nx_0}{c_0}$
    (B) Time taken (t) by yellow light to travel distance x_0 in refractive index n can be $t \ge \frac{nx_0}{c}$
    (C) Since wavelength of yellow light increases in refractive index n its frequency must decreases.
    (D) None of the above

12. Which of the following statement is false?
    (A) Light is a electromagnetic wave
    (B) Speed of light of each colour is same in vacuum
    (C) Time to cover distance x_0 in a medium is same for each colour
    (D) As frequency of light increases then refractive index of glass increases

Comprehension 5

The figure shows a convex lens of focal length 15 cm . A point object is placed on the principle axis of lens at a distance 20 cm from it as shown. On the other side of the lens two observer eyes O_1 and O_2 are situated at a distance 100 cm from the lens at some distance above and below the principle axis.



Now half position of lens below principle axis is painted black. Based on above information, answer the following questions.

13. In initial setup (before painting the lens) which of the following statement is correct.
    (A) Observer O_1 will see a real image at 60 cm from the lens but observer O_2 will not be able to see it
    (B) Observer O_2 will see a real image at 60 cm from the lens but observer O_1 will not be able to see it
    (C) Both the observers will see a real image at 60 cm from lens irrespective the positions of O_1 and O_2
    (D) Both the observers may or may not be able to see the image at 60 cm from lens depending on the positions of O_1 and O_2

14. After painting the lens, which of the following observer will not be able to see the image of object, if before this activity both were seeing the image

(A) O_1 (B) O_2

(C) Both O_1 and O_2 (D) Neither O_1 nor O_2

15. After painting the lens, for which observer the intensity of image will be reduced to half?

(A) For O_1

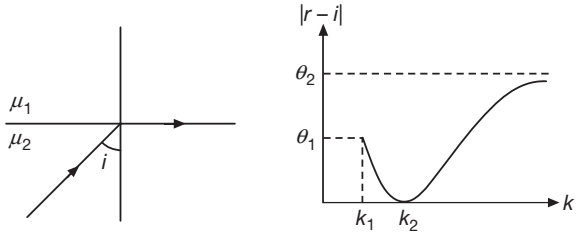
(B) For O_2

(C) Both for O_1 and O_2

(D) Neither for O_1 nor for O_2

Comprehension 6

The figure shows a ray incident at an angle $i = \frac{\pi}{3}$. If the plot drawn shows the variation of $|r - i|$ versus $\frac{\mu_1}{\mu_2} = k$, where r is the angle of refraction, then based on above information, answer the following questions.



16. The value of k_1 is

(A) $\frac{2}{\sqrt{3}}$ (B) 1

(C) $\frac{1}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2}$

17. The value of θ_1 is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{6}$ (D) ZERO

18. The value of k_2 is

(A) 1 (B) 2

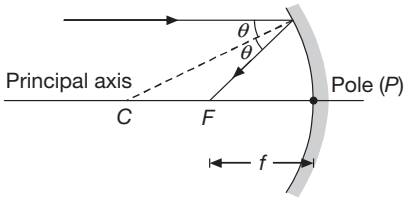
(C) $\frac{1}{2}$ (D) None of these

Comprehension 7

Spherical aberration in spherical mirrors is a defect which is due to dependence of focal length f on angle of incidence θ as shown in figure is given by

$$f = R - \frac{R}{2}\sec\theta$$

where R is radius of curvature of mirror and θ is the angle of incidence. The rays which are closed to principal axis are called paraxial rays and the rays far away from principal axis are called marginal rays. As a result of above dependence different rays are brought to focus at different points and the image of a point object is not a point.



Based on above information, answer the following questions.

19. If f_p and f_m represent the focal length of paraxial and marginal rays respectively, then correct relationship is

(A) $f_p = f_m$ (B) $f_p > f_m$

(C) $f_p < f_m$ (D) None of these

20. If angle of incidence is $60°$, then focal length of this marginal ray is

(A) R (B) $\frac{R}{2}$

(C) $2R$ (D) 0

21. The total deviation suffered by the ray falling on mirror at an angle of incidence equal to $60°$ is

(A) $60°$

(B) $90°$

(C) $30°$

(D) Cannot be determined

Comprehension 8

The XY plane is the boundary between two transparent media. Medium-I with $z \geq 0$ has a refractive index $\sqrt{2}$ and medium-II with $z \leq 0$ has a refractive index $\sqrt{3}$. A ray of light in medium-I given by $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. Based on the above facts, answer the following questions.

22. The vector representing the incident ray has a magnitude of

(A) 5 units (B) 10 units
(C) 15 units (D) 20 units

23. The angle of incidence is
(A) 30° (B) 45°
(C) 60° (D) 90°

24. The angle of refraction is
(A) 30° (B) 45°
(C) 60° (D) 90°

25. The refracted ray is represented by the vector given by
(A) $6\sqrt{3}i - 8\sqrt{3}\,\hat{j} + 10\sqrt{3}\,\hat{k}$
(B) $8\sqrt{3}\hat{j}$
(C) $-10\sqrt{3}\hat{k}$
(D) $6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\sqrt{3}\hat{k}$

26. The vector representing the refracted ray has a magnitude of
(A) $\sqrt{6}$ units (B) 10 units
(C) $10\sqrt{6}$ units (D) $20\sqrt{6}$ units

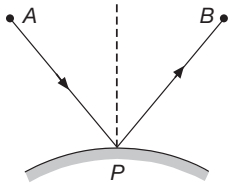
27. The unit vector along refracted ray is
(A) $\frac{3}{5\sqrt{2}}\hat{i} + \frac{4}{5\sqrt{2}}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}$
(B) $3\hat{i} + 4\hat{j} - 5\hat{k}$
(C) $\frac{3}{5\sqrt{2}}\hat{i} - \frac{4}{5\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$
(D) $2\hat{i} - 3\hat{k}$

Comprehension 9

The lens governing the behaviour of the rays namely rectilinear propagation, laws of reflection and refraction can be summarised in one fundamental law known as Fermat's Principle. According to this principle a ray of light travels from one point to another such that the time taken is at a stationary value (maximum or minimum). If c is the velocity of light in a vacuum, the velocity in a medium of refractive index n is $\frac{c}{n}$, hence time taken to travel a distance l is $\frac{nl}{c}$. If the light passes through a number of media, the total time taken is $\left(\frac{1}{c}\right)\sum nl$ or $\frac{1}{c}\int ndl$ if refractive index varies continuously. Now, $\sum nl$ is the total optical path, so that Fermat's Principle states then the path of a ray is such that the optical path in at a stationary value. This principle is obviously in agreement with the fact that the ray are straight lines in a homogeneous isotropic medium. It is found that it also agrees with the classical laws of reflection and refraction. Based on above information, answer the following questions.

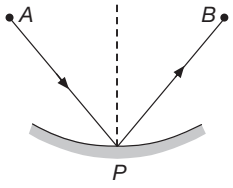
28. If refractive index of a slab varies as $m = 1 + x^2$ where x is measured from one end, then optical path length of a slab of thickness 1 m is
(A) $\frac{4}{3}$ m (B) $\frac{3}{4}$ m
(C) 1 m (D) None of these

29. The optical path length followed by ray from point A to B, given that laws of reflection are obeyed as shown in figure is



(A) Maximum (B) Minimum
(C) Constant (D) None of these

30. The optical path length followed by ray from point A to B, given that laws of reflection are obeyed as shown in figure is



(A) Maximum (B) Minimum
(C) Constant (D) None of these

Comprehension 10

Consider an equiconvex lens of radius R, made of a material of refractive index μ. Its focal length is f_1 when any one face is silvered. Now consider another plano-convex lens of radius R, made of same material having focal length, f_2 when no face is silvered, f_3 when plane face is silvered and f_4 when curved surface is silvered. Based on above information, answer the following questions.

31. f_1 equals

(A) $\frac{R}{(2\mu-1)}$ (B) $\frac{2R}{(2\mu-1)}$

(C) $\frac{R}{2(2\mu-1)}$ (D) $\frac{2R}{2(2\mu+1)}$

32. f_2 equals

(A) R (B) $\frac{R}{2\mu-1}$

(C) $\frac{R}{\mu-1}$ (D) $\frac{R}{\mu+1}$

33. f_3 equals

(A) $\frac{R}{2\mu}$ (B) $\frac{R}{2(\mu-1)}$

(C) $\frac{2R}{\mu}$ (D) $2R\mu$

34. f_4 equals

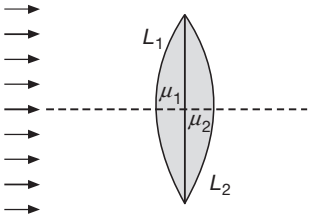
(A) $\frac{R}{\mu}$ (B) $\frac{2R}{\mu}$

(C) $2R\mu$ (D) $\frac{R}{2\mu}$

Comprehension 11

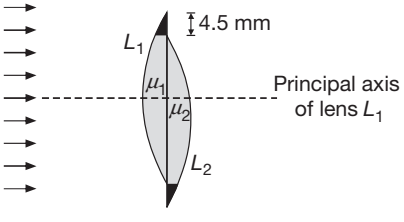
SITUATION-I

Two identical plano-convex lenses L_1 and L_2 having radii of curvature $R = 20$ cm and refractive indices $\mu_1 = 1.4$ and $\mu_2 = 1.5$ are placed as shown in the figure.



SITUATION-II

Now, the second plano-convex lens is shifted vertically downward by a small distance of 4.5 mm and the extended parts of L_1 and L_2 are blackened as shown in figure.



Based on above information, answer the following questions.

35. In SITUATION-I, the position of the image of the parallel beam of light relative to the common principal axis is

(A) $\frac{100}{9}$ cm (B) $\frac{100}{3}$ cm

(C) $\frac{200}{3}$ cm (D) $\frac{200}{9}$ cm

36. In SITUATION-II, the new position of the image of the parallel beam is

(A) $\frac{200}{9}$ cm in front of the lens 2 mm below the principal axis of L_1.

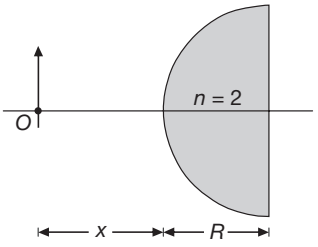
(B) $\frac{100}{9}$ cm behind the lens 2 mm below the principal axis of L_1.

(C) $\frac{200}{9}$ cm behind the lens 2.5 mm below the principal axis of L_1.

(D) $\frac{200}{9}$ cm in front of the lens 2.5 mm below the principal axis of L_1.

Comprehension 12

A small object O is placed in air at the principal axis at a distance x from the pole of the curved surface of a transparent hemisphere having refractive index 2 and radius R as shown. Based on above information, answer the following questions.



37. The value of x, for which the final image of the object at O will be virtual is

(A) $2R$ (B) $3R$

(C) $\frac{R}{3}$ (D) $1.5R$

38. The nature of final image of the object when $x = 2R$ is

(A) Erect and magnified

(B) Inverted and magnified

(C) Erect and same size
(D) Inverted and same size

39. It is observed that for $x = R$, a ray starting from O strikes the spherical surface at grazing incidence. The angle with the normal at which the ray emerges from the plane surface is

(A) 90° (B) 0°
(C) 30° (D) 60°

Comprehension 13

The refractive indices of the crown glass for blue and red lights are 1.51 and 1.49 respectively and those of the flint glass are 1.77 and 1.73 respectively. An isosceles prism of angle 6° is made of crown glass. A beam of white light is incident at a small angle on this prism. The other flint glass isosceles prism is combined with the crown glass prism such that there is no deviation of the incident light. Based on the above facts, answer the following questions.

40. The refractive index of crown glass for yellow colour is

(A) 1.51 (B) 1.49
(C) 1.50 (D) 1.59

41. The refractive index of flint glass for yellow colour is

(A) 1.70 (B) 1.72
(C) 1.73 (D) 1.75

42. The refracting angle of flint glass prism is

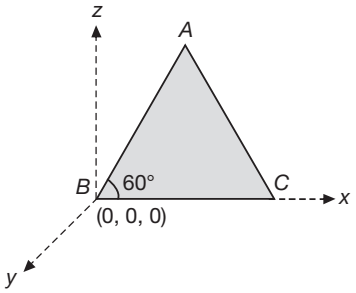
(A) +2° (B) +4°
(C) –2° (D) –4°

43. The net dispersion produced by the combined system is

(A) 0.02° (B) –0.02°
(C) +0.04° (D) –0.04°

Comprehension 14

An equilateral prism ABC is placed in air with its base side BC lying horizontally along x-axis as shown in figure. A ray of light represented by equation $\sqrt{3}z + x = 10$ is incident at a point P on the face AB of prism. Based on above information, answer the following questions.



44. The value of μ for which the ray grazes the face AC is

(A) $\frac{3}{2}$ (B) $\frac{4}{3}$
(C) $\frac{2}{\sqrt{3}}$ (D) $\frac{\sqrt{5}}{2}$

45. The direction of the finally refracted ray for $\mu = \frac{3}{2}$ is

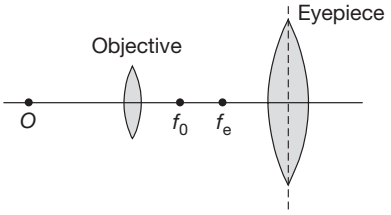
(A) parallel to x-axis
(B) parallel to z-axis
(C) parallel to y-axis
(D) parallel to face AB

46. The equation of ray emerging out of prism, if the bottom BC is silvered is

(A) $z + \sqrt{3}x = 10$ (B) $\sqrt{3}z + x = 10$
(C) $z + \sqrt{3}x = 20$ (D) $x + z = 10\sqrt{3}$

Comprehension 15

The schematic diagram of a compound microscope is shown in the adjacent figure. Its main components are two convex lenses: one acts as the main magnifying lens and is referred to as the objective, and another lens called the eyepiece. The two lenses act independently of each other when bending light rays.



f_0 = focal point of objective

f_e = focal point of eyepiece

Light from the object (O) first passes through the objective and enlarged, inverted first image is formed. The eyepiece then magnifies this image. Usually the magnification of the eyepiece is fixed (either × 10 or × 15) and three rotating objective lenses are used : × 10, × 40 and × 60. Angular magnification is defined as the angle subtended by the final image at the eye to the angle subtended by the object placed at least distance of distinct vision $(\approx 25\text{ cm})$ when viewed by the naked eye. Based on above information, answer the following questions.

47. The type of image that would have to be produced by the objective is

(A) Either virtual or real
(B) Virtual
(C) Real
(D) It depends on the focal length of the lens.

48. Where would the first image have to be produced by the objective relative to the eyepiece such that a second, enlarged image would be generated on the same side of the eyepiece as the first image? Assume that the first image distance is d_i from the eyepiece.

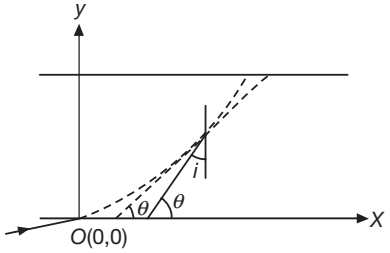
(A) $d_i < f_e$ (B) $d_i = f_e$
(C) $f_e < d_i < 2f_e$ (D) $d_i > 2f_e$

49. Two compound microscopes A and B were compared. Both had objectives and eyepieces with the same magnification but A gave an overall magnification that was greater than that of B. Which of the following is a possible explanation?

(A) The distance between object and eyepiece in A is greater than the corresponding distance in B.
(B) The distance between object and eyepiece in A is less than the corresponding distance in B.
(C) The eyepiece and objective positions were reversed in A.
(D) The eyepiece and objective positions were reversed in B.

Comprehension 16

A ray of light travelling in air is incident at grazing angle on a long rectangular slab of a transparent medium of thickness $t = 1.0$ m. The point of incidence O is the origin $(0, 0)$. The medium has a variable index of refraction $\mu(y)$ given by $\mu(y) = \left[Ky^{3/2} + 1\right]^{1/2}$ where $K = 1.0(m)^{-3/2}$. The refractive index of air is 1.0.



Based on the above facts, answer the following questions.

50. The relation between the slope of the trajectory of the ray at the point $B(x, y)$ in the medium and the angle of incidence (i) at that point is given by

(A) $\tan\theta = \sin i$ (B) $\tan\theta = 2\sin i$
(C) $\tan\theta = \cot i$ (D) $\tan\theta = 2\cot i$

51. The equation for the trajectory $y(x)$ of the ray in the medium is

(A) $y = \frac{x^2}{16}$ (B) $y = \frac{x^3}{16}$
(C) $y = \frac{x^4}{16}$ (D) $y = \frac{x^4}{256}$

52. The co-ordinates (x, y) of the point where the ray intersects the upper surface of the slab-air boundary are

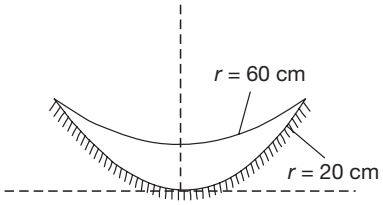
(A) $(1, 1)$ m (B) $(2, 1)$ m
(C) $(3, 1)$ m (D) $(4, 1)$ m

53. The ray finally emerges

(A) parallel to the incident ray
(B) perpendicular to the incident ray
(C) at an angle of $30°$ to the incident ray
(D) at an angle of $45°$ to the incident ray

Comprehension 17

The convex surface of a thin concavo–convex lens of glass of refractive index 1.5 has a radius of curvature of 20 cm. The concave surface has a radius of curvature of 60 cm. The convex side is silvered and placed on a horizontal surface as shown in the figure.



Based on above information, answer the following questions.

54. The focal length of the combination has the magnitude

(A) 1.5 cm (B) 15 cm
(C) 7.5 cm (D) 8.6 cm

55. The combination behaves like

(A) a convex mirror
(B) a concave mirror
(C) a convex lens
(D) a concave lens

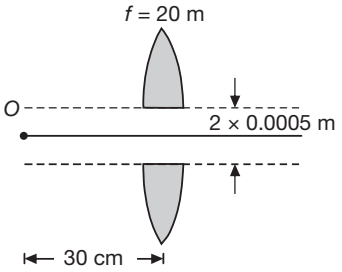
56. A small object is placed on the principal axis of the combination, at a distance of 30 cm in front of the mirror. The magnification of the image is

(A) $-\frac{1}{3}$ (B) $\frac{3}{4}$

(C) 5 (D) $-\frac{1}{4}$

Comprehension 18

A point object O is placed at a distance of 0.3 m from a convex lens (focal length 0.2 m) cut into two halves each of which is displaced by 0.0005 m as shown in the figure.



Based on above information, answer the following questions.

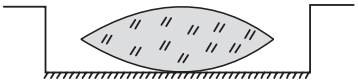
57. The position at which the image is formed is
(A) 30 cm, right of lens (B) 40 cm, left of lens
(C) 60 cm, right of lens (D) 70 cm, left of lens

58. The total number of images generated by the arrangement is/are
(A) 1 (B) 2
(C) 4 (D) 6

59. The spacing between the images so formed is
(A) 0.1 cm (B) 0.3 cm
(C) 0.5 cm (D) 1 cm

Comprehension 19

A thin biconvex lens of refractive index $\frac{3}{2}$ is placed on a horizontal plane mirror as shown in figure.



The space between the lens and the mirror is then filled with water of refractive index $\frac{4}{3}$. It is found that when a point object is placed 15 cm above the lens on its principal axis, the object coincides with its own image. On repeating with another liquid, the object and the image again coincide at a distance 25 cm from the lens. Based on the above facts, answer the following questions.

60. The radius of curvature of both the surfaces of the convex lens is R, then the focal length of the convex lens is
(A) R (B) $\frac{R}{2}$
(C) $\frac{R}{4}$ (D) $\frac{R}{8}$

61. When the space between the lens and mirror is filled with water, the focal length of water concave lens is
(A) R (B) $2R$
(C) $3R$ (D) $4R$

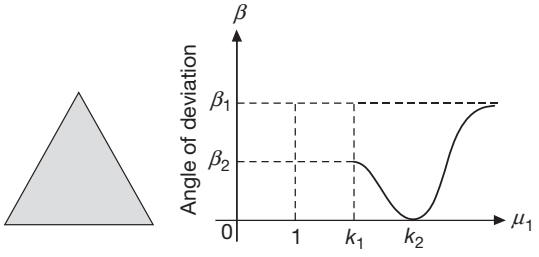
62. The radius of curvature R of each surface of convex lens is
(A) 2 cm (B) 5 cm
(C) 10 cm (D) 15 cm

63. The focal length of the liquid concave lens is
(A) $\frac{20}{3}$ cm (B) $\frac{40}{3}$ cm
(C) $\frac{50}{3}$ cm (D) $\frac{70}{3}$ cm

64. The refractive index of the liquid is
(A) 1.1 (B) 1.2
(C) 1.4 (D) 1.6

Comprehension 20

The diagram shows an equilateral prism. The medium on one side of the prism has refractive index μ_1. The refractive index of the prism is $\mu = \frac{4}{\sqrt{3}}$. The diagram shows variation of magnitude of angle of deviation with respect to μ_1. Consider the light ray to be incident normally on the first face.



Based on above information, answer the following questions.

65. Value of k_2 is
(A) $\frac{6}{\sqrt{3}}$ (B) $\frac{4}{\sqrt{3}}$
(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{8}{\sqrt{3}}$

66. Value of k_1 is

(A) $\frac{4}{3}$ (B) $\frac{5}{3}$

(C) $\frac{8}{3}$ (D) $\frac{10}{3}$

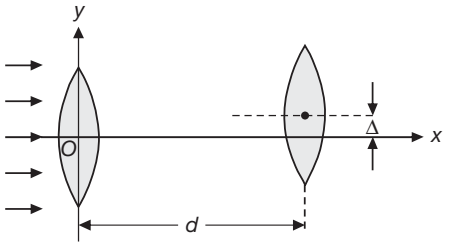
67. Value of $\beta_1 - \beta_2$ is

(A) 20° (B) 30°

(C) 60° (D) 90°

Comprehension 21

Two thin convex lenses of focal lengths f_1 and f_2 are separated by a horizontal distance $d\,(d < f_1$ and $d < f_2)$ and their centers are displaced by a vertical separation as shown in the figure. Take the origin of coordinates O at the center of first lens. For a parallel beam of light coming from the left, as shown in figure.



Based on above information, answer the following questions.

68. The x-coordinate of the focal point of this lens system is

(A) $\frac{d(f_1-d)+f_1f_2}{f_1+f_2-d}$ (B) $\frac{f_1f_2}{f_1+f_2-d}$

(C) $\frac{d(f_1-d)}{f_1+f_2-d}$ (D) $\frac{2d(f_1+d)-f_1f_2}{f_1+f_2-d}$

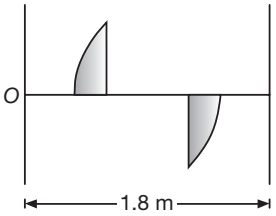
69. The y-coordinate of the focal point of this lens system is

(A) $\frac{(f_1+d)\Delta}{(f_1+f_2-d)}$ (B) $\frac{2(f_1+d)}{(f_1+f_2-d)}$

(C) $\frac{2(f_1-d)\Delta}{(f_1+f_2+d)}$ (D) $\frac{(f_1-d)\Delta}{(f_1+f_2-d)}$

Comprehension 22

A thin plano-convex lens of focal length f is split into two halves. One of the halves is shifted along the optical axis. The separation between the object and image planes is 1.8 m. The magnification of the image formed by one of the half lenses is 2.



Based on above information, answer the following questions.

70. The focal length of the lens used is

(A) 0.4 m (B) 0.6 m

(C) 1 m (D) 2 m

71. The separation between the two halves of the thin plano-convex lens is

(A) 0.2 m (B) 0.4 m

(C) 0.6 m (D) 0.8 m

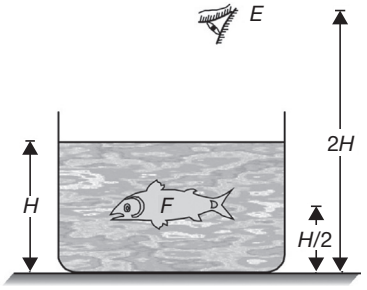
72. The magnification for the second half lens is

(A) 0.5 (B) –0.5

(C) 0.4 (D) –2

Comprehension 23

Consider a beaker filled with water (of refractive index μ) to a height H. A fish F is at a height $\frac{H}{2}$ from the transparent base of the beaker which lies on a surface that happens to be a mirror. An observer whose eye E is at a height $2H$ from the base of beaker is also there. Based on above information, answer the following questions.



73. The distance from itself at which the fish will see the image of the eye by direct observation is

(A) $H\left(\mu+\frac{1}{2}\right)$ (B) $\frac{H}{2}\left(1+\frac{\mu}{2}\right)$

(C) $H\left(\frac{\mu}{2}+1\right)$ (D) $\frac{H}{2}\left(\mu+\frac{1}{2}\right)$

74. The distance from itself at which the fish sees the image of eye by viewing in the mirror is

(A) $H\left(\mu+\frac{1}{2}\right)$ (B) $H\left(\mu+\frac{3}{2}\right)$

(C) $2H\left(\mu+\frac{1}{2}\right)$ (D) $2H\left(\mu+\frac{3}{2}\right)$

75. The distance from itself at which the eye sees the image of the fish by directly observing the fish is

(A) $2H\left(1+\frac{1}{2\mu}\right)$ (B) $2H\left(\frac{1}{2}+\frac{1}{\mu}\right)$

(C) $H\left(1+\frac{1}{2\mu}\right)$ (D) $H\left(\frac{1}{2}+\frac{1}{\mu}\right)$

76. The distance from itself at which the eye sees the image of the fish by viewing in the mirror is

(A) $2\left(H+\frac{3}{2\mu}\right)$ (B) $H+\frac{3}{2\mu}$

(C) $H\left(\frac{1}{2}+\frac{3}{\mu}\right)$ (D) $H\left(1+\frac{3}{2\mu}\right)$

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

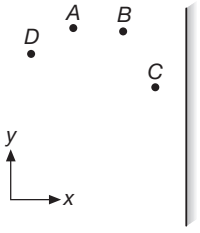
If the correct matches are A → (p, s, t); B → (q, r); C → (p, q); and D → (s, t); then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	●	●	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. For a real object, match the magnification situations in **COLUMN-I**, with their respective matches in **COLUMN-II**.

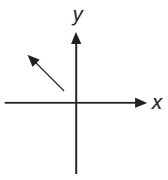
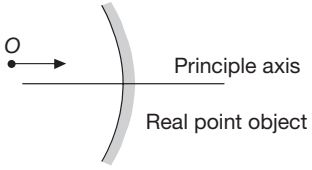
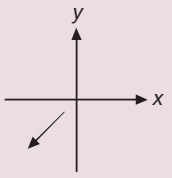
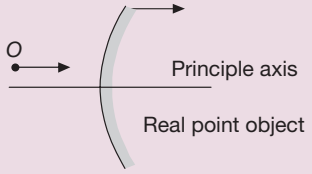
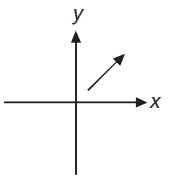
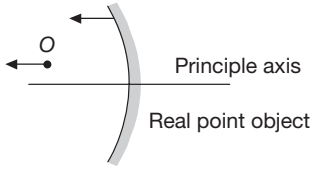
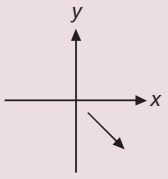
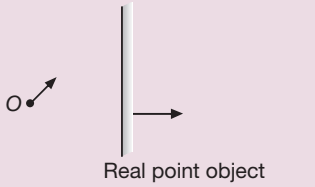
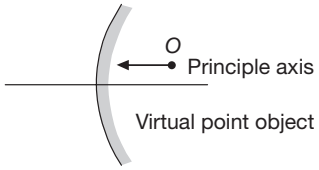
COLUMN-I	COLUMN-II
(A) $m<0$	(p) Plane mirror
(B) $m>0$	(q) Convex mirror
(C) $\lvert m\rvert<1$	(r) Concave mirror
(D) $\lvert m\rvert\geq 1$	(s) Convex lens
	(t) Concave lens

2. Four particles are moving with different velocities in front of stationary plane mirror that lies in the y-z plane. At $t=0$, velocity of A is $\vec{v}_A=\hat{i}$, velocity of B is $\vec{v}_B=-\hat{i}+3\hat{j}$, velocity of C is $\vec{v}_C=5\hat{i}+6\hat{j}$, velocity of D is $\vec{v}_D=3\hat{i}-\hat{j}$. The acceleration of particle A is $\vec{a}_A=2\hat{i}+\hat{j}$ and acceleration of particle C is $\vec{a}_C=2\hat{i}+\hat{j}$, whereas the particle B and D move with uniform velocity. Assume no collision to take place till $t=2$ s, all quantities to be in SI units, the relative velocity of image of object A with respect to object A to be denoted by $\vec{v}_{A'A}$. If velocity of images relative to corresponding objects are given in **COLUMN-I** and their values at $t=2$ s are given in **COLUMN-II**, then match the quantities in **COLUMN-I** with the corresponding values in **COLUMN-II**.

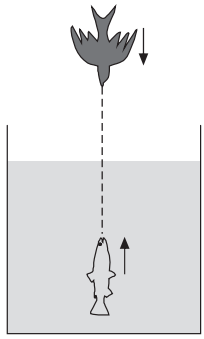


COLUMN-I	COLUMN-II
(A) $\vec{v}_{A'A}$	(p) $2\hat{i}$
(B) $\vec{v}_{B'B}$	(q) $-6\hat{i}$
(C) $\vec{v}_{C'C}$	(r) $-12\hat{i}+4\hat{j}$
(D) $\vec{v}_{D'D}$	(s) $-10\hat{i}$
	(t) Perpendicular to the plane of mirror

3. The **COLUMN-I** shows some probable directions of velocity of images formed due to system shown in **COLUMN-II**. Match the quantities of **COLUMN-I** with the respective possibilities shown in **COLUMN-II.**

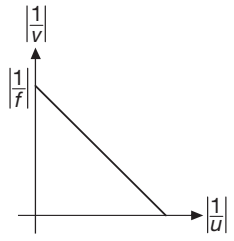
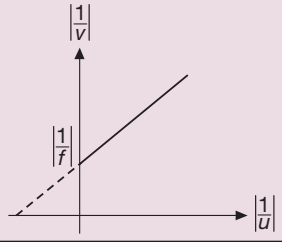
COLUMN-I	COLUMN-II
(A) 	(p) 
(B) 	(q) 
(C) 	(r) 
(D) 	(s) 
	(t) 

4. A bird in air is diving vertically over a tank with a speed of 6 cms^{-1}. The base of the tank is silvered. A fish in the tank is rising upward along the same line with a speed of 8 cms^{-1}. Taking $\mu_{water} = \frac{4}{3}$, match the quantities in **COLUMN-I** with their respective values in **COLUMN-II**.

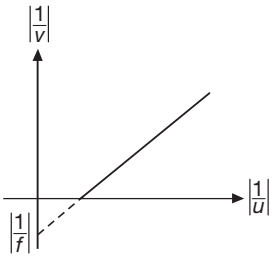
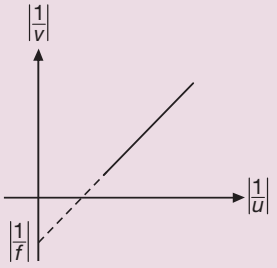


COLUMN-I	COLUMN-II
(A) Speed of the image of fish, in cms^{-1} as seen by the bird directly	(p) 16
(B) Speed of the image of fish, in cms^{-1} formed after reflection from the mirror as seen by the bird	(q) 0
(C) Speed of image of bird, in cms^{-1} relative to the fish looking upwards	(r) 12
(D) Speed of image of bird, in cms^{-1} relative to the fish looking downwards in the mirror	(s) 8

5. Match the descriptions in **COLUMN-I** with corresponding plot(s) in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) In convex mirror, when object is real and image is virtual	(p) 
(B) In convex mirror, when object is virtual and image is real.	(q) 

(*Conitnued*)

COLUMN-I	COLUMN-II
(C) In concave mirror, when object is real and image is virtual.	(r) 
(D) In concave mirror, when object is real or virtual and image is real.	(s) 

6. If (μ_1, λ_1, v_1) and (μ_2, λ_2, v_2) are the refractive indices, wavelengths and speeds of two light waves respectively, then match the entries of **COLUMN-I** with the entries of **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) $\mu_1 > \mu_2$	(p) $v_1 < v_2$
(B) $\mu_1 < \mu_2$	(q) $v_1 > v_2$
(C) $\mu_1 \neq \mu_2$	(r) $\lambda_1 = \lambda_2$
(D) $\mu_1 = \mu_2$	(s) $\lambda_1 < \lambda_2$

7. Match the descriptions in **COLUMN-I** to corresponding details in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) In refraction from a rarer to a denser medium.	(p) Speed of wave does not change.
(B) In refraction.	(q) Wavelength must be decreased.
(C) In reflection from a denser medium.	(r) Frequency does not change.
(D) In reflection.	(s) The reflected ray suffers an additional path change of $\frac{\lambda}{2}$.

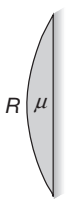
8. Light rays are incident on devices which may cause either reflection or refraction or both. The nature of the incident light and the devices are described in **COLUMN–I**. Some possible results of this on the rays are given in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) A ray of white light passes from an optically denser medium to an optically rarer medium.	(p) Divergent beam
(B) A parallel beam of monochromatic light passes symmetrically through a glass lens.	(q) Total internal reflection
(C) A ray of white light is incident at an angle on a thick glass sheet.	(r) Lateral shift
(D) A ray of white light is incident on one face of an equivalent glass prism.	(s) Dispersion

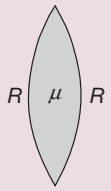
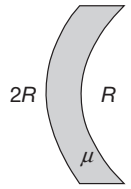
9. For a real object, match the descriptions in **COLUMN-I** to the corresponding details in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Convex mirror	(p) Virtual image
(B) Concave mirror	(q) Real image
(C) Convex lens	(r) Enlarged image
(D) Concave lens	(s) Diminished image

10. Match the details of **COLUMN-I** with the respective name and nature described in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) 	(p) Converging

(*Continued*)

COLUMN-I	COLUMN-II
(B) 	(q) Concavo-convex
(C) 	(r) Convexo-concave
(D)	(s) Diverging

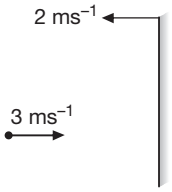
11. Match the following

COLUMN-I	COLUMN-II
(A) Concave mirror, virtual object	(p) Real image
(B) Convex mirror, virtual object	(q) Virtual image
(C) Convex lens, real object	(r) Magnified image
(D) Concave lens, real object	(s) Diminished image

12. For a concave mirror of focal length 20 cm, match the object distances in **COLUMN-I** to the corresponding details of images formed in **COLUMN-II.**

COLUMN-I	COLUMN-II
(A) 10 cm	(p) Magnified, inverted and real
(B) 30 cm	(q) Equal size, inverted and real
(C) 40 cm	(r) Smaller, inverted and real
(D) 50 cm	(s) Magnified, erect and virtual

13. A point object is placed in front of a plane mirror as shown and moving with velocity $3\ ms^{-1}$ towards mirror. Mirror is moving with speed $2\ ms^{-1}$ towards object, then

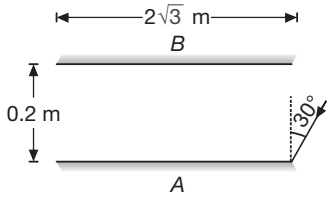


COLUMN-I	COLUMN-II
(A) Speed of image w.r.t. ground	(p) $10\ ms^{-1}$
(B) Speed of image w.r.t. mirror	(q) $5\ ms^{-1}$
(C) Speed of image w.r.t. object	(r) $14\ ms^{-1}$
(D) Speed of mirror w.r.t. object	(s) $7\ ms^{-1}$

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle $30°$ at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. Find the maximum number of times the ray undergoes reflections (including the first one) before it emerges out.



2. Where should an object be placed, in cm, in front of a concave mirror of focal length 30 cm so the image size is 5 times the object size?

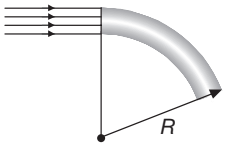
3. A rod of length 20 cm is placed along the optical axis of a concave mirror of focal length 30 cm. One end of the rod is at the centre of curvature and the other end lies between F and C. Find the magnitude of the linear magnification of the rod

4. A point source of light S is placed on the major optical axis of concave mirror at a distance of 60 cm. At what distance, in cm, from the concave mirror should a flat mirror be placed for the rays to converge again at the point S having been reflected from the concave mirror and then from the flat one? Will the position of the point where the rays meet change if they are first reflected from the flat mirror? The radius of the concave mirror is 80 cm.

5. A concave mirror forms on a screen a real image of thrice the linear dimensions of the object. Object and screen are moved until the image is twice the size of the object. If the shift of the object is 6 cm, find the shift of the screen and the focal length of the mirror (both in cm).

6. A fish is rising up vertically inside a pond with velocity $4\ \text{cms}^{-1}$ and notices a bird, which is diving vertically downward and its velocity appears to be $16\ \text{cms}^{-1}$ (to the fish). What is the actual velocity of the diving bird, in cms^{-1}, if refractive index of water is $4/3$.

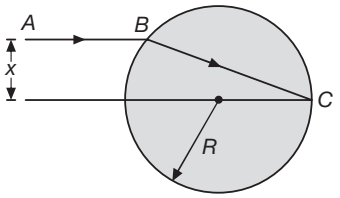
7. A portion of a straight glass rod of diameter 4 cm and refractive index 1.5 is bent into an arc of radius R cm and a parallel beam of light is incident on it as shown in figure. Find the smallest R, in cm, which permits all the light to pass around the arc.



8. A man of height 2 m is standing on level road where because of temperature variation the refractive index of air is varying as $\mu = \sqrt{1+ay}$, where y is height from road. If $a = 2\times10^{-6}\ \text{m}^{-1}$. Then find the maximum distance, in km, till which he can see on the road.

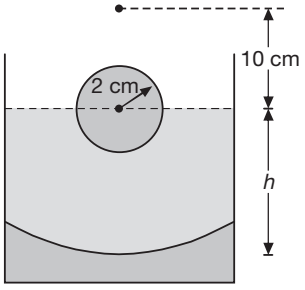
9. A ray of light falls on a glass sphere of refractive index $\sqrt{3}$ such that the directions of the incident ray and emergent ray when produced meet the surface at the same point on the surface. Draw the ray diagram and find the value of angle of incidence, in degree.

10. A spherical ball of transparent material has index of refraction μ. A narrow beam of light AB is aimed as shown. What must the index of refraction be in order that the light is focussed at the point C on the opposite end of the diameter from where the light entered? Given that $x \ll R$.

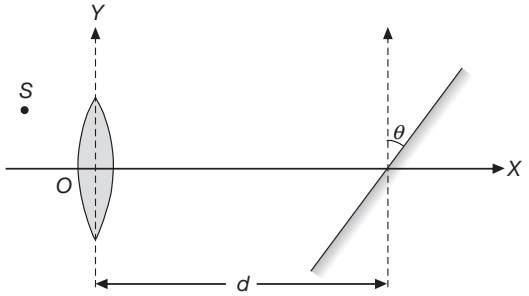


11. The perpendicular faces of a right isosceles prism are coated with silver. The rays incident at an arbitrary angle on the hypotenuse face emerge from the prism after suffering a deviation of x degree. Find x.

12. A transparent solid sphere of radius 2 cm and density ρ floats in a transparent liquid of density 2ρ kept in a beaker. The bottom of the beaker is spherical in shape with radius of curvature 8 cm and is silvered to make it concave mirror as shown in the figure. When an object is placed at a distance of 10 cm directly above the centre of the sphere its final image coincides with it. Find h (as shown in figure), the height of the liquid surface in the beaker, in cm, from the apex of the bottom. Consider the paraxial rays only. The refractive index of the sphere is $\frac{3}{2}$ and that of the liquid is $\frac{4}{3}$.



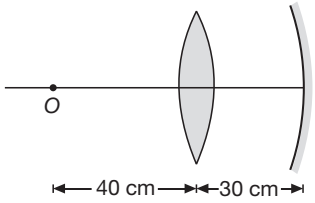
13. A thin converging lens of focal length $f = 1.5$ m is placed along y-axis such that its optical centre coincides with the origin. A small light source S is placed at $(-2,\ 0.1)$ m. A plane mirror inclined at an angle θ, (where $\tan\theta = 0.3$) is placed as shown in figure, such that y co-ordinate of final image is 0.3 m. Find the distance d, in metre. Also find the x co-ordinate of final image, in metre.



14. An object is placed 12 cm to the left of a diverging lens of focal length -6 cm . A converging lens with a focal length of 12 cm is placed at a distance d to the right of the diverging lens. Find the distance d , in cm , that corresponds to a final image at infinity.

15. Determine the position of the image, in cm, produced by an optical system consisting of a concave mirror with a focal length of 10 cm and a convergent lens with a focal length of 20 cm . The distance from the mirror to the lens is 30 cm and from the lens to the object 40 is cm. Consider only two steps.

16. The figure shows an arrangement of an equiconvex lens of focal length 20 cm and a concave mirror of radius of curvature 80 cm . A point object O is placed on the principal axis at a distance 40 cm from the lens such that the final image is also formed at the position of the object. Find the distance d , in cm . Also draw the ray diagram.



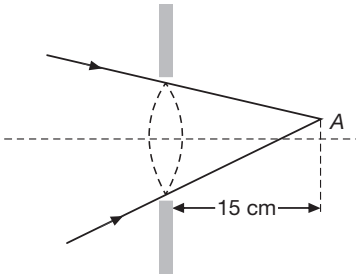
17. A converging beam of rays is incident on a diverging lens. After passing through the lens the rays intersect at a point 15 cm from the lens. If the lens is removed, the point where the rays meet, move 5 cm closer to the mounting that holds the lens. Find the focal length of the lens, in cm.

18. A lens with a focal length of $f = 30$ cm produces on a screen a sharp image of an object that is at a distance of $a = 40$ cm from the lens. A plane-parallel glass plate having $\mu = 1.8$ and a thickness of $d = 9$ cm is placed between the lens and the object perpendicular to the optical axis of the lens. Through what distance, in cm, should the screen be shifted for the image of the object to remain distinct?

19. The height of a candle flame is 5 cm. A lens produces an image of this flame 15 cm high on a screen. Without touching the lens, the candle is moved over a distance of $\ell = 1.5$ cm away from the lens and a sharp image of the flame 10 cm high is obtained again after shifting the screen. Calculate the focal length of the lens, in cm.

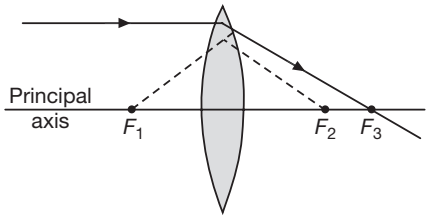
20. The focal length of a convex lens in air is 10 cm . Find its focal length, in cm , in water. Given that $\mu_g = \frac{3}{2}$ and $\mu_w = \frac{4}{3}$.

21. A converging beam of rays passes through a round aperture in a screen as shown in figure. The apex of the beam A is at a distance of 15 cm from the screen. How will the distance from the focus of the rays to the screen change, in cm, if a convergent lens is inserted in the aperture with a focal length of 30 cm ? Plot the path of the rays after the lens is fitted.



22. A lens with a focal length of 16 cm produces a sharp image of an object in two positions which are 60 cm apart. Find the distance, in cm, from the object to the screen.

23. An intense beam parallel to the principal axis is incident on a convex lens. Multiple extra images F_1 , F_2 , … are formed due to feeble internal reflections, called flare spots as shown in the figure. The rardii of curvature of the lens are 30 cm and 60 cm and the refractive index is 1.5. Find the position of the first flare spot, in cm.



24. A parallel beam of rays is incident on a convergent lens with a focal length of 40 cm . Where should a divergent lens with a focal length of 15 cm be placed

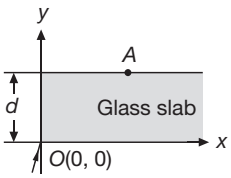
for the beam of rays to remain parallel after passing through the two lenses. Give your answer in cm.

25. One side of radius of curvature $R_2 = 120$ cm of a convex lens of material of refractive index $\mu = 1.5$ and focal length $f_1 = 40$ cm is silvered. It is placed on a horizontal surface with silvered surface in contact with it. Another convex lens of focal length $f_2 = 20$ cm is fixed coaxial $d = 10$ cm above the first lens. A luminous point object O on the axis gives rise to an image coincident with it. Find its height, in cm, above the upper lens.

26. A source of light is located from a convergent lens of focal length $f = 30$ cm at a distance double the focal length of the convergent lens. At what distance from the lens should a flat mirror be placed so that the rays reflected from the mirror are parallel after passing through the lens for the second time? Give your answer in cm.

27. A long rectangular slab of transparent medium of thickness d is placed on a table with length parallel to x-axis and width parallel to the y-axis as shown.

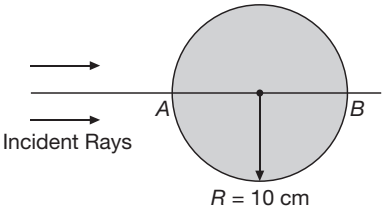
A ray of light is grazing along y-axis and hits the interphase separating the two media at origin. The refractive index μ of the medium varies with x as $\mu = \sqrt{1 + e^{\frac{x}{d}}}$. The refractive index of the air is 1.



(a) The x-coordinate of the point A, where the ray intersects the upper surface of the slab-air boundary is $x = d\log_e(\alpha)$. Find α.

(b) The refractive index of the medium at A is $\sqrt{\beta}$. Find β.

28. A parallel paraxial beam of light is incident on a glass sphere of radius 10 cm along its diameter AB from one side as shown. If all the rays after refraction converge at the point B then calculate the refractive index of the glass sphere.



29. An object of height 4 cm is kept to the left of and on the axis of a converging lens of focal length 10 cm at a distance of 15 cm from the lens. A plane mirror is placed inclined at 45° to the lens axis, 10 cm to the right of the lens. Find the position and size of the image (in cm) formed by the lens and mirror combination. Trace the path of the rays forming the image.

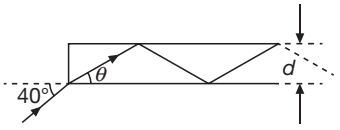
30. A point object is placed at a distance of 25 cm from a convex lens of focal length 20 cm. If a glass slab of thickness t and refractive index 1.5 is inserted between the lens and the object, the image is formed at infinity. Find the thickness t of slab (in cm).

31. The focal lengths of the objective and the eye-piece of an astronomical telescope are 0.25 m and 0.025 m, respectively. The telescope is focussed on an object 5 m from the objective, the final image being formed 0.25 m from the eye of the observer. Calculate the tube length (in centimetre) of the telescope to the nearest integer and $10M$, where M is the magnifying power of the telescope.

ARCHIVE: JEE MAIN

1. **[Online April 2019]**

In figure, the optical fiber is $l = 2$ m long and has a diameter of $d = 20\ \mu$m. If a ray of light is incident on one end of the fiber at angle $\theta_1 = 40°$, the number of reflections it makes before emerging from the other end is close to (refractive index of fiber is 1.31 and $\sin 40° = 0.64$)



(A) 66000 (B) 55000
(C) 45000 (D) 57000

2. **[Online April 2019]**
An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be

(A) 20 cm from the convergent mirror, twice the size of the object

(B) 20 cm from the convergent mirror, same size as the object

(C) 40 cm from the convergent lens, same as the size of the object

(D) 40 cm from the convergent mirror, twice the size as the object

3. **[Online April 2019]**
Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star.

(A) 457.5×10^{-9} radian (B) 305×10^{-9} radian

(C) 152.5×10^{-9} radian (D) 610×10^{-9} radian

4. **[Online April 2019]**
A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself, produces a virtual image would be

(A) 30 cm (B) 25 cm

(C) 20 cm (D) 10 cm

5. **[Online April 2019]**
A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is

(A) 0.32 m (B) 0.24 m

(C) 1.60 m (D) 0.16 m

6. **[Online April 2019]**
A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and x_2 $(x_1 > x_2)$ from the lens. The ratio of x_1 and x_2 is

(A) 3 : 1 (B) 2 : 1

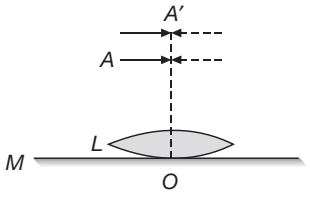
(C) 4 : 3 (D) 5 : 3

7. **[Online April 2019]**
Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600 nm. Coming from a distant object, the limit of resolution of the telescope is close to

(A) 1.5×10^{-7} rad (B) 3.0×10^{-7} rad

(C) 2.0×10^{-7} rad (D) 4.5×10^{-7} rad

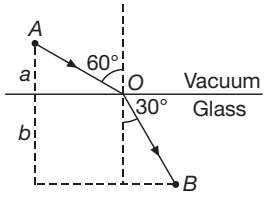
8. **[Online April 2019]**
A thin convex lens L (refractive index $= 1.5$) is placed on a plane mirror M. When a pin is placed at A, such that $OA = 18$ cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_ℓ is put between the lens and the mirror, the pin has to be moved to A', such that $OA' = 27$ cm, to get its inverted real image at A' itself. The value of μ_ℓ will be



(A) $\sqrt{2}$ (B) $\frac{4}{3}$

(C) $\sqrt{3}$ (D) $\frac{3}{2}$

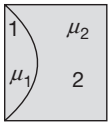
9. **[Online April 2019]**
A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in the figure. The optical path length of light ray from A to B is



(A) $\frac{2\sqrt{3}}{a}+2b$ (B) $2a+\frac{2b}{\sqrt{3}}$

(C) $2a+\frac{2b}{3}$ (D) $2a+2b$

10. **[Online April 2019]**
One plano-convex and one plano-concave lens of same radius of curvature R but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is

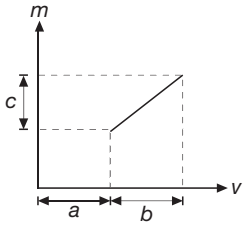


(A) $\dfrac{R}{2(\mu_1-\mu_2)}$ (B) $\dfrac{R}{2-(\mu_1-\mu_2)}$

(C) $\dfrac{R}{\mu_1-\mu_2}$ (D) $\dfrac{2R}{\mu_1-\mu_2}$

11. [Online April 2019]

The graph shows how the magnification m produced by a thin lens varies with image distance v. What is the focal length of the lens used?



(A) $\dfrac{b^2}{ac}$ (B) $\dfrac{b^2c}{a}$

(C) $\dfrac{a}{c}$ (D) $\dfrac{b}{c}$

12. [Online April 2019]

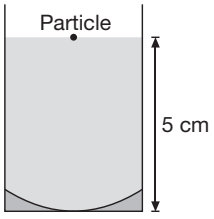
The value of numerical aperture of the objective lens of a microscope is 1.25. If light of wavelength 5000 Å is used, the minimum separation between two points, to be seen as distinct, will be

(A) 0.24 μm (B) 0.38 μm

(C) 0.48 μm (D) 0.12 μm

13. [Online April 2019]

A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to: (Refractive index of water = 1.33)

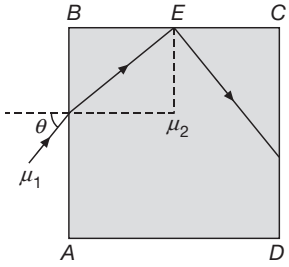


(A) 11.7 cm (B) 6.7 cm

(C) 13.4 cm (D) 8.8 cm

14. [Online April 2019]

A transparent cube of side d, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1(\mu_1<\mu_2)$. A ray is incident on the face AB at an angle θ (shown in the figure). Total internal reflection takes place at point E on the face BC.



Then θ must satisfy

(A) $\theta<\sin^{-1}\left(\dfrac{\mu_1}{\mu_2}\right)$ (B) $\theta<\sin^{-1}\left(\sqrt{\dfrac{\mu_2^2}{\mu_1^2}-1}\right)$

(C) $\theta>\sin^{-1}\left(\dfrac{\mu_1}{\mu_2}\right)$ (D) $\theta>\sin^{-1}\left(\sqrt{\dfrac{\mu_2^2}{\mu_1^2}-1}\right)$

15. [Online January 2019]

A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance d. Then d is

(A) 1.1 cm away from the lens

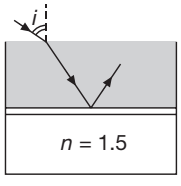
(B) 0.55 cm towards the lens

(C) 0

(D) 0.55 cm away from the lens

16. [Online January 2019]

Consider a tank made of glass (refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is



(A) $\frac{4}{3}$ (B) $\sqrt{\frac{5}{3}}$

(C) $\frac{3}{\sqrt{5}}$ (D) $\frac{5}{\sqrt{3}}$

17. [Online January 2019]

Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from the second mirror (M_2) parallel to the first mirror (M_1). The angle between the two mirrors will be

(A) 75° (B) 45°

(C) 90° (D) 60°

18. [Online January 2019]

A plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as

(A) $2\mu_1 - \mu_2 = 1$ (B) $3\mu_2 - 2\mu_1 = 1$

(C) $2\mu_2 - \mu_1 = 1$ (D) $\mu_1 + \mu_2 = 3$

19. [Online January 2019]

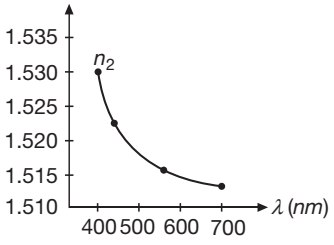
The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34, Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

(A) 4.0 cm (B) 1 cm

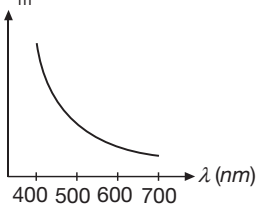
(C) 3.1 cm (D) 2 cm

20. [Online January 2019]

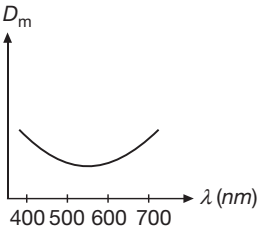
The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation?



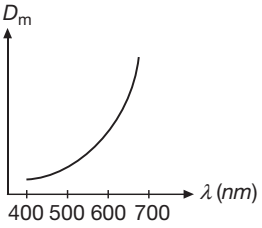
(A)



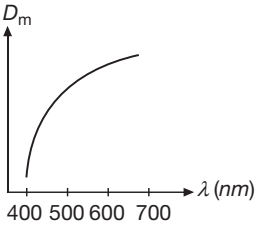
(B)



(C)



(D)



21. [Online January 2019]

An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 ms^{-1}, the speed and direction of the image will be

(A) 0.92×10^{-3} ms^{-1} away from the lens

(B) 2.26×10^{-3} ms^{-1} away from the lens

(C) 1.16×10^{-3} ms^{-1} towards the lens

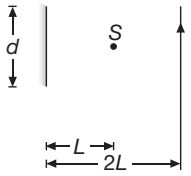
(D) 3.22×10^{-3} ms^{-1} towards the lens

22. [Online January 2019]

A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is

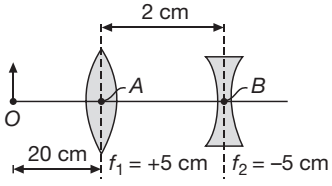
(A) 90° (B) 30°
(C) 45° (D) 60°

23. [Online January 2019]
A point source of light, S is placed at a distance L in front of the centre of plane mirror of width d which is hanging vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance $2L$ as shown below. The distance over which the man can see the image of the light source in the mirror is



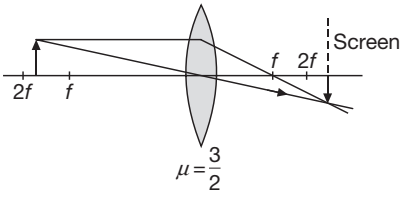
(A) $\frac{d}{2}$ (B) $3d$
(C) $2d$ (D) d

24. [Online January 2019]
What is the position and nature of image formed by lens combination shown in figure? (f_1, f_2 are focal lengths)



(A) $\frac{20}{3}$ cm from point B at right; real
(B) 70 cm from point B at right; real
(C) 40 cm from point B at right; real
(D) 70 cm from point B at left; virtual

25. [Online January 2019]
Formation of real image using a biconvex lens is shown below



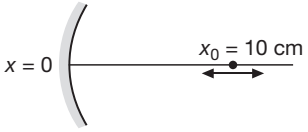
If the whole set up is immersed in water $\mu = \frac{4}{3}$ without disturbing the object and the screen positions, what will one observe on the screen?

(A) Erect real image (B) No change
(C) Image disappears (D) Magnified image

26. [Online January 2019]
A plano-convex lens (focal length f_2, refractive index μ_2, radius of curvature R) fits exactly into a plano-concave lens (focal length f_1, refractive index μ_1, radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be

(A) $f_1 - f_2$ (B) $\frac{R}{\mu_2 - \mu_1}$
(C) $\frac{2f_1f_2}{f_1 + f_2}$ (D) $f_1 + f_2$

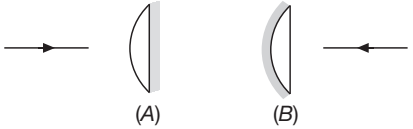
27. [Online 2018]
A particle is oscillating on the x-axis with an amplitude 2 cm about the point $x_0 = 10$ cm, with a frequency ω. A concave mirror of focal length 5 cm is placed at the origin (see figure). Identify the correct statements.



(1) The image executes periodic motion.
(2) The image executes non-periodic motion.
(3) The turning points of the image are asymmetric w.r.t. the image of the point at $x = 10$ cm.
(4) The distance between the turning points of the oscillation of the image is $\frac{100}{21}$ cm

(A) 2, 4 (B) 2, 3
(C) 1, 3, 4 (D) 1, 4

28. [Online 2018]
A planoconvex lens becomes an optical system of 28 cm focal length when its plane surface is silvered and illuminated from left to right as shown in figure (A). If the same lens is instead silvered on the curved surface and illuminated from other side as shown in figure (B), it acts like an optical system of focal length 10 cm. The refractive index of the material of lens is



(A) 1.75 (B) 1.51
(C) 1.55 (D) 1.50

29. [Online 2018]

A convergent doublet of separated lenses, corrected for spherical aberration, has resultant focal length of 10 cm. The separation between the two lenses is 2 cm . The focal lengths of the component lenses are

(A) 18 cm , 20 cm (B) 12 cm , 14 cm

(C) 16 cm , 18 cm (D) 10 cm , 12 cm

30. [Online 2018]

A ray of light is incident at an angle of $60°$ on one face of a prism of angle $30°$. The emergent ray of light makes an angle of $30°$ with incident ray. The angle made by the emergent ray with second face of prism will be

(A) $0°$ (B) $45°$

(C) $90°$ (D) 30

31. [2017]

A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm . A beam of parallel light falls on the diverging lens. The final image formed is

(A) real and at a distance of 40 cm from convergent lens.

(B) virtual and at a distance of 40 cm from convergent lens.

(C) real and at a distance of 40 cm from the divergent lens.

(D) real and at a distance of 6 cm from the convergent lens.

32. [Online 2017]

Let the refractive index of a denser medium with respect to a rarer medium be n_{12} and its critical angle θ_C . At an angle of incidence A when light is travelling from denser medium to rarer medium, a part of the light is reflected and the rest is refracted and the angle between reflected and refracted rays is $90°$. Angle A is given by

(A) $\dfrac{1}{\tan^{-1}(\sin\theta_C)}$ (B) $\dfrac{1}{\cos^{-1}(\sin\theta_C)}$

(C) $\tan^{-1}(\sin\theta_C)$ (D) $\cos^{-1}(\sin\theta_C)$

33. [2016]

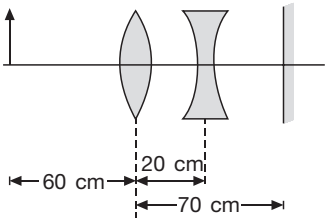
An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears

(A) 10 times taller (B) 10 times nearer

(C) 20 times taller (D) 20 times nearer

34. [Online 2016]

A convex lens, of focal length 30 cm , a concave lens of focal length 120 cm , and a plane mirror are arranged as shown. For an object kept at a distance of 60 cm from the convex lens, the final image formed by the combination is a real image at a distance of



(A) 60 cm from the convex lens

(B) 60 cm from the concave lens

(C) 70 cm form the convex lens

(D) 70 cm from the concave lens

35. [Online 2016]

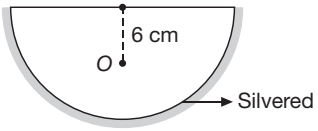
To determine refractive index of glass slab using a travelling microscope, minimum number of readings required are

(A) Two (B) Four

(C) Three (D) Five

36. [Online 2016]

A hemispherical glass body of radius 10 cm and refractive index 1.5 is silvered on its curved surface. A small air bubble is 6 cm below the flat surface inside it along the axis. The position of the image of the air bubble made by the mirror is seen



(A) 14 cm below flat surface

(B) 20 cm below flat surface

(C) 16 cm below flat surface

(D) 30 cm below flat surface

37. [Online 2016]

Two stars are 10 light years away from the earth. They are seen through a telescope of objective diameter 30 cm . The wavelength of light is 600 nm . To see the stars just resolved by the telescope, the minimum distance between them should be (1 light year $= 9.46\times10^{15}$ m) of the order of

(A) 10^8 km (B) 10^{10} km

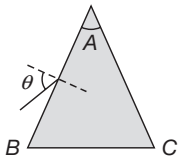
(C) 10^{11} km (D) 10^6 km

38. [2015]
Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is

(A) $100\ \mu$m (B) $300\ \mu$m

(C) $1\ \mu$m (D) $30\ \mu$m

39. [2015]
Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided



(A) $\theta > \cos^{-1}\left(\mu\sin\left(A+\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right)$

(B) $\theta < \cos^{-1}\left(\mu\sin\left(A+\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right)$

(C) $\theta > \sin^{-1}\left(\mu\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right)$

(D) $\theta < \sin^{-1}\left(\mu\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right)$

40. [Online 2015]
You are asked to design a shaving mirror assuming that a person keeps it 10 cm from his face and views the magnified image of the face at the closest comfortable distance of 25 cm. The radius of curvature of the mirror would then be

(A) 30 cm (B) 24 cm

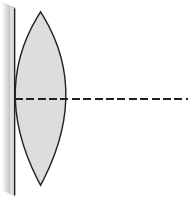
(C) 60 cm (D) –24 cm

41. [Online 2015]
A telescope has an objective lens of focal length 150 cm and an eyepiece of focal length 5 cm. If a 50 m tall tower at a distance of 1 km is observed through this telescope in normal setting, the angle formed by the image of the tower is θ, then θ is close to

(A) 1° (B) 15°

(C) 30° (D) 60°

42. [Online 2015]
A thin convex lens of focal length f is put on a plane mirror as shown in the figure. When an object is kept at a distance a from the lens-mirror combination, its image is formed at a distance $\frac{a}{3}$ in front of the combination. The value of a is



(A) f (B) $2f$

(C) $3f$ (D) $\frac{3}{2}f$

43. [2014]
A thin convex lens made from crown glass $\left(\mu=\frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is

(A) f_1 and f_2 both becomes negative

(B) $f_1 = f_2 < f$

(C) $f_1 > f$ and f_2 becomes negative

(D) $f_2 > f$ and f_1 becomes negative

44. [2014]
A green light is incident from the water to the air-water interface at the critical angle (θ). Select the correct statement.

(A) The entire spectrum of visible light will come out of the water at various angles to the normal.

(B) The entire spectrum of visible light will come out of the water at an angle of $90°$ to the normal.

(C) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium

(D) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.

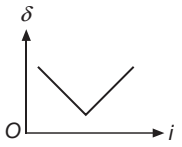
45. [2013]
Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 ms^{-1}, the focal length of the lens is

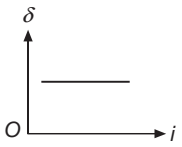
(A) 10 cm (B) 15 cm

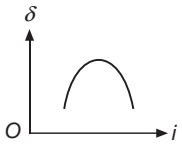
(C) 20 cm (D) 30 cm

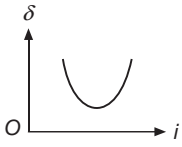
46. [2013]

The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by

(A) 

(B) 

(C) 

(D) 

47. [2012]

An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object be shifted to be in sharp focus on film?

(A) 2.4 m (B) 3.2 m

(C) 5.6 m (D) 7.2 m

48. [2011]

Let the x-z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $x < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is

(A) 30° (B) 45°

(C) 60° (D) 75°

49. [2011]

A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 ms^{-1}. The speed of the image of the second car as seen in the mirror of the first one is

(A) $\frac{1}{10}$ ms^{-1} (B) $\frac{1}{15}$ ms^{-1}

(C) 10 ms^{-1} (D) 15 ms^{-1}

Directions: *Questions number 50-52 are based on the following paragraph.*

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$. where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

50. [2010]

The initial shape of the wavefront of the beam is

(A) planar

(B) convex

(C) concave

(D) convex near the axis and concave near the periphery

51. [2010]

The speed of light in the medium is

(A) maximum on the axis of the beam

(B) minimum on the axis of the beam

(C) the same everywhere in the beam

(D) directly proportional to the intensity I

52. [2010]

As the beam enters the medium, it will

(A) travel as a cylindrical beam

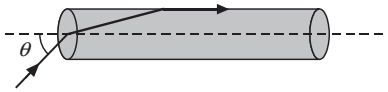
(B) diverge

(C) converge

(D) diverge near the axis and converge near the periphery

53. [2009]

A transparent solid cylindrical rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure.



The incident angle θ for which the light ray grazes along the wall of the rod is

(A) $\sin^{-1}\left(\frac{1}{2}\right)$ (B) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(C) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (D) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

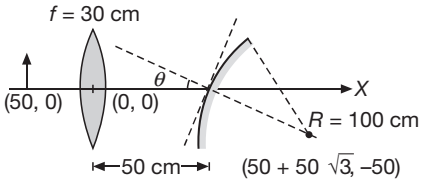
ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct)

1. [JEE (Advanced) 2016]

A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle $\theta = 30°$ to the axis of the lens, as shown in the figure.



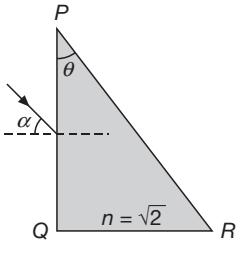
If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are

(A) $\left(25,\ 25\sqrt{3}\right)$ (B) $\left(\frac{125}{3},\ \frac{25}{\sqrt{3}}\right)$

(C) $\left(50-25\sqrt{3},\ 25\right)$ (D) $(0,\ 0)$

2. [JEE (Advanced) 2016]

A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n=\sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of $45°$. The angle θ of the prism is

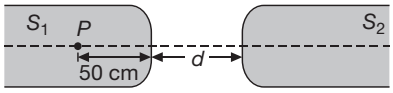


(A) 15° (B) 22.5°

(C) 30° (D) 45°

3. [JEE (Advanced) 2015]

Two identical glass rods S_1 and S_2 (refractive index 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is

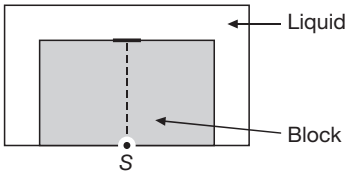


(A) 60 cm (B) 70 cm

(C) 80 cm (D) 90 cm

4. [JEE (Advanced) 2014]

A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is



(A) 1.21 (B) 1.30

(C) 1.36 (D) 1.42

5. [JEE (Advanced) 2013]

A ray of light ravelling in the direction $\frac{1}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$. The angle of incidence is

(A) 30° (B) 45°

(C) 60° (D) 75°

6. [JEE (Advanced) 2013]

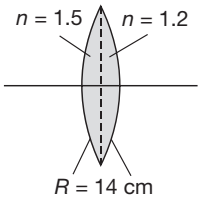
The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the cured surface of the lens is

(A) 1 m (B) 2 m

(C) 3 m (D) 6 m

7. [IIT-JEE 2012]

A bi-convex lens is formed with two thin plano convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature $R = 14$ cm. For this bi-convex lens, for the object distance of 40 cm, the image distance will be



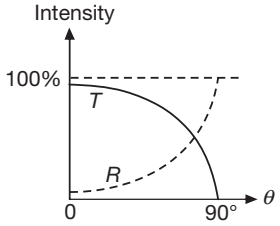
(A) –280 cm (B) 40 cm

(C) 21.5 cm (D) 13.3 cm

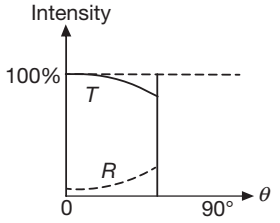
8. [IIT-JEE 2010]

A light ray travelling in glass medium is incident on glass-air interference at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is

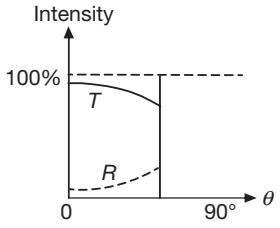
(A)



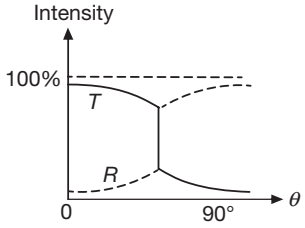
(B)



(C)



(D)



9. [IIT-JEE 2010]

A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is

(A) virtual and at a distance of 16 cm from the mirror

(B) real and at a distance of 16 cm from the mirror

(C) virtual and at a distance of 20 cm from the mirror

(D) real and at a distance of 20 cm from the mirror

10. [IIT-JEE 2009]

A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is $\left(\frac{4}{3}\right)$. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as

(A) 9 ms^{-1} (B) 12 ms^{-1}

(C) 16 ms^{-1} (D) 21.33 ms^{-1}

11. [IIT-JEE 2008]

Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be

(A) 30° for both the colours

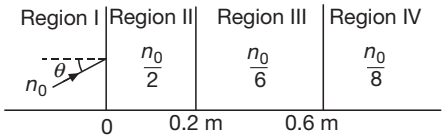
(B) greater for the violet colour

(C) greater for the red colour

(D) equal but not 30° for both colours

12. [IIT-JEE 2008]

A light beam is travelling from Region I to Region IV (Refer Figure). The refractive index in Regions I, II, III and IV are n_0, $\frac{n_0}{2}$, $\frac{n_0}{6}$ and $\frac{n_0}{8}$, respectively. The angle of incidence θ for which the beam just misses entering Region IV is



(A) $\sin^{-1}\left(\frac{3}{4}\right)$ (B) $\sin^{-1}\left(\frac{1}{8}\right)$

(C) $\sin^{-1}\left(\frac{1}{4}\right)$ (D) $\sin^{-1}\left(\frac{1}{3}\right)$

13. [IIT-JEE 2007]

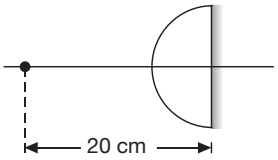
A ray of light travelling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then there will be

(A) only a reflected ray and no refracted ray
(B) only a refracted ray and no reflected ray
(C) a reflected ray and a refracted ray and the angle between them would be less than $180° - 2\theta$
(D) a reflected ray and a refracted ray and the angle between them would be greater than $180° - 2\theta$

14. [IIT-JEE 2007]
In an experiment to determine the focal length (f) of a concave mirror by the u-v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then

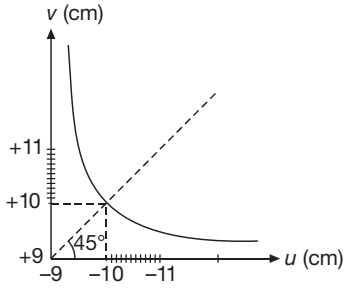
(A) $x < f$ (B) $f < x < 2f$
(C) $x = 2f$ (D) $x > 2f$

15. [IIT-JEE 2006]
A point object is placed at distance of 20 cm from a thin planoconvex lens of focal length 15 cm. The plane surface of the lens is now silvered. The image created by the system is at



(A) 60 cm to the left of the system
(B) 60 cm to the right of the system
(C) 12 cm to the left of the system
(D) 12 cm to the right of the system

16. [IIT-JEE 2006]
The graph between object distance u and image distance v for a lens is given below. The focal length of the lens is



(A) (5 ± 0.1) cm (B) (5 ± 0.05) cm
(C) (0.5 ± 0.1) cm (D) (0.5 ± 0.05) cm

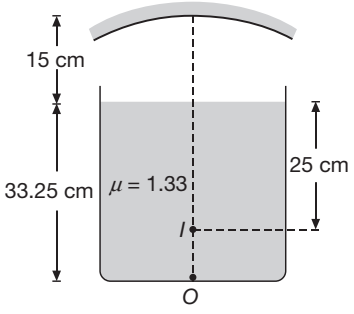
17. [IIT-JEE 2006]
A biconvex lens of focal length f forms a circular image of radius r of sun in focal plane. Then which option is correct?

(A) $\pi r^2 \propto f$
(B) $\pi r^2 \propto f^2$
(C) If lower half part is covered by black sheet, then area of the image is equal to $\frac{\pi r^2}{2}$
(D) If f is doubled, intensity will increase

18. [IIT-JEE 2005]
A convex lens is in contact with concave lens. The magnitude of the ratio of their focal length is $\frac{2}{3}$. Their equivalent focal length is 30 cm. What are their individual focal lengths?

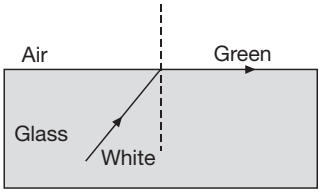
(A) –75 , 50 (B) –10 , 15
(C) 75, 50 (D) –15 , 10

19. [IIT-JEE 2005]
A container is filled with water $(\mu = 1.33)$ upto a height of 33.25 cm. A concave mirror is placed 15 cm above the water level and the image of an object placed at the bottom is formed 25 cm below the water level. The focal length of the mirror is



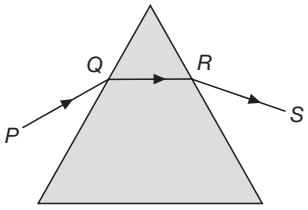
(A) 10 cm (B) 15 cm
(C) 20 cm (D) 25 cm

20. [IIT-JEE 2004]
White light is incident on the interface of glass and air as shown in the figure. If green light is just totally internally reflected then the emerging ray in air contains



(A) yellow, orange, red
(B) violet, indigo, blue
(C) all colours
(D) all colours except green

21. [IIT-JEE 2004]
A ray of light is incident on an equilateral glass prism placed on a horizontal table. For minimum deviation which of the following is true?



(A) PQ is horizontal
(B) QR is horizontal
(C) RS is horizontal
(D) Either PQ or RS is horizontal

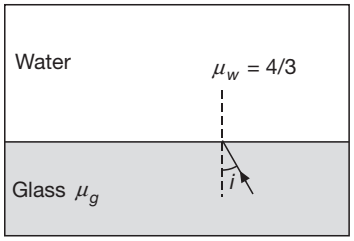
22. [IIT-JEE 2004]
A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5 . The distance of the virtual image from the surface of the sphere is

(A) 2 cm (B) 4 cm
(C) 6 cm (D) 12 cm

23. [IIT-JEE 2003]
The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 2 cm . If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, calculate the new size of the image

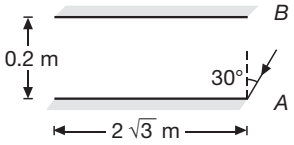
(A) 1.25 cm (B) 2.5 cm
(C) 1.05 cm (D) 2 cm

24. [IIT-JEE 2003]
A ray of light is incident at the glass-water interface at an angle i, it emerges finally parallel to the surface of water, then the value of μ_g would be



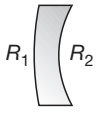
(A) $\left(\frac{4}{3}\right)\sin i$ (B) $\frac{1}{\sin i}$
(C) $\frac{4}{3}$ (D) 1

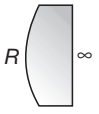
25. [IIT-JEE 2002]
Two plane mirrors A and B are alligned parallel to each other, as shown in figure. A light ray is incident at an angle of 30° at a point just inside one end of A. The plane of incidence coincides with the plane of figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is



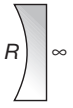
(A) 28 (B) 30
(C) 32 (D) 34

26. [IIT-JEE 2002]
Which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surfaces of the lenses are as given in the diagrams

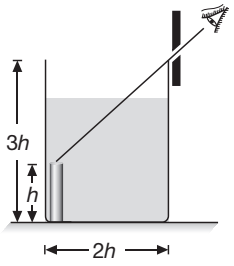
(A) 

(B) 

(C)

(D) 

27. [IIT-JEE 2002]
An observer can see through a pin hole, the top of a thin rod of height h, placed as shown in figure. The beaker's height is $3h$ and its radius is h. When the beaker is filled with a liquid upto a height $2h$, he can see the lower end of the rod. Then the refractive index of liquid must be

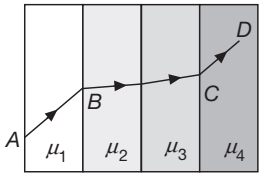


(A) $\frac{5}{2}$ (B) $\sqrt{\frac{5}{2}}$

(C) $\sqrt{\frac{3}{2}}$ (D) $\frac{3}{2}$

28. [IIT-JEE 2001]

A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in figure. The surface of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have

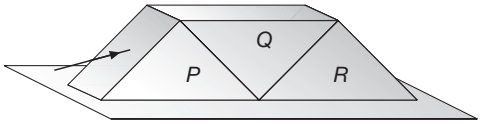


(A) $\mu_1 = \mu_2$ (B) $\mu_2 = \mu_3$

(C) $\mu_3 = \mu_4$ (D) $\mu_4 = \mu_1$

29. [IIT-JEE 2001]

A given ray of light suffers minimum deviation in an equilateral prism P. Additional prisms Q and R of identical shape and of same material as P are now added as shown in figure. The ray will now suffer



(A) greater deviation.
(B) no deviation.
(C) same deviation as before.
(D) total internal reflection.

30. [IIT-JEE 2000]

In a compound microscope, the intermediate image is
(A) virtual, erect and magnified
(B) real, erect and magnified
(C) real, inverted and magnified
(D) virtual, erect and reduced

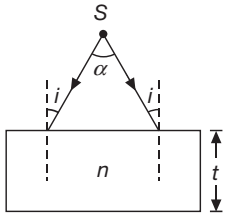
31. [IIT-JEE 2000]

A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids L_1 or L_2 having refractive indices n_1 and n_2 respectively $(n_2 > n_1 > 1)$. The lens will diverge a parallel beam of light if it is filled with

(A) air and placed in air.
(B) air and immersed in L_1.
(C) L_1 and immersed in L_2.
(D) L_2 and immersed in L_1.

32. [IIT-JEE 2000]

A diverging beam of light from a point source S having divergence angle α falls symmetrically on a glass slab as shown. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is t and the refractive index n, then the divergence angle of the emergent beam is

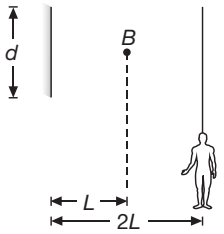


(A) zero (B) α

(C) $\sin^{-1}\left(\frac{1}{n}\right)$ (D) $2\sin^{-1}\left(\frac{1}{n}\right)$

33. [IIT-JEE 2000]

A point source of light B is placed at a distance L in front of the centre of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown. The greatest distance over which he can see the image of the light source in the mirror is

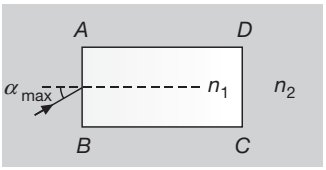


(A) $\frac{d}{2}$ (B) d

(C) $2d$ (D) $3d$

34. [IIT-JEE 2000]

A rectangular glass slab $ABCD$ of refractive index n_1, is immersed in water of refractive index $n_2 (n_1 > n_2)$. A ray of light is incident at the surface AB of the slab as shown. The maximum value of the angle of incidence α_{max}, such that the ray comes out only from the other surface CD is given by



(A) $\sin^{-1}\left\{\frac{n_1}{n_2}\cos\left(\sin^{-1}\left(\frac{n_2}{n_1}\right)\right)\right\}$

(B) $\sin^{-1}\left\{n_1\cos\left(\sin^{-1}\left(\frac{1}{n_2}\right)\right)\right\}$

(C) $\sin^{-1}\left(\frac{n_1}{n_2}\right)$

(D) $\sin^{-1}\left(\frac{n_2}{n_1}\right)$

35. [IIT-JEE 1999]

A concave lens of glass, refractive index 1.5, has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a

(A) convergent lens of focal length $3.5R$.

(B) convergent lens of focal length $3.0R$.

(C) divergent lens of focal length $3.5R$.

(D) divergent lens of focal length $3.0R$.

36. [IIT-JEE 1998]

A real image of a distant object is formed by a plano-convex lens on its principal axis. Spherical aberration

(A) is absent.

(B) is smaller if the curved surface of the lens faces the object.

(C) is smaller if the plane surface of the lens faces the object.

(D) is the same whichever side of the lens faces the object.

37. [IIT-JEE 1998]

A concave mirror is placed on a horizontal table, with its axis directed vertically upwards. Let O be the pole of the mirror and C its centre of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be

(A) real and will remain at C.

(B) real and located at a point between C and ∞.

(C) virtual and located at a point between C and O.

(D) real and located at a point between C and O.

38. [IIT-JEE 1998]

A spherical surface of radius of curvature R separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at the point O and PO = OQ. The distance PO is equal to

(A) $5R$ (B) $3R$

(C) $2R$ (D) $1.5R$

39. [IIT-JEE 1997]

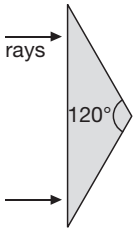
An eye specialist prescribes spectacles having combination of convex lens of focal length 40 cm in contact with a concave lens of focal length 25 cm. The power of this lens combination in diopters is

(A) +1.5 (B) –1.5

(C) +6.67 (D) –6.67

40. [IIT-JEE 1995]

An isosceles prism of angle $120°$ has a refractive index 1.44. Two parallel of monochromatic light enter the prism parallel to each other in air as shown. The rays emerge from the opposite face



(A) are parallel to each other

(B) are diverging

(C) make an angle $2\left(\sin^{-1}(0.72) - 30°\right)$ with each other

(D) make an angle $2\sin^{-1}(0.72)$ with each other

41. [IIT-JEE 1995]

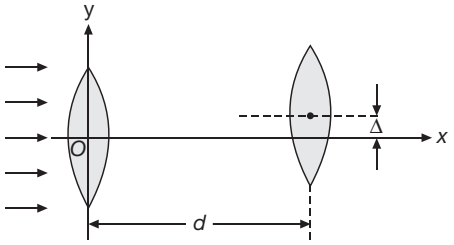
The focal lengths of the objective and the eye piece of a compound microscope are 2.0 cm and 3.0 cm respectively. The distance between the objective and the eye piece is 15.0 cm. The final image formed by the eye piece is at infinity. The two lenses are thin. The distance in cm of the object and the image produced by the objective, measured from the objective lens, are respectively

(A) 2.4 and 12.0 (B) 2.4 and 15.0
(C) 2.0 and 12.0 (D) 2.0 and 3.0

42. [IIT-JEE 1995]
A diminished image of an object is to be obtained on a screen 1.0 m from it. This can be achieved by appropriate placing
(A) a concave mirror of suitable focal length
(B) a convex mirror of suitable focal length
(C) a convex lens of focal length less than 0.25 m
(D) a convex lens of suitable focal length

43. [IIT-JEE 1994]
Spherical aberration in a thin lens can be reduced by
(A) using a monochromatic light.
(B) using a doublet combination.
(C) using a circular annular mask over the lens.
(D) increasing the size of the lens.

44. [IIT-JEE 1993]
Two thin convex lenses of focal lengths f_1 and f_2 are separated by a horizontal distance d (where, $d < f_1$, $d < f_2$) and their centres are displaced by a vertical separation Δ as shown in the figure



Taking the origin of coordinates, O, at the centre of the first lens, the x and y-coordinates of the focal point of this lens system, for a parallel beam of rays coming from the left, are given by

(A) $x = \dfrac{f_1 f_2}{f_1 + f_2}$, $y = \Delta$

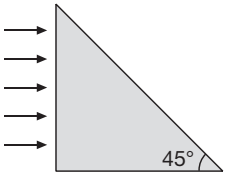
(B) $x = \dfrac{f_1(f_2 + d)}{f_1 + f_2 - d}$, $y = \dfrac{\Delta}{f_1 + f_2}$

(C) $x = \dfrac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$, $y = \dfrac{\Delta(f_1 - d)}{f_1 + f_2 - d}$

(D) $x = \dfrac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$, $y = 0$

45. [IIT-JEE 1990]
A thin prism P_1 with angle 4° and made from glass of refractive index is 1.54 is combined with another thin prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. The angle of the prism P_2 is
(A) 5.33° (B) 4°
(C) 3° (D) 2.6°

46. [IIT-JEE 1989]
A beam of light consisting of red, green and blue colours is incident on a right angled prism. The refractive indices of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will



(A) separate the red colour from the green and blue colours
(B) separate the blue colour from the red and green colours
(C) separate all the three colours from one another
(D) not separate even partially any colour from the other two colours

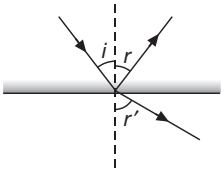
47. [IIT-JEE 1989]
An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eye piece is 36 cm and the final image is formed at infinity. The focal length f_o of the objective and the focal length f_e of the eye piece are
(A) $f_o = 45$ cm and $f_e = -9$ cm
(B) $f_o = 50$ cm and $f_e = 10$ cm
(C) $f_o = 7.2$ cm and $f_e = 5$ cm
(D) $f_o = 30$ cm and $f_e = 6$ cm

48. [IIT-JEE 1988]
A short linear object of length b lies along the axis of a concave mirror or focal length f at a distance u from the pole of the mirror. The size of the image is approximately equal to

(A) $b\left(\dfrac{u-f}{f}\right)^{\frac{1}{2}}$ (B) $b\left(\dfrac{f}{u-f}\right)^{\frac{1}{2}}$

(C) $b\left(\dfrac{u-f}{f}\right)$ (D) $b\left(\dfrac{f}{u-f}\right)^{2}$

49. [IIT-JEE 1983]
A ray of light from a denser medium strikes a rarer medium at an angle of incidence i (shown in figure). The reflected and refracted rays make an angle of 90° with each other. The angles of reflection and refraction are r and r'. The critical angle is



(A) $\sin^{-1}(\tan r)$ (B) $\sin^{-1}(\cot i)$

(C) $\sin^{-1}(\tan r')$ (D) $\tan^{-1}(\sin i)$

50. [IIT-JEE 1982]

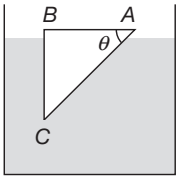
A convex lens of focal length 40 cm is in contact with a concave lens of focal length 25 cm. The power of the combination in dioptre is

(A) –1.5 (B) –6.5

(C) +6.5 (D) +6.67

51. [IIT-JEE 1981]

A glass prism of refractive index 1.5 is immersed in water (refractive index $\frac{4}{3}$). A light beam incident normally on the face AB is totally reflected to reach the face BC, if



(A) $\sin\theta > \frac{8}{9}$ (B) $\sin\theta \le \frac{2}{3}$

(C) $\frac{2}{3} < \sin\theta < \frac{8}{9}$ (D) None of these

52. [IIT-JEE 1980]

When a ray of light enters a glass slab from air

(A) its wavelength decreases.

(B) its wavelength increases.

(C) its frequency increases.

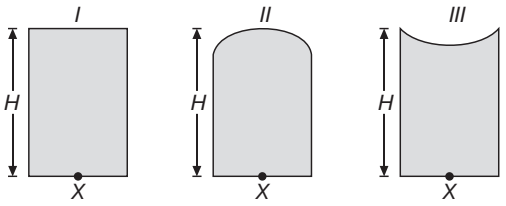
(D) neither its wavelength nor its frequency changes.

Multiple Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct)

1. [JEE (Advanced) 2019]

Three glass cylinders of equal height $H = 30$ cm and same refractive index $n = 1.5$ are placed on a horizontal surface as shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same ($R = 3$ m). If H_1, H_2 and H_3 are the apparent depths of a point X on the bottom of the three cylinders, respectively, the correct statement(s) is/are



(A) $0.8 \text{ cm} < (H_2 - H_1) < 0.9 \text{ cm}$

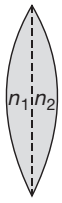
(B) $H_2 > H_3$

(C) $H_3 > H_1$

(D) $H_2 > H_1$

2. [JEE (Advanced) 2019]

A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surfaces are equal. f is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n \ll (n-1)$ and $1 < n < 2$, the correct statement(s) is/are



(A) $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

(B) The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surfaces of the same radius of curvature.

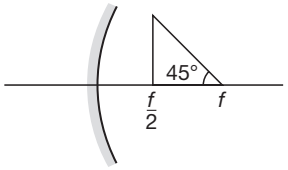
(C) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

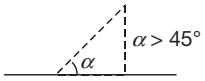
(D) For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20$ cm, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place)

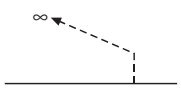
3. [JEE (Advanced) 2017]

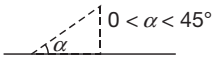
A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s)

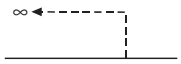
the shape of the image of the bent wire? (These figures are not to scale).



(A) 

(B) 

(C) 

(D) 

4. [JEE (Advanced) 2017]

For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct?

(A) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism

(B) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = \left(\frac{i_i}{2}\right)$

(C) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\left(\frac{A}{2}\right)-1} - \cos A\right]$

(D) For this prism, the refractive index μ and the angle prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

5. [JEE (Advanced) 2016]

A plano-convex lens is made of material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statements(s) is (are) true?

(A) The refractive index of the lens is 2.5

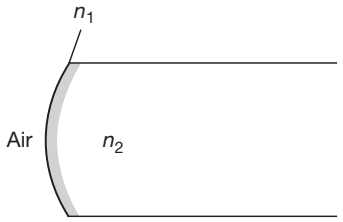
(B) The radius of curvature of the convex surface is 45 cm

(C) The faint image is erect and real

(D) The focal length of the lens is 20 cm

6. [JEE (Advanced) 2014]

A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then

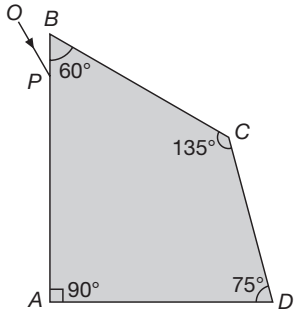


(A) $|f_1| = 3R$ (B) $|f_1| = 2.8R$

(C) $|f_2| = 2R$ (D) $|f_2| = 1.4R$

7. [IIT-JEE 2010]

A ray OP of monochromatic light is incident on the face AB of prism $ABCD$ near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct?



(A) The ray gets totally internally reflected at face CD

(B) The ray comes out through face AD

(C) The angle between the incident ray and the emergent ray is 90°

(D) The angle between the incident ray and the emergent ray is 120°

8. [IIT-JEE 2009]

A student performed the experiment of determination of focal length of a concave mirror by u-v method using an optical bench of length 1.5 m. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are: (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that cannot come from experiment and is (are) incorrectly recorded, is (are)

(A) (42, 56) (B) (48, 48)
(C) (66, 33) (D) (78, 39)

9. [IIT-JEE 1986]
A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen
(A) half of the image will disappear
(B) complete image will be formed
(C) intensity of the image will increase
(D) intensity of the image will decrease

10. [IIT-JEE 1992]
A planet is observed by an astronomical refracting telescope having an objective of focal length 16 m and an eye piece of focal length 2 cm .
(A) The distance between objective and eye piece is 16.02 m .
(B) The angular magnification of the planet is -800 .
(C) The image of the planet is inverted.
(D) The objective is larger than the eye piece.

11. [IIT-JEE 1996]
Which of the following form(s) the virtual and erect image for all positions of object ?
(A) concave mirror (B) convex lens
(C) convex mirror (D) concave lens

12. [IIT-JEE 1998]
A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence $45°$. The ray undergoes total internal reflection. If n is the refractive index of the medium with respect to air, select the possible value(s) of n from the following
(A) 1.3 (B) 1.4
(C) 1.5 (D) 1.6

Reasoning Based Questions

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as
Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

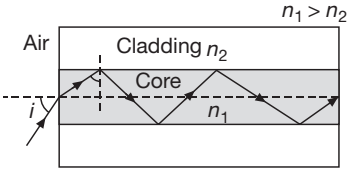
1. [IIT-JEE 2007]
Statement-1: The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature.
Statement-2: Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.

Comprehension Type Questions

Comprehension 1

Light guidance in an optical fibre can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as $\sin i_m$.

1. [JEE (Advanced) 2015]
For two structures namely S_1 with $n_1 = \frac{\sqrt{45}}{4}$ and $n_2 = \frac{3}{2}$, and S_2 with $n_1 = \frac{8}{5}$ and $n_2 = \frac{7}{5}$ and taking the refractive index of water to be $\frac{4}{3}$ and that to air to be 1 , the correct options is/are



(A) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$
(B) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same as that of S_2 immersed in water
(C) NA of S_1 placed in air is the same as that S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$
(D) NA of S_1 placed in air is the same as that of S_2 placed in water

2. [JEE (Advanced) 2015]

If two structures of same cross-sectional area, but different numerical apertures NA_1 and $NA_2\,(NA_2 < NA_1)$ are joined longitudinally, the numerical aperture of the combined structure is

(A) $\dfrac{NA_1NA_2}{NA_1+NA_2}$ (B) NA_1+NA_2

(C) NA_1 (D) NA_2

Comprehension 2

Most materials have the refractive index, $n>1$. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\dfrac{\sin\theta_1}{\sin\theta_2}=\dfrac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n=\left(\dfrac{c}{v}\right)=\pm\sqrt{\varepsilon_r\mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

In normal materials, both ε_r and μ_r are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one most choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behaviour, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

3. [IIT-JEE 2012]

Choose the correct statement.

(A) The speed of light in the meta-material is $v=c|n|$

(B) The speed of light in the meta-material is $v=\dfrac{c}{|n|}$

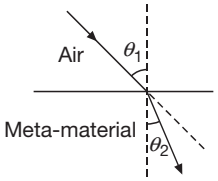
(C) The speed of light in the meta-materials is $v=c$

(D) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m=\lambda_{\text{air}}|n|$, where λ_{air}

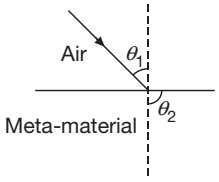
4. [IIT-JEE 2012]

For light incident from air on a meta-material, the appropriate ray diagram is

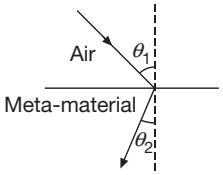
(A)



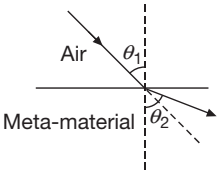
(B)



(C)



(D)



Matrix Match/Column Match Type Questions

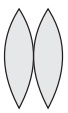
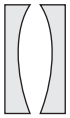
Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following :

	p	q	r	s	t
A	Ⓟ	Ⓠ	Ⓡ	Ⓢ	Ⓣ
B	Ⓟ	Ⓠ	Ⓡ	Ⓢ	Ⓣ
C	Ⓟ	Ⓠ	Ⓡ	Ⓢ	Ⓣ
D	Ⓟ	Ⓠ	Ⓡ	Ⓢ	Ⓣ

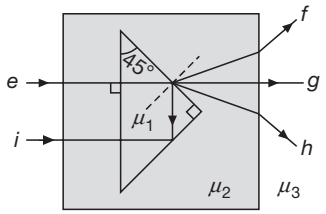
1. [JEE (Advanced) 2014]

Four combinations of two thin lenses are given in **COLUMN-I.** The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in **COLUMN-I** with their focal length in **COLUMN-II** and select the correct answer using the codes given below the lists.

COLUMN-I	COLUMN-II
A. 	p. $2r$
B.	q. $\dfrac{r}{2}$
C. 	r. $-r$
D.	s. r

2. [JEE (Advanced) 2013]

A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure. A ray of light e enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2 and μ_3, it takes one of the four possible paths ef, eg, eh or ei.

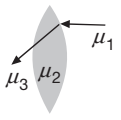
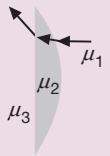
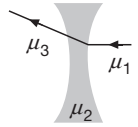
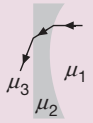
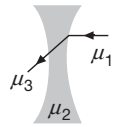


Match the paths in **COLUMN-I** with conditions of refractive indices in **COLUMN-II** and select the correct answer using the codes given below the lists.

COLUMN-I	COLUMN-II
A. $e \to f$	p. $\mu_1 > \sqrt{2}\mu_2$
B. $e \to g$	q. $\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
C. $e \to h$	r. $\mu_1 = \mu_2$
D. $e \to i$	s. $\mu_2 < \mu_1 < \sqrt{2}\mu_2$ and $\mu_2 > \mu_3$

3. [IIT-JEE 2010]

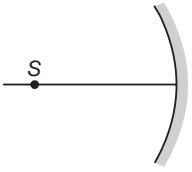
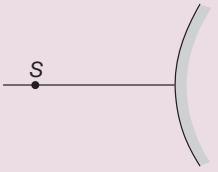
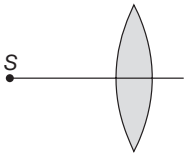
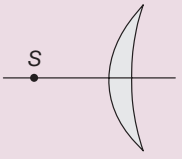
Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in **COLUMN-II**. A ray traversing these media is also shown in the figures. In **COLUMN-I** different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagram shown in **COLUMN-II**.

COLUMN-I	COLUMN-II
A. $\mu_1 < \mu_2$	p. 
B. $\mu_1 > \mu_2$	q. 
C. $\mu_2 = \mu_3$	r. 
D. $\mu_2 > \mu_3$	s. 
	t. 

(*Continued*)

4. [IIT-JEE 2008]

An optical component and an object S placed along its optic axis are given in **COLUMN-I**. The distance between the object and the component can be varied. The properties of images are given in **COLUMN-II**. Match all the properties of images from **COLUMN-II** with the appropriate components given in **COLUMN-I**.

COLUMN-I	COLUMN-II
(A) 	(p) Real image
(B) 	(q) Virtual image
(C) 	(r) Magnified image
(D) 	(s) Image at infinity

5. [IIT-JEE 2006]

Some laws/processes are given in **COLUMN-1.** Match these with the physical phenomena given in **COLUMN-II.**

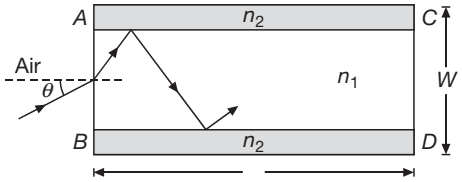
COLUMN-I	COLUMN-II
A. Intensity of light received by lens	p. radius of aperture (R)
B. Angular magnification	q. dispersion of lens
C. Length of telescope	r. focal length f_0, f_e
D. Sharpness of image	s. spherical aberration

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

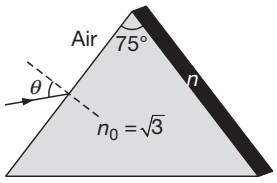
1. [JEE (Advanced) 2019]

A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If $L \gg W$, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure, For $L = 9.6$ m, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}$ s, where t is _____ (Speed of light $c = 3 \times 10^8$ ms^{-1})



2. [JEE (Advanced) 2019]

A monochromatic light is incident from air on a refracting surface of a prism of angle $75°$ and refractive index $n_0 = \sqrt{3}$. The other refracting surface of the prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \le 60°$. The value of n^2 is _____.

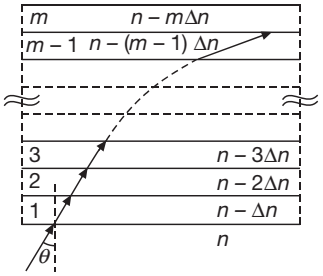


3. [JEE (Advanced) 2018]

Sunlight of intensity 1.3 kWm^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, kWm^{-2}, at a distance 22 cm from the lens on the other side is _____.

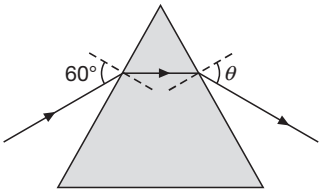
4. [JEE (Advanced) 2017]

A monochromatic light is travelling in a medium of refractive index $n = 1.6$. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the mth slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)$th and mth slabs from the right side of the stack. What is the value of m?



5. [JEE (Advanced) 2015]

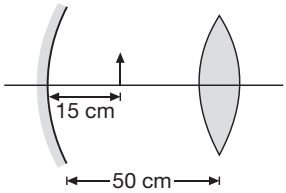
A monochromatic beam of light is incident at $60°$ on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see figure). For $n = \sqrt{3}$ the value of θ is $60°$ and $\frac{d\theta}{dn} = m$. The value of m is



6. [JEE (Advanced) 2015]

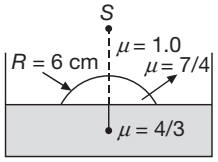
Consider a concave mirror and a convex lens (refractive index $= 1.5$) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index $= 1$) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1.

When the set-up is kept in a medium of refractive index $\frac{7}{6}$, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is



7. [IIT-JEE 2011]

Water (with refractive index $=\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature $R = 6$ cm as shown. Consider oil to act as a thin lens. An object S is placed 24 cm above water surface. The location of its image is at x cm above the bottom of the tank. Then x is.



8. [IIT-JEE 2010]

Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$m to $\frac{50}{7}$m in 30 s. What is the speed of the object in kmh^{-1}?

9. [IIT-JEE 2010]

The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is

10. [IIT-JEE 2010]

A large glass slab $\left(\mu = \frac{5}{3}\right)$ of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R?

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Reflection at Plane Surfaces)

1. $30°$
2. 4 cm
3. $\dfrac{u\cos\alpha(\tan\alpha-\tan\theta)}{g}$
4. (a) $60°$
   (b) $240°$ CCW
5. $60°$
6. $100°$ (CW)
7. 20 cm , 60 cm , 80 cm , 100 cm and 140 cm
8. 12
9. $3d$
10. (a) $12\text{ cm}\times 8\text{ cm}$
    (b) $12\text{ cm}\times 4\text{ cm}$
12. $-5\left(1+\sqrt{3}\right)\hat{i}+5\hat{j}$
13. $\alpha=2\theta$
14. $30°$
15. $50°$

Test Your Concepts-II (Based on Reflection at Curved Surfaces)

1. 5 cm inverted
2. Concave, 6.67 m
4. 7.5 cm , 12.5 cm
5. (a) 8 cm
   (b) 16 cm
   (c) 48 cm
6. 15 cm
7. (a) $\left(\dfrac{2+\cos\omega t}{1+\cos\omega t}\right)f$
   (b) $x=0$
   (c) $m\to\infty$
8. (a) Concave
9. $\left(\dfrac{\sqrt{3}+1}{2}\right)R$ from the convex mirror
11. 10 cm
12. $-\dfrac{3}{2}, -\dfrac{3}{4}$
13. $40\sqrt{2}$ cm , $10\sqrt{2}$ cm

Test Your Concepts-III (Based on General Refraction)

1. 75 cm
2. 7.5 cm
3. $\Delta v=0.55$ cm , $\dfrac{m_2}{m_1}\approx 1.1$
4. 2×10^8 ms^{-1} , 4000 Å yellow
5. 3.5 cm
6. 6.6 cm
7. 2.88 m
9. 0.7 r
10. $\sin^{-1}\left(\dfrac{L}{R^2}\left(\sqrt{\mu^2R^2-L^2}-\sqrt{R^2-L^2}\right)\right)$
11. 12 cm
12. $y=\dfrac{x^2}{4}$

Test Your Concepts-IV (Based on Total Internal Reflection (TIR))

1. (a) $54.34°$
   (b) Yes
2. (a) 2.81 m
   (b) 0.23 m
3. (b) $\sqrt{2}$
4. (a) $26.8°$
   (b) Yes
5. $\dfrac{h}{\sqrt{\mu^2-1}}$
6. (a) $40.54°$
   (b) $26.6°$
7. $67.3°$
8. $\dfrac{4}{3}$ cm
9. $\sqrt{2}$
10. $OP\geq\sqrt{\dfrac{2}{3}}R$

Test Your Concepts-V (Based on Prism)

1. (a) $30°$
   (b) $\sqrt{\dfrac{7}{3}}$

2. (a) $157.2°$
(b) $128.4°$
3. $A_{max} = 83.62°$
4. (a) $\sqrt{5}$
(b) $58.8°$
5. (a) $34.2°$
(b) $8.4°$
6. $\sin^{-1}(\mu \sin\alpha) - \alpha$
7. $\sqrt{2}$
8. $0°$, $\sqrt{3}$
9. $\delta_V - \delta_R = 4.5°$
10. $22°$, $56°$
11. $0°$
12. $19°$
13. $10.1°$
14. $\delta_{red} = 30.6°$, $\delta_{violet} = 33.4°$
15. 2, 10

Test Your Concepts-VI (Based on Refraction at Curved Surfaces)

1. 2.5 D
2. Final image is formed at pole of the mirror
3. $x \approx 0.75R$
4. 7.42 cm
5. (a) 80 cm
(b) $u < 12$ cm
6. Final image is formed at 65 cm from first face on the same side of the object.
7. 3.33 cm, infinity
9. 3.84 mms^{-1}
10. $\dfrac{R(2-\mu)}{2(\mu-1)}$
11. $\dfrac{2d}{3}$
12. $\dfrac{3+\sqrt{5}}{2}$
13. 8.57 cm

Test Your Concepts-VII (Based on Lens Formula)

1. 6 cm from either of the object
2. 12 cm
3. $\dfrac{4}{3}$
4. Concave mirror of focal length 15 cm
5. (a) 90 cm
(b) 102 cm
6. 12.5 cm in front of the silvered lens
7. 10 cm
8. 2.14 cm
9. 1.37
10. 7.5 cm
11. $\dfrac{4f_1^2}{f_2}$
12. 2 m, 1 m
13. (a) $\dfrac{t}{t-f_1}$
(b) No Shift
14. 7.5 cm
17. $(m+1)$ times smaller
18. (a) Convex
19. 2.4 cm
20. (a) 1.4
21. Rays will become parallel to the optic axis.
22. $I = \dfrac{3-2\mu_0}{2a}$
23. $\dfrac{R}{2(\mu n + \mu - 1)}$
24. (a) x_1
(b) $\dfrac{x_1 x_2}{x_1 - x_2}$
(c) $\dfrac{x_1}{x_1 - x_2}$
25. 0.6 m
26. 1.7
27. $(5f, 2d)$

Test Your Concepts-VIII (Based on Aberrations, Human Eye and Optical Instruments)

1. 20 D to 24 D
2. 0.0325
3. –0.5 D
4. (i) 98 cm × 98 cm (ii) 2401
5. 0.2575 m
6. 9 cm away from objective lens
7. –327.5
8. 0.31852 m , 27.39
9. 55 cm, 2.63 cm, 10

Single Correct Choice Type Questions

1. C	2. C	3. A	4. D	5. B	6. C	7. C	8. D	9. A	10. D
11. C	12. B	13. B	14. A	15. C	16. B	17. B	18. D	19. B	20. D
21. C	22. C	23. C	24. D	25. D	26. D	27. B	28. D	29. A	30. B
31. C	32. D	33. A	34. B	35. C	36. D	37. D	38. C	39. B	40. B
41. C	42. B	43. A	44. B	45. C	46. D	47. C	48. B	49. D	50. C
51. C	52. B	53. A	54. A	55. A	56. A	57. C	58. D	59. D	60. C
61. D	62. B	63. A	64. B	65. A	66. B	67. D	68. A	69. B	70. C
71. D	72. A	73. D	74. C	75. A	76. B	77. C	78. C	79. C	80. A
81. B	82. B	83. C	84. A	85. C	86. C	87. A	88. C	89. C	90. C
91. D	92. A	93. B	94. B	95. B	96. A	97. C	98. C	99. D	100. A
101. C	102. C	103. A	104. D	105. C	106. B	107. B	108. C	109. D	110. C
111. C	112. B	113. B	114. D	115. C	116. C	117. C	118. B	119. D	120. B
121. B	122. A	123. B	124. C	125. C	126. C	127. B	128. D	129. C	130. C
131. C	132. B	133. C	134. B	135. A	136. A	137. A	138. C	139. B	140. D
141. C	142. C	143. D	144. B	145. C	146. A	147. C	148. C	149. A	150. D
151. C	152. C	153. A	154. D	155. C	156. C	157. C	158. A	159. D	160. B
161. D	162. A	163. C	164. C	165. A	166. B	167. C	168. C	169. C	170. D
171. A	172. C	173. D	174. C	175. D	176. B	177. C	178. D	179. D	180. C
181. B	182. C	183. C	184. A	185. C	186. C	187. B	188. B	189. D	190. A
191. C	192. B	193. A	194. A	195. B	196. B	197. D	198. B	199. C	200. C
201. C	202. C	203. C	204. A	205. B	206. A	207. B	208. B	209. A	210. A
211. D	212. B	213. D	214. D	215. A	216. A	217. D	218. D	219. C	220. D
221. C	222. C	223. B	224. D	225. B	226. A	227. B	228. C	229. C	230. C
231. C	232. C	233. C	234. C	235. D	236. B	237. D	238. A	239. B	240. D
241. D	242. A	243. B	244. D	245. A	246. D	247. B	248. B	249. B	250. A
251. B	252. B	253. B	254. B	255. C	256. D	257. D	258. D	259. A	260. B
261. B	262. C	263. A	264. A	265. D	266. C	267. D	268. B	269. D	270. B
271. B	272. C	273. D	274. A	275. D	276. C	277. B	278. C	279. B	280. D
281. A	282. A	283. A	284. B	285. D	286. C	287. A	288. B	289. C	290. D
291. B	292. D	293. D	294. A	295. A	296. C	297. C	298. A	299. D	300. B
301. C	302. B	303. B	304. A	305. C	306. C	307. A	308. A	309. A	310. D
311. A	312. C	313. A	314. D	315. B	316. A	317. C	318. D	319. A	320. A
321. C	322. D	323. B	324. C	325. D					

Multiple Correct Choice Type Questions

1. B, C	2. A	3. A, D	4. A, B	5. B, D
6. A, C	7. A, C, D	8. B, C	9. B, C, D	10. A
11. B, C	12. A, D	13. B, C	14. A, B, CD	15. B, D
16. A, C, D	17. A, D	18. A, C	19. B, C, D	20. A, B, C
21. A, B	22. B, C	23. A, D	24. A, D	25. B, C
26. B, D	27. C, D	28. A, C	29. B, D	30. A, C, D

31. A, C	**32.** B, C	**33.** B	**34.** B	**35.** A, D
36. A, D	**37.** A, C	**38.** A, C	**39.** A, C	**40.** C
41. A, B	**42.** B, C	**43.** A, B, C	**44.** B, C	**45.** A, B, C
46. B, C	**47.** A, C	**48.** B, D	**49.** B, C	**50.** A, B, C

Reasoning Based Questions

1. D	**2.** B	**3.** A	**4.** B	**5.** A	**6.** C	**7.** D	**8.** B	**9.** C	**10.** D
11. A	**12.** A	**13.** D	**14.** D	**15.** A	**16.** B	**17.** A	**18.** D	**19.** A	**20.** C
21. B	**22.** A	**23.** B	**24.** D	**25.** D	**26.** B	**27.** D	**28.** C	**29.** C	**30.** C

Linked Comprehension Type Questions

1. B	**2.** C	**3.** B	**4.** D	**5.** A	**6.** D	**7.** C	**8.** A	**9.** D	**10.** B
11. D	**12.** C	**13.** D	**14.** A	**15.** D	**16.** D	**17.** C	**18.** A	**19.** C	**20.** D
21. A	**22.** D	**23.** C	**24.** B	**25.** D	**26.** C	**27.** A	**28.** A	**29.** B	**30.** A
31. C	**32.** C	**33.** B	**34.** D	**35.** D	**36.** C	**37.** C	**38.** D	**39.** C	**40.** C
41. D	**42.** D	**43.** D	**44.** C	**45.** B	**46.** B	**47.** C	**48.** A	**49.** A	**50.** C
51. D	**52.** D	**53.** A	**54.** C	**55.** B	**56.** A	**57.** C	**58.** B	**59.** B	**60.** A
61. C	**62.** C	**63.** C	**64.** D	**65.** B	**66.** C	**67.** B	**68.** A	**69.** D	**70.** A
71. C	**72.** D	**73.** A	**74.** B	**75.** C	**76.** D				

Matrix Match/Column Match Type Questions

1.	A → (r, s)	B → (p, q, r, s, t)	C → (q, r, s, t)	D → (p, r, s)
2.	A → (s, t)	B → (p, t)	C → (s, t)	D → (q, t)
3.	A → (s)	B → (p, q, r)	C → (q, r, s)	D → (t)
4.	A → (r)	B → (q)	C → (p)	D → (p)
5.	A → (p, q)	B → (r)	C → (s)	D → (p, q)
6.	A → (p, s)	B → (q)	C → (p, q, s)	D → (r)
7.	A → (q, r)	B → (r)	C → (p, r, s)	D → (p, r)
8.	A → (p, q, s)	B → (p, q)	C → (r)	D → (p, q, s)
9.	A → (p, s)	B → (p, q, r, s)	C → (p, q, r, s)	D → (p, s)
10.	A → (p)	B → (p)	C → (r, s)	D → (q, p)
11.	A → (p, s)	B → (p, q, r, s)	C → (p, q, r, s)	D → (q, s)
12.	A → (s)	B → (p)	C → (q)	D → (r)
13.	A → (s)	B → (q)	C → (p)	D → (q)

Integer/Numerical Answer Type Questions

1. 30.	**2.** 24, 36	**3.** 3	**4.** 90	**5.** 36
6. 9	**7.** 12	**8.** 2	**9.** 60	**10.** 2
11. 180	**12.** 15	**13.** 5, 4	**14.** 8	**15.** 5
16. 30	**17.** 30	**18.** 60	**19.** 9	**20.** 40
21. 5	**22.** 100	**23.** 12	**24.** 25	**25.** 10
26. 45	**27.** (a) 4 (b) 5	**28.** 2	**29.** 30 (Right of lens), 8	**30.** 15
31. 29 cm, 116				

ARCHIVE: JEE MAIN

1. D	**2.** C	**3.** B	**4.** D	**5.** A	**6.** A	**7.** B	**8.** B	**9.** D	**10.** C
11. D	**12.** A	**13.** D	**14.** B	**15.** D	**16.** C	**17.** D	**18.** A	**19.** C	**20.** A
21. C	**22.** D	**23.** B	**24.** B	**25.** C	**26.** B	**27.** C	**28.** C	**29.** A	**30.** C
31. A	**32.** C	**33.** D	**34.** A	**35.** C	**36.** B	**37.** A	**38.** D	**39.** C	**40.** C
41. D	**42.** B	**43.** C	**44.** C	**45.** D	**46.** D	**47.** C	**48.** B	**49.** B	**50.** A
51. B	**52.** C	**53.** D							

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. A	**2.** A	**3.** B	**4.** C	**5.** A	**6.** C	**7.** B	**8.** C	**9.** B	**10.** C
11. A	**12.** B	**13.** C	**14.** B	**15.** C	**16.** B	**17.** B	**18.** D	**19.** C	**20.** A
21. B	**22.** C	**23.** B	**24.** B	**25.** B	**26.** C	**27.** B	**28.** D	**29.** C	**30.** C
31. D	**32.** B	**33.** D	**34.** A	**35.** A	**36.** B	**37.** D	**38.** A	**39.** B	**40.** C
41. A	**42.** C	**43.** C	**44.** C	**45.** C	**46.** A	**47.** D	**48.** D	**49.** A	**50.** A
51. A	**52.** A								

Multiple Correct Choice Type Problems

1. B, D	**2.** B, C, D	**3.** D	**4.** A, B, C	**5.** A, D
6. A, C	**7.** A, B, C	**8.** C, D	**9.** B, D	**10.** A, B, C, D
11. C, D	**12.** C, D			

Reasoning Based Questions

1. C

Comprehension Type Questions

1. A	**2.** D	**3.** B	**4.** C

Matrix Match/Column Match Type Questions

1.	A → (p)	B → (s)	C → (r)	D → (p)
2.	A → (q)	B → (r)	C → (s)	D → (p)
3.	A → (p, r)	B → (q, s, t)	C → (p, r, t)	D → (q, s)
4.	A → (p, q, r, s)	B → (q)	C → (p, q, r, s)	D → (p, q, r, s)
5.	A → (p)	B → (r)	C → (r)	D → (p, q, r)

Integer/Numerical Answer Type Questions

1. 50	**2.** 1.5	**3.** 130	**4.** 8	**5.** 2
6. 7	**7.** 2	**8.** 3	**9.** 6	**10.** 6

CHAPTER

2 Wave Optics

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Wave nature of light
(b) Huygen's Principle
(c) Interference
(d) Young's Double Slit Experiment (along with its variations)
(e) Diffraction phenomenon
(f) Resolving power
(g) Fresnel's distance and Polarisation

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

INTRODUCTION

The phenomenon of interference, diffraction and polarisation exhibited by light could not be explained on the basis of Newton's Corpuscular Theory. In 1678, Huygen suggested that light propagates in the form of waves. The first historic experiment in favour of wave theory was done by Focault, who in 1850 found experimentally that velocity of light in denser medium is less than that in the rarer medium which was contrary to Newton's Corpuscular Theory.

NEWTON'S CORPUSCULAR THEORY

Newton proposed that light is made up of tiny, light and elastic particles called **corpuscles** which are emitted by a luminous body. These corpuscles travel with speed equal to the speed of light in all directions in straight lines and carry energy with them. When the corpuscles strike the retina of the eye, they produce the sensation of vision. The corpuscles of different colour are of different sizes (red corpuscles larger than blue corpuscles).

The corpuscular theory explains that light carry energy and momentum, light travels in a straight line, Propagation of light in vacuum, Laws of reflection and refraction. However, it fails to explain the phenomenon of interference, diffraction and polarization.

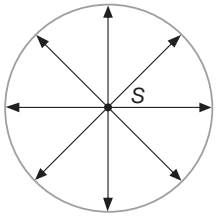
WAVE OPTICS

Wave optics is the study of the wave nature of light. Interference and diffraction are two main phenomena giving convincing evidence that light is a wave.

WAVEFRONTS AND RAYS

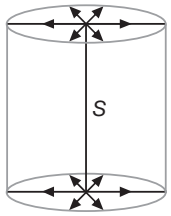
The locus of all the points vibrating in same phase of oscillation is called a **wavefront** (WF) i.e. a wavefront is defined as a surface joining the points vibrating in the same phase. The direction of propagation of light (ray of light) is along the normal to the Wavefront. The speed with which the wavefront moves onwards from the source is called the phase velocity or wave velocity. The energy travels outwards along straight lines emerging from the source, normally to the wavefront, that is, along the radii of the spherical wavefront. These lines are called the **rays**.

For a **point source** in a homogeneous medium the wavefront is **spherical**.



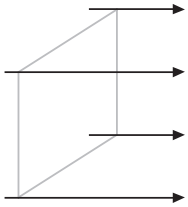
Spherical wavefront

For a **linear** source of light, the wavefront is **cylindrical**.



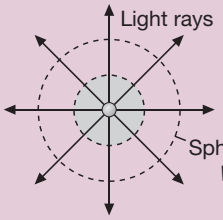
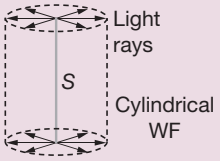
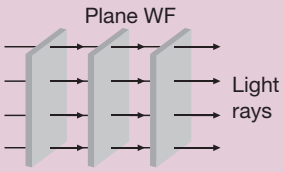
Cylindrical wavefront

A small part of a spherical or cylindrical wavefront from a distant source will appear plane and is, therefore, called a **plane wavefront**.



Plane wavefront

Conceptual Note(s)

Types of Wavefront (WF)	Intensity	Amplitude
Spherical WF due to a point source or a spherical source S. 	$I \propto \frac{1}{r^2}$	$A \propto \frac{1}{r}$
Cylindrical WF due to a line source or a cylindrical source S. 	$I \propto \frac{1}{r}$	$A \propto \frac{1}{\sqrt{r}}$
Plane WF 	$I \propto r^0$	$A \propto r^0$

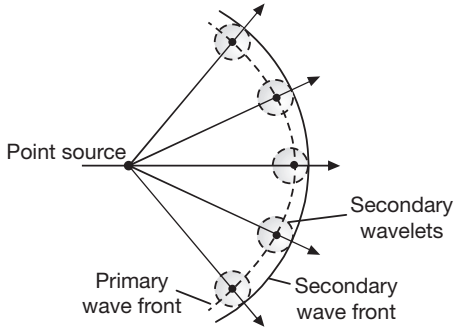
(*Continued*)

HUYGEN'S PRINCIPLE

This principle is useful for determining the position of a given wavefront at any further time if its present position is known. The principle may be stated in three parts.

(a) Every point on the given wavefront may be regarded as the source of the new disturbance.

(b) The new disturbances from each point spread out in all directions with the velocity of light in the same manner as the original source of light does and these new disturbances are called **secondary wavelets**.

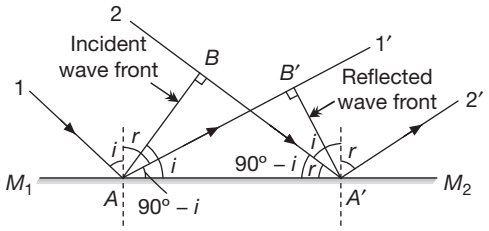


(c) The surface of tangency to the secondary wavelets in forward direction at any time gives the position of the new wavefront at that time. This new wavefront is called the Secondary Wavefront.

This principle explained successfully, reflection, refraction, total internal reflection, interference and diffraction but failed to explain the rectilinear propagation light.

LAWS OF REFLECTION ON THE BASIS OF HUYGEN'S THEORY

Let AB be the plane wave front incident on a plane mirror M_1M_2 at $\angle BAA' = i$, where 1, 2 are the corresponding incident rays perpendicular to AB.



According to Huygen's principle every point on AB is a source of secondary wavelets, so

$BA' = ct$, where c is speed of light

The secondary wavelets from A will travel the same distance ct in the same time. So, $AB' = ct$

Now, $\angle AA'B = 90 - i$, so that $\angle A'AB = i$, $(0 < i < 90°)$

Also, $\angle A'AB' = 90° - r$, so that $\angle AA'B' = r$, $(0 < r < 90°)$

From $\Delta AB'A'$, we have

$$\sin r = \frac{AB'}{AA'} = \frac{ct}{AA'} \qquad \ldots(1)$$

From $\Delta A'BA$, we have

$$\sin i = \frac{A'B}{AA'} = \frac{ct}{AA'} \qquad \ldots(2)$$

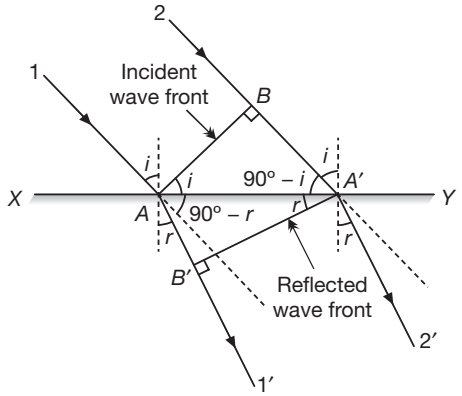
From equation (1) and (2), we get

$\sin i = \sin r$

$\angle i = \angle r$ which is the Law of Reflection

LAW OF REFRACTION ON THE BASIS OF HUYGEN'S THEORY

XY is a plane surface that separates a denser medium of refractive index μ from a rarer medium.



If v_1 is velocity of light in rarer medium and v_2 is velocity of light in denser medium, then by definition

$$\mu = \frac{v_1}{v_2} \qquad \ldots(1)$$

AB is a plane wave front incident on XY at $\angle BAA' = \angle i$, where 1, 2 are the corresponding incident rays normal to AB

According to Huygen's principle

$$BA' = v_1 t \qquad \ldots(2)$$

The secondary wavelets from A travel in the denser medium with a velocity v_2 and would cover a distance $AB' = v_2 t$ in the same time.

So, from $\Delta ABA'$ and $\Delta AB'A'$

$$\sin i = \frac{BA'}{AA'} \text{ and } \sin r = \frac{AB'}{AA'}$$

$$\Rightarrow \quad \frac{\sin i}{\sin r} = \frac{BA'}{AA'} \times \frac{AA'}{AB'} = \frac{BA'}{AB'} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2}$$

So, from equation (1), we get

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \mu$$

which is the Snell's Law of Refraction.

INTERFERENCE

When two waves of same frequency, nearly same amplitude and constant initial phase difference travel in the same direction along same straight line, they superimpose in such a way that in the region of superposition, the intensity is maximum at some points and minimum at some other points. This modification in intensity in the region of superposition is called **Interference**. The sources having the same

frequency and constant initial phase difference are called **coherent sources**. The phenomenon of interference is based on the Law of Conservation of Energy.

SUSTAINED INTERFERENCE

The interference pattern in which the positions of maxima and minima remain fixed is called a **sustained interference**.

Conditions for Sustained Interference

(a) The fundamental condition for sustained interference is that the two sources should be coherent i.e., the initial phase difference between the two interfering waves must remain constant with time.
(b) The amplitudes of the two waves should be equal or nearly equal. This will give good contrast between bright and dark fringes.
(c) The two sources should be very closely spaced, otherwise the fringes will be too close for the eye to resolve.
(d) The sources should be monochromatic, otherwise there will be overlapping of interference patterns due to different wavelengths, which will reduce are contrast.
(e) The frequencies of the two interfering waves must equal.
(f) The **sources should be narrow.**

Since two independent sources cannot be coherent, a sustained interference pattern can be obtained only if the two sources simultaneously and, therefore, the phase difference between them remains constant.

COHERENT SOURCES

Two sources which emit light of the same wavelength with zero or a constant phase difference are called **coherent sources**.

Unlike sound waves, two independent sources of light cannot be coherent. Sound is a bulk property of matter. So, two independent sources of sound can produce coherent waves. However, two independent sources of light cannot be coherent. The emission of light from any source is from a very large number of atoms and the emission from each atom is random and independent of each other. Therefore, there is no stable phase relationship between radiations from two independent sources. So, for two sources to be coherent, they must be derived from the same parent source.

In practice, coherent sources are obtained either by dividing the wavefront (as in the case of Young's Double Slit Experiment, Fresnel's biprism, Lloyd mirror, etc.) or by dividing the amplitude (as in the case of thin films, Newton rings, etc.) of the incoming waves from a single source.

A **laser** discovered in 1960, is different from common light sources. Its atoms act in a cooperative manner so as to produce intense, monochromatic, unidirectional and coherent light. Thus, two independent laser beams can produce observable interference on a screen.

METHODS OF PRODUCING COHERENT SOURCES

Division of Wavefront

In this method the wavefront is divided into two parts by the use of mirrors, or lenses or prisms. Well known methods are Young's double slit arrangement, Fresnel's biprism and Lloyd's single mirror.

Division of Amplitude

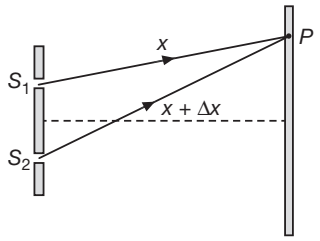
In this method the amplitude of the incoming beam is divided into two parts by means of partial reflection of refraction. These divided parts travel different paths and are finally brought together to produce interference. This class of interference requires broad sources of light. The common examples of such interference of light are the brilliant colours seen when a thin film of transparent material like soap bubble or thin film of kerosene oil spread on the surface of water is exposed to an extended source of light. This kind of interference exists in two types.

(a) Interference due to waves reflected from both the front and back surfaces of the film.
(b) Interference due to transmitted waves.

INTERFERENCE: MATHEMATICAL TREATMENT

Two waves (whether sound or light) of equal frequencies travelling almost in the same direction show interference. Consider two waves coming from

sources S_1 and S_2. These reach point P with a path difference Δx, having amplitude A_1 and A_2.



$$y_1 = A_1 \sin(\omega t - kx) \quad \ldots(1)$$

and $y_2 = A_2 \sin[\omega t - k(x + \Delta x)]$

$$\Rightarrow \quad y_2 = A_2 \sin(\omega t - kx - \phi) \quad \ldots(2)$$

where $\phi = k\Delta x = \left(\frac{2\pi}{\lambda}\right)\Delta x$ and Δx is the path difference.

By Principle of Superposition, the resultant wave at P is

$$y = y_1 + y_2 = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx - \phi)$$

$$\Rightarrow \quad y = (A_1 + A_2 \cos\phi)\sin(\omega t - kx) - (A_2 \sin\phi)\cos(\omega t - kx) \quad \ldots(3)$$

Substituting $A_1 + A_2 \cos\phi = A\cos\theta$ and $A_2 \sin\phi = A \sin\theta$, we get

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\phi \quad \ldots(4)$$

Equation (3), becomes

$$y = A\sin(\omega t - kx - \theta) \quad \ldots(5)$$

where, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$ and

$$\tan\theta = \frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$$

The intensity of the resultant wave is

$$I = \frac{1}{2}\rho v\omega^2 A^2 = KA^2 = K\left[A_1^2 + A_2^2 + 2A_1A_2\cos\phi\right]$$

$$\Rightarrow \quad I = I_1 + I_2 + \underbrace{2\sqrt{I_1I_2}\cos\phi}_{\text{Interference term}} \quad \ldots(6)$$

Thus, when interference of two waves of equal intensities occur, the intensity of maxima becomes 4 times that of single wave and that of minima becomes zero.

CONDITION FOR MAXIMA: CONSTRUCTIVE INTERFERENCE

From equation (6), I is maximum, when

$$\cos\phi = +1$$

$$\Rightarrow \quad \phi = 0,\ 2\pi,\ 4\pi, \ldots.$$

$$\Rightarrow \quad \phi = 2n\pi$$

$$\Rightarrow \quad \phi = \left(\frac{2\pi}{\lambda}\right)x = 2n\pi, \text{ where } n = 0,\ 1,\ 2,\ 3,\ .....$$

$$\Rightarrow \quad \Delta x = (2n)\frac{\lambda}{2}, \text{ where } n = 0,\ 1,\ 2,\ 3,\ .....$$

So, intensity will be maximum when phase difference ϕ is an even multiple of π or path difference Δx is an even multiple of $\frac{\lambda}{2}$.

$$\Rightarrow \quad I_{max} = I_1 + I_2 + 2\sqrt{I_1I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = (A_1 + A_2)^2$$

CONDITION FOR MINIMA: DESTRUCTIVE INTERFERENCE

Intensity I will be minimum, when

$$\cos\phi = -1$$

$$\Rightarrow \quad \phi = \pi,\ 3\pi,\ 5\pi, \ldots.$$

$$\Rightarrow \quad \phi = (2n+1)\pi$$

$$\Rightarrow \quad \phi = \left(\frac{2\pi}{\lambda}\right)x = (2n+1)\pi, \text{ where } n = 0,\ 1,\ 2,\ 3,\ .....$$

$$\Rightarrow \quad \Delta x = (2n+1)\frac{\lambda}{2}, \text{ where } n = 0,\ 1,\ 2,\ 3,\ .....$$

So, intensity will be maximum when phase difference ϕ is an odd multiple of π or path difference Δx is an odd multiple of $\frac{\lambda}{2}$.

$$\Rightarrow \quad I_{min} = I_1 + I_2 - 2\sqrt{I_1I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = K(A_1 \sim A_2)^2$$

The ratios $\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2} = \frac{(A_1 + A_2)^2}{(A_1 \sim A_2)^2}$

If $I_1 = I_2 = I_0$ (i.e., $A_1 = A_2$), we have

$$I_{max} = 4I_0 \text{ and } I_{min} = 0$$

Thus, when interference of two waves of equal intensities occur, the intensity of maxima becomes four times that of single wave and that of minima becomes zero.

In **Young's Double Slit Experiment** popularly known as $YDSE$, usually the intensities I_1 and I_2 are equal, so $I_1 = I_2 = I_0$

Since, $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$, so we get

$$I = 2I_0(1+\cos\phi)$$

$$\Rightarrow \quad I = 4I_0\cos^2\left(\frac{\phi}{2}\right)$$

PHASE DIFFERENCE AND PATH DIFFERENCE

If two waves travel different lengths of path to reach a point, they may not be in phase with each other. The phase difference depends on the path difference as $\phi = \frac{2\pi}{\lambda}\Delta x$, where Δx is the difference in length of path traversed by the waves.

The phase difference between two light waves can change if the waves travel through different materials having different refractive indices.

Suppose, we have two waves having identical wavelengths λ, initially in phase, in air. One of the waves travel through medium 1 of refractive index μ_1 and length L and other wave travels through same length L in another medium of refractive index μ_2. As wavelength differs in a medium, the two waves may not remain in phase.

The path difference after crossing through the medium is given by

$$\Delta x = (n_1 - n_2)\lambda$$

where n_1 is number of wavelengths in medium 1 and

n_2 is number of wavelengths in medium 2

$$\Rightarrow \quad \Delta x = \left(\frac{L}{\lambda_1} - \frac{L}{\lambda_2}\right)\lambda = \left(\frac{\lambda}{\lambda_1} - \frac{\lambda}{\lambda_2}\right)L$$

Since $\frac{\lambda}{\lambda_1} = \mu_1$ and $\frac{\lambda}{\lambda_2} = \mu_2$, so

$$\Delta x = (\mu_1 - \mu_2)L$$

ILLUSTRATION 1

Two light rays, initially in phase and having wavelength 6×10^{-7} m, go through different plastic layers of the same thickness, 7×10^{-6} m. The indices of refraction are 1.65 for one layer and 1.49 for the other.

(a) What is the equivalent phase difference between the rays when they emerge?

(b) If those two rays then reach a common point, does the interference result in complete darkness, maximum brightness, intermediate illumination but closer to complete darkness, or intermediate illumination but closer to maximum brightness?

SOLUTION

(a) $\Delta x = (\mu_1 - \mu_2)t = (1.65 - 1.49)(7\times10^{-6})$

$\Delta x = 1.12\times10^{-6}$ m

Since, Phase difference $\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x)$

$$\Rightarrow \quad \phi = \left(\frac{2\pi}{6\times10^{-7}}\right)(1.12\times10^{-6})$$

$$\Rightarrow \quad \phi = 11.72 \text{ radian}$$

(b) To discuss this case, two options arise

Option 1: Waves are in phase, then using

$I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$, we get

$$I = I_{max}\cos^2\left(\frac{11.72}{2}\right) = 0.8I_{max}$$

This value is intermediate illumination closer to maximum brightness.

Option 2: Waves are out of phase, then

$$\phi_{net} = 11.72 \pm \pi$$

$$\Rightarrow \quad \phi_{net} = 14.86 \text{ rad}$$

$$\Rightarrow \quad I = I_{max}\cos^2\left(\frac{14.86}{2}\right)$$

$$\Rightarrow \quad I = 0.17I_0$$

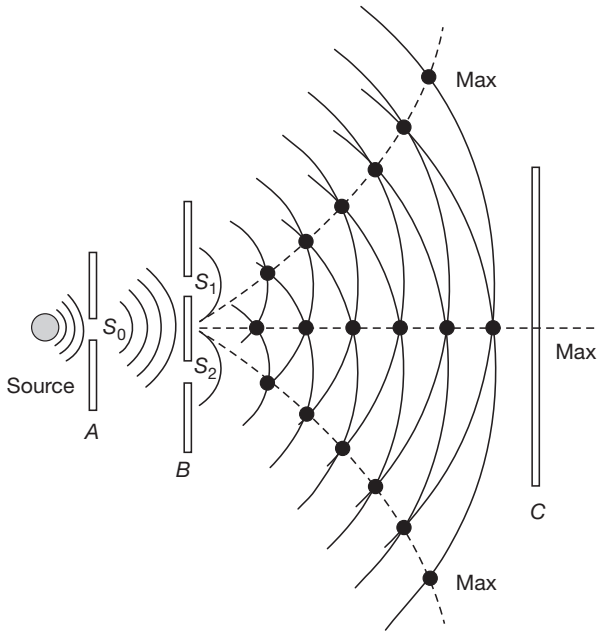
This value is intermediate illumination closer to darkness.

THEORY OF INTERFERENCE: MAXIMA AND MINIMA

Theory of Division of Wavefront: Young's Double Slit Experiment

The phenomenon of interference of light waves arising from two sources was first demonstrated by Thomas Young in 1801.

Light is incident on screen A, which is provided with a narrow slit, S_0.

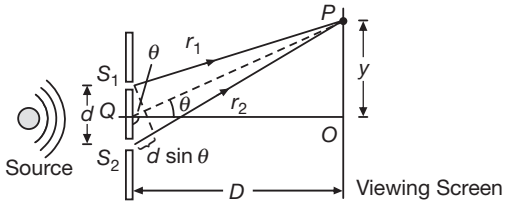


Schematic diagram of young's double-slit experimet. The narrow slits act as sources of cylindrical waves. Slits S_1 and S_2 behave as coherent sources which produce an interference pattern on screen C.

The cylindrical waves emerging from this slit arrive at screen B, which contains two narrow, parallel slits, S_1 and S_2. Light emerges from these two slits as cylindrical waves. In effect, slits S_1 and S_2 act as individual light sources that are in phase as they originate from the same cylindrical wavefront. The light from the two slits produces a visible pattern on screen C. The pattern consists of a series of bright and dark parallel bands called **fringes**. The overall light amplitude at a given point on the screen is the result of the superposition of the two wave amplitudes from S_1 and S_2. Two waves that add constructively give a bright fringe, and any two waves that add destructively produce a dark fringe.

YDSE (QUANTITATIVE TREATMENT): METHOD 1

Consider a point P on the viewing screen located a perpendicular distance D from the two identical slits S_1 and S_2, which are separated by a distance d.



Geometric construction for describing young's double-slit experiment. Note that the path difference between the two rays is $r_2 - r_1 = d \sin\theta$

Let us assume that the source is equidistant from the two slits and is monochromatic, that is, emitting light of a single wavelength λ. Under these assumptions, the waves emerging from slits S_1 and S_2 have the same frequency and amplitude and are in phase. The light intensity on the screen at P is the resultant of light coming from both slits. Note that a wave from the lower slit travels farther than a wave from the upper slit by an amount equal to $d\sin\theta$. This distance is called the **path difference**, x, where

$$x = r_2 - r_1 = d\sin\theta \qquad \ldots(1)$$

The value of this path difference will determine whether or not the two waves are in phase when they arrive at P. If the path difference is either zero or some integral multiple of the wavelength, the two waves are in phase at P and constructive interference results. Therefore, the condition for bright fringes, or constructive interference, at P is given by

$$x = d\sin\theta = n\lambda \qquad (n = 0, \pm 1, \pm 2, \pm 3\ldots) \qquad \ldots(2)$$

The index number n is called the **order number** of the **fringe**. The central bright fringe at $\theta = 0 (n = 0)$ is called the **zeroth order maximum**. The first maximum on either side, when $n = \pm 1$, is called the **first order maximum**, etc.

Similarly, when the path difference is an odd multiple of $\frac{\lambda}{2}$, the two waves arriving at P will be opposite in phase and will give rise to destructive

interference. Therefore, the condition for dark fringes, or destructive interference, at P is given by

$$x = d\sin\theta = (2n+1)\frac{\lambda}{2} \quad (n = 0, \pm 1, \pm 2...) \quad ...(3)$$

It is useful to obtain expressions for the positions of the bright and dark fringes measured vertically from O to P. We shall assume that $D > d$ and consider only points P that are close to O. In this case, θ is small, and so we can use the approximation $\sin\theta \approx \tan\theta$. From the large triangle OPQ in Figure, we see that

$$\sin\theta \approx \tan\theta = \frac{y}{D} \quad ...(4)$$

Using this result together with equation (2), we see that the positions of the bright fringes measured from O are given by

$$y_{\text{bright}} = n\left(\frac{\lambda D}{d}\right) \quad ...(5)$$

From this expression, we find that the separation between any two adjacent bright fringes called **Fringe Width** is equal to $\frac{\lambda D}{d}$, that is,

$$\beta = y_{n+1} - y_n = \frac{\lambda D}{d}(n+1) - \frac{\lambda D}{d}n = \frac{\lambda D}{d} \quad ...(6)$$

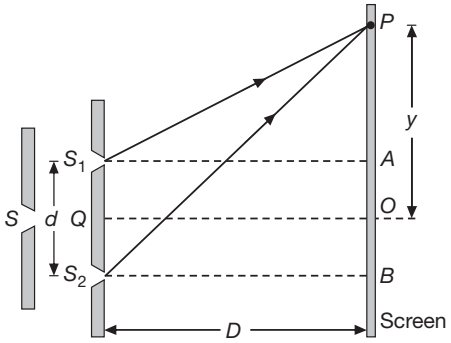
Similarly, using equation (3) and (4), we find that the dark fringes are located at

$$y_{\text{dark}} = (2n+1)\frac{\lambda D}{2d} \quad ...(7)$$

This result shows that the separation between adjacent dark fringes is also equal to $\beta = \frac{\lambda D}{d}$. Since the quantities D and d are both measurable, we see that the double-slit interference pattern, together with equation (6), provides a direct determination of the wavelength λ. Young used this technique to make the first measurement of the wavelength of light.

YDSE (QUANTITATIVE TREATMENT): METHOD 2

Consider that two coherent sources of light S_1 and S_2 are placed at a distance d apart and a screen is placed at a distance D from the plane of the two sources.



Let P be a point on the screen at a distance y from a point O exactly opposite to the centre of the two sources S_1 and S_2. If x is path difference between the light waves reaching point P from the sources S_1 and S_2, then

$$x = S_2P - S_1P$$

In right angled ΔS_2BP, we have

$$S_2P^2 = S_2B^2 + BP^2 = D^2 + \left(y + \frac{d}{2}\right)^2$$

Also, in right angled ΔS_1AP, we have

$$S_1P^2 = S_1A^2 + AP^2 = D^2 + \left(y - \frac{d}{2}\right)^2$$

$$\Rightarrow \quad S_2P^2 - S_1P^2 = \left(D^2 + \left(y + \frac{d}{2}\right)^2\right) - \left(D^2 + \left(y - \frac{d}{2}\right)^2\right)$$

$$\Rightarrow \quad (S_2P + S_1P)(S_2P - S_1P) = \left(y + \frac{d}{2}\right)^2 - \left(y - \frac{d}{2}\right)^2$$

Since $S_2P - S_1P = x$ (the path difference between the two light waves), the above equation becomes

$$(S_2P + S_1P)x = 4y\left(\frac{d}{2}\right) = 2yd$$

$$\Rightarrow \quad x = \frac{2yd}{S_2P + S_1P}$$

In practice, the point P lies very close to the centre of screen, so we have

$$S_2P = S_1P = D$$

$$\Rightarrow \quad x = \frac{2yd}{D+D} = \frac{2yd}{2D}$$

$$\Rightarrow \quad x = \frac{yd}{D} \quad ...(1)$$

For Maxima, we know that path difference x must be an even multiple of $\frac{\lambda}{2}$, so

$$x = (2n)\frac{\lambda}{2}, \text{ where } n = 0, 1, 2, \ldots..$$

$$\Rightarrow \quad \frac{yd}{D} = n\lambda, \text{ where } n = 0, 1, 2, \ldots..$$

$$\Rightarrow \quad y_n = n\left(\frac{\lambda D}{d}\right), \text{ where } n = 0, 1, 2, \ldots..$$

So, $y_{n^{\text{th}}\text{ bright}} = n\left(\frac{\lambda D}{d}\right)$; $n = 0, 1, 2, 3, 4, 5, \ldots..$

At $y = 0$ (i.e., for $n = 0$) we get a Central Bright Fringe.

For Minima, we know that path difference x must be an odd multiple of $\frac{\lambda}{2}$, so

$$x = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, \ldots..$$

$$\Rightarrow \quad \frac{yd}{D} = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, \ldots..$$

$$\Rightarrow \quad y_n = (2n-1)\frac{\lambda D}{2d}, \text{ where } n = 1, 2, \ldots..$$

$$\Rightarrow \quad y_n = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d}, \text{ where } n = 1, 2, \ldots..$$

$$y_{n^{\text{th}}\text{ dark}} = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d} \text{ ; } n = 1, 2, 3, 4, 5, \ldots..$$

Conceptual Note(s)

For central bright fringe $n = 0$, $y = 0$ and $\Delta x = 0$

For n^{th} bright fringe the distance from centre of central right fringe is $n\beta = y_{n^{\text{th}}\text{ bright}}$.

For n^{th} dark fringe the distance from centre of central bright fringe is $\left(n - \frac{1}{2}\right)\beta = y_{n^{\text{th}}\text{ bright}}$.

ILLUSTRATION 2

If the maximum intensity in $YDSE$ is I_0, find the intensity at a point on the screen where

(a) the phase difference between the two interfering beams is $\frac{\pi}{3}$.

(b) the path difference between the two interfering beams is $\frac{\lambda}{4}$.

SOLUTION

(a) Since, $I = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$

where $I_{\max}$ is I_0 i.e., intensity due to independent sources is $\frac{I_0}{4}$. Therefore, at

$$\phi = \frac{\pi}{3}$$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{6}$$

$$\Rightarrow \quad I = I_0\cos^2\left(\frac{\pi}{6}\right) = \frac{3}{4}I_0$$

(b) Phase difference corresponding to the given path difference Δx is given by

$$\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x$$

$$\Rightarrow \quad \phi = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)$$

$$\Rightarrow \quad \phi = \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{4}$$

$$\Rightarrow \quad I = I_0\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$$

ILLUSTRATION 3

In $YDSE$, the interference pattern is found to have an intensity ratio between the bright and dark fringes as 9. Find the ratio of

(a) intensities.

(b) amplitudes of the two interfering waves.

SOLUTION

In case of interference, we have

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

(a) $I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

and $I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2$

Since, $\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}\sim\sqrt{I_2}\right)^2} = \frac{9}{1}$

$$\Rightarrow \frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}} = \frac{3}{1}$$

Solving, we get

$$\frac{I_1}{I_2} = \frac{4}{1} = 4$$

(b) Since, $I \propto A^2$,

$$\Rightarrow \frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2$$

$$\Rightarrow \left(\frac{A_1}{A_2}\right)^2 = 4$$

$$\Rightarrow \frac{A_1}{A_2} = 2$$

ILLUSTRATION 4

The intensity of the light coming from one of the slits in $YDSE$ is double the intensity from the other slit. Find the ratio of the maximum intensity to the minimum intensity in the interference fringe pattern observed.

SOLUTION

Since, we know that

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$$

According to the problem, we have

$$I_1 = 2I_0 \text{ and } I_2 = I_0$$

$$\Rightarrow \frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 = 34$$

ILLUSTRATION 5

In a Young's double-slit experiment the distance between the slits is 1 mm and the distance of the screen from the slits is 1 m. If light of wavelength 6000 Å is used, find the distance between the second dark fringe and the fourth bright fringe.

SOLUTION

The position of the second dark fringe is given by

$$y_2(\text{dark}) = (2n-1)\frac{\lambda D}{2d} = (4-1)\frac{\lambda D}{2d} = \frac{3}{2}\left(\frac{\lambda D}{d}\right)$$

The position of the 4^{th} bright fringe is given by

$$y_2(\text{bright}) = \frac{n\lambda D}{d} = \frac{4\lambda D}{d}$$

Therefore, the separation is given by

$$\Delta y = y_4(\text{bright}) - y_2(\text{dark}) = \left(4-\frac{3}{2}\right)\frac{\lambda D}{d}$$

$$\Rightarrow \Delta y = \frac{5}{2}\times\frac{6000\times10^{-10}\times1}{10^{-3}} = 1.5\times10^{-3} = 1.5 \text{ mm}$$

ILLUSTRATION 6

In a $YDSE$, the separation between the coherent sources is 6 mm, the separation between coherent sources and the screen is 2 m. If light of wavelength 6000 Å is used, then

(a) find the fringe width.
(b) find the position of the third maxima.
(c) find the position of the second minima.

SOLUTION

(a) Since fringe width is given by $\beta = \frac{\lambda D}{d}$, so we have

$$\beta = \frac{\lambda D}{d} = \frac{(6000\times10^{-10})(2)}{6\times10^{-3}} = 0.2 \text{ mm}$$

(b) Position of third maxima is obtained by substituting $n = 3$ in the equation $y_n = n\left(\frac{\lambda D}{d}\right)$, so we get

$$y_3 = \frac{3\lambda D}{d} = 3\beta = 3(0.2) = 0.6 \text{ mm}$$

(c) Position of second minima is obtained by putting $n = 2$ in the equation $y_n = (2n-1)\frac{\lambda D}{2d}$, so we get

$$y_2 = \left(2-\frac{1}{2}\right)\frac{\lambda D}{d} = \frac{3}{2}\beta = \frac{3}{2}(0.2) = 0.3 \text{ mm}$$

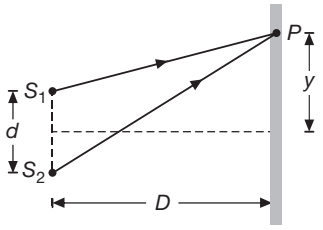
ILLUSTRATION 7

Young's double slit experiment is carried out using microwaves of wavelength $\lambda = 3$ cm. Distance between the slits is $d = 5$ cm and the distance between the plane of slits and the screen is $D = 100$ cm. Find the number of maximas and their positions on the screen.

SOLUTION

$$\begin{pmatrix}\text{The maximum path}\\ \text{difference that}\\ \text{can be produced}\end{pmatrix} = \begin{pmatrix}\text{Distance between the}\\ \text{coherent sources}\\ \text{i.e., 5 cm}\end{pmatrix}$$

Thus, in this case we can have only three maximas, one central maxima and two on its either side (for a path difference of $\lambda = 3$ cm)



For maximum intensity at P, we have

$$S_2P - S_1P = \lambda$$

$$\Rightarrow \quad \sqrt{\left(\frac{y+d}{2}\right)^2 + D^2} - \sqrt{\left(\frac{y-d}{2}\right)^2 + D^2} = \lambda^2$$

Substituting $d = 5$ cm, $D = 100$ cm and $\lambda = 3$ cm and solving the equation, we get

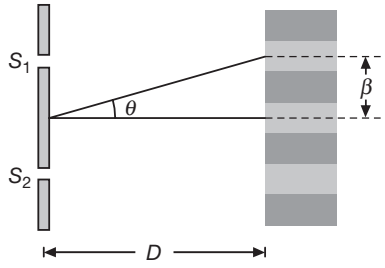
$$y = \pm 75 \text{ cm}$$

Thus, the three maximas will be at

$$y = 0 \text{ and } y = \pm 75 \text{ cm}$$

FRINGE WIDTH AND ANGULAR FRINGE WIDTH

The separation between two consecutive bright (or dark) fingers is called the **fringe width** (β), given by



$$\beta = y_{n+1} - y_n = \frac{\lambda D}{d}$$

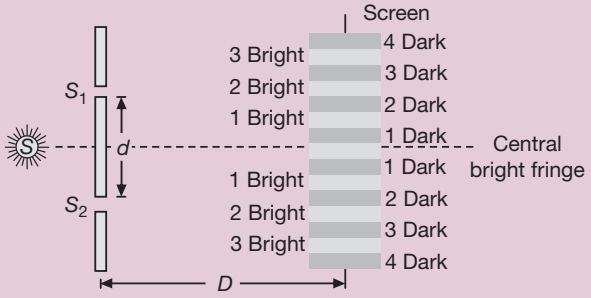
Note that fringe width β is independent of n. That is, the interference fringes have same width throughout.

The angular fringe width is given by

$$\Delta\phi = \frac{\beta}{D} = \frac{\lambda}{d}$$

Conceptual Note(s)

In YDSE alternate bright and dark bands obtained on the screen. These bands are called Fringes.



d = Distance between slits
D = Distance between slits and screen
λ = Wavelength of monochromatic light emitted from source

(a) Central fringe is always bright, because at central position the path difference, $x = 0$ and hence the phase difference, $\phi = 0°$ and hence the Central Bright Fringe is also called the zeroth maxima.

(b) The nth minima come before the nth maxima.

(c) The fringe pattern obtained due to a slit is more bright than that due to a point.

(d) If the slit widths are unequal, the minima will not be complete dark. For very large width uniform illumination occurs.

(e) If one slit is illuminated with red light and the other slit is illuminated with blue light, no interference pattern is observed on the screen.

(f) If the two coherent sources consist of object and it's reflected image, the central fringe is dark instead of bright one.

(g) $y_{n^{\text{th}}\text{ bright}} = n\beta$ and $y_{n^{\text{th}}\text{ dark}} = \left(n - \frac{1}{2}\right)\beta$.

ILLUSTRATION 8

In a $YDSE$ bi-chromatic light of wavelengths **400 nm** and **560 nm** are used. The distance between the slits is **0.1 mm** and the distance between the plane of the slits and the screen is **1 m**. Find the minimum distance between two successive regions of complete darkness.

SOLUTION

Let n_1 minima of 400 nm coincides with n_2 minima of 560 nm, then

$$(2n_1-1)400=(2n_2-1)560$$

$$\Rightarrow \quad \frac{2n_1-1}{2n_2-1}=\frac{7}{5}=\frac{14}{10}=\frac{21}{15}$$

i.e., 4th minima of 400 nm coincides with 3rd minima of 560 nm

The location of this minima is

$$y_4=\frac{7(1000)(400\times10^{-6})}{2\times0.1}=14 \text{ mm}$$

Next, 11th minima of 400 nm will coincide with 8th minima of 560 nm

Location of this minima is

$$y_{11}=\frac{21(1000)(400\times10^{-6})}{2\times0.1}=42 \text{ mm}$$

So, required separation is

$$\Delta y=42-14=28 \text{ mm}$$

ILLUSTRATION 9

Light from a source consists of two wavelength $\lambda_1=6500$ Å and $\lambda_2=5200$ Å. If the separation between the sources from each other is 6.5 mm and that from the screen is **2 m**, find the minimum value of $y(\neq 0)$ where the maxima of both the wavelengths coincide.

SOLUTION

Let n_1 maxima of λ_1 coincides with n_2 maxima of λ_2. Then, $y_{n_1}=y_{n_2}$

$$\Rightarrow \quad \frac{n_1\lambda_1 D}{d}=\frac{n_2\lambda_2 D}{d}$$

$$\Rightarrow \quad \frac{n_1}{n_2}=\frac{\lambda_2}{\lambda_1}=\frac{5200}{6500}=\frac{4}{5}$$

$$\Rightarrow \quad 4\lambda_1=5\lambda_2$$

Thus, fourth maxima of λ_1 coincides with fifth maxima of λ_2. The minimum value of $y(\neq 0)$ is given by

$$y=\frac{4\lambda_1 D}{d}=\frac{4(0.65\times10^{-6})(2)}{6.5\times10^{-3}}=0.8 \text{ mm}$$

ILLUSTRATION 10

Two coherent sources are 0.3 mm apart. They are 0.9 m away from the screen. The second dark fringe is at a distance of 0.3 cm from the centre. Find the distance of fourth bright fringe from the centre. Also find the wavelength of light used.

SOLUTION

Second dark fringe $n=2$ will be obtained at

$$y_n=\left(n-\frac{1}{2}\right)\beta$$

$$\Rightarrow \quad y=(2n-1)\frac{\lambda D}{2d}=\frac{3\lambda D}{2d} \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{\lambda D}{d}=\frac{2}{3}y$$

$$\Rightarrow \quad \beta=\frac{\lambda D}{d}=\frac{2}{3}(0.3)=0.2 \text{ m}$$

Fourth bright fringe from the centre will be obtained at

$$y_4=4\beta=0.8 \text{ cm}$$

From equation (1), we get

$$\lambda=\frac{2yd}{3D}=\frac{2\times0.3\times10^{-3}\times0.3\times10^{-2}}{3\times0.9}$$

$$\Rightarrow \quad \lambda=6.67\times10^{-7} \text{ m}$$

ILLUSTRATION 11

In a Young's double slit experiment, two narrow slits **0.8 mm** apart are illuminated by a source of light of wavelength 4000 Å. How far apart are the adjacent bright bands in the interference pattern observed on a screen 2 m away?

SOLUTION

Since, $d=0.8\times10^{-3}$ m, $\lambda=4000$ Å $=4000\times10^{-10}$ m, $D=2$ m

The distance between the adjacent bright bands or the fringe width is given by

$$\beta=\frac{\lambda D}{d}=\frac{4000\times10^{-10}\times2}{0.8\times10^{-3}} \text{ m}=1 \text{ mm}$$

ILLUSTRATION 12

A beam of light consisting of two wavelengths, 6500 Å and 5200 Å, is used to obtain interference fringes in a Young's double slit experiment. The distance between the slits is 2 mm and the distance between the plane of the slits and the screen is 120 cm .

(a) Find the distance of the third bright fringe on the screen from the central maximum for the wavelength 6500 Å

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

SOLUTION

According to the problem, we have

$$\lambda_1 = 6500 \text{ Å} = 6500\times10^{-10} \text{ m}$$

$$\lambda_2 = 5200 \text{ Å} = 5200\times10^{-10} \text{ m}$$

$$d = 2 \text{ mm} = 2\times10^{-3} \text{ m}$$

$$D = 120 \text{ cm} = 1.2 \text{ m}$$

(a) $y_n = \dfrac{n\lambda D}{d}$

$$\Rightarrow\ y_3 = \frac{3\times6500\times10^{-10}\times1.2}{2\times10^{-3}}$$

$$\Rightarrow\ y_3 = 1.17\times10^{-3} \text{ m} = 1.17 \text{ mm}$$

(b) The least distance from the central maximum where the bright fringes due to both the wavelength coincide corresponds to that value of n for which

$$\frac{n\lambda_1 D}{d} = \frac{(n+1)\lambda_2 D}{d}$$

$$\Rightarrow\ n\lambda_1 = (n+1)\lambda_2$$

$$\Rightarrow\ \frac{n+1}{n} = \frac{6500}{5200} = \frac{5}{4}$$

$$\Rightarrow\ n = 4$$

$$\Rightarrow\ y_4 = \frac{n\lambda D}{d} = \frac{4\times6500\times10^{-10}\times1.2}{2\times10^{-3}}$$

$$\Rightarrow\ y_4 = 1.56\times10^{-3} \text{ m} = 1.56 \text{ mm}$$

ILLUSTRATION 13

A convergent lens with a focal length of $f = 10$ cm is cut into two halves that are then moved apart to a distance of $d = 0.5$ mm (a double lens). Find the fringe width on screen at a distance of 60 cm behind the lens if a point source of monochromatic light $(\lambda = 5000 \text{ Å})$ is placed in front of the lens at a distance of $a = 15$ cm from it.

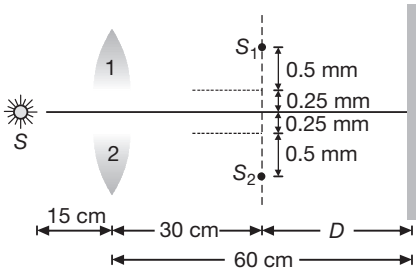
SOLUTION

Applying lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v} + \frac{1}{15} = \frac{1}{10}$$

$$\Rightarrow\ v = 30 \text{ cm}$$

Since, $m = \dfrac{v}{u} = \dfrac{30}{-15} = -2$



Distance between two slits is $d = 1.5$ mm

Distance between slits and screen is $D = 30$ cm

Fringe width $\beta = \dfrac{\lambda D}{d} = \dfrac{(5\times10^{-7})(0.3)}{(1.5\times10^{-3})} = 10^{-4}$ m

$$\beta = 0.1 \text{ mm}$$

INTERFERENCE EXPERIMENT IN WATER

In water (liquid), of refractive index μ the wavelength decreases from λ to $\lambda' = \dfrac{\lambda}{\mu}$. Therefore, if interference experiment is performed in water the fringe width decreases from β to β', such that

$$\beta = \frac{\lambda D}{d} \text{ and } \beta' = \frac{\lambda' D}{d} = \frac{\lambda D}{\mu d}$$

$$\Rightarrow\ \beta' = \frac{\beta}{\mu}$$

ILLUSTRATION 14

A Young's double slit arrangement produces interference fringes for sodium light $(\lambda = 5890\ \text{Å})$ that are $0.20°$ apart. What is the angular fringe separation if the entire arrangement is immersed in water? Refractive index of water is $\frac{4}{3}$.

SOLUTION

The wavelength of light in water is $\lambda_w = \frac{\lambda}{\mu}$

Angular fringe-width in air, $\theta_a = \frac{\lambda}{d}$

Angular fringe-width in water, $\theta_w = \frac{\lambda_w}{d}$

So, $\theta_w = \frac{\lambda_w}{d} = \frac{\lambda}{\mu d} = \frac{\theta_a}{\mu} = \frac{0.20°}{\frac{4}{3}} = 0.15°$

FRINGE VISIBILITY (*V*)

With the help of the concept of visibility, the knowledge about coherence, fringe contrast and interference pattern is obtained. Fringe visibility V is defined as

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$$

If $I_{min} = 0$, $V = 1$ (maximum) i.e., fringe visibility will be the best.

Also, if $I_{max} = 0$ then $V = -1$

and if $I_{max} = I_{min}$, then $V = 0$

INTENSITY DISTRIBUTION

When two coherent light waves of intensity I_1 and I_2 with a constant phase difference ϕ superimpose, then the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

In $YDSE$, usually the intensities I_1 and I_2 are equal, so $I_1 = I_2 = I_0$

For maxima, $\phi = 2n\pi$ i.e., $\cos\phi = +1$

$\Rightarrow \quad I_{max} = 4I_0$

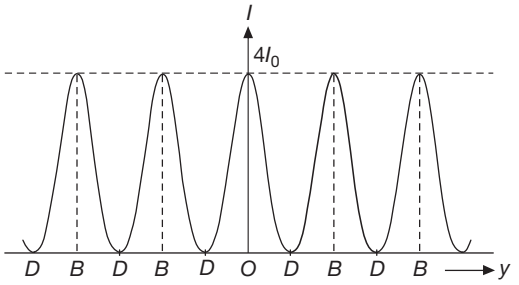
For minima, $\phi = (2n+1)\pi$ i.e., $\cos\phi = -1$

$\Rightarrow \quad I_{min} = 0$

Since, $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$, so we get

$$I = 2I_0(1+\cos\phi)$$

$$\Rightarrow \quad I = 4I_0\cos^2\left(\frac{\phi}{2}\right) = 4I_0\cos^2\left(\frac{\pi x}{\lambda}\right) = I_{max}\cos^2\left(\frac{\pi x}{\lambda}\right)$$



Intensity distribution on the screen as a function of y in YDSE $I_{max} = 4I_0$ for bright fringe and $I_{min} = 0$ for dark fringe.

Problem Solving Technique(s)

(a) Interference occurs due to Law of Conservation of Energy. Actually, redistribution of energy takes place.

(b) If w_1 and w_2 are the widths of the slits and I_1 and I_2 is the intensity of light (with respective amplitudes a_1 and a_2) passing through slits, then

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{w_1}{w_2}$$

(c) $\frac{I_{min}}{I_{max}} = \left(\frac{a_1 - a_2}{a_1 + a_2}\right)^2 = \left(\frac{\sqrt{w_1} - \sqrt{w_2}}{\sqrt{w_1} + \sqrt{w_2}}\right)^2$

$$\Rightarrow \quad \frac{I_{min}}{I_{max}} = \left(\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right)^2$$

(d) If point source is used to illuminate the two slits, the intensity emerging from the slit is proportional to area of exposed part of slit. In case of identical slits.

$I_1 = I_2$

$\Rightarrow \quad a_1 = a_2$

(e) When white light is used to illuminate the slit, we obtain an interference pattern consisting of a central white fringe having on both sides symmetrically a few coloured fringes and then a uniform illumination.

(f) If ϕ is the phase difference between two waves of intensities I_1 and I_2, then

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi, \text{ where } \phi = \Delta\phi = \phi_2 - \phi_1$$

(g) If x is the path difference, then

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\frac{2\pi}{\lambda}x\right)$$

where $x = \Delta x = x_2 - x_1$

(h) In *YDSE*, if n_1 fringes are visible in a field of view with light of wavelength λ_1, while n_1 with light of wavelength λ_2 in the same field, then $n_1\lambda_1 = n_2\lambda_2$.

(i) Separation (Δx) between fringes

(a) Between n^{th} bright and m^{th} bright fringes $(n > m)$

$$\Delta x = (n-m)\beta$$

(b) Between n^{th} bright and m^{th} dark fringe

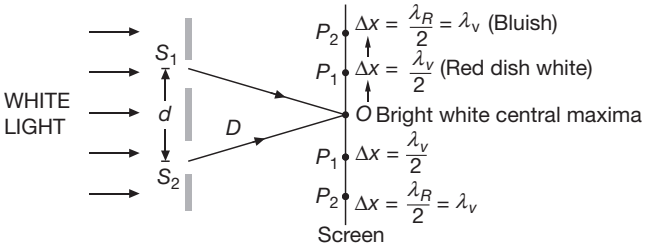
(i) If $n > m$ then $\Delta x = \left(n - m + \frac{1}{2}\right)\beta$

(ii) If $n < m$ then $\Delta x = \left(m - n - \frac{1}{2}\right)\beta$

(j) Identification of central bright fringe: To identify central bright fringe, monochromatic light is replaced by white light. Due to overlapping central maxima will be white with red edges. On the other side of it we shall get a few coloured band and then uniform illumination.

USE OF WHITE LIGHT IN YDSE

Let us discuss the effect of using white light in YDSE setup. The slits S_1 and S_2 are illuminated by white light. White light is made of seven colours where approximate wavelength of violet colour is $\lambda_v \cong 4000$ Å and approximate wavelength of red colour is $\lambda_R \cong 8000$ Å (actually $\lambda_v = 4200$ Å and $\lambda_R = 7900$ Å), so $\lambda_R \cong 2\lambda_v$ when white light illuminates two slits, then both the slits act as source of white light. At the centre of slits on the screen, all colours travel equal path from S_1 to O and from S_2 to O, so that they interfere at O with zero path difference so as to give constructive interference at O and hence a bright white fringe is obtained at O.



Now as we start moving away from centre of screen, a path difference is introduced and since λ_v is minimum, so at some point $(\text{say } P_1)$ we see that the path difference is $\frac{\lambda_v}{2}$ and at this point destructive interference of violet light occurs (i.e. violet colour is absent but all others present). At this point, the white light has violet colour absent and due to which a slightly reddish in colour or reddish white (but not red because it is white minus violet colour).

So, this point being closest to central maxima, the closest edge of this bright white central maxima will appear reddish in colour.

Now as we move further away from this point, various colours will be absent and there will be a point where path difference will be equal to $\frac{\lambda_{red}}{2} = \frac{\lambda_R}{2} = \lambda_v$ (because $\lambda_R \cong 2\lambda_v$).

So, at this point, $(\text{say } P_2)$ not only destructive interference of red colour is obtained but constructive interference of violet is also obtained. In this region, light red colour is absent from white light and violet is interfering constructively. So, at this point the bluish fringe is obtained (not blue, because in white light red colour is absent and violet colour is dominating) and then onwards there is no point where any prominent colour is obtained (because all colours will mix on screen). So in the outer region, we can say whitish fringes merged into each other are obtained and we cannot distinguish between these fringes.

To conclude, we observe that at the centre, a permanent bright white central fringe is obtained whose closer edges are reddish white and farther edges are bluish and then onwards whitish fringes are obtained. The same effect is observed in the pattern below central maxima.

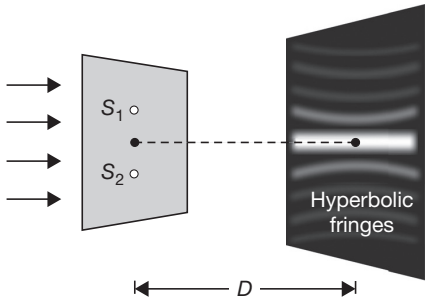
SHAPE OF INTERFERENCE FRINGES DUE TO DIFFERENT TYPES OF SOURCES (IN YDSE SETUP)

In *YDSE* setup, it is observed that the fringes obtained are straight and parallel to the slits. This is because, at all the points on screen where the path difference of the light waves from the two sources is same, will have same intensity and hence fringes of same intensity are obtained.

Let us now consider different cases of two light sources which produce interference pattern on a screen and analyse the shape of fringes obtained in the resulting interference pattern.

WHEN TWO POINT SOURCES IN A LINE ARE PLACED PARALLEL TO SCREEN

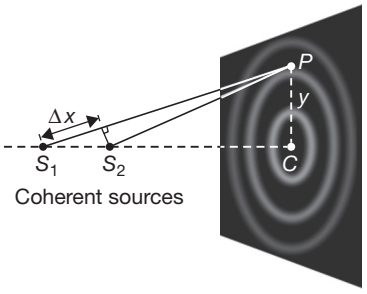
Let us consider a cardboard with two holes in a line and a screen placed in front of the cardboard and parallel to it as shown in figure.



When a light beam incident on the board illuminates the two holes, then these two holes will act like point sources S_1 and S_2 due to which interference pattern is obtained on the screen. We observe that the shape of fringes obtained on the screen is hyperbolic because a hyperbola is the locus of all the points on a plane which are at a constant distance from two points in space.

TWO POINT SOURCES IN A LINE PLACED NORMAL TO THE SCREEN

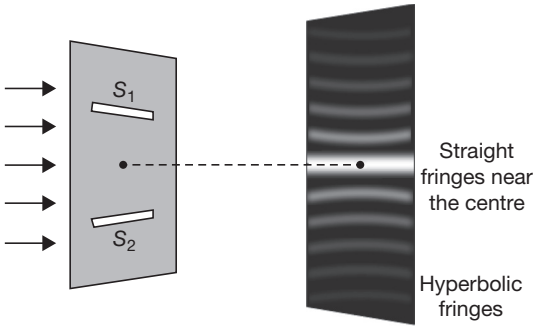
Let us consider two coherent point sources S_1 and S_2 along a line normal to the screen placed at some distance from the screen as shown in figure.



In this situation, if we consider a point P on the screen at a distance y from centre C of the screen, then due to the point sources, the path difference in light waves reaching at P remains constant and hence alternate bright and dark circular fringes are obtained.

TWO RECTANGULAR SLIT SOURCES IN A PLANE PARALLEL TO SCREEN

The fringe pattern obtained in *YDSE* setup when the light waves from two rectangular slit sources S_1 and S_2 interfere on screen is shown in figure.



Due to length of the rectangular slits, we observe that close to the middle region of the screen, fringes are straight and parallel. However, after moving away from the centre of the screen, the shape of fringes will be approximately hyperbolic (along the length of slits) because as discussed earlier, a hyperbola is the locus of all the points on a plane which are at a constant distance from two points in space.

ILLUSTRATION 15

In *YDSE*, light of wavelength 60 nm is used. The separation between the sources is 6 mm and between the sources and the screen is 2 m. Find the positions of a point lying between third maxima and third minima where the intensity is three-fourth of the maximum intensity on the screen.

SOLUTION

Since, $I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$

where $I = \frac{3}{4}(4I_0) = 3I_0$

$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \frac{\phi}{2} = n\pi \pm \frac{\pi}{6}$

$\Rightarrow \quad \phi = 2n\pi \pm \frac{\pi}{3}$

Since, $\phi = \frac{2\pi}{\lambda}\Delta x$ where $\Delta x = \frac{y_n d}{D}$

$\Rightarrow \quad \frac{2\pi}{\lambda}\frac{y_n d}{D} = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow \quad y_n = \left(n \pm \frac{1}{6}\right)\frac{\lambda D}{d}$

For the point lying between third minima and third maxima, we have

$n = 3$

$\Rightarrow \quad y_3 = \left(3 - \frac{1}{6}\right)\frac{\lambda D}{d}$

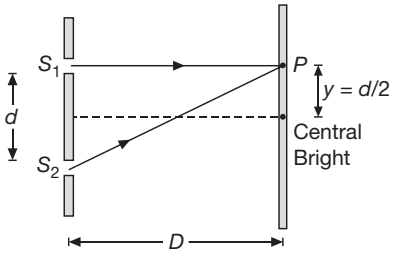
$\Rightarrow \quad y_3 = \frac{17}{6}\left(\frac{\lambda D}{d}\right)$

Substituting $\lambda = 0.6 \times 10^{-6}$ m, $D = 2$ m, $d = 6$ mm, we get

$$y_3 = \frac{17}{6}\frac{(0.6 \times 10^{-6})(2)}{6 \times 10^{-3}} = 5.67 \text{ mm}$$

MISSING WAVELENGTH(S) IN FRONT OF ANY ONE SLIT IN YDSE

Suppose P is a point of observation in front of slit S_1 as shown.



The wavelengths missing are the ones obtained by using the condition of destructive interference, i.e.,

$$\Delta x = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3,.....$$

Now $\Delta x = \sqrt{D^2 + d^2} - D$

$$\Rightarrow \quad \Delta x = D\left[\left(1 + \frac{d^2}{D^2}\right)^{\frac{1}{2}} - 1\right] \qquad \ldots(1)$$

Since, $D \gg d$, so $\left(1 + \frac{d^2}{D^2}\right)^{\frac{1}{2}} \cong 1 + \frac{d^2}{2D^2}$

So, (1) becomes

$$\Delta x = \frac{d^2}{2D} = (2n-1)\frac{\lambda}{2}$$

We can also find the missing wavelengths by using the following steps.

Since $d\sin\theta = \left(n - \frac{1}{2}\right)\lambda$, where $\sin\theta = \frac{y}{D}$

$$\Rightarrow \quad \frac{d\left(\frac{d}{2}\right)}{D} = \left(n - \frac{1}{2}\right)\lambda$$

$$\Rightarrow \quad \lambda = \frac{d^2}{(2n-1)D}, \; n = 1, 2, 3, 4, 5,.....$$

$$\Rightarrow \quad \lambda_{\text{missing}} = \frac{d^2}{(2n-1)D}, \text{ where } n = 1, 2, 3,...$$

By putting $n = 1, 2, 3...$, the missing wavelengths are

$$\lambda = \frac{d^2}{D}, \; \frac{d^2}{3D}, \; \frac{d^2}{5D}....$$

ILLUSTRATION 16

White light is used in a $YDSE$ with separation between the sources to be 0.9 m and separation between the sources and the screen to be 1 m. Light reaching the screen at position $y = 1$ mm is passed through a prism and its spectrum is obtained. Find the missing lines in the visible region of this spectrum.

SOLUTION

Path difference is given by

$$\Delta x = \frac{yd}{D} = (9 \times 10^{-4})(1 \times 10^{-3}) = 900 \text{ nm}$$

For minima $\Delta x = (2n-1)\frac{\lambda}{2}$, where $n = 1, 2, 3, \ldots$.

$$\Rightarrow \quad \lambda = \frac{2\Delta x}{(2n-1)} = \frac{1800}{(2n-1)}$$

$$\Rightarrow \quad \lambda_{\text{missing}} = \frac{1800}{1}, \frac{1800}{3}, \frac{1800}{5}, \frac{1800}{7} \ldots..$$

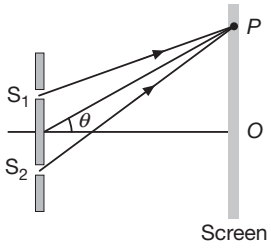
Of these, 600 nm and 360 nm lie in the visible range, so these will be missing lines in the visible spectrum.

ORDER OF FRINGES

When Slits are Vertical

If the slits are vertical, as shown in figure, path difference is,

$$\Delta x = d \sin\theta$$

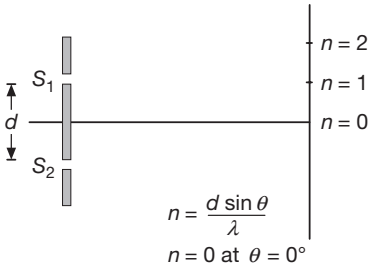


This path difference increases as θ increases.
The order of fringe n is given by

$$d \sin\theta = n\lambda$$

$$\Rightarrow \quad n = \frac{d \sin\theta}{\lambda}$$

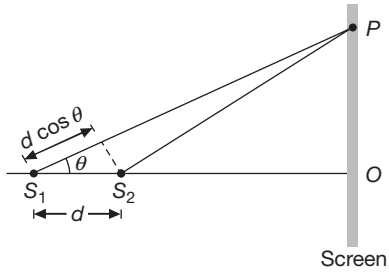
The order of fringe increases as we move away from point O on the screen.



When Slits are Horizontal

If the slits are horizontal, as shown in figure, then the path difference is

$$\Delta x = d \cos\theta$$



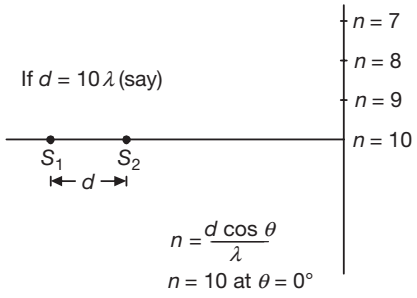
This path difference decreases as θ increases
The order of fringe n is given by

$$d \cos\theta = n\lambda$$

$$\Rightarrow \quad n = \frac{d \cos\theta}{\lambda}$$

The order of fringe decreases as we move away from point O.

Central maxima $(\text{i.e., } \Delta x = 0)$ obtained when $\theta \to \frac{\pi}{2}$ i.e., point of central maxima at far off distance from S_1 and S_2.



Conceptual Note(s)

To calculate the number of maximas or minimas that can be obtained on the screen, we use the fact that value of $\sin\theta$ (or $\cos\theta$) can never be greater than 1. For example in the first case when the slits are vertical.

$$\sin\theta = \frac{n\lambda}{d} \qquad \{\text{for maximum intensity}\}$$

Since, $\sin\theta \not> 1$

$$\Rightarrow \quad \frac{n\lambda}{d} \not> 1$$

$$\Rightarrow \quad n \not> \frac{d}{\lambda}$$

Suppose in some question $\frac{d}{\lambda}$ comes out say 3.6 then total number of maximas on the screen will be 7. Corresponding to $n = 0, \pm1, \pm2, \pm3$.
So, highest order of interference maxima is

$$n_{max} = \left[\frac{d}{\lambda}\right]$$

where [ ] represents the greatest integer function.
So, total number of maximas obtained are

$$N = 2n_{max} + 1$$

Similarly, highest order of interference minima is

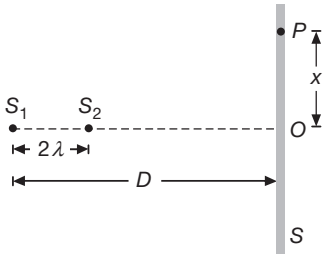
$$n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right]$$

where [ ] represents the greatest integer function.
So, total number of minimas obtained are

$$N = 2n_{min}$$

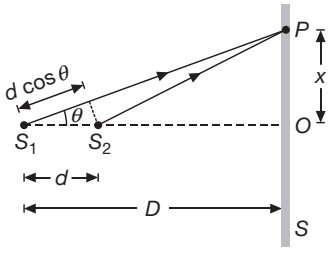
ILLUSTRATION 17

Two coherent narrow slits emitting light of wavelength λ in the same phase are placed parallel to each other at a small separation of 2λ. The light is collected on a screen S which is placed at a distance $D(\gg \lambda)$ from the slit S_1 as shown in figure. Find the finite distance x such that the intensity at P is equal to intensity at O.



SOLUTION

Path difference for waves reaching at O is $S_1O - S_2O = 2\lambda$ i.e., maximum intensity is obtained at O.



Next maxima will be obtained at point P where,

$$S_1P - S_2P = \lambda$$
$$\Rightarrow \quad d\cos\theta = \lambda$$
$$\Rightarrow \quad (2\lambda)\cos\theta = \lambda$$
$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$
$$\Rightarrow \quad \theta = 60^\circ$$

Now in ΔS_1PO

$$\frac{PO}{S_1O} = \tan\theta$$
$$\Rightarrow \quad \frac{x}{D} = \tan 60^\circ = \sqrt{3}$$
$$\Rightarrow \quad x = \sqrt{3}D$$

Conceptual Note(s)

At point O, path difference is 2λ i.e. we obtain second order maxima. At point P, where path difference is λ (so, $x = \sqrt{3}D$) we get first order maxima. The next, i.e., zero order maxima (central maxima) will be obtained where path difference, $d\cos\theta = 0$

$$\Rightarrow \quad \theta = 90^\circ$$
$$\Rightarrow \quad x \to \infty$$

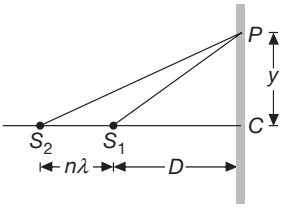
So, our answer, i.e., finite distance of x should be $x = \sqrt{3}D$, corresponding to first order maxima.

ILLUSTRATION 18

Two point sources are $d = n\lambda$ apart. A screen is held at right angles to the line joining the two sources at a distance D from the nearest source. Calculate the distance of the point on the screen, where the first bright fringe (excluding the centre one) is observed. Assume $D \gg d$.

SOLUTION

At point C path difference is $n\lambda$. Therefore, nth bright fringe will be observed.



Next bright fringe is observed where path difference is $(n-1)\lambda$, so

$$S_2P - S_1P = (n-1)\lambda \quad \ldots(1)$$

Now, $S_1P = \left(D^2 + y^2\right)^{1/2} = D\left(1 + \frac{y^2}{D^2}\right)^{1/2} \approx D + \frac{y^2}{2D}$ …(2)

Similarly, $S_2P = (D+d) + \frac{y^2}{2(D+d)}$ …(3)

Substituting (2) and (3) in (1) i.e., $S_2P - S_1P = (n-1)\lambda$, we get

$$d - \frac{y^2 d}{2D(D+d)} = (n-1)\lambda$$

Since, $d = n\lambda$

$$\Rightarrow \quad n\lambda - \frac{y^2(n\lambda)}{2D(D+n\lambda)} = n\lambda - \lambda$$

$$\Rightarrow \quad \frac{ny^2}{2D(D+n\lambda)} = 1$$

$$\Rightarrow \quad y = \sqrt{\frac{2D(D+n\lambda)}{n}}$$

OPTICAL PATH

It is defined as distance travelled by light in vacuum in the same time in which it travels a given path length in a medium. If light travels a path length d in a medium at speed v, the time taken by it will be $t = \frac{d}{v}$. So, the optical path length is

$$OPL = ct = c\left(\frac{d}{v}\right) = \left(\frac{c}{v}\right)d = \mu d \qquad \left\{\because \ \mu = \frac{c}{v}\right\}$$

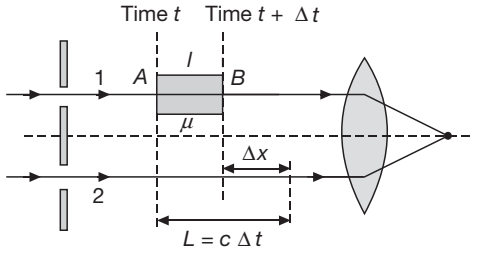
Since, for all media $\mu > 1$, optical path length is always greater than geometrical path length.

When two light waves arrive at a point by travelling different distances in different media, the phase difference between the two is related by their optical path difference instead of simple path difference. So,

$$\text{Phase Difference} = \frac{2\pi}{\lambda}(OPL)$$

PATH DIFFERENCE BETWEEN TWO PARALLEL WAVES DUE TO A DENSER MEDIUM IN PATH OF ONE BEAM

Consider two coherent light rays (thin beams) from a single source of light travelling in same direction parallel to each other. If in path of first beam a glass slab of refractive index μ having width l is placed as shown in figure.



The light ray 1 will slow down after it enters in slab at point A and its speed will reduce to $\frac{c}{\mu}$. When this ray 1 comes out of the slab at point B, then in this duration, the ray 2 which was travelling in space would have travelled a longer path because the ray 2 travels at a speed c.

The time taken by ray 1 in travelling through the glass slab is

$$\Delta t = \left(\frac{l}{\frac{c}{\mu}}\right) = \frac{\mu l}{c}$$

Path length covered by ray 2 in space while ray 1 was travelling in slab is

$$L = c\Delta t = c\left(\frac{\mu l}{c}\right) = \mu l$$

Thus, path difference between the two rays is given by

$$\Delta x = L - l = \mu l - l = l(\mu - 1)$$

If these two light waves (rays) are brought to focus on a converging lens as shown, then the two waves will interfere with a phase difference given as

$$\phi = \Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\left[l(\mu-1)\right]$$

If each of the light wave in the two thin light beams (rays) have intensity I_0 then at the focal point of the lens the resulting intensity of light is given as

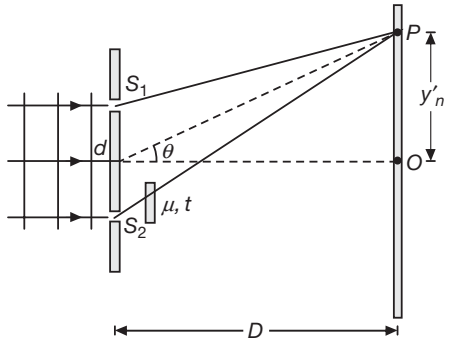
$$I_R = 4I_0\cos^2\left(\frac{\phi}{2}\right) = 4I_0\cos^2\left(\frac{\pi l(\mu-1)}{\lambda}\right)$$

DISPLACEMENT OR SHIFTING OF FRINGE PATTERN IN YDSE

When a transparent film of thickness t and refractive index μ is introduced in front of one of the slits, the fringe pattern shifts in the direction where the film is placed.

How much is the fringe shift?

Consider the $YDSE$ arrangement shown in the figure.



A film of thickness t and refractive index μ is placed in front of the lower slit.

The optical path difference is given by

$$x = \left[(S_2P - t) + \mu t\right] - S_1P$$

$$\Rightarrow \quad x = (S_2P - S_1P) + t(\mu - 1)$$

Since $S_2P - S_1P = d\sin\theta$

$$\Rightarrow \quad x = d\sin\theta + t(\mu - 1)$$

Since $\sin\theta \approx \tan\theta = \frac{y'_n}{D}$

$$\Rightarrow \quad x = \frac{dy'_n}{D} + t(\mu - 1)$$

The maxima will be obtained when the path difference is an even multiple of $\frac{\lambda}{2}$ i.e.,

$$x = (2n)\frac{\lambda}{2}$$

$$\Rightarrow \quad (2n)\frac{\lambda}{2} = \frac{dy'_n}{D} + t(\mu - 1)$$

$$\Rightarrow \quad y'_n = \frac{n\lambda D}{d} - (\mu - 1)\frac{tD}{d}$$

In the absence of film, the position of the nth maxima is given by equation

$$y_n = \frac{n\lambda D}{d}$$

Therefore, the fringe shift (FS) is given by

$$FS = y_n - y'_n = \frac{D}{d}(\mu - 1)t = \frac{\beta}{\lambda}(\mu - 1)t \quad \left\{\because \beta = \frac{\lambda D}{d}\right\}$$

Note that the shift is in the direction where the film is introduced.

Conceptual Note(s)

(a) The entire pattern shifts towards the side where the plate is introduced and there is no other change in the pattern.

(b) To measure this shift white light must be used because with monochromatic light all the fringes will exactly be similar and hence no shift can be observed.

(c) The effective path in air is increased by an amount $(\mu - 1)t$ due to introduction of the plate i.e., the additional path difference is $(\mu - 1)t$.

(d) If shift is equivalent to n fringes then $n = \frac{(\mu - 1)t}{\lambda}$ or $t = \frac{n\lambda}{(\mu - 1)}$

(e) The shift Δx is independent of the order of fringe n, i.e. Shift of zero order maxima = Shift of n^{th} order maxima.

(f) Shift is independent of wavelength.

ILLUSTRATION 19

Interference fringes are produced by a double slit arrangement and a piece of plane parallel glass of refractive index 1.5 is interposed in one of the interfering beam. If the fringes are displaced through 30 fringe widths for light of wavelength 6×10^{-5} cm, find the thickness of the plate.

SOLUTION

Path difference due to the introduction of glass slab is

$$\Delta x = (\mu - 1)t$$

Thirty fringes are displaced due to the introduction of slab. So,

$$\Delta x = 30\lambda$$

$$\Rightarrow \quad (\mu - 1)t = 30\lambda$$

$$\Rightarrow \quad t = \frac{30\lambda}{\mu - 1} = \frac{30 \times 6 \times 10^{-5}}{1.5 - 1}$$

$$\Rightarrow \quad t = 3.6 \times 10^{-3} \text{ cm}$$

ILLUSTRATION 20

In Young's double slit experiment using monochromatic light the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the monochromatic light used in the experiment.

SOLUTION

Shifting of fringes due to introduction of slab in the path of one of the slits is given by

$$\Delta y = \frac{(\mu - 1)tD}{d} \qquad \ldots(1)$$

Now, the distance between the screen and slits is doubled.

Hence, the new fringe width will become

$$\beta' = \frac{\lambda(2D)}{d} \qquad \ldots(2)$$

Given, $\Delta y = \beta'$

$$\Rightarrow \quad \frac{(\mu - 1)tD}{d} = \frac{\lambda(2D)}{d}$$

$$\Rightarrow \quad \lambda = \frac{(\mu - 1)t}{2} = \frac{(1.6 - 1)(1.964 \times 10^{-6})}{2}$$

$$\Rightarrow \quad \lambda = 0.5892 \times 10^{-6} \text{ m} = 5892 \text{ Å}$$

ILLUSTRATION 21

In a $YDSE$, the two coherent sources are separated from each other by 6 mm and from the screen by 2 m. A light of wavelength 6000 Å is used. A film of refractive index 1.5 is introduced in front of the lower slit such that the third maxima shifts to the origin.

(a) Find the thickness of the film.
(b) Find the positions of the fourth maxima.

SOLUTION

(a) Since third minima shifts to the origin, therefore, the fringe shift (FS) is equal to three fringe widths i.e., 3β, so we have

$$FS = y_3 = 3\left(\frac{\lambda D}{d}\right)$$

Since we know that the fringe shift (FS) is given by

$$FS = (\mu - 1)\frac{tD}{d}$$

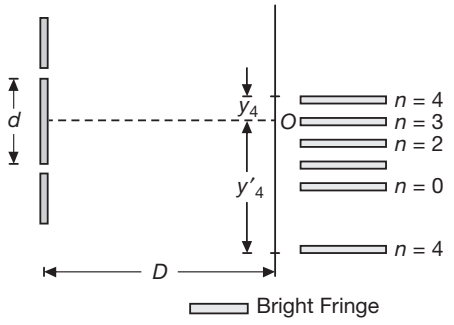
$$\Rightarrow \quad (\mu - 1)\frac{tD}{d} = 3\frac{\lambda D}{d}$$

$$\Rightarrow \quad t = \frac{3\lambda}{\mu - 1}$$

Since, $\lambda = 0.6 \times 10^{-6}$ m, $\mu = 1.5$

$$\Rightarrow \quad t = \frac{(3)(0.6 \times 10^{-6})}{1.5 - 1} = 3.6 \ \mu\text{m}$$

(b) There are two positions of fourth maxima, one above and the other below the origin. So, we have



$$y_4 = 1\beta = \frac{\lambda D}{d} = 0.2 \text{ mm and}$$

$$y_4' = -7\beta = -7\left(\frac{\lambda D}{d}\right) = -1.4 \text{ mm}$$

ILLUSTRATION 22

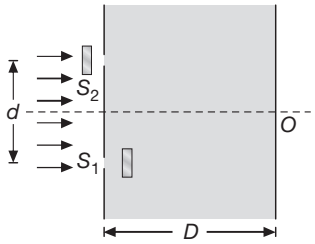
A Young double slit apparatus is immersed in a liquid of refractive index μ_1. The slit plane touches the liquid surface. A parallel beam of monochromatic

light of wavelength λ (in air) is incident normally on the slits.

(a) Find the fringe width

(b) If one of the slits (say S_2) is covered by a transparent slab of refractive index μ_2 and thickness t as shown, find the new position of central maxima.

(c) Now the other slit S_1 is also covered by a slab of same thickness and refractive index μ_3 as shown in figure due to which the central maxima recovers its position find the value of μ_3.



(d) Find the ratio of intensities at O in the three conditions (a), (b) and (c).

SOLUTION

(a) Fringe width is given by

$$\beta = \frac{\lambda' D}{d} = \frac{\lambda D}{\mu_1 d} \qquad \left\{\because \lambda' = \frac{\lambda}{\mu}\right\}$$

(b) Position of central maximum is shifted upwards by a distance

$$\Delta y = \frac{(\mu_2 - 1)tD}{d} \qquad \ldots(1)$$

(c) Downward shift is now given by

$$\Delta y' = \frac{\left(\frac{\mu_3}{\mu_1} - 1\right)tD}{d} \qquad \ldots(2)$$

Since the central maxima recovers its position, so

$$\Delta y = \Delta y'$$

So, from (1) and (2), we get

$$\frac{(\mu_1 - 1)tD}{d} = \frac{\left(\frac{\mu_3}{\mu_1} - 1\right)tD}{d}$$

$$\Rightarrow \frac{\mu_3}{\mu_1} = \mu_2$$

$$\Rightarrow \mu_3 = \mu_1\mu_2$$

(d) Since, $I = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$

where $\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x$

$$\Rightarrow \frac{\phi}{2} = \left(\frac{\pi}{\lambda}\right)\Delta x$$

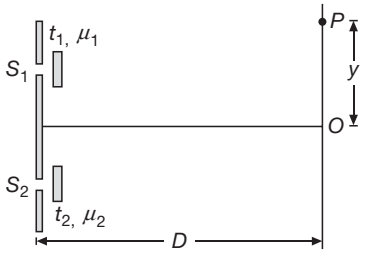
$$\Rightarrow I \propto \cos^2\left(\frac{\phi}{2}\right)$$

In the first and third case, $\Delta x = 0$ while in second case, $\Delta x = (\mu_2 - 1)t$. Therefore, the desired ratio is,

$$I_1 : I_2 : I_3 = 1 : \cos^2\left(\frac{\pi(\mu_2 - 1)t}{\lambda}\right) : 1$$

ILLUSTRATION 23

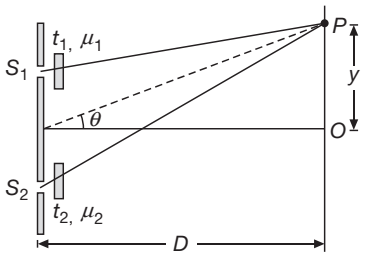
Two transparent sheets of thickness t_1 and t_2 and refractive indexes μ_1 and μ_2 are placed in front of the slits in $YDSE$ setup as shown in figure.



If D is the distance of the screen from the slits, then find the distance of zero order maxima from the centre of the screen. What is the condition that zero order maxima is formed at the centre O?

SOLUTION

The two waves from the two slits reaching the point P on the screen are shown in figure.



If this point is the position of zero order maxima and the distance of P from the centre O of the screen is

y_0, so the optical path of light waves from source S_1 is given as

$$x_1 = S_1P + (\mu_1 - 1)t_1$$

The optical path of light waves from source S_2

$$x_2 = S_2P + (\mu_2 - 1)t_2$$

The path difference between the two waves reaching at P is

$$\Delta x = x_2 - x_1$$

$$\Delta x = (S_2P - S_1P) + (\mu_2 - 1)t_2 - (\mu_1 - 1)t_1$$

Physically the path difference from sources to a point P on the screen is given by

$$S_2P - S_1P = d\sin\theta \approx \frac{yd}{D}$$

$$\Rightarrow \quad \Delta x = \frac{yd}{D} + (\mu_2 - 1)t_2 - (\mu_1 - 1)t_1$$

For zero order maxima, $\Delta x = 0$

$$\Rightarrow \quad 0 = \frac{yd}{D} + (\mu_2 - 1)t_2 - (\mu_1 - 1)t_1$$

$$\Rightarrow \quad y = \frac{D\left[(\mu_1 - 1)t_1 - (\mu_2 - 1)t_2\right]}{d}$$

For zero order maxima to be form at the centre O, we have

$$y = 0$$

$$\Rightarrow \quad 0 = \frac{D\left[(\mu_1 - 1)t_1 - (\mu_2 - 1)t_2\right]}{d}$$

$$\Rightarrow \quad (\mu_1 - 1)t_1 = (\mu_2 - 1)t_2$$

YDSE FOR SOURCE NOT PLACED AT THE CENTRAL LINE

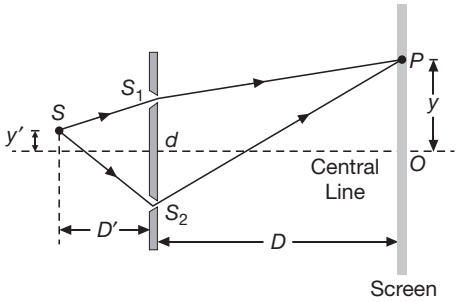
If the source of light is not placed at the central line but a little beyond (slightly up or down) the central line, then the waves reaching S_1 and S_2 from S will already have an initial path difference.

Path difference of waves meeting at the point P is given by

$$\Delta x = (SS_2 + S_2P) - (SS_1 + S_1P)$$

$$\Rightarrow \quad \Delta x = (SS_2 - SS_1) + (S_2P - S_1P)$$

$$\Rightarrow \quad \Delta x = \frac{y'd}{D'} + \frac{yd}{D} = \left(\frac{y'}{D'} + \frac{y}{D}\right)d$$



where, y' is the distance of the source S above or below the central line and D' is the distance of S from S_1 and S_2.

Similarly, here once we calculated the path difference, then

FOR MAXIMA, $\Delta x = (2n)\frac{\lambda}{2}$, $n = 0, 1, 2, 3 \ldots..$

FOR MINIMA, $\Delta x = (2n+1)\frac{\lambda}{2}$, $n = 0, 1, 2, 3 \ldots..$

Problem Solving Technique(s)

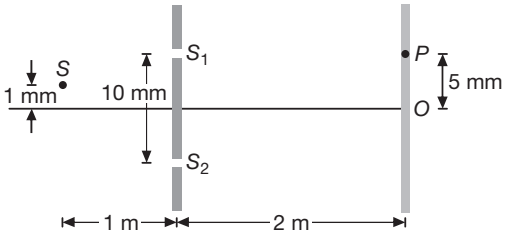
At the position of central maxima, we have

$$\Delta x = 0 \text{ i.e., } \frac{y'}{D'} + \frac{y}{D} = 0$$

i.e., if S is above central line, then central maxima is below the central line and vice versa.

ILLUSTRATION 24

In the Young's Double Slit experiment a point source of $\lambda = 5000$ Å is placed slightly off the central axis as shown in the figure.



(a) Find the nature and order of the interference at the point P.

(b) Find the nature and order of the interference at O.

(c) Where should we place a film of refractive index $\mu = 1.5$ and what should be its thickness so that a maxima of zero order is placed at O.

SOLUTION

(a) The optical path difference between the two waves arriving at P is

$$\Delta x = \frac{y_1 d}{D_1} + \frac{y_2 d}{D_2} = \frac{(1)(10)}{10^3} + \frac{(5)(10)}{2\times 10^3}$$

$$\Rightarrow \Delta x = 3.5\times 10^{-2} \text{ mm} = 0.035 \text{ mm}$$

To calculate the order of interference, we shall calculate

$$n = \frac{\Delta x}{\lambda}$$

$$\Rightarrow n = \frac{0.035\times 10^{-3} \text{ m}}{5000\times 10^{-10} \text{ m}}$$

$$\Rightarrow n = 70$$

$$\Rightarrow \Delta x = 70\lambda$$

So, 70^{th} order maxima is obtained at P.

(b) At O, $\Delta x = \frac{y_1 d}{D_1} = 10^{-2}$ mm $= 0.01$ mm

Now, we observe that $\Delta x = 20\lambda$

So, 20^{th} order maxima is obtained at O.

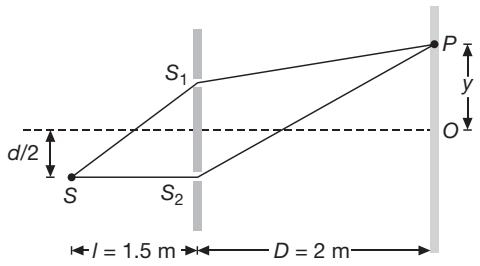
(c) $(\mu - 1)t = 0.01$ mm

$$\Rightarrow t = \frac{0.01}{1.5-1} = 0.02 \text{ mm} = 20\ \mu\text{m}$$

Since the pattern has to be shifted upwards, therefore, the film must be placed in front of S_1.

ILLUSTRATION 25

In $YDSE$, the monochromatic source of wavelength λ is placed at a distance $\frac{d}{2}$ from the central axis (as shown in the figure), where d is the separation between the two slits S_1 and S_2.



(a) Find the position of the central maxima.

(b) Find the order of interference formed at O.

(c) Find the minimum thickness of the film of refractive index $\mu = 1.5$ to be placed in front of S_2 so that intensity at O becomes $\frac{3}{4}$th of the maximum intensity.

Given $\lambda = 6000$ Å and $d = 6$ mm.

SOLUTION

(a) $(\Delta x)_{\text{net}} = 0$

$$\Rightarrow \frac{y_1 d}{D_1} = \frac{y_2 d}{D_2}$$

$$\Rightarrow \frac{d/2}{1.5} = \frac{y}{2}$$

$$\Rightarrow y = \frac{d}{1.5} = \frac{6}{1.5} = 4 \text{ mm}$$

(b) At O, net path difference is given by

$$\Delta x = \frac{y_1 d}{D_1} = \frac{\left(\frac{d}{2}\right)(d)}{D_1} = \frac{(6\times 10^{-3})^2}{2\times 1.5} \quad \{\because S_1O = S_2O\}$$

$$\Rightarrow \Delta x = 12\times 10^{-6} \text{ m}$$

$$\Rightarrow \Delta x = 120\times 10^{-7} \text{ m}$$

Since, $\lambda = 6000$ Å $= 6\times 10^{-7}$ m

$$\Rightarrow \Delta x = 20\lambda$$

So at O, the bright fringe of order 20 will be obtained.

(c) $I = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$

$$\Rightarrow \frac{3}{4}I_{\max} = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$$

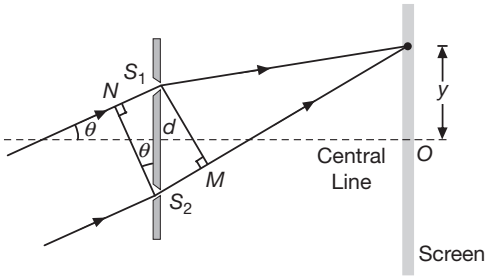
$$\Rightarrow \frac{\phi}{2} = \frac{\pi}{6}$$

$$\Rightarrow \phi = \frac{\pi}{3} = \left(\frac{2\pi}{\lambda}\right)(\mu - 1)t$$

$$\Rightarrow t = \frac{\lambda}{6(\mu-1)} = \frac{6000}{6(1.5-1)} = 2000 \text{ Å}$$

YDSE WHEN INCIDENT RAYS ARE NOT PARALLEL TO CENTRAL LINE

In this case, the rays reaching S_1 and S_2 already have an initial path difference.



So, net path difference between the rays reaching the point P is given by

$$\Delta x = (NS_1 + S_1P) - S_2P$$

$$\Rightarrow \quad \Delta x = |NS_1 - (S_2P - S_1P)| = |NS_1 - MS_2|$$

Now, $NS_1 = d\sin\theta$ and

$$MS_2 = S_2P - S_1P = \frac{yd}{D} \text{ (as done earlier)}$$

$$\Rightarrow \quad \Delta x = \left| d\sin\theta - \frac{yd}{D} \right|$$

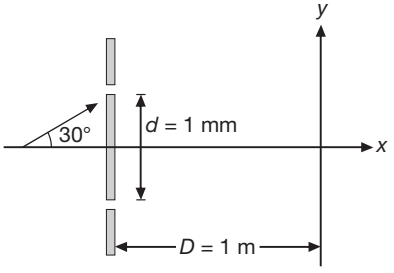
Once the path difference Δx is known, then

FOR MAXIMA, $\Delta x = (2n)\dfrac{\lambda}{2}$, $n = 0, 1, 2, 3, \ldots..$

FOR MINIMA, $\Delta x = (2n+1)\dfrac{\lambda}{2}$, $n = 0, 1, 2, 3, \ldots..$

ILLUSTRATION 26

A coherent parallel beam of microwaves of wavelength $\lambda = 0.5$ mm falls on a Young's double slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of 1.0 m from it as shown in the figure.

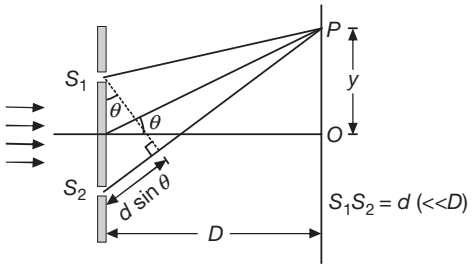


(a) If the incident beam falls normally on the double slit apparatus, find the y-coordinates of all the interference minima on the screen.

(b) If the incident beam makes an angle of 30° with the x-axis (as in the dotted arrow shown in figure), find the y-coordinates of the first minima on either side of the central maximum.

SOLUTION

(a) Given $\lambda = 0.5$ mm, $d = 1$ mm, $D = 1$ m



When the incident beam falls normally, path difference between the two rays S_2P and S_1P is

$$\Delta x = S_2P - S_1P \approx d\sin\theta$$

For minimum intensity,

$$d\sin\theta = (2n-1)\frac{\lambda}{2}, \quad n = 1,\ 2,\ 3, \ldots.$$

$$\Rightarrow \quad \sin\theta = \frac{(2n-1)\lambda}{2d} = \frac{(2n-1)0.5}{2\times 1} = \frac{2n-1}{4}$$

Since, $\sin\theta \le 1$

$$\Rightarrow \quad \frac{(2n-1)}{4} \le 1$$

$$\Rightarrow \quad n \le 2.5$$

So, n can be either 1 or 2

When $n = 1$, we have $\sin\theta_1 = \dfrac{1}{4}$

$$\Rightarrow \quad \tan\theta_1 = \frac{1}{\sqrt{15}}$$

When, $n = 2$, we have $\sin\theta_2 = \dfrac{3}{4}$

$$\Rightarrow \quad \tan\theta_2 = \frac{3}{\sqrt{7}}$$

Since, $y = D\tan\theta = \tan\theta \ (D = 1 \text{ m})$

So, the position of minima will be

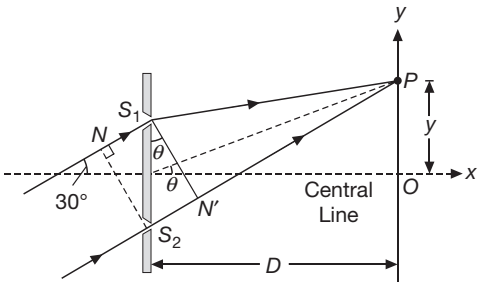
$$y_1 = D\tan\theta_1 = \frac{1}{\sqrt{15}} \text{ m}$$

$$y_2 = D\tan\theta_2 = \frac{3}{\sqrt{7}} \text{ m}$$

Since, minima can be on either side of centre O, so there will be four minimas at positions $\pm\frac{1}{\sqrt{15}}$ m and $\pm\frac{3}{\sqrt{7}}$ m on the screen.

(b) Path difference between the rays before entering the slits S_1 and S_2 is

$$NS_1 = \Delta x_1 = d\sin(30°) = \frac{d}{2} \qquad \ldots(1)$$



Path difference between the rays after passing through the slits S_1 and S_2 is

$$S_2P - S_1P = \Delta x_2 = d\sin\theta$$

So, net path difference is given by

$$\Delta x = |NS_1 + S_1P - S_2P| = |NS_1 - (S_2P - S_1P)|$$

$$\Rightarrow \Delta x = |\Delta x_1 - \Delta x_2| = \left|\frac{d}{2} - d\sin\theta\right|$$

For first minima, we have

$$\Delta x = \frac{\lambda}{2}$$

$$\Rightarrow \left|\frac{d}{2} - d\sin\theta\right| = \frac{\lambda}{2}$$

$$\Rightarrow \frac{d}{2} - d\sin\theta = \pm\frac{\lambda}{2}$$

$$\Rightarrow \frac{1}{2} - \sin\theta = \pm\frac{\lambda}{2d}$$

$$\Rightarrow \sin\theta = \frac{1}{2} \mp \frac{\lambda}{2d}$$

Since $d = 1$ mm, $\lambda = 0.5$ mm

$$\Rightarrow \sin\theta = \frac{1}{2} \mp \frac{0.5}{2}$$

$$\Rightarrow \sin\theta = \frac{1}{2} \mp \frac{1}{4}$$

$$\Rightarrow \sin\theta = \frac{1}{4} \text{ and } \sin\theta = \frac{3}{4}$$

Since the position of first minima on either side of central maxima is

$$\tan\theta = \frac{y}{D}$$

$$\Rightarrow y = D\tan\theta$$

For $\sin\theta = \frac{1}{4}$, we have $\tan\theta = \frac{1}{\sqrt{15}}$

$$\Rightarrow y = \frac{1}{\sqrt{15}} \text{ m}$$

For $\sin\theta = \frac{3}{4}$, we have $\tan\theta = \frac{3}{\sqrt{7}}$

$$\Rightarrow y = \frac{3}{\sqrt{7}} \text{ m}$$

Problem Solving Technique(s)

If two thin plates are also inserted just after S_1 and S_2, then our first task is to find the path difference. In the figure shown, path of ray 1 is more than path of ray 2 by a distance,

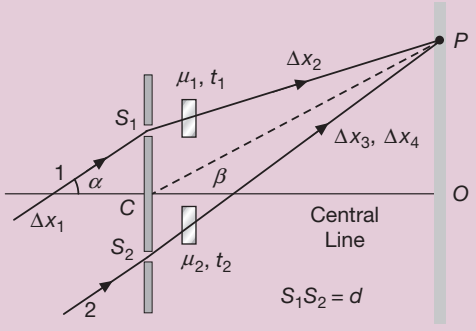
$$\Delta x_1 = d\sin\alpha \text{ and } \Delta x_2 = (\mu_1 - 1)t_1$$

and path of ray 2 is greater than path of ray 1 by a distance.

$$\Delta x_3 = d\sin\beta \text{ and } \Delta x_4 = (\mu_2 - 1)t_2$$

Therefore, net path difference is,

$$\Delta x = (\Delta x_1 + \Delta x_2) \sim (\Delta x_3 + \Delta x_4)$$



Once, we know the path difference Δx, then

FOR MAXIMA, $\Delta x = (2n)\frac{\lambda}{2}$, $n = 0, 1, 2, \ldots..$

FOR MINIMA, $\Delta x = (2n+1)\frac{\lambda}{2}$, $n = 0, 1, 2, \ldots..$

ILLUSTRATION 27

A large opaque sheet placed parallel to the yz plane at $x = 0.03$ m. The region $x \geq 0$ is filled with a transparent liquid of refractive index $\frac{3}{2}$. A wide monochromatic beam of light of wavelength 900 nm falls on the yz-plane at $x = 0$ making an angle of 30° with the x-axis. The sheet has two slits parallel to z-axis at $y = \pm 0.9$ mm. The intensity of the wave is measured on a screen placed at $x = 1.03$ m parallel to the sheet.

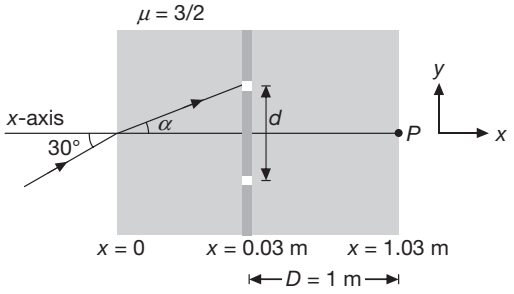
(a) Find the intensity at a point P on the screen where $y = z = 0$.

(b) The lower slit is covered by a transparent strip of refractive index 1.4 and thickness 4.2 mm. Now find the intensity at point P.

Given that $\tan(20°) = \frac{1}{2\sqrt{2}}$

SOLUTION

(a) The situation in the problem is shown in figure.



We observe that separation between the sources is

$$d = 2 \times 0.9 = 1.8 \text{ mm}$$

Since, we know that $\lambda' = \frac{\lambda}{\mu} = \frac{900}{\frac{3}{2}} = 600$ nm

Applying Snell's Law at the $x = 0$ interface, we get

$$\frac{3}{2} = \frac{\sin(30°)}{\sin\alpha}$$

$$\Rightarrow \sin\alpha = \frac{1}{3}$$

$$\Rightarrow \alpha = 20° \qquad \{\text{Using Trigonometry}\}$$

Initial path difference is

$$\Delta x = d\sin\alpha = (1.8)\left(\frac{1}{3}\right) = 0.6 \text{ mm}$$

$$\Rightarrow \phi = \frac{2\pi}{\lambda'}\Delta x = \frac{2\pi}{600 \times 10^{-9}} \times 0.6 \times 10^{-3} = 2000\pi$$

$$\Rightarrow I = I_{\max}\cos^2\left(\frac{\phi}{2}\right) = I_{\max}$$

(b) Net path difference at P is now

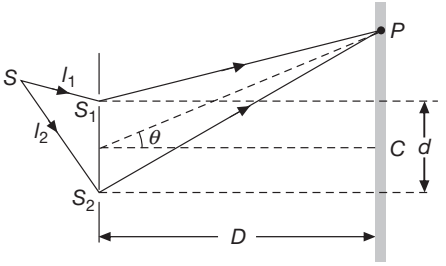
$$\Delta x = (0.6 \text{ mm}) + \left(\frac{1.5}{1.4} - 1\right)(4.2 \text{ mm}) = 0.3 \text{ mm}$$

$$\Rightarrow \phi = 1000\pi$$

$$\Rightarrow I = I_{\max}$$

ILLUSTRATION 28

In a Young's Double slit Experiment, the light source is at distance $l_1 = 20$ μm and $l_2 = 40$ μm from the slits. The light of wavelength $\lambda = 500$ nm is incident on slits separated at a distance 10 μm. A screen is placed at a distance $D = 2$ m away from the slits as shown in figure.



(a) Find the values of θ relative to the central line where maxima appear on the screen?

(b) How many maxima will appear on the screen?

(c) What should be minimum thickness of a slab of refractive index 1.5 be placed on the path of one of the ray so that minima occurs at C?

SOLUTION

(a) The optical path difference between the beams arriving at P is given by

$$\Delta x = (l_2 - l_1) + d\sin\theta$$

The condition for maximum intensity is,

$$\Delta x = n\lambda \text{, where } n = 0, \pm 1, \pm 2, \ldots\ldots$$

$$\Rightarrow \sin\theta = \frac{1}{d}\left(\Delta x - (l_2 - l_1)\right) = \frac{1}{d}\left(n\lambda - (l_2 - l_1)\right)$$

$$\Rightarrow \sin\theta = \frac{1}{10 \times 10^{-6}}\left(n \times 500 \times 10^{-9} - 20 \times 10^{-6}\right)$$

$$\Rightarrow \sin\theta = 2\left(\frac{n}{40} - 1\right)$$

$$\Rightarrow \sin\theta = \frac{n}{20} - 2$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{n}{20} - 2\right)$$

(b) Since we know that

$$|\sin\theta| \le 1$$

$$\Rightarrow -1 \le \left(\frac{n}{20} - 2\right) \le 1$$

$$\Rightarrow -20 \le (n - 40) \le 20$$

$$\Rightarrow 20 \le n \le 60$$

Hence, number of maxima obtained is

$$N = 60 - 20 = 40$$

(c) At C, phase difference, $\phi = \frac{2\pi}{\lambda}\Delta x = \left(\frac{2\pi}{\lambda}\right)(l_2 - l_1)$

$$\Rightarrow \phi = \left(\frac{2\pi}{500\times10^{-9}}\right)(20\times10^{-6}) = 80\pi$$

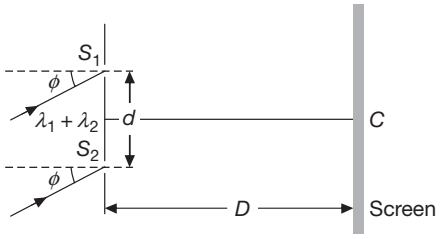
Hence, maximum intensity will appear at C.
For minimum intensity at C, we have

$$(\mu - 1)t = \frac{\lambda}{2}$$

$$\Rightarrow t = \frac{\lambda}{2(\mu-1)} = \frac{500\times10^{-9}}{2\times0.5} = 500 \text{ nm}$$

ILLUSTRATION 29

In a Young's double slit experiment a parallel beam containing wavelengths $\lambda_1 = 4000$ Å and $\lambda_2 = 5600$ Å incident at an angle $\phi = 30°$ on a diaphragm having narrow slits at a separation $d = 2$ mm.



The screen is placed at a distance $D = 40$ cm from slits. A mica slab of thickness $t = 5$ mm is placed in front of one of the slits and whole the apparatus is submerged in water. If the central bright fringe is observed at C, calculate

(a) the refractive index of the slab.
(b) the distance of the first black line from C. Both wavelengths are in air. Take $\mu_w = \frac{4}{3}$.

SOLUTION

(a) To observe bright fringe at C, the mica slab should be placed in front of S_2. In that case, net path difference at C is,

$$\Delta x = d\sin\phi - (\mu_r - 1)t \qquad \left\{\because \mu_r = \frac{\mu_{\text{slab}}}{\mu_w}\right\}$$

For central bright at C we have

$$\Delta x = 0$$

$$\Rightarrow d\sin\phi = (\mu_r - 1)t$$

$$\Rightarrow (\mu_r - 1) = \frac{d\sin\phi}{t} = \frac{(2\times10^{-3})\sin(30°)}{5\times10^{-3}} = 0.2$$

$$\Rightarrow \mu_r = 1.2$$

$$\Rightarrow \frac{\mu_{\text{slab}}}{\mu_w} = 1.2$$

$$\Rightarrow \mu_{\text{slab}} = 1.2\mu_w = 1.2\left(\frac{4}{3}\right) = 1.6$$

(b) A black line is formed at the position where both the wavelengths interfere destructively. Distance of nth dark fringe from C is given by

$$y = \frac{(2n-1)\lambda D}{2d}$$

For black line, $\frac{(2n_1 - 1)\lambda_1' D}{2d} = \frac{(2n_2 - 1)\lambda_2' D}{2d}$

where λ_1' and λ_2' are wavelengths in water.

$$\Rightarrow \frac{\lambda_1'}{\lambda_2'} = \frac{\frac{\lambda_1}{\mu_w}}{\frac{\lambda_2}{\mu_w}} = \frac{\lambda_1}{\lambda_2} = \frac{4000}{5600}$$

Substituting these values in equation (1), we get

$$\frac{2n_1 - 1}{2n_2 - 1} = \frac{7}{5}$$

For minimum value $n_1 = 4$ and $n_2 = 3$
Hence, distance of first black line is given by

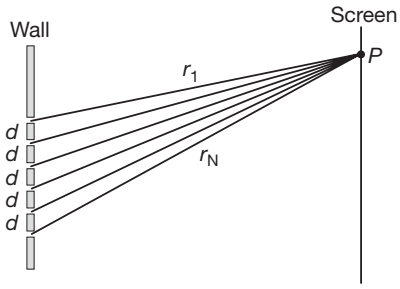
$$y = \frac{(2\times4-1)(4000\times10^{-10})40\times10^{-2}\times3}{2\times2\times10^{-3}\times4}$$

$$\Rightarrow y = 2.1\times10^{-4} \text{ m}$$

$$\Rightarrow y = 0.21 \text{ mm}$$

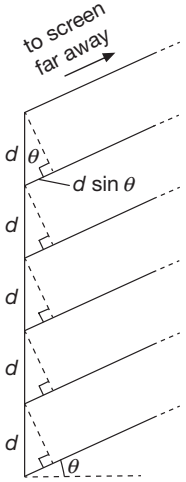
MULTIPLE SLIT INTERFERENCE PATTERN

Let us now look at the case where we have a general number N of equally spaced slits, instead of two equally spaced slits. As an assumption, we have shown in figure a set-up of six equally spaced slits.



Set up for 6 equally spaced slits

Similar to the $N = 2$ case discussed already, we will make the far-field assumption that the distance of the sources from the screen is much larger than the total span of the slits, which is $(N-1)d$. We can then say, as we did in the $N = 2$ case, that all the paths to a given point P on the screen have approximately the same length in a multiplicative (but not additive) sense, which implies that the amplitudes of the interfering waves are all essentially equal and we can also say that all the paths are essentially parallel (because of far-field assumption). A close-up version near the slits is shown in figure. Also, each path length is $d\sin\theta$ longer than the one just above it. So the lengths take the form of $r_n = r_1 + (n-1)d\sin\theta$.



To find the total wave at a given point at an angle θ on the screen, we need to add up the N individual waves. The procedure is the same as in the $N = 2$ case, except that now we simply have more terms in the sum. If a be the amplitude due to an individual source, then the equations of waves interfering at the point P' are given by

$$y_1 = a\sin(\omega t)$$

$$y_2 = a\sin(\omega t + \phi)$$

$$y_3 = a\sin(\omega t + 2\phi)$$

$$y_4 = a\sin(\omega t + 3\phi)$$

$$\vdots$$

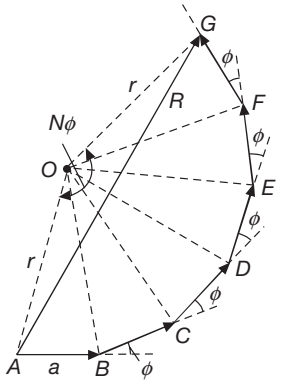
$$y_n = a\sin[\omega t + (N-1)\phi]$$

At angle θ, the path difference between any two successive slits is $\Delta x = d\sin\theta$. So, the corresponding phase difference ϕ is given by

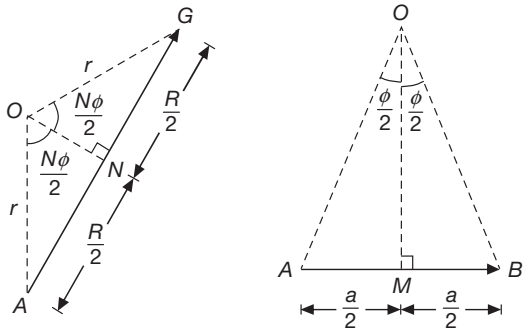
$$\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x = \frac{2\pi}{\lambda}(d\sin\theta)$$

RESULTANT WAVE AMPLITUDE (USING PHASORS)

The above set of equations can be represented by phasor diagram shown in figure (for a set of six sources generalised to N sources).



If R be the amplitude of the resultant of N interfering waves, then from above phasor diagram, on extracting triangles OAG and OAB, we get following figures to be used for evaluation of R.



OAG and OAB are isosceles triangles so for triangle OAG, we have

$$\frac{R}{2} = r\sin\left(\frac{N\phi}{2}\right) \quad \ldots(1)$$

Triangle OAB, we have

$$\frac{a}{2} = r\sin\left(\frac{\phi}{2}\right) \quad \ldots(2)$$

Dividing (1) and (2), we get

$$\frac{R}{a} = \frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}$$

So, the resultant amplitude R is given by

$$R = a\left[\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right] \quad \ldots(3)$$

If I_R be the resultant intensity, then

$$I_R = R^2$$

$$\Rightarrow \quad I_R = a^2\left[\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right]^2$$

If I_0 be the intensity due to an individual source, then

$$I_R = I_0\left[\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right]^2 \quad \ldots(4)$$

CHECK POINT

For $N = 2$, we get

$$I_R = I_0\left[\frac{\sin\phi}{\sin\left(\frac{\phi}{2}\right)}\right]^2$$

Since $\sin\phi = 2\sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right)$

$$\Rightarrow \quad I_R = 4I_0\cos^2\left(\frac{\phi}{2}\right) = 4a^2\cos^2\left(\frac{\phi}{2}\right)$$

RESULTANT WAVE EQUATION

To find the resultant wave equation, we shall be using the concept of complex numbers. From our knowledge of complex numbers, we know that

$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$

$$\Rightarrow \quad \text{Im}\left(e^{i\omega t}\right) = \sin(\omega t) \text{ and } \text{Re}\left(e^{i\omega t}\right) = \cos(\omega t)$$

Since the resultant wave equation is obtained by adding individual waves, so we get

$$y = a\left[\sin(\omega t) + \sin(\omega t + \phi) + \ldots.. + \sin\left(\omega t + (N-1)\phi\right)\right]$$

$$\Rightarrow \quad y = a\,\text{Im}\left[e^{i\omega t} + e^{i(\omega t+\phi)} + \ldots.. + e^{i(\omega t+(N-1)\phi)}\right]$$

$$\Rightarrow \quad y = a\,\text{Im}\left[e^{i\omega t}\left(1 + e^{i\phi} + \ldots.. + e^{i(N-1)\phi}\right)\right]$$

Since, $1 + r + r^2 + \ldots.. + r^{N-1} = 1\left(\frac{1-r^n}{1-r}\right)$

$$\Rightarrow \quad y = a\,\text{Im}\left[e^{i\omega t}\left(\frac{1-e^{iN\phi}}{1-e^{i\phi}}\right)\right]$$

$$\Rightarrow \quad y = a\,\text{Im}\left[e^{i\omega t}\left(\frac{1-e^{i\left(\frac{N\phi}{2}\right)}e^{i\left(\frac{N\phi}{2}\right)}}{1-e^{i\left(\frac{\phi}{2}\right)}e^{i\left(\frac{\phi}{2}\right)}}\right)\right]$$

$$\Rightarrow \quad y = a\,\text{Im}\left[\left(e^{i\omega t}\right)\left(\frac{e^{i\left(\frac{N\phi}{2}\right)}}{e^{i\left(\frac{\phi}{2}\right)}}\right)\left(\frac{e^{i\left(\frac{N\phi}{2}\right)} - e^{-i\left(\frac{N\phi}{2}\right)}}{e^{i\left(\frac{\phi}{2}\right)} - e^{-i\left(\frac{\phi}{2}\right)}}\right)\right]$$

$$\Rightarrow \quad y = a\,\text{Im}\left[e^{i\left(\omega t + (N-1)\frac{\phi}{2}\right)}\left(\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right)\right]$$

But $\text{Im}\left[e^{i\left(\omega t+(N-1)\frac{\phi}{2}\right)}\right]=\sin\left(\omega t+(N-1)\frac{\phi}{2}\right)$

$$\Rightarrow \quad y=a\left[\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right]\sin\left(\omega t+(N-1)\frac{\phi}{2}\right)$$

Again here, we have directly come across the amplitude of the resultant wave given by

$$R=a\left[\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right]$$

$$\Rightarrow \quad I_R=I_0\left[\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right]^2, \text{ where } I_0=a^2$$

We observe that at the centre of screen, I_R is indeterminate. So, the maximum intensity at the midpoint of the screen i.e., at $\theta=0°$ is obtained by taking the limit when $\phi\to 0°$. (Please note that since $\phi=\frac{2\pi}{\lambda}(d\sin\theta)$, so $\phi\to 0°$ when $\theta\to 0°$). So,

$$I_{\max}=\lim_{\phi\to 0}\left[I_0\frac{\sin^2\left(\frac{N\phi}{2}\right)}{\sin^2\left(\frac{\phi}{2}\right)}\right]$$

For $\phi\to 0$, we have $\sin\phi\approx\phi$

$$\Rightarrow \quad \sin^2\left(\frac{N\phi}{2}\right)\approx\frac{N^2\phi^2}{4} \text{ and } \sin^2\left(\frac{\phi}{2}\right)=\frac{\phi^2}{4}$$

$$\Rightarrow \quad I_{\max}=I_0\left[\frac{\left(\frac{N^2\phi^2}{4}\right)}{\left(\frac{\phi^2}{4}\right)}\right]$$

$$\Rightarrow \quad I_{\max}=N^2I_0$$

LOCATION OF SECONDARY MINIMA(S)

I_R has zero values, when $\sin^2\left(\frac{N\phi}{2}\right)=0$

$$\Rightarrow \quad \frac{N\phi}{2}=\text{Integral Multiple of }\pi$$

$$\Rightarrow \quad \frac{N\phi}{2}=m\pi, \text{ where } m=0, 1, 2, 3,....$$

$$\Rightarrow \quad \phi=(2m)\frac{\pi}{N}$$

$$\Rightarrow \quad \phi=\text{Even Multiple of }\frac{\pi}{N} \qquad ...(5)$$

However, one exception to this is when $\frac{\phi}{2}$ is also an integral multiple of π, because the denominator in equation (4) is also zero.

So, $\frac{\phi}{2}=m'\pi$, where $m'=0, 1, 2, 3, ...$

$$\Rightarrow \quad \phi=(2m')\pi$$

$$\Rightarrow \quad \phi=\text{Even multiple of }\pi \qquad ...(6)$$

So, from (5) and (6), we conclude that $I_R=0$, when

$$\boxed{\phi=(2m)\frac{\pi}{N} \text{ excluding } \underbrace{\phi=0, 2\pi, 4\pi, 6\pi,.....}_{\text{Positions of Primary Maxima}}}$$

i.e., $\phi=(\text{Even Multiple})\frac{\pi}{N}$, excluding Positions of Primary Maxima (located at 0, 2π, 4π, 6π,....)

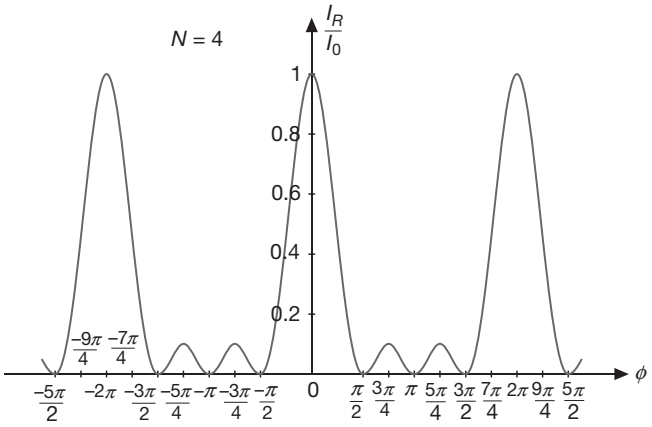
LOCATION OF SECONDARY MAXIMA(S)

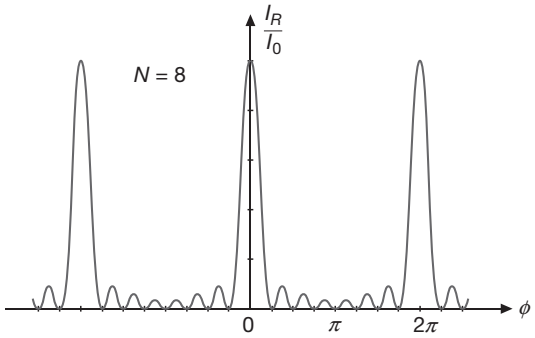
To find the locations of the secondary maxima (i.e., small bumps) we have to find the local maxima of I_R by taking the derivative I_R w.r.t. $\phi\left(\text{i.e., }\frac{dI_R}{d\phi}\right)$ and then equating it to zero.

So, $\frac{dI_R}{d\phi}=0$

$$\Rightarrow \quad N\tan\left(\frac{\phi}{2}\right)=\tan\left(\frac{N\phi}{2}\right)$$

This equation has to be solved numerically. However, for large N, the solutions of ϕ are generally very close to odd multiples of $\frac{\pi}{N}$ excluding the values of $\phi=2\pi\pm\frac{\pi}{N}$, because these values will be lying well within the primary maxima region. Just to make you understand, we are plotting $\frac{I_R}{I_0}$ (with ϕ for $N=4$ and $N=8$).





Conceptual Note(s)

(a) It is customary not to deal with the resultant intensity alone, but rather to deal with the resultant intensity relative to the maximum intensity i.e., $\frac{I_R}{I_{max}}$

$$\Rightarrow \frac{I_R}{I_{max}} = \frac{I_R}{I_0} = \left[\frac{\sin\left(\frac{N\phi}{2}\right)}{N\sin\left(\frac{\phi}{2}\right)}\right]^2 \quad ...(1)$$

(b) $\lim_{\theta\to 0^\circ}\left(\frac{I_R}{I_{max}}\right) = \lim_{\phi\to 0}\left(\frac{I_R}{I_{max}}\right) = 1$

(c) $\frac{I_R}{I_{max}}$ has a periodicity of 2π in ϕ i.e., repeats itself for integral multiples of 2π.

(d) The number of zeros between the main peaks is $(N-1)$, where N is the number of slits used.

(e) The number of secondary maxima (little bumps) between the main peaks is $(N-2)$, where N is the number of slits used.

For $N = 2$ (for two slits), we get ZERO Secondary Maxima.

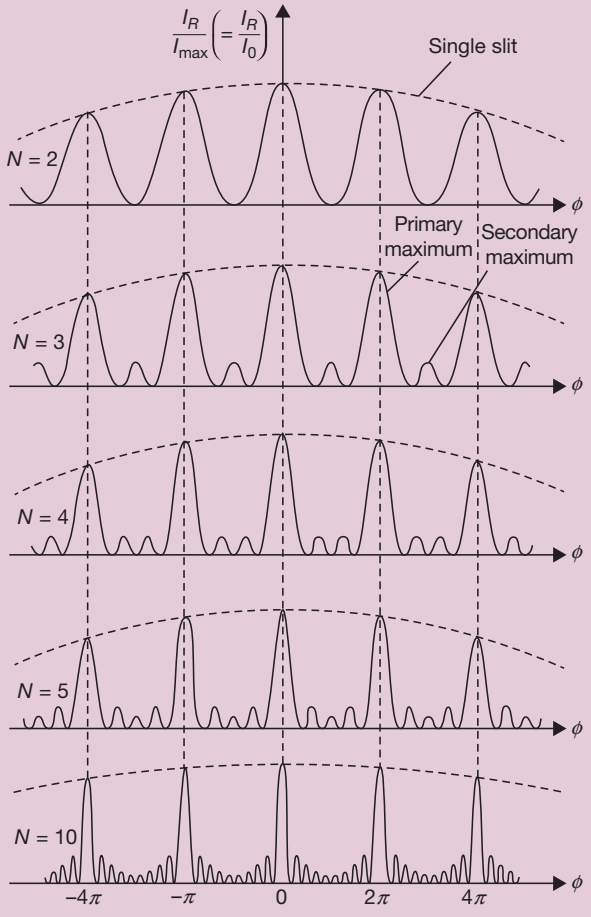
For $N = 3$ (for 3 slits), we get One Secondary Maxima.

For $N = 4$ (for 4 slits), we get Two Secondary Maxima and so on.

(f) A point worth noting here is that the height of the secondary maxima (little bumps) i.e., the bump sizes are symmetric around $\phi = \pi$ (or in general any multiple of π). Also, we know that since

$$I_R = I_0\left[\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right]^2$$

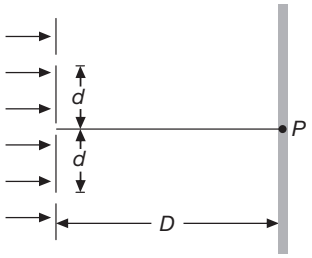
Hence, the bump size is shortest at $\phi = \pi$, because then the denominator in the equation (1), will be having a maximum value at $\phi = \pi$, due to which I_R becomes the least at $\phi = \pi$. Furthermore, the bump size grows as they get closer to the main peaks, as shown for various slits taken.



(g) As N, the number of slits, is increased, the primary maxima (the tallest peaks in each graph) become narrower but remain fixed in position and the number of secondary maxima increases. For any value of N, the decrease in intensity in maxima to the left and right of the central maximum, indicated by the blue dashed arcs, is due to diffraction patterns from the individual slits, which can be neglected here.

ILLUSTRATION 30

A light wave of wavelength 500 nm falls upon three slits a distance 0.5 mm each from one another. A screen is placed at a distance 2 m from slits. Find



(a) the distances from P where intensity reduces to zero.

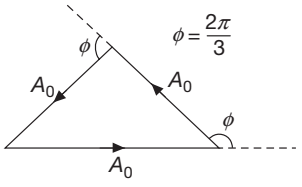
(b) the distances from P where next bright fringe are observed.

(c) the ratio of intensities of bright fringes observed on the screen.

SOLUTION

(a) In case of three slits, intensity becomes zero, when phase difference between any two waves is,

$$\phi = 2n\pi + \frac{2\pi}{3}, \text{ where } n = 0, 1, 2, \ldots.$$



The corresponding path difference, $\Delta x = \left(\frac{\lambda}{2\pi}\right)\phi$

$$\Rightarrow \quad d\sin\theta = \left(\frac{\lambda}{2\pi}\right)\left(\frac{2\pi}{3} + 2\pi n\right) = n\lambda + \frac{\lambda}{3}$$

$$\Rightarrow \quad \sin\theta = \frac{\lambda}{d}\left(n + \frac{1}{3}\right)$$

For small angles, $\sin\theta \approx \tan\theta = \frac{y}{D}$

$$\Rightarrow \quad \frac{y}{D} = \frac{\lambda}{d}\left(n + \frac{1}{3}\right)$$

$$\Rightarrow \quad y = \frac{\lambda D}{d}\left(n + \frac{1}{3}\right)$$

Substituting the values, we have

$$y = \frac{500\times10^{-9}\times 2}{0.5\times10^{-3}}\left(n + \frac{1}{3}\right) = 2\times10^{-3}\left(n + \frac{1}{3}\right)\text{ m}$$

$$\Rightarrow \quad y = 2\left(n + \frac{1}{3}\right)\text{ mm, where } n = 0, 1, 2, \ldots..$$

$$\Rightarrow \quad y = \frac{2}{3}\text{ mm for } n = 0,\ y = \frac{8}{3}\text{ mm for } n = 1 \text{ etc.}$$

(b) Bright fringes are obtained on the screen where

(i) $\phi = 2n\pi,\ n = 1, 2, 3, \ldots.$

$$\Delta x = \left(\frac{\lambda}{2\pi}\right)(\phi) = n\lambda$$

$$\Rightarrow \quad d\sin\theta = n\lambda$$

$$\Rightarrow \quad \sin\theta = \frac{n\lambda}{d}$$

For small angles,

$$\sin\theta \approx \tan\theta = \frac{y}{D}$$

$$\Rightarrow \quad \frac{y}{D} = \frac{n\lambda}{d}$$

$$\Rightarrow \quad y = \frac{n\lambda D}{d} = \frac{n\left(500\times10^{-9}\right)(2)}{\left(0.5\times10^{-3}\right)}$$

$$\Rightarrow \quad y = 2n\times10^{-3}\text{ m}$$

$$\Rightarrow \quad y = (2n)\text{ mm} \quad \{\text{where } n = 1, 2, 3, \ldots. \text{ etc.}\}$$

There are called **primary** maximas.

(ii) $\phi = (2n+1)\pi,\ n = 1, 2, \ldots.$

Proceeding in the similar manner, we get

$$y = 2\left(n + \frac{1}{2}\right)\text{ mm} \quad \{\text{where } n = 1, 2, 3, \ldots.\}$$

These are called **secondary** maximas.

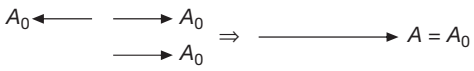
Note that $y = 0$ is also a secondary maxima because at P, $\phi = \pi$.

(c) At principal maximas, we have $\phi = 2\pi, 4\pi, ...,$ etc.

Resultant amplitude $R = 3A_0$

$\Rightarrow I_R = 9I_0 \quad \{\because I \propto A^2\}$

While at secondary maximas, $(\phi = \pi,\ 3\pi,\ 5\pi.....)$



Resultant amplitude, $R' = A_0$

$\Rightarrow I'_R = I_0$

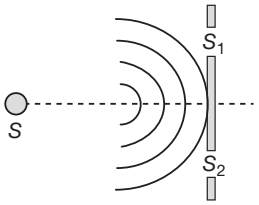
So, the desired ratio is therefore, $9:1$

COHERENT SOURCES BY DIVISION OF WAVEFRONT

When two or more waves travel through a medium simultaneously, the resultant intensity at any point, in the medium depends on whether they interfere constructively or destructively which, in turn, depends upon the phase difference between them. Resultant intensity, at any point, remains constant with time if the phase difference between them does not change. Two independent sources can never have same phase or a constant phase difference, because if we try to have interference with two independent sources, then net intensity at any point undergoes a continuous change due to a change in the phase difference between them. As a result of this no fixed interference pattern can be observed. The interference pattern of such sources is so short-lived that its photograph with the fastest available camera cannot be obtained. To obtain a fixed interference pattern we must have two sources which either have no phase difference or have a constant difference of phase. These sources are called **coherent sources**. It has been generally observed that coherent sources are obtained when they are derived from the same parent source. The methods for obtaining coherent sources (derived from the same parent source) are given below.

Double Slit Method

Light from a source S is limited to a narrow beam with the help of a slit. The emergent light is made to fall upon a screen containing two slits S_1 and S_2 placed symmetrically with respect to the slit.

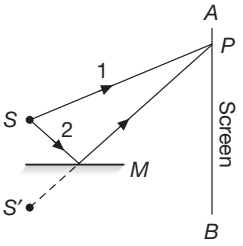


Coherent sources by double slit method

Here, both S_1 and S_2 are illuminated by the same wavefront. Therefore, the beams of light coming out from S_1 and S_2 have no phase difference. Thus S_1 and S_2 can be treated as the coherent sources. Young used this technique in his famous Young's double slit experiment.

A Source and its Own Virtual Image

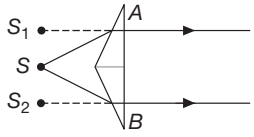
Light from a source S is made to fall on a plane mirror M. Point of observation P on a screen AB receives direct light as well as light reflected from M.



To an observer, reflected light appears to come from a source S' (virtual image of S). So, interference at P takes place between waves coming from S and S'. Since S' is not an independent source, being the virtual image of S, it will have the same phase as S. Hence the two are taken to be coherent sources. Lloyd made use of this arrangement in Lloyd single mirror experiment.

Biprism Method

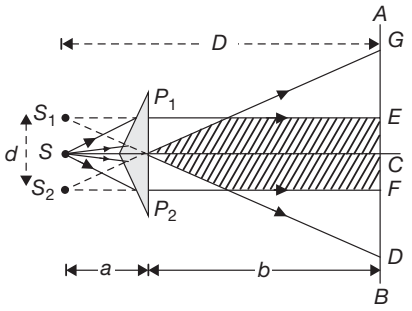
Light from a source S is made to fall on an assembly of two right angled prisms A and B joined base to base as shown in Figure.



S_1 and S_2 are the virtual image of S produced by refraction through prisms A and B respectively. Being virtual images of the same source, S_1 and S_2 have same phase and hence can be treated as the coherent sources. This type of arrangement is made use of in Fresnel's biprism experiment.

FRESNEL'S BIPRISM

It is one of the convenient laboratory arrangements for producing interference fringes. It consists of a combination of two right angled prisms with their bases joined together so that their faces are inclined to each other at angle of 179° 20′. Source of light is taken in the form of a narrow slit S, illuminated by the monochromatic light and is held symmetrically at a distance of about 5 cm from the biprism.



Light from S gets refracted by prism P_1 and P_2, thereby, producing virtual images S_1 and S_2, which can be taken as two coherent sources producing interference. Light beams from S_1 and S_2 strike the screen in the regions ED and FG respectively. EF is the common region where both the beams can be found. Therefore, interference pattern can be observed in the region EF.

The separation between these sources may be found by using the formula for deviation caused by a thin prism. If α is the small angle of biprism, μ refractive index of material of biprism and a the separation of source S from biprism, then deviation caused by prism.

$$\delta = (\mu - 1)\alpha$$

From Figure, $d = 2a\delta = 2a(\mu - 1)\alpha$

DETERMINATION OF λ

Biprism method can be used to determine the wave length of light. The fringe width β for the interference pattern obtained is given by,

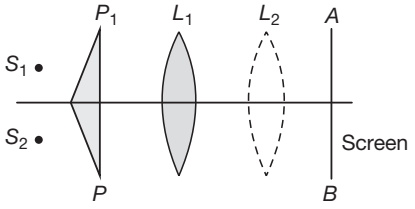
$$\beta = \frac{\lambda D}{d}$$

$$\Rightarrow \quad \lambda = \frac{\beta d}{D} \qquad \ldots(1)$$

(a) Determination of *D*: It is the distance between source and screen. It can be measured with an ordinary metre rod.

(b) Determination of β: A low power travelling microscope is used to find the total separation x between a number of fringes, say 20 and hence $\beta = \frac{x}{20}$.

(c) Determination of *d*: d can be calculated by using displacement method. A convex lens is placed in between the biprism and the screen.



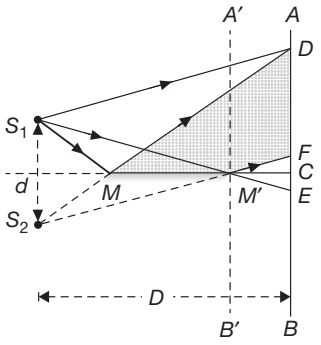
It is observed that for two positions L_1 and L_2 of the lens, the images of S_1 and S_2 can be focussed on the screen AB. Let x and y be the distances between these images when the lens is at L_1 and L_2 respectively. Then,

$$d = \sqrt{xy}$$

Substituting for β, D and d in equation (1), λ can be calculated.

LLOYD'S SINGLE MIRROR

This experimental set-up for producing interference fringes, was devised by Dr. Lloyd in 1834. Light from a source S_1 in the form of narrow slit is held in such a way that the light is incident, at almost grazing incidence, upon a mirror MM' which is blackened at the back to avoid internal reflections. S_2 is the virtual image of source S_1 obtained after reflection from MM'.



Experimental set up for Lloyd's single mirror

Screen AB is placed to receive light coming directly from S_1 as well as that reflected from the mirror. Reflected light can be supposed to be coming from source S_2. DF is the common region on the screen where both the beams are received and hence interference is obtained in region DF.

The point C lies symmetrically w.r.t. S_1 and S_2 and also lies outside the interference region, zero order fringe is not visible. It can be seen by moving the screen to position $A'B'$ so that it just touches the mirror. It will be observed that the zero order fringe at M' is dark instead of being bright as demanded by the theory of interference fringes since at M' path difference is zero. This indicates that the beam which suffers reflection from MM' undergoes in phase of π-radian. Llyod's single mirror can be used to determine the wave length of light.

If, a is the height of source S_1 above MM', then

$$d = 2a$$

If, D is the distance of source S_1 from screen AB, then fringe width β is given by

$$\beta = \frac{\lambda D}{d}$$

$$\Rightarrow \quad \lambda = \frac{\beta d}{D} = \frac{\beta(2a)}{D}$$

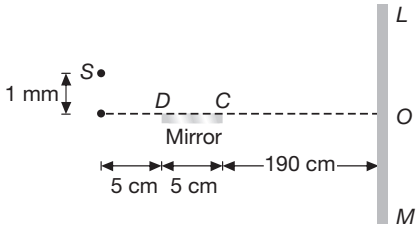
β can be determined, experimentally, by using a low power microscope. Knowing β, a and D value of λ can be calculated.

Conceptual Note(s)

Central spot, in case of Lloyd's single mirror is a dark one instead of being bright. This proves that there is a phase change of π-radian when a transverse wave (light) is reflected from a denser medium.

ILLUSTRATION 31

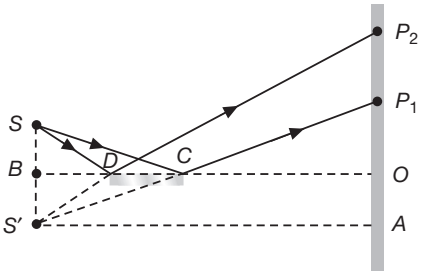
The arrangement for a mirror experiment is shown in the figure. S is a point source of frequency 6×10^{14} Hz. D and C represent the two ends of a mirror placed horizontally and LOM represents the screen.



Determine the position of the region where the fringes will be visible and calculate the number of fringes.

SOLUTION

Fringes will be observed in the region between P_1 and P_2 because the reflected rays lie only in this region.



From similar triangles BDS' and $S'P_2A$, $\frac{AP_2}{BS'} = \frac{AS'}{BD}$

$$\Rightarrow \quad AP_2 = \frac{(AS')(BS')}{BD} = \frac{(190+5+5)(0.1)}{5} = 4 \text{ cm}$$

Similarly, in triangles BCS' and $S'P_1A$, we have

$$\frac{AP_1}{BS'} = \frac{AS'}{BC}$$

$$\Rightarrow \quad AP_1 = \frac{(AS')(BS')}{BC} = \frac{(190+5+5)(0.1)}{10} = 2 \text{ cm}$$

$$\Rightarrow \quad P_1P_2 = AP_2 - AP_1 = 2 \text{ cm}$$

Wavelength of the light $\lambda = \frac{c}{f} = \frac{3\times10^8}{6\times10^{14}} = 5\times10^{-7}$ m

Fringe width $\beta = \frac{\lambda D}{d}$

Since, $D = S'A = (190+5+5) = 200 \text{ cm} = 2 \text{ cm}$,

$d = SS' = 2 \text{ mm} = 2\times10^{-3}$ m

$$\Rightarrow \quad \beta = \frac{(5\times10^{-7})(2)}{2\times10^{-3}} = 5\times10^{-4}\ \text{m} = 0.05\ \text{cm}$$

Number of fringes is

$$N = \frac{P_1P_2}{\beta} = 40$$

ILLUSTRATION 32

A narrow slit S is transmitting light of wavelength λ and it is placed at a distance d above a large plane mirror as shown in figure.

The light coming directly from the slit and that coming after reflection interfere at a screen placed at a distance $D(D \gg d)$ from the slit.

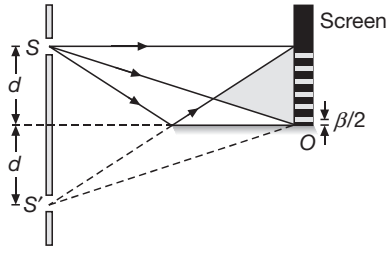
(a) What will be the intensity at a point just above the mirror at point O?

(b) At what distance from O does the first maximum will occur?

SOLUTION

(a) Just above the point O, direct waves from source and reflected waves have a phase difference π (due to reflection from mirror). So these waves interfere destructively due to which a dark fringe is obtained at O. Hence intensity of light at O will be zero.

(b) From the figure, we can see that the distance of first maximum (first bright fringe) will be located at half fringe width above O.

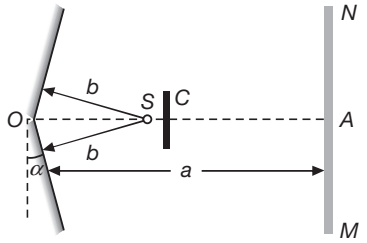


So we have $y = \frac{\beta}{2} = \left(\frac{\frac{D\lambda}{2d}}{2}\right)$

$$\Rightarrow \quad y = \frac{D\lambda}{4d}$$

ILLUSTRATION 33

Two flat mirrors form an angle close to $180°$. A source of light S is placed at equal distances b from the mirrors. Find the interval between adjacent interference bands on screen MN at a distance $OA = a$ from the point of intersection of the mirror. The wavelength of the light wave is known and equal to λ. Shield C does not allow the light to pass directly from the source to the screen.

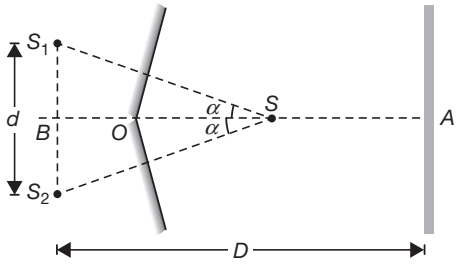


SOLUTION

Fringe width is given by

$$\beta = \frac{\lambda D}{d}$$

where $D = AB \approx a + b$ and $d = S_1S_2$



In ΔS_1SB, we have

$$\frac{d}{2} = 2b\frac{\alpha}{2}$$

$$\Rightarrow \quad d = 2b\alpha$$

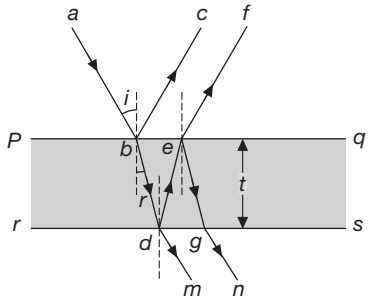
$$\Rightarrow \quad \beta = \frac{\lambda(a+b)}{2b\alpha}$$

THEORY OF DIVISION OF AMPLITUDE

Reflected Light

If μ is the refractive index of material of film of thickness t, then path difference between the waves abc and $abdef$ is $2\mu t\cos r$

Additional path difference due to reflection at denser medium (at b) is $\frac{\lambda}{2}$



So, effective path difference is

$$x = 2\mu t\cos r + \frac{\lambda}{2}$$

For maxima or constructive interference to take place, we have

$$2\mu t\cos r + \frac{\lambda}{2} = (2n)\frac{\lambda}{2}$$

$$\Rightarrow \quad 2\mu t\cos r = (2n-1)\frac{\lambda}{2},\ n = 1, 2, 3, \ldots$$

For minima or destructive interference to take place

$$2\mu t\cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \quad 2\mu t\cos r = n\lambda,\ n = 1, 2, 3, \ldots$$

Transmitted Light

In transmitted light system there is no phase difference or path difference due to reflection or transmission as all reflections take place from rarer medium. So, the effective path difference is

$$x = 2\mu t\cos r$$

For maxima

$$2\mu t\cos r = n\lambda$$

and **for minima**

$$2\mu t\cos r = (2n-1)\frac{\lambda}{2},\ n = 1, 2, 3, \ldots$$

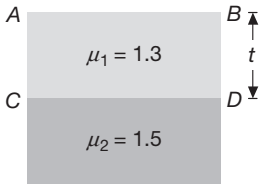
Obviously, the conditions of interference in reflected and transmitted lights are opposite to each other, therefore if the film appears dark in reflected light, it will appear bright in transmitted light and vice versa. With the use of white light, the colours visible in reflected light will be complementary to those visible in transmitted light, i.e., the colours absent in one system will be present in the other system; the sum of two constituting the white light.

ILLUSTRATION 34

A thick glass slab $(\mu = 1.5)$ is to be viewed in reflected white light. It is proposed to coat the slab with a thin layer of a material having refractive index 1.3 so that the wavelength 6000 Å is suppressed. Find the minimum thickness of the coating required.

SOLUTION

Optical path difference for the reflected light from coating and slab is $\Delta x = 2\mu t$



For minimum intensity, $2\mu_1 t = \frac{\lambda}{2}$

$$\Rightarrow \quad t = \frac{\lambda}{4\mu_1} = \frac{6000}{4\times 1.3}$$

$$\Rightarrow \quad t = 1154 \text{ Å}$$

Conceptual Note(s)

Both reflected rays (one from *AB* and the another from *CD*) get a phase change of π.

ILLUSTRATION 35

A parallel beam of white light falls on a thin film whose refractive index is equal to $\frac{4}{3}$. The angle of incidence $i = 53°$. What must be the minimum film thickness if the reflected light is to be coloured yellow (λ of yellow $= 0.6\ \mu$m) most intensively? Given $\tan 53° = \frac{4}{3}$.

SOLUTION

According to Snell's Law, we have $\mu = \dfrac{\sin i}{\sin r}$

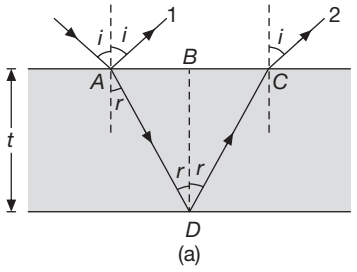
Given that $\tan 53° = \dfrac{4}{3}$, so $\sin 53° = \dfrac{4}{5}$

$$\Rightarrow \quad \frac{4}{3} = \frac{\sin(53°)}{\sin r} = \frac{\frac{4}{5}}{\sin r}$$

$$\Rightarrow \quad \sin r = \frac{3}{5}$$

$$\Rightarrow \quad r = 37°$$

From Figure (a):



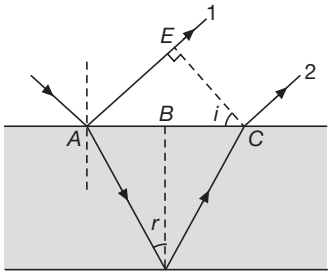
(a)

Path difference between 2 and 1 is $\Delta x_1 = 2(AD)$

$$\Rightarrow \quad \Delta x_1 = 2BD\sec r = 2t\sec r$$

Their optical path corresponding to Δx_1 is $2\mu t\sec r$

From Figure (b):



(b)

Path difference between 1 and 2 is given by

$$\Delta x_2 = AC\sin i = (2t\tan r)\sin i$$

$$\Rightarrow \quad (\Delta x)_{\text{net}} = \Delta x_1 - \Delta x_2 = 2\mu t\sec r - 2t(\tan r)(\sin i)$$

$$\Rightarrow \quad \Delta x_{\text{net}} = 2\times\frac{4}{3}\times t\times\frac{5}{4} - 2\times t\times\frac{3}{4}\times\frac{4}{5}$$

$$\Rightarrow \quad \Delta x_{\text{net}} = \frac{32}{15}t$$

Since reflection takes place at the surface of denser medium, so phase difference between 1 and 2 is π. So, for constructive interference, we have

$$\frac{32}{15}t = \frac{\lambda}{2}$$

$$\Rightarrow \quad t = \frac{15\lambda}{64} = \frac{15\times 0.6}{64} = 0.14\ \mu\text{m}$$

Test Your Concepts-I

Based on Interference

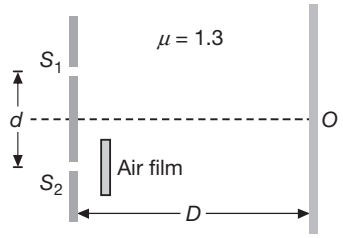
(Solutions on page H.121)

1. In a Young's Double Slit Experiment carried out in a liquid of refractive index $\mu = 1.3$, a thin film of air is formed in front of the lower slit as shown in the figure. If a maxima of third order is formed at the origin *O*, find the

(a) thickness of the air film.

(b) positions of the fourth maxima.

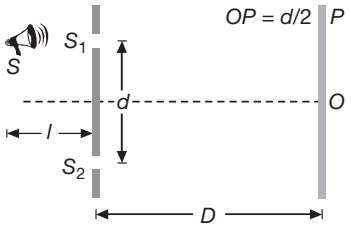
The wavelength of light in air is $\lambda_0 = 0.78\ \mu$m and $\dfrac{D}{d} = 1000$.



2. In *YDSE*, if light of wavelength 5000 Å is used, find the thickness of a glass slab ($\mu = 1.5$) which should be placed before the upper the upper slit S_1 so that

the central maximum now lies at a point where 5th bright fringe was lying earlier (before inserting the slab).

3. A source S of wavelength λ is kept directly behind the slit S_1 in a double slit apparatus. Find the phase difference at a point O which is equidistant from S_1 and S_2. If $D \gg d$, what will be the phase difference at P if a liquid of refractive index μ is filled
(a) between the screen and the slits?
(b) between the slits and the source S?



4. In solar cells, a silicon solar cell ($\mu = 3.5$) is coated with a thin film of silicon monoxide $SiO(\mu = 1.45)$ to minimize reflective losses from the surface. Determine the minimum thickness of SiO that produces the least reflection at a wavelength of 550 nm, near the centre of the visible spectrum.

5. A parallel beam of green light of wavelength 546 nm passes through a slit of width 0.4 mm. The transmitted light is collected on a screen 40 cm away. Find the distance between the two first order minima.

6. Calculate the minimum thickness of a soap bubble film ($\mu = 1.33$) that results in constructive interference in the reflected light if the film is illuminated with light whose wavelength in free space is $\lambda = 600$ nm.

7. Monochromatic light of wavelength 5000 Å is used in $YDSE$, with slit separation 1 mm, distance between screen and slits 1 m. If intensity at the two slits are, $I_1 = 4I_0$, $I_2 = I_0$, find
(a) fringe width β.
(b) distance of 5th minima from the central maxima on the screen.
(c) intensity at $y = \frac{1}{3}$ mm.
(d) distance of the 1000th maxima.
(e) distance of the 5000th maxima.

8. S_1 and S_2 are two point sources of radiation that are radiating waves in phase with each other of wavelength 400 nm. The sources are located on x-axis at $x = 6.5$ μm and $x = -6$ μm, respectively.
(a) Determine the phase difference (in radian) at the origin between the radiation from S_1 and the radiation from S_2.
(b) Suppose a slab of transparent material with thickness 1.5 μm and index of refraction $\mu = 1.5$ is placed between $x = 0$ and $x = 1.5$ μm. What then is the phase difference (in radian) at the origin between the radiation from S_1 and the radiation from S_2?

9. In a Young's double slit experiment using monochromatic light the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 micron is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the monochromatic light used in the experiment.

10. In Young's experiment a thin glass plate is placed in the path of one of the interfering rays. This causes the central light band to shift into a position which was initially occupied by the fifth bright band (not counting the central one). The ray falls onto the plate perpendicularly. The refractive index of the plate is 1.5. The wavelength is 6×10^{-7} m. What is the thickness of the plate?

11. In a double slit pattern ($\lambda = 6000$ Å), the first order and tenth order maxima fall at 12.50 mm and 14.75 mm from a particular reference point. If λ is changed to 5500 Å, find the position of zero order and tenth order fringes, other arrangements remaining the same.

12. In $YDSE$, the two slits are separated by 0.1 mm and they are 0.5 m from the screen. The wavelength of light used is 5000 Å. Find the distance between 7th maxima and 11th minima on the screen.

13. What is the effect on the interference fringes in a YDSE due to each of the following operations?
(a) The screen is moved away from the plane of the slits
(b) The (monochromatic) source is replaced by another (monochromatic) source of shorter wavelength

(c) The separation between the two slits is increased

(d) The monochromatic source is replaced by source of white light

(e) The whole experiment is carried out in a medium of refractive index μ

14. In a Young's double slit experiment, the slits are 2 mm apart and are illuminated with a mixture of two wavelengths $\lambda_1 = 750$ nm and $\lambda_2 = 900$ nm. At what minimum distance from the common central bright fringe on a screen 2 m from the slits will a bright fringe from one interference pattern coincide with a bright fringe from the other?

15. Bi-chromatic light is used in *YDSE* having wavelengths $\lambda_1 = 400$ nm and $\lambda = 700$ nm. Find minimum order of λ_1 which overlaps with λ_2.

16. In Young's double slit experiment set-up with light of wavelength $\lambda = 6000$ Å, distance between the two slits is 2 mm and distance between the plane of slits and the screen is 2 m. The slits are of equal intensity. When a sheet of glass of refractive index 1.5 (which permits only a fraction η of the incident light to pass through) and thickness 8000 Å is placed in front of the lower slit, it is observed that the intensity at a point P, 0.15 mm above the central maxima does not change. Find the value of η.

17. In a Young's double slit experiment set up, the wavelength of light used is 546 nm. The distance of screen from slits is 1 metre. The slit separation is 0.3 mm.

(a) Compare the intensity at a point P distant 10 mm from the central fringe where the intensity is I_0.

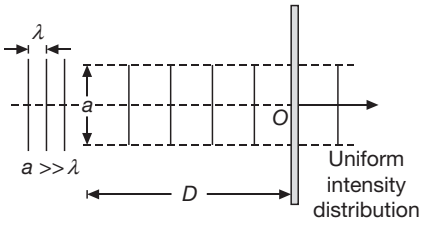
(b) Find the number of bright fringes between P and the central fringe.

18. In a Young's double slit experiment, two wavelengths of 500 nm and 700 nm were used. What is the minimum distance from the central maximum where their maximas coincide again? Take $\frac{D}{d} = 10^3$. Symbols have their usual meanings.

19. When a thin sheet of a transparent material of thickness 7.2×10^{-4} cm is introduced in the path of one of the interfering beams, the central fringe shift to a position occupied by the sixth bright fringe. If $\lambda = 6\times10^{-5}$ cm, find the refractive index of the sheet.

DIFFRACTION: INTRODUCTION AND CLASSIFICATION

When light waves pass through a small aperture, an interference pattern is observed rather than a sharp spot of light cast by the aperture. This shows that light spreads in various directions beyond the aperture into regions where a shadow would be expected if light travelled in straight lines.

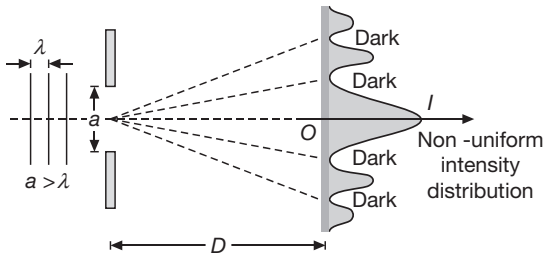


Light passing through two slits does not produce two distinct bright areas on a screen. Instead, an interference pattern is observed on the screen which shows that the light has deviated from a straight-line path and has entered the otherwise shadowed region. Other waves, such as sound waves and water waves, also have this property of being able to bend around corners.

This deviation of light from a straight-line path is called **diffraction**. Diffraction results from the interference of light from many coherent sources. In principle, the intensity of a diffraction pattern at a given point in space can be computed using Huygens' principle, where each point on the wavefront acts as the source emitting waves as the original source does.

The phenomenon of bending of light around the corners of an obstacle/aperture of the size of the wave length of light is called ***diffraction****.*

The phenomenon resulting from the superposition of secondary wavelets originating from different parts of the same wave front is define as diffraction of light. Diffraction is the characteristic of all types of waves. Greater the wave length of wave higher will be it's degree of diffraction.



Conceptual Note(s)

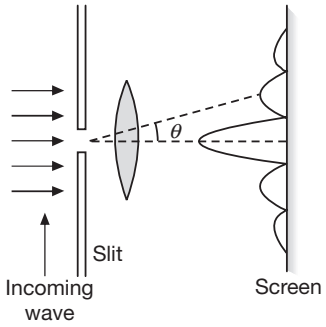
Diffraction, can be regarded as a consequence of interference from many coherent wave sources. In other words, the phenomena of diffraction and interference are basically equivalent.

TYPES OF DIFFRACTION

Diffraction phenomena are usually classified as being one of two types, which are named after the men who first explained them. The first type is called **Fraunhofer Diffraction** and the second is called **Fresnel's Diffraction**.

Fraunhofer Diffraction

This occurs when the rays reaching a point are approximately parallel i.e. when both the source and screen are effectively at infinite distance from the diffracting device. In this case, the incident light is a plane wave so that the phase of the light at each point in the aperture is the same. This can be achieved experimentally either by placing the observing screen at a large distance from the aperture or by using a converging lens to focus parallel rays on the screen, as in Figure.



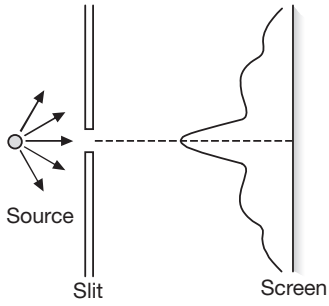
Note that a bright fringe is observed along the axis at $\theta = 0$, with alternating bright and dark fringes on either side of the central bright fringe.

Common examples: Diffraction at single slit, double slit and diffraction grating.

Fresnel's Diffraction

When the observing screen is placed at a finite distance from the slit and no lens is used to focus parallel rays, the observed pattern is called a **Fresnel Diffraction Pattern**. Fresnel diffraction is rather complex to treat quantitatively.

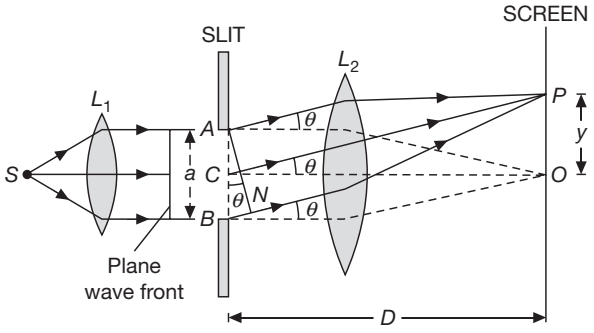
Common examples: Diffraction at a straight edge, narrow wire or small opaque disc etc.



A fresnel diffraction pattern of a single slit is observed when the incident rays are not parallel and the observing screen is at a finite distance from the slit.

FRAUNHOFER DIFFRACTION AT A SINGLE SLIT

Consider that a monochromatic source of light S, emitting light waves of wavelength λ, is placed at the principal focus of the convex lens L_1. A parallel beam of light i.e., a plane wavefront gets incident on a narrow slit AB of width a as shown in figure.



The diffraction pattern is obtained on a screen lying at a distance D from the slit and at the focal plane of the convex lens L_2.

According to rectilinear propagation of light, a bright image of the slit is expected at the centre O of the screen. But in practice, we get a diffraction pattern i.e., a central maximum at the centre O flanked by a number of dark and bright fringes called secondary maxima and minima on either side of the point O.

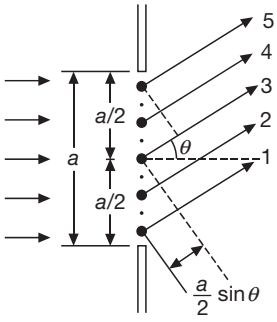
The diffraction pattern is obtained on the screen, which lies at the focal plane of the convex lens L_2. It is found that

(i) the width of the central maximum is twice as that of a secondary maximum and

(ii) the intensity of the secondary maxima goes on decreasing with the order of maxima. These observations are explained on the basis of the phenomenon of diffraction using the following mathematical treatment.

EXPLANATION AND MATHEMATICAL TREATMENT

Consider Fraunhofer diffraction by a single slit as shown in Figure. Important features of this problem can be deduced by examining waves coming from various portions of the slit. According to Huygens' principle, each portion of the slit acts as a source of waves. Hence, light from one portion of the slit can interfere with light from another portion, and the resultant intensity on the screen will depend on the direction θ.



Diffraction of light by a narrow slit of width a. Each portion of the slit acts as a point source of waves. The path difference between rays 1 and 3 or between rays 2 and 4 is equal to $(a/2)\sin\theta$

To analyze the resultant diffraction pattern, it is convenient to divide the slit in two halves as in Figure. All the waves that originate from the slit are initially in phase. Consider waves 1 and 3, which originate from the bottom and center of the slit, respectively. Wave 1 travels farther than wave 3 by an amount equal to the path difference $\left(\frac{a}{2}\right)\sin\theta$, where a is the width of the slit. Similarly, the path difference between waves 2 and 4 is also equal to $\left(\frac{a}{2}\right)\sin\theta$.

If this path difference is exactly one half of a wavelength (corresponding to a phase difference of $180°$), the two waves cancel each other and destructive interference results. This is true, in fact, for any two waves that originate at points separated by half the slit width, since the phase difference between two such points is $180°$. Therefore, waves from the upper half of the slit interfere destructively with waves from the lower half of the slit. when

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

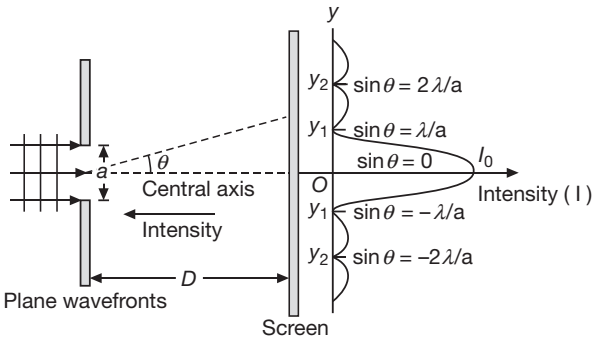
$$\Rightarrow \quad \sin\theta = \frac{\lambda}{a}$$

Similarly, destructive interference (minima) occurs when the path difference $\left(\frac{a}{2}\right)\sin\theta$ equals $\frac{\lambda}{2}$, $\frac{3\lambda}{2}$, $\frac{5\lambda}{2}$, etc. These points occur at progressively larger values of θ. Therefore, the general condition for destructive interference is

$$\sin\theta = n\frac{\lambda}{a} \qquad (n = \pm 1, \pm 2, \pm 3, \ldots) \qquad \ldots(1)$$

where $|n| \le \frac{a}{\lambda}$ $\qquad \{\because |\sin\theta| \le 1\}$

Equation (1) gives the values of θ for which the diffraction pattern has zero intensity. However, it tells us nothing about the variation in intensity along the screen. The general features of the intensity distribution along the screen are shown in Figure.



Position of the various minima for the Fraunhofer diffraction pattern of a single slit of width a.

A broad central bright fringe is observed, flanked by much weaker alternating maxima. The central bright fringe corresponds to those points opposite the slit for which the path difference is zero, or $\theta = 0$. All waves originating from the slit reach this region in phase, hence constructive interference results. The various dark fringes (points of zero intensity) occur at the values of θ that satisfy equation (1). The positions of the weaker maxima lie approximately halfway between the dark fringes. Note that the central bright fringe is twice as wide as the weaker maxima (which are narrower).

Angular width of central maxima is $\frac{2\lambda}{a}$ and width of central maxima is $\frac{2\lambda D}{a}$, where D is the distance of the screen from the slit.

The intensity distribution of the **diffraction pattern** is quite different from the interference pattern produced due to superposition of light from two coherent sources. The point O on the central axis is the brightest. The **angular position** (θ) of n^{th} diffraction **minima** is given by

$$a\sin\theta = n\lambda \qquad n = 1, 2, 3, 4, \ldots\ldots$$

For secondary maxima, we have

$$a\sin\theta = (2n+1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, 4, \ldots\ldots$$

$$\Rightarrow \quad \theta = (2n+1)\frac{\lambda}{2a};\ n = 1, 2, 3, 4, \ldots\ldots$$

$$\Rightarrow \quad y = (2n+1)\frac{\lambda D}{2a},\ n = 1, 2, 3, 4, \ldots\ldots$$

i.e., angular position of secondary maxima is

$$\frac{3\lambda}{2a}, \frac{5\lambda}{2a}, \frac{7\lambda}{2a}, \ldots\ldots$$

Position of secondary maxima is

$$\frac{3\lambda D}{2a}, \frac{5\lambda D}{2a}, \frac{7\lambda D}{2a}, \ldots\ldots$$

For small angle θ, we have $\sin\theta \approx \theta$. Thus, as shown in the figure, the angular position of the 1^{st}, 2^{nd}, 3^{rd}, … minima are $\frac{\lambda}{a}, \frac{2\lambda}{a}, \frac{3\lambda}{a}$, ….. respectively on either side of the central axis. A maximum is approximately halfway between two adjacent minima.

Note that as the slit width a increases, the width of the central diffraction maximum decreases. That is, there is less spreading out of the light by the slit. The secondary maxima also decreases in width and becomes weaker. When a becomes much greater than λ, the secondary maxima disappear.

The **intensity** I of the diffraction pattern as a function of θ is given as

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$

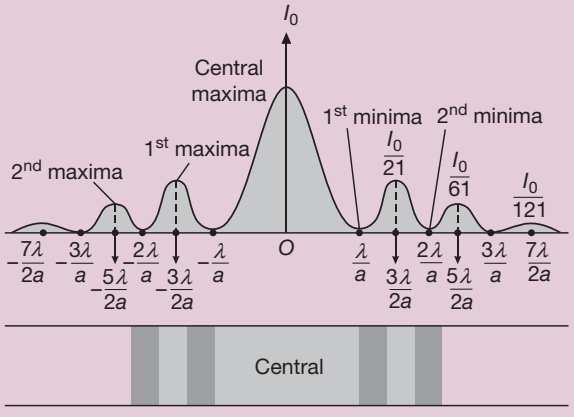
where $\alpha = \frac{\pi a\sin\theta}{\lambda}$

Since $\sin\theta \approx \frac{y}{D}$, so we get $\alpha \approx \frac{\pi}{\lambda}\left(\frac{ya}{D}\right)$

The intensity of secondary maxima is much less. Compared to the intensity of central maximum (I_0), the intensity of the first of the secondary maxima is only 4.5%, of the second is only 1.6%, of the third is merely 0.83%…… The successive secondary maxima decrease rapidly in intensity.

Conceptual Note(s)

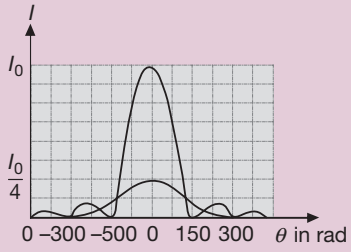
(a) If the intensity of the central maxima is I_0 then the intensity of the first and second secondary maxima are found to be $\frac{I_0}{21}$ and $\frac{I_0}{61}$. Thus, diffraction fringes are of unequal width and unequal intensities. Hence the ratio of the intensities of secondary maxima to central maxima are $1:\frac{1}{21}:\frac{1}{61}:\frac{1}{121}$....



(b) As the slit width increases (relative to wavelength) the width of the control diffraction maxima decreases, that is, the light undergoes less flaring by the slit. The secondary maxima also decrease in width (and becomes weaker).

(c) If $a \gg \lambda$, the secondary maxima due to the slit disappear; we then no longer have single slit diffraction.

(d) When the slit width is reduced by a factor of 2, the amplitude of the wave at the centre of the screen is reduced by a factor of 2, so the intensity at the centre is reduced by a factor of 4.



DIFFRACTION MAXIMA DUE TO SINGLE SLIT

The angular positions of diffraction minima can be given by the equation

$$\sin\theta = n\frac{\lambda}{a} \qquad (n = \pm 1, \pm 2, \pm 3, \ldots)$$

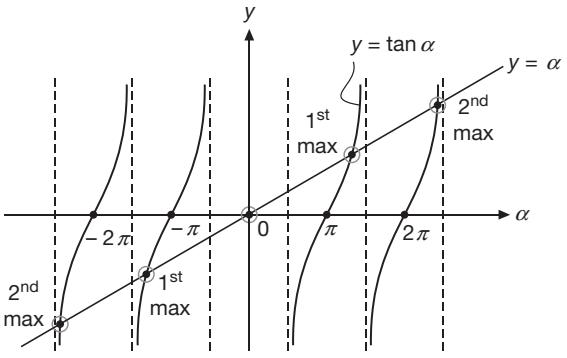
However, to find the angular positions of diffraction maxima other than central maxima we differentiate equation $I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$, w.r.t. α, where $\alpha = \frac{\pi a \sin\theta}{\lambda}$ and equate to zero.

$$\Rightarrow \quad \frac{dI(\theta)}{d\alpha} = \frac{\alpha^2(\alpha\sin\alpha\cos\alpha) - (\sin\alpha)(2\alpha)}{\alpha^4} = 0$$

$$\Rightarrow \quad \tan\alpha = \alpha \qquad \ldots(1)$$

In equation (1), $\alpha = 0$ corresponds to central maxima.

All other values of α satisfying equation (1) will correspond to higher order diffraction maxima in the diffraction pattern which can be calculated graphically by finding the intersection points of the curves $y = \tan\alpha$ and $y = \alpha$ as shown in figure below.



In above figure we can see that successive higher order maxima are not located at the mid points of all the minima's.

ILLUMINATION PATTERN DUE TO DIFFRACTION BY A SINGLE SLIT

Since the first minima in a single slit diffraction pattern is obtained at an angular position given by

$$\theta = \sin^{-1}\left(\frac{\lambda}{a}\right) \qquad \ldots(1)$$

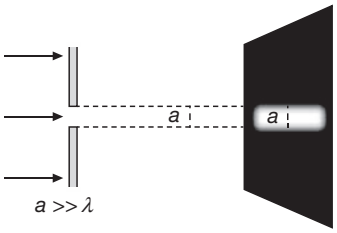
The above angle in equation (1) gives the edges of central diffraction maxima which is most prominent in the illumination pattern of single slit diffraction pattern. Now for different wavelengths and slit widths let us discuss the following cases.

CASE-1: When $a \gg \lambda$

When slit width a is very large compared to wavelength λ of light, then from equation (1) we get

$$\theta = \sin^{-1}\left(\frac{\lambda}{a}\right) \to 0$$

Which simply shows the rectilinear propagation of light because the light being does not flare out of the region beyond $\theta = 0$. This is shown in figure below in which the central maxima will just be the projection of light on screen which of width equal to that of the slit.



CASE-2: When $a > \lambda$

When slit width a is more than the wavelength of light, then first minima and other higher order minima and maxima can also be obtained as discussed earlier.

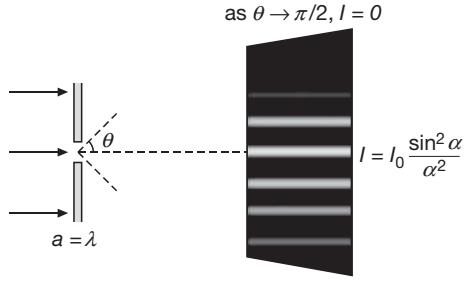
CASE-3: When $a = \lambda$

In this case we can see from equation (1) we get

$$\theta = \sin^{-1}(1) = \frac{\pi}{2}$$

Thus, the central maxima will spread on the whole screen as shown in figure and as we move away from centre of screen the intensity of light gradually decreases with the function given by equation

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2, \text{ w.r.t. } \alpha, \text{ where } \alpha = \frac{\pi a \sin\theta}{\lambda}$$



CASE-4: When $a < \lambda$

When slit width is less than the wavelength of light, then equation (1) is not valid and we observe that no minima is obtained anywhere and on screen and hence there will be almost uniform illumination near of the centre of screen.

ILLUSTRATION 36

A slit of width a is illuminated by white light.

(a) For what value of a, will the first minimum for red light of $\lambda = 6500$ Å be at $\theta = 15°$?

(b) What is the wavelength λ' of the light whose first side-maximum is at $\theta = 15°$, thus coinciding with the first minimum for the red light?

SOLUTION

(a) The angular position θ_n of n^{th} minimum is given by

$$a\sin\theta_n = n\lambda$$

Here, $n = 1$, $\lambda = 6500\times10^{-10}$ m, $\theta = 15°$

$$\Rightarrow\ a = \frac{n\lambda}{\sin\theta_n} = \frac{1\times6500\times10^{-10}}{\sin 15°} \approx 2.5\ \mu\text{m}$$

(b) This maximum is approximately halfway between the first and second minima produced with light of wavelength λ'. Thus, by putting $n = 1.5$, we get

$$a\sin\theta = 1.5\lambda'$$

$$\Rightarrow\ \lambda' = \frac{a\sin\theta}{1.5} = \frac{2.5\times10^{-6}\times\sin(15°)}{1.5}$$

$$\Rightarrow\ \lambda' = 430 \text{ mm} = 4300 \text{ Å}$$

This is the wavelength of violet light. Note that the first side-maximum for light of $\lambda' = 4300$ Å will always coincide with the first minimum for light of $\lambda = 6500$ Å, no matter what the slit-width is.

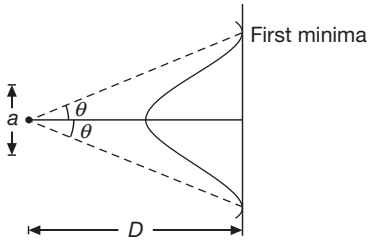
ILLUSTRATION 37

Angular width of central maximum in the Fraunhofer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, the angular width decreases by 30%. Calculate the wavelength of this light. The same decrease in the angular width of central maximum is obtained when the original apparatus is immersed in a liquid. Find refractive index of the liquid.

SOLUTION

(a) Given $\lambda = 6000$ Å

Let a be the width of slit and D the distance between screen and slit.



First minima is obtained at $a\sin\theta = \lambda$

$$\Rightarrow\ a\theta = \lambda\sin\theta \approx \theta$$

$$\Rightarrow\ \theta = \frac{\lambda}{a}$$

Angular width of first maxima $= 2\theta = \frac{2\lambda}{a} \propto \lambda$

Angular width will decrease by 30% when λ is also decreased by 30%.
Therefore, new wavelength

$$\lambda' = \left\{(6000) - \left(\frac{30}{100}\right)6000\right\} = 4200 \text{ Å}$$

(b) When the apparatus is immersed in a liquid of refractive index μ, the wavelength is decreased μ times.

$$\Rightarrow \quad 4200 \text{ Å} = \frac{6000 \text{ Å}}{\mu}$$

$$\Rightarrow \quad \mu = \frac{6000}{4200}$$

$$\Rightarrow \quad \mu = 1.429 \approx 1.43$$

ILLUSTRATION 38

Angular width of central maximum in the Fraunhofer diffraction pattern of a slit is measured. The slit is illuminated by another wavelength, the angular width decreases by 30%. Calculate the wavelength of this light. The same decrease in angular width of central maximum is obtained when the original apparatus is immersed in a liquid. Find the refractive index of the liquid.

SOLUTION

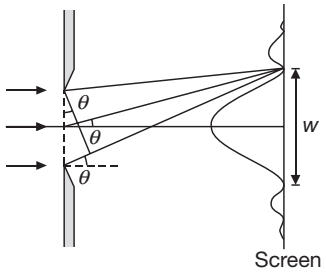
For diffraction minima on screen, we use

$$a\sin\theta = n\lambda \text{, where } n = 1, 2, 3, \ldots$$

Angular width of central maxima is 2θ for $n = 1$

$$\Rightarrow \quad a\sin\theta = \lambda$$

For small θ, we have $\sin\theta \approx \theta$, so $a\theta = \lambda$



So, angular width of central maxima is given by

$$w = 2\theta = \frac{2\lambda}{a}$$

$$\Rightarrow \quad w = \frac{2\times 6000\times 10^{-10}}{a}$$

When the wavelength is changed the angular width of central maxima is reduced by 30%. Thus new angular width is given by

$$w' = w - 0.3w = 0.7w$$

$$\Rightarrow \quad 0.7w = \frac{2\lambda'}{a}$$

$$\Rightarrow \quad \lambda' = (0.7w)\left(\frac{a}{2}\right) = 0.7\left(\frac{2\times 6000\times 10^{-10}}{a}\right)\left(\frac{a}{2}\right)$$

$$\Rightarrow \quad \lambda' = 0.7\times 6000\times 10^{-10} = 4200\times 10^{-10} \text{ m}$$

$$\Rightarrow \quad \lambda' = 4200 \text{ Å}$$

When the setup is submerged in a liquid, then also the angular width of central maxima decreases by 30%, which indicates that the wavelength of light decreases by 30%, so

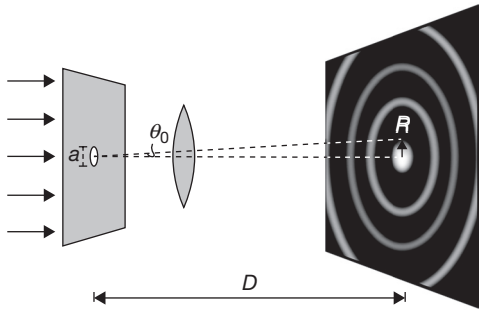
$$\lambda' = \frac{\lambda}{\mu}$$

$$\Rightarrow \quad 4200 = \frac{6000}{\mu}$$

$$\Rightarrow \quad \mu = \frac{6000}{4200} = \frac{10}{7} \approx 1.43$$

FRAUNHOFER DIFFRACTION AT A CIRCULAR APERTURE

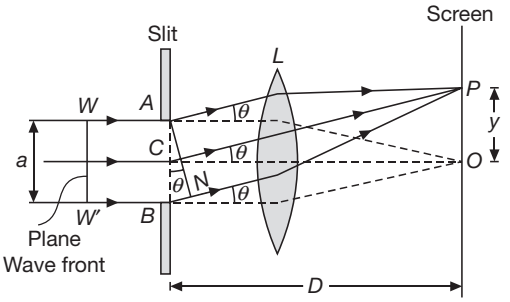
In Fraunhofer diffraction at a circular aperture or disc, the diffraction pattern has intermediate dark and bright fringes with a central bright circular spot.



This circular spot formed at the centre is known as Airy disc which is the description of best spot of light that a perfect lens of circular aperture can make.

Nearby, the circular patterns formed are those known as Airy patterns. These are named after George Biddle Airy. The concentric circular rings will get fainter as it moves from the central spot.

The problem of diffraction at a circular aperture was first solved by Airy in 1835. A circular aperture of diameter a is shown as AB in figure.



Fraunhofer diffraction at a circular aperture

A plane wave front WW' is incident normally on this aperture. Every point on the plane wave front in the aperture acts as a source of secondary wavelets. The secondary wavelets spread out in all directions as diffracted rays in the aperture. These diffracted secondary wavelets are converged on the screen by placing a convex lens L (of focal length f) between the aperture and the screen. The screen is at the focal plane of the convex lens. The diffracted rays travelling normal to the plane of aperture i.e. along CO get converged at O.

All these waves travel some distance to reach the point O and there is no path difference between these rays. Hence a bright spot is formed at O. This bright spot is known as Airy's disc. The point O corresponds to the central maximum.

Next consider the secondary waves travelling at an angle θ with respect to the direction of CO. All these secondary waves travel in the form of a cone and hence, they form a diffracted ring on the screen. The radius of that ring is y and its centre is at O. Now consider a point P on the ring at a distance y from O. The intensity of light at P depends on the path difference between the waves at A and B to reach P. The path difference between the waves from A and B arriving at the point P is

$$BN = AB\sin\theta = a\sin\theta$$

The diffraction due to a circular aperture is similar to the diffraction due to a single slit. Hence, the intensity at P depends on the path difference $a\sin\theta$.

If the path difference is an integral multiple of λ then intensity at P is minimum. On the other hand, if the path difference is an odd multiple of $\frac{\lambda}{2}$, then the intensity is maximum. So,

for minima, $a\sin\theta = n\lambda$ …(1)

for maxima, $a\sin\theta = (2n-1)\frac{\lambda}{2}$ …(2)

where $n = 1, 2, 3,.....$ and $n = 0$ corresponds to the central maximum.

The Airy disc is surrounded by alternate bright and dark concentric rings called the Airy's rings.

The intensity of the dark ring is zero and the intensity of the bright ring decreases as we go radially from O on the screen. If the collecting lens L is placed very near to the circular aperture (or the screen is at a large distance from the lens), then, $D \approx f$. So, we have

$$\sin\theta \approx \theta \approx \frac{y}{D} \approx \frac{y}{f} \quad \ldots(3)$$

where f is the focal length of the lens.

Also, from the condition for first secondary minimum i.e. from equation (1), we get

$$\sin\theta \approx \theta \approx \frac{\lambda}{a} \quad \ldots(4)$$

Equating (3) and (4), we get

$$\frac{y}{f} = \frac{\lambda}{a} \quad \ldots(5)$$

But according to Airy, the exact value of radius or Airy disc is R given by

$$R = \frac{1.22 f\lambda}{a} \quad \ldots(6)$$

Using equation (6), the radius of Airy's disc can be obtained. Also, from this equation, we observe that the radius of Airy's disc is inversely proportional to the diameter a of the aperture. Hence on decreasing the diameter of aperture, the size of Airy's disc increases.

The examples for circular apertures are the eyes and optical instruments like camera, microscope, telescope and so on. These are diffraction limited. Diffraction limited is the ability to produce images with angular resolution limited by aperture resolution. Rayleigh creation is used to calculate this resolution.

ILLUSTRATION 39

A convex lens of diameter 8.0 cm is used to focus a parallel beam of light of wavelength 6200 Å. If the light be focused at a distance of 20 cm from the lens, what would be the radius of the central bright spot formed?

SOLUTION

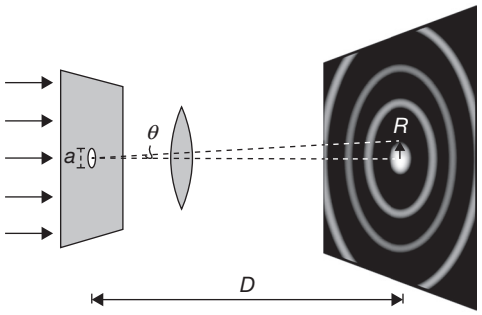
The angular spread of the central bright spot is given by

$$\sin\theta = \frac{1.22\lambda}{a}$$

$$\Rightarrow \quad \sin\theta = \frac{1.22\times 620\times 10^{-9}}{0.08}$$

$$\Rightarrow \quad \sin\theta = 9.455\times 10^{-6} \text{ rad}$$

Since θ is small, so $\sin\theta \approx \theta = 9.455\times 10^{-6}$ rad



Also, $\theta \approx \frac{R}{D}$

$$\Rightarrow \quad \frac{R}{D} = 9.45\times 10^{-6}$$

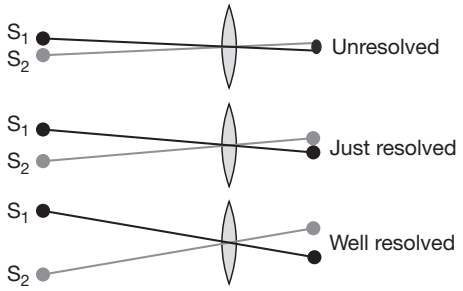
$$\Rightarrow \quad R = 9.45\times 10^{-6}\times 0.20$$

$$\Rightarrow \quad R = 1.89\times 10^{-6} \text{ m}$$

RESOLVING POWER AND RAYLEIGH'S CRITERION

The image of a point object, formed by a converging lens, is not a point image. Rather, it is a diffraction disc surrounded by a few alternate bright and dark fringes of sharply decreasing intensity. The size of the disc depends on the aperture of the lens and the wavelength of light used.

It two bright point objects S_1 and S_2 lying very close to each other are seen with a naked eye or with the help of an optical instrument (a microscope or a telescope), then they may or may not be seen as two separate distinct objects due to the overlapping of their diffraction patterns.



The smallest separation (linear or angular) between two point objects at which they appear just separated is called **the limit of resolution of an optical instrument** and the reciprocal of the limit of resolution is called its **resolving power**.

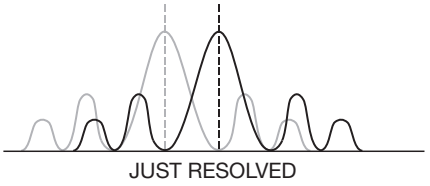
Whether the two objects are seen as two separate point objects or not, depends on the separation between the centres of the bright discs of the images of the two objects. Therefore, when two objects are seen with a naked eye or with the help of an optical instrument the two objects may be just resolved, well resolved or unresolved as explained by Rayleigh's Criterion.

RAYLEIGH'S CRITERION

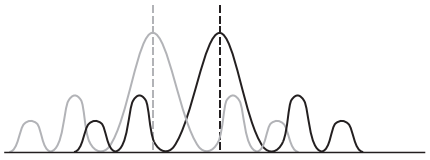
(a) Two objects are said to be **just resolved**, if the separation between the central maxima of the objects is just equal to the distance between the central maximum and the first minimum of any of the two. In other words, two images are said to be just resolved when central maxima of one diffraction pattern falls on first minima of other.

Limit of resolution of a telescope is

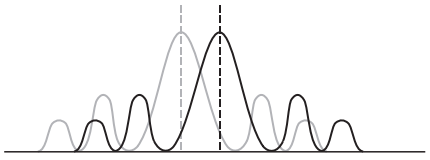
$$\theta = \frac{1.22\lambda}{a}$$



(b) Two objects are said to **well resolved**, if the separation between the central maxima of the objects is greater than the distance between the central maximum and the first minimum of either of them.



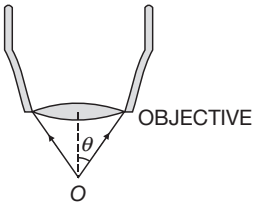
(c) Two objects are said to be **unresolved**, if separation between the central maxima of the objects is less than the distance between the central maximum and the first minimum of either of them.



These results are called **Rayleigh's Criterion of Limiting Resolution**. Therefore, it follows that diffraction limits the resolving power of an optical instrument.

RESOLVING POWER OF A MICROSCOPE

Resolving power of a microscope is defined as the reciprocal of the least separation between two close objects, so that they appear just separated, when seen through the microscope.



Consider a point object to be illuminated with the light of wavelength λ and seen through a microscope. The rays of light scattered from the object enter the objective of the microscope in a cone of semi-vertical angle θ. The least separation between the two objects, so that they appear just separated is given by

$$d = \frac{\lambda}{2\mu \sin\theta} \qquad \ldots(1)$$

where μ is the refractive index of the medium between the objective of the microscope and the object.

This least separation between the two objects is called the **limit of resolution** of the microscope.

From definition, the resolving power of the microscope is given by

$$\frac{1}{d} = \frac{2\mu \sin\theta}{\lambda} \qquad \ldots(2)$$

Following conclusions can be drawn from the above expression.

(a) The resolving power of a microscope increases with decrease in the value of the wavelength of the light used to illuminate the object.

(b) The resolving power of a microscope increases with increase in the value of the refractive index of the medium between its objective and the object. For this reason, **oil immersion objective microscopes** are used to achieve high resolving power.

Since wavelength of ultraviolet light is less than that of the visible light, the microscopes employing ultraviolet light for illuminating the objects are used to achieve high resolving power. Such microscopes are called **ultra microscopes**. Still higher resolving power can be obtained in an **electron microscope.**

RESOLVING POWER OF A TELESCOPE

Resolving power of a telescope is the reciprocal of the smallest angular separation between two distant objects, so that they appear just separated, when seen through the telescope.

Let two distant objects be observed through a telescope, whose objective is of diameter a. Let λ be the wavelength of the light, in which objects are observed. The smallest angular separation between the two objects, so that they appear just separated is found to be

$$d\theta = \frac{1.22\lambda}{a} \qquad \ldots(3)$$

$$\Rightarrow \quad \text{Resolving Power of Telescope} = \frac{1}{d\theta} = \frac{a}{1.22\lambda} \ldots(4)$$

So, we observe that the resolving power of a telescope increases, when objective of larger diameter is used or light of smaller wavelength is used to see the objects.

HUMAN EYE

In case of the human eye, two points can be seen distinctly if angle subtended by them at the eye is about one minute. This is the angular limit of resolution of the eye and the reciprocal of this is the resolving power.

The limit of resolution of eye lens is nearly $\left(\frac{1}{60}\right) = 1'$

VALIDITY OF GEOMETRICAL OPTICS AND FRESNEL'S DISTANCE (Z_F)

When a slit of width a is illuminated by a parallel beam of light, the angular spread of diffracted light is approximately $\frac{\lambda}{a}$. Therefore, after travelling a distance D, the diffracted beam acquires a width $\frac{D\lambda}{a}$. Geometrical optics is based on rectilinear propagation of light, which is just an approximation. We can say that geometrical optics is valid, if the width $\frac{D\lambda}{a}$ of the diffracted beam is less than the size of the slit, that is

$$\frac{D\lambda}{a} < a$$

$$\Rightarrow \quad D < \frac{a^2}{\lambda}$$

This distance from a slit or an obstacle upto which the spreading of light due to diffraction can be ignore (i.e. light goes straight and hence ray optics or geometrical optics can be applied) is called Fresnel's distance $Z_F = \frac{a^2}{\lambda}$.

Since λ is very small so Z_F is fairly large (in most of the cases) and so diffraction spreading can be neglected up to a fairly large distance.

Therefore, geometrical optics is valid for $Z_F < \frac{a^2}{\lambda}$ i.e. beyond Z_F spreading of light becomes significance and ray optics cannot be applied. Theoretically when $\lambda \to 0$, then $Z_F \to \infty$.

FRESNEL'S ZONE

If we expect a beam to travel a distance D without too much broadening by diffraction, we must have

$$D < Z_F$$

$$\Rightarrow \quad D < \frac{a^2}{\lambda}$$

$$\Rightarrow \quad a > \sqrt{\lambda D}$$

$\sqrt{\lambda D}$ is called the **size of the Fresnel's zone**, a_F

$$\Rightarrow \quad a_F = \sqrt{\lambda D}$$

ILLUSTRATION 40

For what distance is the ray optics a good approximation, if the slit is 3 mm wide and the wavelength of light is 5000 Å?

SOLUTION

For ray optics to be valid $D < Z_F \left(= \frac{a^2}{\lambda}\right)$

$$\Rightarrow \quad D < \frac{a^2}{\lambda} = \frac{(3\times10^{-3})^2}{5000\times10^{-10}} = 18 \text{ m}$$

Thus, upto a distance of 18 m, we can assume rectilinear propagation of light to a good approximation.

INTERFERENCE AND DIFFRACTION: A COMPARISON

INTERFERENCE	DIFFRACTION
It results from interaction of light coming from two different wave fronts originating from two coherent sources.	It results from interaction of light coming from different parts of the same wavefront.
Here, the fringes are of the same width.	Here the fringes are always of varying width.
The fringes of minimum intensity are dark (or perfectly dark when waves are of same amplitude).	The fringes of minimum intensity are not perfectly dark.
All bright fringes possess the same intensity.	The intensity of all the bright fringes is not same. It is maximum for central fringe and decreases sharply for first, second etc. bright fringes.
An interference pattern consists a good contrast between the dark and bright fringes.	In diffraction pattern the contrast between the bright and dark fringes is comparatively poor.

Test Your Concepts-II

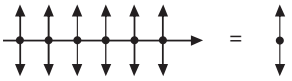
Based on Diffraction

(Solutions on page H.124)

1. A slit of width 0.025 m is placed in front of a lens of focal length 50 cm. The slit is illuminated with light of wavelength 5900 Å. Calculate the distance between the centre and first dark band of diffraction pattern obtained on a screen placed at the focal plane of the lens.
2. Two spectral lines of sodium D_1 and D_2 have wavelengths of approximately 5890 Å and 5896 Å. A sodium lamp sends incident plane wave onto a slit of width 2 micrometre. A screen is located 2 m from the slit. Find the spacing between the first maxima of two sodium lines as measured on the screen.
3. In Young's double slit experiment, the distance d between the slits S_1 and S_2 is 1 mm. What should the width of each slit be so as to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern?
4. Estimate the distance for which ray optics is a good approximation for an aperture of 4 mm and wavelength 400 nm.
5. Two towers on the top of two hills are 40 km apart. The line joining them passes 50 m above a hill half way between the towers. What is the longest wavelength of radio waves which can be sent between the towers without appreciable diffraction effects?
6. A slit of width d is illuminated by white light. For what value of d will the first minimum for red light ($\lambda = 6500$ Å) fall at an angle $\theta = 30°$?
7. A screen is placed 2 m away from a single narrow slit. Calculate the slit width if the first minimum lies 5 mm on either side of central maximum. Incident plane waves have a wavelength of 5000 Å.
8. Determine the angular separation between central maximum and first order maximum of the diffraction pattern due to a single slit of width 0.25 mm when light of wavelength 5890 Å is incident on it normally.
9. Parallel light of wavelength 5000 Å falls normally on a single slit. The central maximum spreads out to 30° on either side of the incident light. Find the width of the slit. For what width of the slit the central maximum would spread out to 90° from the direction of the incident light?
10. A laser light beam of power 20 mW is focused on a target by a lens of focal length 0.05 m. If the aperture of the laser be 1 mm and the wavelength of its light 7000 Å, calculate the angular spread of the laser, the area of the target hit by it and the intensity of the impact on the target.

POLARIZATION OF LIGHT

According to Maxwell, light possesses electromagnetic nature. An electromagnetic wave consists of varying electric and magnetic fields, such that the two fields are mutually perpendicular to each other and to the direction of propagation of waves. The optical phenomena i.e., phenomena concerning light may primarily be attributed to the vibrations of electric field vector in a direction perpendicular to the direction of propagation of light. In ordinary or unpolarised light, the vibrations of electric field vector are regularly or symmetrically distributed in a plane perpendicular to the direction of the propagation of the light.

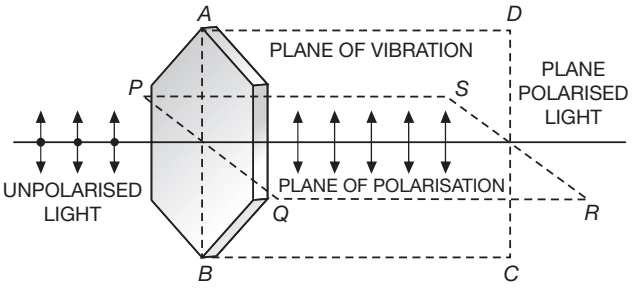


UNPOLARISED LIGHT (REPRESENTATION)

In an ordinary ray of light, the electric vibrations are in all the directions but perpendicular to the direction of propagation of the light. Such a ray of light is called a **ray of ordinary or unpolarised light**. It is schematically represented as shown. The arrows represent vibrations in the plane of the paper, while the dots represent vibrations in a direction perpendicular to the plane of the paper.

The phenomenon, due to which the vibrations of light are restricted to a particular plane, is called the **polarisation of light**.

When ordinary light i.e. unpolarised light passes through a tourmaline crystal, out of all the vibrations which are symmetrical about the direction of propagation, only those passes through it, which are parallel to its crystallographic axis AB. Therefore, on emerging through the crystal, the vibrations no longer remain symmetrical about the direction of propagation but are confined to a single plane (see Figure).



PLANE OF VIBRATION

The plane $(ABCD)$, which contains the vibrations of plane polarised light, is called the **plane of vibration**.

PLANE OF POLARISATION

The plane $(PQRS)$ perpendicular to the plane of vibrations is called the **plane of polarisation**.

PLANE POLARISED LIGHT

It may be defined as the light, in which the vibrations of the light (vibrations of the electric vector) are restricted to a particular plane.

In a plane polarised light, the vibrations are restricted to a fixed plane, so that vibrations are perpendicular to direction of propagation of light. Figure (a) represents plane polarised light having vibrations in the plane of the paper and Figure (b) represents the plane polarised light having vibrations in a plane perpendicular to the plane of the paper.

POLARISED LIGHT (REPRESENTATION)

Problem Solving Technique(s)

The vibrations in plane polarised light are perpendicular to the plane of polarisation.

POLARIZATION BY REFLECTION

Polarized light may also be obtained by the process of reflection. When an unpolarized light beam is reflected, light is completely polarized, partially polarized, or unpolarized, depending on the angle of incidence. If the angle of incidence is either 0 or $90°$ (normal or grazing angles), the reflected beam is unpolarized. However, for intermediate angles of incidence, the reflected light is polarized to some extent.

Suppose an unpolarized light beam is incident on a surface as in figure. The beam can be described by two electric field components, one parallel to the surface (the dots) and the other perpendicular to the first and to the direction of propagation (the arrows). It is found that the parallel component reflects more strongly than the other component, and this results in a partially polarized beam. Furthermore, the refracted ray is also partially polarized.

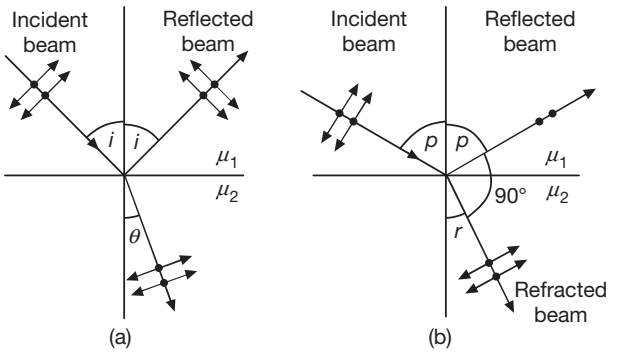
Now suppose the angle of incidence, i, is varied until the angle between the reflected and refracted beams is $90°$. At this particular angle of incidence, the reflected beam is completely polarized with its electric field vector parallel to the surface, while the refracted beam is partially polarized. The angle of incidence at which this occurs is called the **polarizing angle,** p.

From figure, we see that at the polarizing angle, $p+90°+r=180°$, so that $r=90°-p$. Using Snell's Law, we have

$$\mu=\frac{\sin p}{\sin r}$$

Since $\sin r=\sin(90°-p)=\cos p$, the expression for μ can be written

$$\mu=\frac{\sin p}{\cos p}=\tan p$$



(a) When unpolarized light is incident on a reflecting surface, the reflected and refracted beams are partially polarized. (b) The reflected beam is completely polarized when the angle of incidence equals the polarizing angle θ_p

This expression is called **Brewster's Law**, and the polarizing angle p is sometimes called **Brewster's Angle**, after its discoverer, Sir David Brewster (1781–1868). For example, the Brewster's angle for crown glass $(\mu = 1.52)$ is $p = \tan^{-1}(1.52) = 56.7°$. Since μ varies with wavelength for a given substance, the Brewster's angle is also a function of the wavelength.

Polarization by reflection is a common phenomenon. Sunlight reflected from water, glass, snow and metallic surfaces is partially polarized. If the surface is horizontal, the electric field vector of the reflected light will have a strong horizontal component. Sunglasses made of polarizing material reduce the glare of reflected light. The transmission axes of the lenses are oriented vertically so as to absorb the strong horizontal component of the reflected light.

ILLUSTRATION 41

A ray of light strikes a glass plate at an angle of 60°. If the reflected and refracted rays are perpendicular to each other, find the refractive index of glass.

SOLUTION

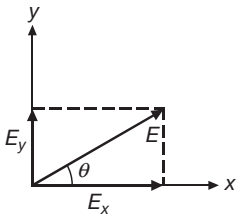
Reflected and refracted rays are mutually perpendicular only when the angle of incidence is equal to polarising angle. So, $i_p = 60°$. Hence the refractive index is given by

$$\mu = \tan i_p = \tan 60° = \sqrt{3} = 1.732$$

LINEARLY, CIRCULARLY AND ELLIPTICALLY POLARISED LIGHT

A wave is said to be linearly polarized if only one of these directions of vibration of E exists at a particular point. (Sometimes such a wave is described as plane-polarized, or simply polarized).

Suppose a light beam travelling in the z direction has an electron field vector that is at an angle θ with the x axis at some instant, as in figure. The vector has components E_x and E_y as shown. Obviously, the light is linearly polarized if one of these components is always zero or if the angle θ remains constant in time. However, if the tip of the vector E rotates in a circle with time, the wave is said to be circularly polarized. This occurs when the magnitudes of E_x and E_y are equal, but differ in phase by $90°$. On the other hand, if the magnitudes of E_x and E_y are not equal, but differ in phase by $90°$, the tip of E moves in an ellipse. Such a wave is said to be elliptically polarized. Finally, if E_x and E_y are, on the average, equal in magnitude, but have a randomly varying phase difference the light beam is unpolarized.



A linearly polarized wave with E at an angle θ to x has components $E_x = E\cos\theta$ and $E_x = E\sin\theta$

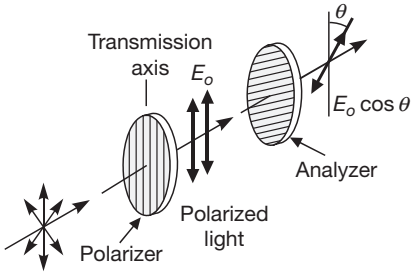
It is possible to obtain a linearly polarized beam from an unpolarized beam by removing all waves from the beam except those whose electric field vectors oscillate in a single plane. Four different physical processes of producing polarized light from unpolarized light are

(a) selective absorption (or dichroism)
(b) reflection
(c) double refraction
(d) scattering

POLARIZATION BY SELECTIVE ABSORPTION

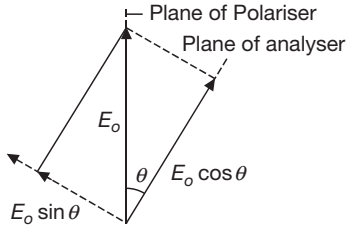
The most common technique for obtaining polarized light is to use a material that will transmit waves whose electric field vectors are parallel to a certain direction and will absorb most other directions of polarization. Any substance that has the property of transmitting light with the electric field vector vibrating in only one direction is called a **dichroic substance**. In 1938, E.H. Land discovered a material, which he called **Polaroid**, that polarizes light through selective absorption by oriented molecules. Long chain hydrocarbon molecules (such as polyvinyl alcohol) in thin-sheet form are aligned in one direction when the sheet is stretched during fabrication. After the sheet is dipped into a solution containing iodine, the molecules become conducting. However, the conduction takes place primarily along the hydrocarbon chains since the valence electrons of the molecules readily absorb light whose electric field vector is parallel to their length and transmit light whose electric field vector is perpendicular to their length. It is common to refer to the direction perpendicular to the molecular chains as the transmission axis. In an ideal polarizer, all light with E parallel to the transmission axis is transmitted, and all light with E perpendicular to the transmission axis is absorbed.

Figure represents an unpolarized light beam incident on the first polarizing sheet, called the **polarizer**, where the transmission axis is indicated by the straight lines on the polarizer.



The light that is passing through this sheet is polarized vertically as shown, where the transmitted electric field vector is E_0. A second polarizing sheet, called the **analyzer**, intercepts this beam with its transmission axis at an angle θ to the axis of the polarizer.

The component of E_0 perpendicular to the axis of the analyzer is completely absorbed. The component of E_0 parallel to the axis of the analyzer is $E_0 \cos\theta$



Two polarizing sheets whose transmission axes make an angle θ with each other. Only a fraction of the polarized light incident on the analyzer is transmitted.

Since the transmitted intensity varies as the square of the transmitted amplitude, we conclude that the transmitted intensity is given by

$$I = I_0 \cos^2\theta$$

where I_0 is the intensity of the polarized wave incident on the analyzer.

This expression, known as **Malus law**, applies to any two polarizing materials whose transmission axes are at an angle θ to each other. From this expression, note that the transmitted intensity is a maximum when the transmission axes are parallel $(\theta = 0 \text{ or } 180°)$. In addition, the transmitted intensity is zero (complete absorption by the analyzer) when the transmission axes are perpendicular to each other.

Conceptual Note(s)

(a) If polarised light of intensity I_0 is passed through an analyser, the intensity of light transmitted is

$$I = I_0 \cos^2\theta \qquad \{\text{Malus Law}\}$$

(b) If incident light is unpolarized (or ordinary light) of intensity I_0, then

$$I = \frac{I_0}{2}\cos^2\theta$$

(c) Analysis of a Given Light Beam

Let a given light beam is made incident on a polaroid (or Nicol) and the polaroid/Nicol is gradually rotated.

(i) If light beam shows no variation in intensity, then given beam is unpolarised.

(ii) If light beam shows variation in intensity but the minimum intensity is non-zero, then given beam is partially polarised.

(iii) If light beam shows variation in intensity and intensity becomes zero twice in a rotation, then given beam of light is plane polarised.

LAW OF MALUS

When a plane polarised light is seen through an analyser, the intensity of transmitted light varies as the analyser is rotated in its own plane about the incident direction. In 1809, E.N. Malus discovered that when a beam of completely plane polarised light is passed through analyser, the intensity I of transmitted light varies directly as the square of the cosine of the angle θ between the transmission directions of polariser and analyser. This statement is known as the **Law of Malus**.

Mathematically, according to Malus Law, we have

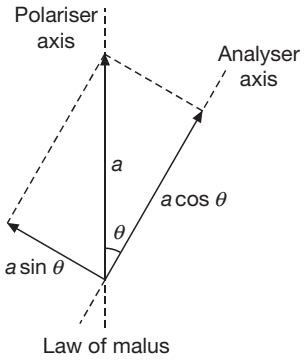
$$I \propto \cos^2\theta$$

$$\Rightarrow \quad I = I_0 \cos^2\theta$$

where I_0 is the maximum intensity of transmitted light. **It may be noted that I_0 is equal to half the intensity of unpolarised light incident on the polariser.**

EXPLANATION OF THE LAW

Let the planes of polariser and analyser are inclined to each other at an angle θ as shown in figure. Let I_0 be the intensity and a the amplitude of the plane polarised light transmitted by the polariser.



The amplitude a of the light incident on the analyser has two rectangular components,

(i) $a\cos\theta$, parallel to the plane of transmission of the analyser, and

(ii) $a\sin\theta$, perpendicular to the plane of transmission of the analyser.

So only the component $a\cos\theta$ is transmitted by the analyser. The intensity of light transmitted by the analyser is

$$I = k(a\cos\theta)^2$$

$$\Rightarrow \quad I = ka^2\cos^2\theta$$

$$\Rightarrow \quad I = I_0\cos^2\theta$$

where $I_0 = ka^2$, is the maximum intensity of light transmitted by the analyser (when $\theta = 0°$). The above equation is the **Law of Malus or Malus Law.**

Conceptual Note(s)

(a) When $\theta = 0°$ **or** $180°$, $\cos\theta = \pm 1$

$\Rightarrow \quad I = I_0$

So, when the transmission directions of polariser and analyser are parallel or antiparallel to each other, the maximum intensity of plane polarised light is transmitted by the analyser and is equal to the intensity emerging from the polariser.

(b) When $\theta = 90°$, $\cos\theta = 0$

$\Rightarrow \quad I = 0$

So, when the transmission directions of polariser and analyser are perpendicular to each other, the intensity of light transmitted through the analyser is zero.

(c) When a beam of unpolarised light is incident on the polariser, then

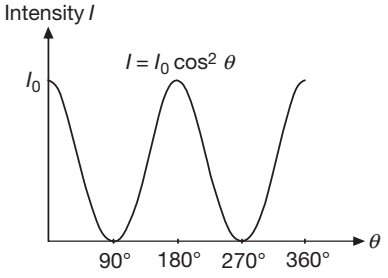
$$I = I_0\langle\cos^2\theta\rangle$$

Since, $\langle\cos^2\theta\rangle = \frac{1}{2}$

$$\Rightarrow \quad I = \frac{I_0}{2}$$

INTENSITY CURVE

As the angle θ between the transmission directions of polariser and analyser is varied, the intensity I of the light transmitted by the analyser varies as a function of $\cos^2\theta$, as shown in figure.



POLARISATION BY SCATTERING

When we look at the blue portion of the sky through a polaroid and rotate the polaroid, the transmitted light shows rise and fall of intensity. This shows that the light from the blue portion of the sky is plane polarised. This is because sunlight gets scattered (i.e., its direction is changed) when it encounters the molecules of the earth's atmosphere. The scattered light seen in a direction perpendicular to the direction of incidence is found to be plane polarised.

Explanation. Figure shows the unpolarised light incident on a molecule. The dots show vibrations perpendicular to the plane of paper and double arrows show vibrations in the plane of paper. The electrons in the molecule begin to vibrate in both of these directions. The electrons vibrating parallel to the double arrows cannot send energy towards an observer looking at 90° to the direction of the sun because their acceleration has no transverse component. The light scattered by the molecules in this direction has only dots. It is polarised perpendicular to the plane of paper. This explains the polarisation of light scattered from the sky.

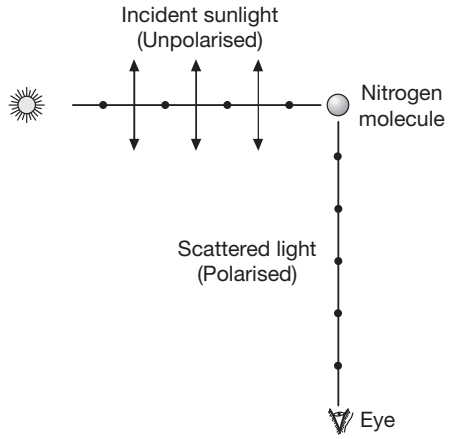


ILLUSTRATION 42

Two 'crossed' polaroids A and B are placed in the path of a light-beam. In between these, a third polaroid C is placed whose polarisation axis makes an angle θ with the polarisation axis of the polaroid A. If the intensity of light emerging from the polaroid A is I_0, then show that the intensity of light emerging from polaroid B will be $\frac{1}{4}I_0 \sin^2(2\theta)$.

SOLUTION

By Malus Law, the intensity of light emerging from the middle polaroid C will be

$$I_1 = I_0 \cos^2\theta$$

This intensity I_1 falls on the polaroid B whose polarisation axis makes an angle of $(90° - \theta)$ with the polarisation axis of the polaroid C. Therefore, the intensity of light emerging from B will be

$$I_2 = I_1 \cos^2(90° - \theta) = \left(I_0 \cos^2\theta\right)\cos^2(90° - \theta)$$

$$\Rightarrow \quad I_2 = I_0 \cos^2\theta \sin^2\theta = \frac{I}{4} I_0 (2\sin\theta\cos\theta)^2$$

$$\Rightarrow \quad I_2 = \frac{1}{4} I_0 \sin^2(2\theta)$$

ILLUSTRATION 43

Two polaroids are placed 90° to each other. What happens when $N-1$ more polaroids are inserted between two crossed polaroids (at 90° to each other). Their axes are equally spaced. How does the transmitted intensity behave for large N?

SOLUTION

Transmitted intensity through first polaroid is

$$I_1 = I_0 \cos^2\theta$$

where I_0 is the original intensity. Similarly, the transmitted intensity through second polaroid will be

$$I_2 = I_1 \cos^2\theta = I_0 \cos^4\theta$$

If N polaroids are used, then

$$I_N = I_0 (\cos\theta)^{2N}$$

As the optic axes of the polaroids are equally inclined, so angle of rotation θ is same for each polaroid. Thus

$$\frac{I_N}{I_0} = (\cos\theta)^{2N}$$

Since, angle between successive polaroids is given by

$$\theta = \frac{90°}{N} = \frac{\pi}{2N} \text{ radian}$$

For large N, θ becomes small, so we get

$$\left(\cos\frac{\pi}{2N}\right)^{2N} = \left(1 - \frac{\pi^2}{8N^2} + \ldots.\right)^{2N} \simeq \left(1 - \frac{2N\pi^2}{8N^2} + \ldots\right)$$

which approaches 1 for large N. So, fractional intensity, is

$$\frac{I_N}{I_0} = 1$$

$$\Rightarrow \quad I_N = I_0$$

ILLUSTRATION 44

A beam of plane-polarised falls normally on a polariser (cross-sectional area 3×10^{-4} m^2) which rotates about the axis of the ray with an angular velocity of 31.4 rads^{-1}. Find the energy of light passing through the polariser per revolution and the intensity of the emergent beam if the flux of energy of the incident ray is 10^{-3} W.

SOLUTION

Cross-sectional area of polaroid, $A = 3\times10^{-4}$ m^2

Angular velocity, $\omega = 3.14$ rads^{-1}

Time taken to complete one revolution,

$$T = \frac{2\pi}{\omega} = \frac{2\times3.14}{31.4} = 0.2 \text{ s}$$

$$(\text{Energy incident/sec}) = 10^{-3} \text{ W}$$

So, intensity of incident polarised beam is given by

$$I_0 = \frac{\text{Energy incident/sec}}{\text{Area}} = \frac{10^{-3}}{3\times10^{-4}} = \frac{10}{3} \text{ Wm}^{-2}$$

Since, $I = I_0\cos^2\theta$ where $\langle\cos^2\theta\rangle = \frac{1}{2}$

So, average intensity transmitted is

$$I_{av} = \frac{I_0}{2} = \frac{10}{3\times2} = 1.67 \text{ Wm}^{-2}$$

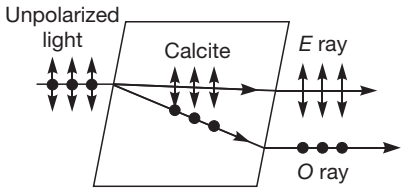
Light energy passing through polariser per revolution is given by

$$E = I_{av}AT = \frac{10}{6}(3\times10^{-4})(0.2) = 10^{-4} \text{ J}$$

POLARIZATION BY DOUBLE REFRACTION

When light travels through an isotropic medium, such as glass, it travels with a speed that is the same in all directions. Such isotropic materials are characterized by a single index of refraction. However, in certain crystals, such as calcite and quartz, the speed of light is not the same in all directions. The fundamental reason for this phenomenon is associated with the complex arrangement of the crystalline structures. Such optically anisotropic materials are characterized by two indices of refraction. Hence, they are often referred to as **double refracting or birefringent materials**.

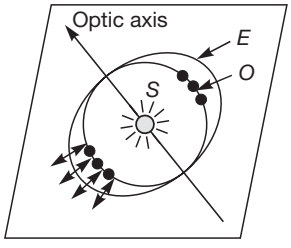
When an unpolarized beam of light enters a calcite crystal, it splits into two plane-polarized rays which travel with different velocities, corresponding to two different angles of refraction, as shown.



(a) When an unpolarized light beam is incident on a calcite crystal, it splits into an ordinary (O) ray and an extraordinary (E) ray. The rays are polarized in mutually perpendicular directions.

The two rays are polarized in two mutually perpendicular directions, as indicated by the dots and arrows. One ray called the **Ordinary (O) ray**, is characterized by an index of refraction μ_O that is the same in all directions, hence the ordinary ray has a spherical wavefront. The second ray, called the **Extraordinary (E) ray**, travels with different speeds in different directions and hence is characterized by an index of refraction μ_E that varies with the direction of propagation.

The wavefronts for the extraordinary ray are ellipsoids of revolution. Figure (b) illustrates the wavefronts associated with the ordinary and extraordinary rays, assuming a point source within the material.



(b) A point source S inside a doubly refracting crystal produces a spherical wavefront corresponding to the O ray and an elliptical wavefront corresponding to the E ray. The two waves propagate with the same velocity along the optic axis.

Note that there is one direction, called the **Optic axis**, along which the O and E rays have the same velocity, corresponding to the direction for which $\mu_O = \mu_E$. The difference in velocity for the two rays is a maximum in the direction perpendicular to the optic axis. For example, in calcite $\mu_O = 1.658$ at a wavelength of 589.3 nm, while μ_E varies from 1.658 along the optic axis to 1.486 perpendicular to the optic axis.

QUARTER WAVE PLATE

Quarter wave plate is a plate of a doubly refracting crystal, whose refracting faces are cut parallel to direction of optic axis and which produces a path difference of $\frac{\lambda}{4}$ between ordinary (O) and extraordinary (E) rays. If t is thickness of such plate, then

$$(\mu_O - \mu_E)t = \frac{\lambda}{4}$$

$$\Rightarrow \quad t = \frac{\lambda}{4(\mu_O - \mu_E)}$$

When a plane polarised light is incident on a quarter wave plate with its vibrations, making an angle of $45°$ with optic axis, the emergent light is circularly polarised. But if the vibrations of incident polarised light do not make an angle of $45°$ with optic axis, the emergent light is elliptically polarised.

HALF WAVE PLATE

Half wave plate is a plate of doubly refracting crystal (quartz or calcite), whose refracting faces are cut parallel to optic axis and whose thickness is such that it produces a path difference of $\frac{\lambda}{2}$ between (ordinary) O and E (extra ordinary) rays. If t is thickness of half wave plate, then

$$(\mu_O - \mu_E)t = \frac{\lambda}{2}$$

$$\Rightarrow \quad t = \frac{\lambda}{2(\mu_O - \mu_E)}$$

When a plane polarised light falls on a half wave plate, the emergent light is also plane polarised, but its direction of vibration is rotated through an angle 2θ with respect to incident light.

Conceptual Note(s)

(a) Polarisation is the property of transverse waves only. It is not shown by longitudinal waves.
(b) Plane of polarisation does not contain any component of vibrations in it.
(c) Extent of polarisation by reflection is maximum if the light is incident at polarising angle.
(d) Tangent of polarising angle is equal to the refractive index of medium upon which the light is incident.
(e) In double refraction, the two beams are polarised in mutually perpendicular planes.

ILLUSTRATION 45

The faces of a half wave plate are parallel to the optical axis of the crystal.

(i) What is the thinnest possible plate that would serve to put the ordinary and extra-ordinary rays of $\lambda = 5890$ Å, a half wave apart on their exist?
(ii) What multiples of this thickness would give the same result? The indices of refraction of quartz are $\mu_E = 1.553$, $\mu_0 = 1.544$.

SOLUTION

Given that $\lambda = 5890$ Å $= 5890 \times 10^{-8}$ cm, $\mu_E = 1.553$, $\mu_0 = 1.544$

(i) The thickness of the half wave plate is given by

$$t_{1/2} = \frac{\lambda}{2|\mu_0 - \mu_E|} = \frac{5890 \times 10^{-8}}{2|1.544 - 1.553|}$$

$$\Rightarrow \quad t_{1/2} = 32.7 \times 10^{-4} \text{ cm}$$

(ii) The other thicknesses which will give the same result are t, $3t$, $5t$, ….., $(2n+1)t$, where n is an integer.

ILLUSTRATION 46

A beam of linearly polarised is changed into circularly polarised light by passing it through a slice of crystal 0.003 cm thick. Calculate the difference in the refractive index of the two rays in the crystal assuming this to be minimum thickness that will produce the effect and that the wavelength is 6×10^{-5} cm.

SOLUTION

To convert a linearly polarised light into a circularly polarised light, a thickness equal to that of a quarter-wave plate is required which is given by

$$t_{1/2} = \frac{\lambda}{4(\mu_0 - \mu_e)}$$

Since $t = 0.003$ cm , $\lambda = 6 \times 10^{-5}$ m

$$\Rightarrow \quad \mu_0 - \mu_e = \frac{\lambda}{4t} = \frac{6 \times 10^{-5}}{4 \times 0.003} = 0.005$$

OPTICAL ACTIVITY AND SPECIFIC ROTATION (α)

The optical activity of pure liquids and solutions is measured by specific rotation α , which is defined as the rotation produced in the plane of vibration of plane polarised light by a decimeter length of solution having unit concentration of optically active substance i.e.,

$$\alpha(\lambda, T) = \frac{\theta}{lc}$$

where c is concentration, (in kgm^{-3}), l is length of solution tube (in metre) and θ is angle of rotation (in radian).

If length of solution tube is in cm, then

$$\alpha(\lambda, T) = \frac{10\theta}{lc}$$

The measured angle of rotation (θ) depends upon

(a) the type or nature of the sample e.g. sugar solution.
(b) concentrations (c) of optical active components.
(c) length (l) of sample tube.
(d) wavelength (λ) of light source.
(e) temperature (T) of the sample.

SI unit of α is $\text{radm}^2\text{kg}^{-1}$. However practically concentration c is measured in gcm^{-3}, length of solution tube l in decimetre (where 1 decimetre = 1 dm = 10 cm) and angle of rotation θ in degree. Therefore, the practical unit of α can also be written as degree cubic centimetre per gram per decimetre shortly written as $^\circ\text{cm}^3\text{g}^{-1}(\text{dm})^{-1}$.

ILLUSTRATION 47

Calculate the thickness of quartz plate cut with its faces perpendicular to the optic axis, which will produce the same rotation as that of a 0.1 m long solution of concentration $400\ \text{kg m}^{-3}$. Given specific rotation of quartz $380\ \text{rad m}^{-1}$ and that of sugar $0.011\ \text{radm}^2\text{kg}^{-1}$.

SOLUTION

Let t be the required thickness of the quartz plate. Then rotation produced by quartz in the plane of polarisation

$$\theta = 380t \qquad \ldots(1)$$

For sugar solution, we have

$$l = 0.1 \text{ m}, s = 0.011 \text{ rad m}^{-1}\ \text{kg}^{-1}\ \text{m}^3,$$

$$c = 400\ \text{kg m}^{-3}$$

$$\Rightarrow \quad \theta = slc = 0.011 \times 0.1 \times 400 \qquad \ldots(2)$$

From (1) and (2), we get

$$380t = 0.011 \times 0.1 \times 400$$

$$\Rightarrow \quad t = \frac{0.011 \times 0.1 \times 400}{380} = 1.6 \times 10^{-3} \text{ m}$$

Test Your Concepts-III

Based on Polarisation

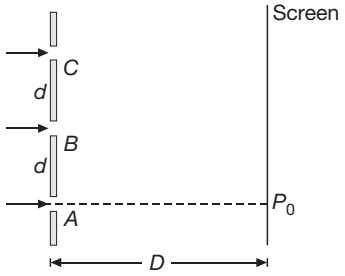
(Solutions on page H.126)

1. Two polarising sheets have their polarising directions parallel so that the intensity of the transmitted light is maximum. Through what angle must the either sheet be turned if the intensity is to drop by one-half?
2. A polariser and an analyser are oriented so that the maximum light is transmitted. What is the fraction of maximum light transmitted when analyser is rotated through (a) 30° (b) 60°?
3. Two polaroids are crossed to each other. If one of them is rotated through 60°, then what percentage of the incident unpolarised light will be transmitted by the polaroids?
4. Two polaroids are placed at 90° to each other and the transmitted intensity is zero. What happens when one more polaroid is placed between these two bisecting the angle between them?
5. A polaroid examines two adjacent plane-polarised light beams *A* and *B* whose planes of polarisation are mutually at right angles. In one position of the polaroid, the beam *B* shows zero intensity. From this position a rotation of 30° shows the two beams of equal intensities. Find the intensity ratio $\frac{I_A}{I_B}$ of the two beams.
6. Show that when a ray of light is incident on the surface of a transparent medium at the polarising angle, the reflected and transmitted rays are perpendicular to each other.
7. Unpolarised light of intensity 32 Wm^{-2} passes through three polarisers such that the transmission axis of the last polariser is crossed with the first, if the intensity of the emerging light is 3 Wm^{-2}, what is the angle between the transmission axes of the first two polarisers? At what angle will the transmitted intensity be maximum?
8. Yellow light is incident on the smooth surface of a block of dense flint glass for which the refractive index is 1.6640. Find the polarising angle. Also find the angle of refraction.
9. Calculate the thickness of (a) a quarter wave plate (b) a half wave plate, given that $\mu_e = 1.533$, $\mu_0 = 1.544$ and $\lambda = 5000$ Å.
10. Calculate the specific rotation if the plane of polarization is turned through 13.2°, traversing a length of 20 cm of 10% sugar solution.
11. A 5% solution taken in a decimetre tube produces an optical rotation of 20°. How much length of 10% solution of the same substance will produce a rotation of 30°?

SOLVED PROBLEMS

PROBLEM 1

Three equidistant slits of equal width being illuminated by a monochromatic parallel beam of light as shown in figure. A point P_0 is taken on the screen directly in front of A. If in this situation path difference $BP_0 - AP_0 = \frac{\lambda}{3}$ and $D \gg \lambda$. Show that the



(a) slit separation is given by $d = \sqrt{\frac{2\lambda D}{3}}$.

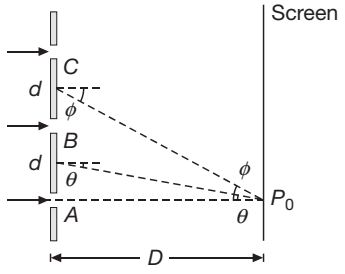
(b) intensity at P_0 is three times the intensity due to any of the three slits individually.

SOLUTION

(a) For calculating the path difference in the given situation, we redraw the arrangement as shown in figure. Since the path difference in the waves reaching from slit A and B to point P_0 is given by $BP_0 - AP_0 = \frac{\lambda}{3}$

Also, $BP_0 - AP_0 = d\sin\theta$

$$\Rightarrow \quad d\sin\theta = \frac{\lambda}{3}$$



For small θ, we can use $\sin\theta \approx \theta$

$$\Rightarrow \quad \theta d = \frac{\lambda}{3} \qquad \ldots(1)$$

From figure, for $D \gg d$, θ is given by

$$\theta = \frac{\frac{d}{2}}{D}$$

So, from equation (1), we get

$$d\left(\frac{d}{2D}\right) = \frac{\lambda}{3}$$

$$\Rightarrow \quad d = \sqrt{\frac{2\lambda D}{3}} \qquad \ldots(2)$$

(b) If we consider Δx_{AB} as the path difference between waves coming from A and B which is given as $\Delta x_{AB} = \frac{\lambda}{3}$. If ϕ_{AB} is the corresponding phase difference, then

$$\phi_{AB} = \frac{2\pi}{\lambda}\Delta x_{AB} = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{3}\right) = \frac{2\pi}{3}$$

So, amplitude of resultant wave obtained at P_0 due to sources A and B (each of amplitude a) is

$$a_{AB} = \sqrt{a^2 + a^2 + 2a^2\cos\left(\frac{2\pi}{3}\right)} = a$$

Similarly, for waves coming from slits B and C to point P_0, we use

$$\Delta x_{BC} = d\sin\phi, \text{ where } \sin\phi \approx \frac{d + \frac{d}{2}}{D} = \frac{3d}{2D}$$

$$\Rightarrow \quad \Delta x_{BC} = d\left(\frac{3d}{2D}\right) = \frac{3d^2}{2D} \qquad \ldots(3)$$

From equation (2), we have $d = \sqrt{\frac{2\lambda D}{3}}$

Substituting this value of d in equation (3), we get

$$\Delta x_{BC} = \frac{3d^2}{2D} = \frac{3}{2D}\left(\frac{2\lambda D}{3}\right) = \lambda$$

Thus the waves from slits B and C will reach point P_0 in same phase, so the resulting amplitude due to superposition of the waves at P_0 from slits B and C will become $a_{BC} = 2a$

Phase difference between waves arriving at P_0 from sources AB and BC is

$$\Delta\phi = \phi_{BC} - \phi_{AB} = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$$

So the resultant wave amplitude of the waves arriving at the point P_0 is given by

$$a_r = \sqrt{(a_{AB})^2 + (a_{BC})^2 + 2(a_{AB})(a_{AB})\cos\left(\frac{4\pi}{3}\right)}$$

where, $a_{AB} = a$, $a_{BC} = 2a$ and $\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$

$$\Rightarrow \quad a_r = \sqrt{a^2 + (2a)^2 + 2(a)(2a)\left(-\frac{1}{2}\right)} = \sqrt{3}a$$

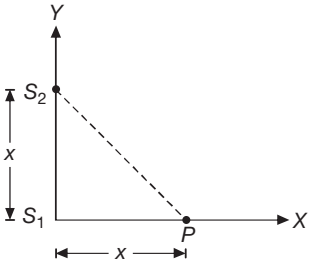
Since intensity of light is directly proportional to the square of amplitude, so we can conclude that intensity at point P_0 will be three times the intensity due to any of the three slits individually.

PROBLEM 2

An interference pattern is observed due to two coherent sources S_1 placed at origin and S_2 placed at $(0,\ 3\lambda,\ 0)$, where λ is the wavelength of the sources. A detector D is moved along the positive x-axis. Find the coordinates on the x-axis (excluding $x = 0$ and ∞) where maximum intensity is observed.

SOLUTION

At $x = 0$, path difference is 3λ. Hence, third order maxima will be obtained. At $x \to \infty$, path difference is zero. Hence, zero order maxima is obtained. In between first and second order maximas will be obtained.



For First order maxima, we have

$$S_2P - S_1P = \lambda$$

$$\Rightarrow \quad \sqrt{x^2 + 9\lambda^2} - x = \lambda$$

$$\Rightarrow \quad \sqrt{x^2 + 9\lambda^2} = x + \lambda$$

Squaring both sides, we get $x^2 + 9\lambda^2 = x^2 + \lambda^2 + 2x\lambda$
Solving this, we get

$$x = 4\lambda$$

For Second order maxima, we have

$$S_2P - S_1P = 2\lambda$$

$$\Rightarrow \quad \sqrt{x^2 - 9\lambda^2} - x = 2\lambda$$

$$\Rightarrow \quad \sqrt{x^2 + 9\lambda^2} = (x + 2\lambda)$$

Squaring both sides, we get

$$x^2 + 9\lambda^2 = x^2 + 4\lambda^2 + 4x\lambda$$

Solving, we get

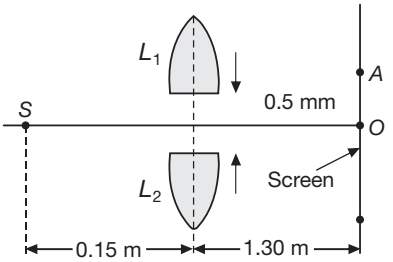
$$x = \frac{5}{4}\lambda = 1.25\lambda$$

Hence, the desired x coordinates are

$$x = 1.25\lambda \text{ and } x = 4\lambda$$

PROBLEM 3

In given figure, S is a monochromatic point source emitting light of wavelength $\lambda = 500$ nm. A thin lens of circular shape and focal length 0.10 m is cut into two identical halves L_1 and L_2 by a plane passing through a diameter. The two halves are placed symmetrically about the central axis SO with a gap of 0.5 mm. The distance along the axis from S to L_1 and L_2 is 0.15 m while that from L_1 and L_2 to O is 1.30 m. The screen at O is normal to SO.



(i) If the third intensity maximum excluding central maximum, occurs at the point A on the screen, find the distance OA.

(ii) If the gap between L_1 and L_2 is reduced from its original value of 0.5 mm, will the distance OA increase, decrease, or remain the same.

SOLUTION

(i) For the lens, $u = -0.15$ m, $f = +0.10$ m

Therefore, using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we get

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{(-0.15)} + \frac{1}{(0.10)}$$

$\Rightarrow \quad v = 0.3$ m

Linear magnification, $m = \frac{v}{u} = \frac{0.3}{-0.15} = -2$

Hence, two images S_1 and S_2 of S will be formed at 0.3 m from the lens as shown in figure. Image S_1 due to part 1 will be formed at 0.5 mm above its optics axis $(m = -2)$. Similarly, S_2 due to part 2 is formed 0.5 mm below the optic axis of this part as shown.

Hence, distance between S_1 and S_2 is $d = 1.5$ mm

Also, $D = 1.30 - 0.30 = 1.0$ m $= 10^3$ mm

and $\lambda = 500$ nm $= 5 \times 10^{-4}$ mm

So, fringe width is given by

$$\beta = \frac{\lambda D}{d} = \frac{(5 \times 10^{-4})(10^3)}{(1.5)} \text{ mm} = \frac{1}{3} \text{ mm}$$

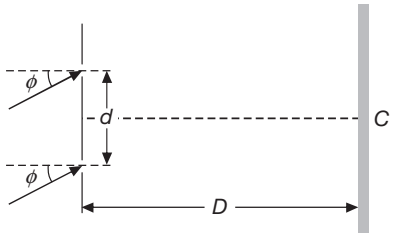
Now, as the point A is at the third maxima

$$\Rightarrow \quad OA = 3\beta = 3\left(\frac{1}{3}\right) = 1 \text{ mm}$$

(ii) If the gap between L_1 and L_2 is reduced, d will decrease. Hence, the fringe width β will increase or the distance OA will increase.

PROBLEM 4

Light of wavelength $\lambda = 500$ nm falls on two narrow slits placed a distance $d = 50 \times 10^{-4}$ cm apart, at an angle $\phi = 30°$ relative to the slits shown in figure. ON the lower slit a transparent slab of thickness 0.1 nm and refractive index $\frac{3}{2}$ is placed. The interference pattern is observed on a screen at a distance $D = 2$ m from the slits.



(a) Locate the position of the central maxima.

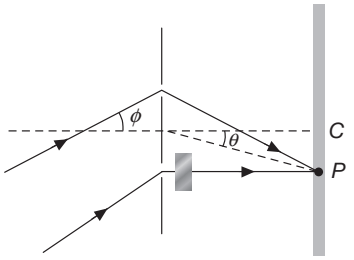
(b) Find the order of minima closest to centre C of screen.

(c) How many fringes will pass over C, if we remove the transparent slab from the lower slit?

SOLUTION

(a) Path difference is given by

$$\Delta x = d \sin\phi + d \sin\theta - (\mu - 1)t$$



For central maxima, $\Delta x = 0$

$$\Rightarrow \quad \sin\theta = \frac{(\mu - 1)t}{d} - \sin\phi$$

$$\Rightarrow \quad \sin\theta = \frac{\left(\frac{3}{2} - 1\right)(0.1)}{50 \times 10^{-3}} - \sin(30°) = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30°$$

(b) At C, $\theta = 0°$, so we get

$$\Delta x = d \sin\phi - (\mu - 1)t$$

$$\Rightarrow \quad \Delta x = (50 \times 10^{-3})\left(\frac{1}{2}\right) - \left(\frac{3}{2} - 1\right)(0.1)$$

$$\Rightarrow \quad \Delta x = 0.025 - 0.05 = -0.025 \text{ mm}$$

Substituting, $\Delta x = n\lambda$, we get

$$n = \frac{\Delta x}{\lambda} = \frac{-0.025}{500 \times 10^{-6}} = -50$$

Hence, at C there will be maxima. Therefore the order of minima closest to the C are -49.

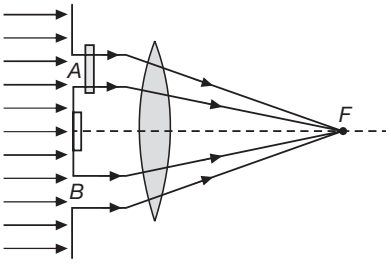
(c) Number of fringes shifted upwards is

$$N = \frac{(\mu - 1)t}{\lambda} = \frac{\left(\frac{3}{2} - 1\right)(0.1)}{500 \times 10^{-6}} = 100$$

PROBLEM 5

In a modified Young's double slit experiment, a monochromatic uniform and parallel beam of light of wavelength 6000 Å and intensity $\left(\frac{10}{\pi}\right)$ Wm^{-2} is

incident normally on two apertures A and B of radii 0.001 m and 0.002 m respectively. A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength of 6000 Å is placed in front of aperture A (shown in figure).



Calculate the power (in W) received at the focal spot F of the lens. The lens is symmetrically placed with respect to the apertures. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot.

SOLUTION

Power received by aperture A is given by

$$P_A = I\left(\pi r_A^2\right) = \frac{10}{\pi}(\pi)(0.001)^2 = 10^{-5} \text{ W} \quad \left\{\because I = \frac{P}{A}\right\}$$

Power received by aperture B is given by

$$P_B = I\left(\pi r_B^2\right) = \frac{10}{\pi}(\pi)(0.002)^2 = 4\times10^{-5} \text{ W}$$

Only 10% of P_A and P_B goes to the original direction, so

Portion of P_A going to original direction is $P_1 = 10^{-6}$ W

Portion of P_B going to original direction is $P_2 = 4\times10^{-6}$ W

Path difference created by slab is given by

$$\Delta x = (\mu - 1)t = (1.5-1)(2000) = 1000 \text{ Å}$$

Corresponding phase difference is given by

$$\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x = \frac{2\pi}{6000}\times1000 = \frac{\pi}{3}$$

Now, resultant power at the focal point is given by

$$P = P_1 + P_2 + 2\sqrt{P_1P_2}\cos\phi$$

$$\Rightarrow \quad P = 10^{-6} + 4\times10^{-6} + 2\sqrt{(10^{-6})(4\times10^{-6})}\cos\left(\frac{\pi}{3}\right)$$

$$\Rightarrow \quad P = 7\times10^{-6} \text{ W}$$

PROBLEM 6

A central portion with a width of $d = 0.5$ mm is cut out of a convergent lens having a focal length of $f = 10$ cm, as shown in figure. Both halves are tightly fitted against each other. The lens receives monochromatic light ($\lambda = 5000$ Å) from a point source at a distance of 5 cm from it.

(a) At what distance should a screen be fixed on the opposite side of the lens to observe three interference bands on it?

(b) What is the maximum possible number of interference bands that can be observed in this installation?

SOLUTION

Applying the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

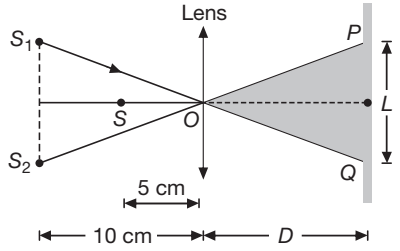
$$\frac{1}{v} + \frac{1}{5} = \frac{1}{10}$$

$$\Rightarrow \quad v = -10 \text{ cm}$$

$$\Rightarrow \quad m = \frac{v}{u} = \frac{-10}{-5} = 2$$

i.e., two virtual sources are formed with distance between them

$$d = 0.5 \text{ mm}$$



Fringe width $\beta = \frac{\lambda(D+10)}{d}$

Fringes are observed between the region P and Q (waves interfere in this region only), where

$$\frac{L}{D} = \frac{d}{10}$$

$$\Rightarrow \quad L = \frac{Dd}{10}$$

Number of fringes that can be observed on the screen is given by

$$N = \frac{L}{\beta} = \frac{d^2 D}{10\lambda(D+10)} \qquad \ldots(1)$$

Substituting the values, we get

$$3 = \frac{(0.05)^2 D}{10 \times 5 \times 10^{-5}(D+10)}$$

Solving this equation, we get

$$D = 15 \text{ cm}$$

From equation (1), we have

$$N = \frac{d^2}{10\lambda\left(1+\frac{10}{D}\right)}$$

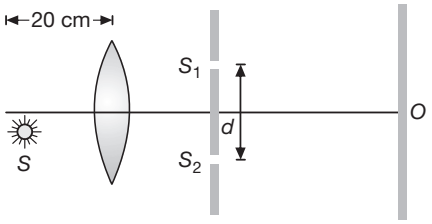
N will be maximum when $D \to \infty$

$$\Rightarrow \quad \frac{10}{D} \to 0$$

$$\Rightarrow \quad N_{\max} = \frac{d^2}{10\lambda} = \frac{(0.05)^2}{10 \times 5 \times 10^{-5}} = 5$$

PROBLEM 7

A point source is placed at a distance $\frac{d}{2}$ below the principal axis of an equiconvex lens of refractive index $\frac{3}{2}$ and radius 20 cm. The emergent light from lens having wavelength $\lambda = 5000$ Å falls on the slits S_1 and S_2 separated by $d = 1$ mm which are placed symmetrically along the principal axis. The resulting interference pattern is observed on the screen kept at a distance $D = 1$ m from the slit plane.



(a) Find the position of central maxima and its width
(b) Find the intensity at point O.

SOLUTION

Using Lens Maker's Formula, we get

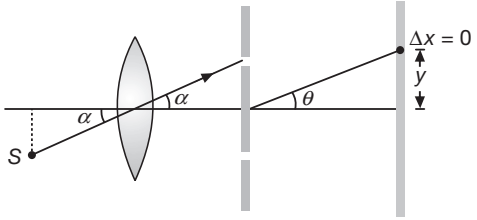
$$\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{1}{20}-\frac{1}{-20}\right)$$

$$\Rightarrow \quad f = 20 \text{ cm}$$

Since source lies in focal plane of lens. So, all the emergent rays will be parallel. So,

$$\tan\alpha = \frac{d/2}{20} = \frac{d}{40} = \frac{1}{400} \approx \sin\alpha$$

Initial path difference $(\Delta x)_{\text{initial}} = d\sin\alpha$



Let the central maxima is obtained at angle θ. Then

$$d\sin\theta = d\sin\alpha$$

$$\Rightarrow \quad \sin\theta = \sin\alpha$$

$$\Rightarrow \quad \tan\theta = \tan\alpha$$

$$\Rightarrow \quad \frac{y}{D} = \frac{d}{40}$$

$$\Rightarrow \quad y = \frac{Dd}{40}$$

$$\Rightarrow \quad y = \frac{100}{40}d = 2.5d = 2.5 \text{ mm}$$

At O, net path difference is given by

$$\Delta x = d\sin\alpha = \frac{1}{400} \text{ mm}$$

Since, $\phi = \frac{2\pi}{\lambda}\Delta x$

$$\Rightarrow \quad \phi = \frac{2\pi}{5000 \times 10^{-10}} \times \frac{1}{400} \times 10^{-3}$$

$$\Rightarrow \quad \phi = 10\pi$$

Since, $I = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$

$$\Rightarrow \quad I = I_{\max}$$

PROBLEM 8

In a given $YDSE$ setup, the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å. It is found that the point P on the screen where the central maximum $(n=0)$ fell before the glass plates were inserted now has $\frac{3}{4}$ the original intensity. It is further observed that what used to be the 5^{th} maximum earlier, lies below the point P while the 6^{th} minimum lies above P. Calculate the thickness of the glass plate. Absorption of light by glass plate may be neglected.

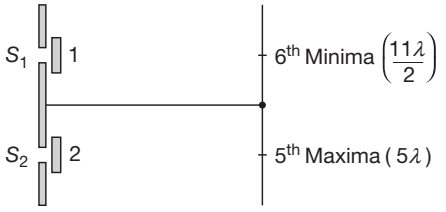
SOLUTION

As the given refractive indices of the glass plates are $\mu_1 = 1.4$ and $\mu_2 = 1.7$ and if t be the thickness of each glass plate then the path difference at the screen centre O, due to insertion of glass plates will be

$$\Delta x = (\mu_2 - \mu_1)t = (1.7 - 1.4)t$$

$$\Rightarrow \quad \Delta x = 0.3t \qquad \ldots(1)$$

As 5^{th} maxima (earlier) lies below point O and 6^{th} minima lies above point O, this path difference must lie between 5λ and 5.5λ. This situation is shown in figure.



Due to the path difference Δx, the phase difference at O will be

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}(5\lambda + \Delta_1)$$

$$\Rightarrow \quad \phi = \left(10\pi + \frac{2\pi}{\lambda}\Delta_1\right) \qquad \ldots(2)$$

In above equation Δ_1 is considered less than $\frac{\lambda}{2}$.

Intensity at point O is given $\frac{3}{4}I_{max}$ so we use the intensity at a point where the phase difference between the two waves is ϕ is given as

$$I(\phi) = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$

$$\Rightarrow \quad \frac{3}{4}I_{max} = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$

$$\Rightarrow \quad \frac{3}{4} = \cos^2\left(\frac{\phi}{2}\right)$$

$$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{6}$$

$$\Rightarrow \quad \phi = \frac{\pi}{3} \qquad \ldots(3)$$

From equation, (2) and (3), we get

$$\Delta_1 = \frac{\lambda}{6}$$

So, path difference at O is given by

$$\Delta x = 5\lambda + \frac{\lambda}{6} = \frac{31}{6}\lambda \qquad \ldots(4)$$

Since from (1), $\Delta x = 0.3t$

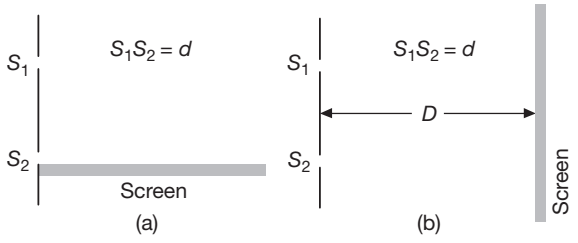
$$\Rightarrow \quad 0.3t = \frac{31\lambda}{6}$$

$$\Rightarrow \quad t = \frac{31\lambda}{6(0.3)} = \frac{(31)(5400\times10^{-10})}{1.8}\ \text{m}$$

$$\Rightarrow \quad t = 9.3\times10^{-6}\ \text{m} = 9.3\ \mu\text{m}$$

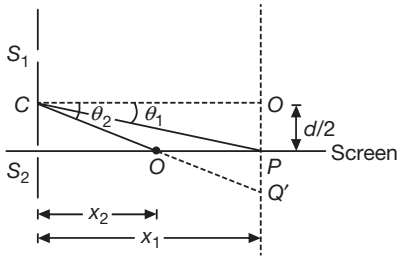
PROBLEM 9

The interference pattern of a Young's double slit experiment is observed in two ways by placing the screen in two possible ways as shown in figure (a) and (b). The distance between two consecutive right most minima on the screen of figure (a) using light of wavelength $\lambda_1 = 4000$ Å is observed to be 600 times the fringe width in the screen of figure (b) using the wavelength $\lambda_2 = 6000$ Å. If D (as shown in figure) is 1 m then find the separation between the coherent sources S_1 and S_2. Given that $d > \frac{3\lambda_1}{2}$.



SOLUTION

Had the screen been perpendicular to S_2P, then P and Q' would have been the positions of first and second minima (last two).



Since, the angular positions of minima do not depend on the position of the screen, so the second minima is formed at Q on the screen.

For right most minima at P, we have

$$d\sin\theta_1 = \frac{\lambda_1}{2} \qquad \ldots(1)$$

For small angles, we have

$$\sin\theta_1 \approx \tan\theta_2 = \frac{d/2}{x_2}$$

Substituting in equation (1), we get

$$x_2 = \frac{d^2}{\lambda_1} \qquad \ldots(2)$$

For next minima at Q, we have

$$d\sin\theta_2 = \frac{3}{2}\lambda_1 \qquad \ldots(3)$$

For small angles, we have

$$\sin\theta_2 \approx \tan\theta_2 = \frac{\frac{d}{2}}{x_2}$$

Substituting in equation (3), we get

$$x_2 = \frac{d^2}{3\lambda_1} \qquad \ldots(4)$$

$$\Rightarrow \quad PQ = x_1 - x_2 = \frac{2d^2}{3\lambda_1} \qquad \ldots(5)$$

In the second case, fringe width is given by

$$\beta = \frac{\lambda_2 D}{d} \qquad \ldots(6)$$

Since it is given that $PQ = 600\beta$

$$\Rightarrow \quad \frac{2d^2}{3\lambda_1} = 600\frac{\lambda_2 D}{d}$$

$$\Rightarrow \quad d^3 = 900\lambda_1\lambda_2 D$$

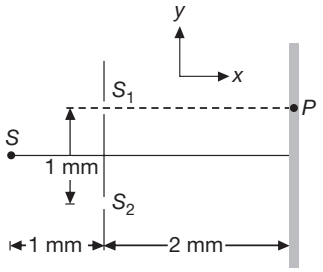
$$\Rightarrow \quad d^3 = 900\times 4000\times 6000\times 10^{-20}\times 1$$

$$\Rightarrow \quad d^3 = 216\times 10^{-12}$$

$$\Rightarrow \quad d = 6\times 10^{-4}\ \text{m} = 0.6\ \text{mm}$$

PROBLEM 10

In a Young's double slit experiment set-up source S of wavelength 5000 Å illuminates two slits S_1 and S_2, which act as two coherent sources. The source S oscillates about its shown position according to the equation $y = 0.5\sin(\pi t)$, where y is in millimetres and t in seconds.



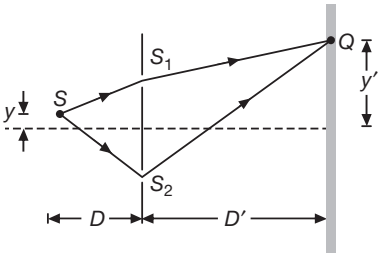
(a) Locate the position of the central maxima as a function of time.

(b) Calculate the minimum value of t for which the intensity at point P on the screen exactly in front of the upper slit becomes maximum.

SOLUTION

(a) Net path difference of the waves reaching at Q, is

$$\Delta x = \frac{yd}{D} + \frac{y'd}{D'}$$



For central maximum, $\Delta x = 0$

$$\Rightarrow \quad y' = -\frac{D'}{D}y'$$

$$\Rightarrow \quad y' = -\left(\frac{2}{1}\right)(0.5\sin(\pi t))$$

$\Rightarrow \quad y' = -\sin(\pi t)$ mm

(b) $y' = \dfrac{d}{2}$, at point P exactly in front of S_1, so we have

$$\Delta x = \left(\frac{yd}{D}\right) + \frac{\left(\frac{d^2}{2}\right)}{D'}$$

For maximum intensity, we have path difference to be an even multiple of $\dfrac{\lambda}{2}$, so

$$\Delta x = (2n)\frac{\lambda}{2} = n\lambda$$

Substituting the values, we get

$$0.5\sin(\pi t) + 0.25 = 0.5n$$

$$\Rightarrow \quad \sin(\pi t) = \frac{0.5n - 0.25}{0.5}$$

For minimum value of t, we have $n = 1$

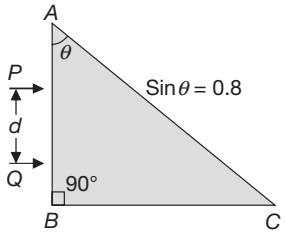
$$\Rightarrow \quad \sin(\pi t) = 0.5$$

$$\Rightarrow \quad \pi t = \frac{\pi}{6}$$

$$\Rightarrow \quad t = \frac{1}{6} = 0.167 \text{ s}$$

PROBLEM 11

Two parallel beams of light P and Q (separation d) containing radiations of wavelengths 4000 Å and 5000 Å (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown in figure. The refractive index of the prism as a function of wavelength is given by the relation, $\mu(\lambda) = 1.20 + \dfrac{b}{\lambda^2}$ where λ is in Å and b is positive constant. The value of b is such that the condition for total reflection at the face AC is just satisfied for one wavelength and is not satisfied for the other.



(a) Find the value of b

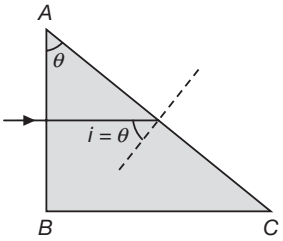
(b) Find the deviation of the beams transmitted through the face AC

(c) A convergent lens is used to bring these transmitted beams into focus. If the intensities of the upper and the lower beams immediately after transmission from the face AC, are $4I$ and I respectively, find the resultant intensity at the focus.

SOLUTION

(a) Total internal reflection (TIR) will take place first for those wavelength for which critical angle is small or μ is large.

From the given expression of μ, it is more for the wavelength for which value of λ is less.



Thus, condition of TIR is just satisfied for 4000 Å

$\Rightarrow \quad i = C$ for 4000 Å

$\Rightarrow \quad \theta = C$

$\Rightarrow \quad \sin\theta = \sin C$

Since $\sin\theta = 0.8$ and $\sin C = \dfrac{1}{\mu}$, so we get

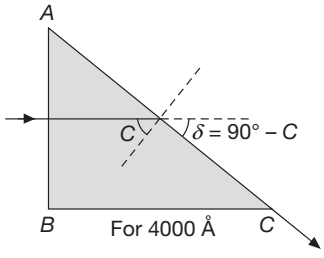
$$\Rightarrow \quad 0.8 = \frac{1}{\mu} \qquad \text{(for 4000 Å)}$$

$$\Rightarrow \quad 0.8 = \frac{1}{1.20 + \dfrac{b}{(4000)^2}}$$

Solving this equation, we get

$$b = 8.0 \times 10^5 \ (\text{Å})^2$$

(b) For, 4000 Å condition of TIR is just satisfied. Hence, it will emerge from AC, just grazingly.

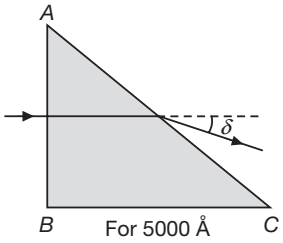


So, deviation for 4000 Å is given by

$$\Rightarrow \quad \delta_{4000\ \text{Å}} = 90 - i = 90 - \sin^{-1}(0.8) \approx 37^\circ$$

For 5000 Å, we have

$$\mu = 1.2 + \frac{b}{\lambda^2} = 1.2 + \frac{8\times10^5}{(5000)^2} = 1.232$$



Applying, $\mu = \dfrac{\sin i_{\text{air}}}{\sin i_{\text{medium}}}$

$$\Rightarrow \quad 1.232 = \frac{\sin i_{\text{air}}}{\sin\theta} = \frac{\sin i_{\text{air}}}{0.8}$$

$$\Rightarrow \quad i_{\text{air}} = 80.26^\circ$$

So, deviation for 5000 Å is given by

$$\delta_{5000\ \text{Å}} = i_{\text{air}} - i_{\text{medium}} = 80.26^\circ - \sin^{-1}(0.8) = 27.13^\circ$$

(c) The intensity of the upper beam (4000 Å) and lower beam (5000 Å) after transmission are $4I$ and I respectively, then

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Since no phase change takes place for the waves refracted from the lens, so $\phi = 0^\circ$.

$$\Rightarrow \quad I_R = 4I + I + 2\sqrt{(4I)I}\cos(0^\circ)$$

$$\Rightarrow \quad I_R = 9I$$

PROBLEM 12

A glass plate of refractive index 1.5 is coated with a thin layer of thickness t and refractive index 1.8. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. Write the condition for their constructive interference. If $\lambda = 648$ nm, obtain the least value of t for which the rays interfere constructively.

SOLUTION

Incident ray AB is partly reflected as ray 1 from the upper surface and partly reflected as ray 2 from the lower surface of the layer of thickness t and refractive index $\mu_1 = 1.8$ as shown in figure. Path difference between the two rays would be

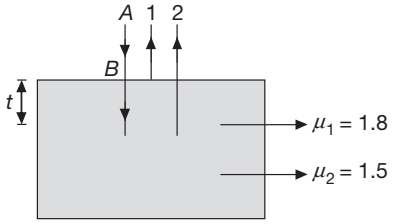
$$\Delta x = 2\mu_1 t = 2(1.8)t = 3.6t$$

Ray 1 is reflected from a denser medium, therefore, it undergoes a phase change of π, whereas the ray 2 gets reflected from a rarer medium, therefore, there is no change in phase of ray 2.

Hence, phase difference between rays 1 and 2 would be $\Delta\phi = \pi$. Therefore, condition of constructive interference will be

$$\Delta x = \left(n - \frac{1}{2}\right)\lambda \qquad \text{where } n = 1, 2, 3, \ldots$$

$$\Rightarrow \quad 3.6t = \left(n - \frac{1}{2}\right)\lambda$$



Least values of t is corresponding to $n = 1$ or

$$t_{\min} = \frac{\lambda}{2\times3.6}$$

$$\Rightarrow \quad t_{\min} = \frac{648}{7.2}\ \text{nm}$$

$$\Rightarrow \quad t_{\min} = 90\ \text{nm}$$

Conceptual Note(s)

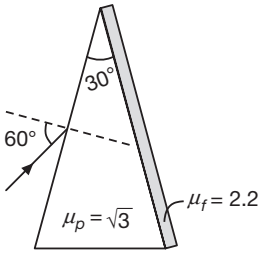
(a) For a wave (whether it is sound or electromagnetic), a medium is denser or rarer is decided from the speed of wave in that medium. In denser medium speed of wave is less. For example, water is rarer for sound, while denser for light compared to air because speed of sound in water is more than in air, while speed of light is less.

(b) In transmission/refraction, no phase change takes place. In reflection, there is a change of phase of π when it is reflected by a denser medium and phase change is zero if it is reflected by a rarer medium.

(c) If two waves in phase interfere having a path difference of Δx; then condition of maximum intensity would be $\Delta x = n\lambda$, $n = 0, 1, 2, ...$
But if two waves, which are already out of phase (a phase difference of π) interfere with path difference Δx, then condition of maximum intensity will be $\Delta x = \left(n - \frac{1}{2}\right)\lambda$, $n = 1, 2, ...$

PROBLEM 13

Shown in the figure is a prism of refracting angle $30°$ and refractive index $\mu_p\,(=\sqrt{3})$. The face AC of the prism is covered with a thin film of refractive index $\mu_f\,(=2.2)$. A monochromatic light of wavelength $\lambda = 550$ nm falls on the face AB at an angle of incidence of $60°$. Calculate



(a) the angle of emergence.
(b) the minimum value of thickness t of the coated film so that the intensity of the emergent ray is maximum.

SOLUTION

(a) Applying Snell's Law, we get

$$\sin i_1 = \mu \sin r_1$$

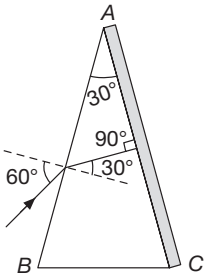
$$\Rightarrow \sin(60°) = \sqrt{3}\,\sin r_1$$

$$\Rightarrow \sin r_1 = \frac{1}{2}$$

$$\Rightarrow r_1 = 30°$$

Since, $r_1 + r_2 = A$

$$\Rightarrow r_2 = A - r_1 = 30° - 30° = 0°$$



Therefore, ray of light falls normally on the face AC and angle of emergence $e = i_2 = 0°$.

(b) Multiple reflection occurs between surfaces of film. Intensity will be maximum if interference takes place in the transmitted wave.

For maximum thickness, we have

$$\Delta x = 2\mu t = \lambda$$

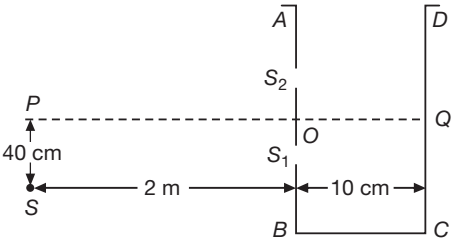
where t is the thickness of coated film

$$\Rightarrow t = \frac{\lambda}{2\mu} = \frac{550}{2 \times 2.2} = 125 \text{ nm}$$

PROBLEM 14

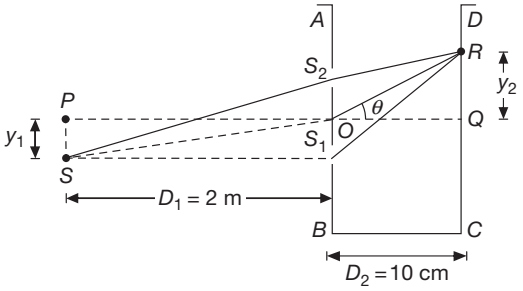
A vessel $ABCD$ of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of S_1 and S_2. A monochromatic light source is kept at S, 40 cm below P and 2 m from the vessel, to illuminate the slits as shown in the figure alongside.

(a) Calculate the position of the central bright fringe on the other wall CD with respect to the line OQ.
(b) Now, a liquid is poured into the vessel and filled upto OQ. The central bright fringe is found to be at Q. Calculate the refractive index of the liquid.



SOLUTION

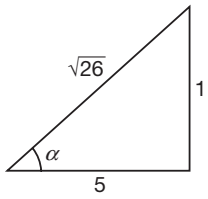
(a) Given $y_1 = 40$ cm, $D_1 = 2$ m $= 200$ cm, $D_2 = 10$ cm



$$\tan\alpha = \frac{y_1}{D_1} = \frac{40}{200} = \frac{1}{5}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{1}{5}\right)$$

$$\Rightarrow \sin\alpha = \frac{1}{\sqrt{26}} \approx \frac{1}{5} = \tan\alpha$$



Path difference between SS_1 and SS_2 is

$$\Delta x_1 = SS_1 - SS_2$$

$$\Rightarrow \Delta x_1 = d\sin\alpha = (0.8 \text{ mm})\left(\frac{1}{5}\right)$$

$$\Rightarrow \Delta x_1 = 0.16 \text{ mm} \quad \ldots(1)$$

Now, let at point R on the screen, central bright fringe is observed (i.e., net path difference $= 0$). Path difference between S_2R and S_1R is

$$\Delta x_2 = S_2R - S_1R$$

$$\Rightarrow \Delta x_2 = d\sin\theta \quad \ldots(2)$$

Central bright fringe will be observed when net path difference is zero.

$$\Rightarrow \Delta x_2 - \Delta x_1 = 0$$

$$\Rightarrow \Delta x_2 = \Delta x_1$$

$$\Rightarrow d\sin\theta = 0.16$$

$$\Rightarrow (0.8)\sin\theta = 0.16$$

$$\Rightarrow \sin\theta = \frac{0.16}{0.8} = \frac{1}{5}$$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{24}}$$

$$\Rightarrow \sin\theta \approx \frac{1}{5}$$

$$\Rightarrow \tan\theta \approx \sin\theta = \frac{y_2}{D_2} = \frac{1}{5}$$

$$\Rightarrow y_2 = \frac{D_2}{5} = \frac{10}{5} = 2 \text{ cm}$$

Therefore, central bright fringe is observed at 2 cm above point Q on side CD.

Alternate solution for (a)

Δx at R will be zero if $\Delta x_1 = \Delta x_2$

$$\Rightarrow d\sin\alpha = d\sin\theta$$

$$\Rightarrow \alpha = \theta$$

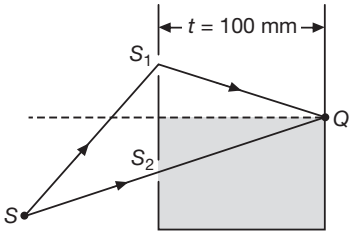
$$\Rightarrow \tan\alpha = \tan\theta$$

$$\Rightarrow \frac{y_1}{D_1} = \frac{y_2}{D_2}$$

$$\Rightarrow y_2 = \left(\frac{D_2}{D_1}\right)y_1 = \left(\frac{10}{200}\right)(40) \text{ cm}$$

$$\Rightarrow y_2 = 2 \text{ cm}$$

(b) The central bright fringe will be observed at point Q, if the path difference created by the liquid slab of thickness $t = 10$ cm or 100 mm is equal to Δx_1, so that the net path difference at Q becomes zero.



$$\Rightarrow (\mu - 1)t = \Delta x_1$$

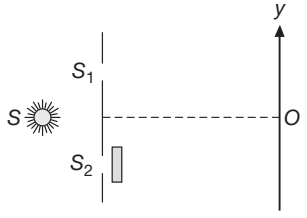
$$\Rightarrow (\mu - 1)(100) = 0.16$$

$$\Rightarrow \mu - 1 = 0.0016$$

$$\Rightarrow \mu = 1.0016$$

PROBLEM 15

The Young's double slit experiment is done in a medium of refractive index $\frac{4}{3}$. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10.4 μm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown in the figure.



(a) Find the location of central maximum (bright fringe with zero path difference) on the y-axis.

(b) Find the light intensity of point O relative to the maximum fringe intensity.

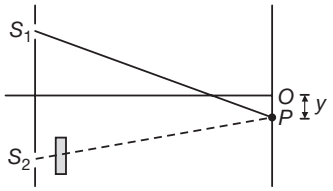
(c) Now, if 600 nm light is replaced by white light of range 400 to 700 nm, find the wavelengths of the light that form maxima exactly at point O.

[All wavelengths in the problem are for the given medium of refractive index $\frac{4}{3}$. Ignore dispersion]

SOLUTION

Given $\lambda = 600 \text{ nm} = 6\times10^{-7} \text{ m}$,

$d = 0.45 \text{ mm} = 0.45\times10^{-3} \text{ m}$ and $D = 1.5 \text{ m}$



Thickness of glass sheet, $t = 10.4\ \mu\text{m} = 10.4\times10^{-6} \text{ m}$

Refractive index of the medium, $\mu_m = \frac{4}{3}$

And refractive index of glass sheet, $\mu_g = 1.5$

(a) Let central maximum is obtained at a distance y below point O.

$$\Rightarrow \Delta x_1 = S_1P - S_2P = \frac{yd}{D}$$

Path difference due to glass sheet is given by

$$\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$

Net path difference will be zero, when we have

$$\Delta x_1 = \Delta x_2$$

$$\Rightarrow \frac{yd}{D} = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$

$$\Rightarrow y = \left(\frac{\mu_g}{\mu_m} - 1\right)t\frac{D}{d}$$

Substituting the values, we get

$$y = \left(\frac{1.5}{4/3} - 1\right)\frac{10.4\times10^{-6}(1.5)}{0.45\times10^{-3}}$$

$$\Rightarrow y = 4.33\times10^{-3} \text{ m}$$

$$\Rightarrow y = 4.33 \text{ mm}$$

(b) At O, $\Delta x_1 = 0$ and $\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$

So, net path difference is

$$\Delta x = \Delta x_2$$

Corresponding phase difference, $\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x$

Substituting the values, we get

$$\phi = \Delta\phi = \frac{2\pi}{6\times10^{-7}}\left(\frac{1.5}{4/3} - 1\right)\left(10.4\times10^{-6}\right)$$

$$\Rightarrow \phi = \left(\frac{13}{3}\right)\pi$$

Now, $I(\phi) = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$

$$\Rightarrow I = I_{\max}\cos^2\left(\frac{13\pi}{6}\right)$$

$$\Rightarrow I = \frac{3}{4}I_{\max}$$

(c) At O, path difference is $\Delta x = \Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$

For maximum intensity at O, we have

$\Delta x = n\lambda$, where $n = 1, 2, 3, \ldots\ldots$

$$\Rightarrow \lambda = \frac{\Delta x}{1}, \frac{\Delta x}{2}, \frac{\Delta x}{3}, \ldots\ldots \text{ and so on}$$

$$\Rightarrow \Delta x = \left(\frac{1.5}{4/3} - 1\right)\left(10.4\times10^{-6} \text{ m}\right)$$

$$\Rightarrow \Delta x = \left(\frac{1.5}{4/3} - 1\right)\left(10.4\times10^{3} \text{ nm}\right) = 1300 \text{ nm}$$

So, maximum intensity will be corresponding to

$$\lambda = 1300 \text{ nm}, \frac{1300}{2} \text{ nm}, \frac{1300}{3} \text{ nm}, \frac{1300}{4} \text{ nm}, \ldots$$

$$\Rightarrow \lambda = 1300 \text{ nm}, 650 \text{ mm}, 433.33 \text{ nm}, 325 \text{ nm}, \ldots.$$

The wavelength in the range 400 nm to 700 nm are 650 nm and 433.33 nm.

PROBLEM 16

In Young's experiment, the source is red light of wavelength 7×10^{-7} m. When a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beams, the central bright fringe shifts by 10^{-3} m to the position previously occupied

by the 5th bright fringe. Find the thickness of the plate. When the source is now changed to green light of wavelength 5×10^{-7} m, the central fringe shifts to a position initially occupied by the 6th bright fringe due to red light. Find the refractive index of glass for green light. Also estimate the change in fringe width due to the change in wavelength.

SOLUTION

Path difference due to the glass slab,

$$\Delta x = (\mu - 1)t = (1.5 - 1)t = 0.5t$$

Due to this slab, 5 red fringes have been shifted upwards. So, we have

$$\Delta x = 5\lambda_{red}$$

$$\Rightarrow \quad 0.5t = (5)(7\times10^{-7}\ \text{m})$$

$$\Rightarrow \quad t = \text{thickness of glass slab} = 7\times10^{-6}\ \text{m}$$

Let μ' be the refractive index for green light, then

$$\Delta x' = (\mu' - 1)t$$

Now the shifting is of 6 fringes of red light. So, we have

$$\Delta x' = 6\lambda_{red}$$

$$\Rightarrow \quad (\mu' - 1)t = 6\lambda_{red}$$

$$\Rightarrow \quad (\mu' - 1) = \frac{(6)(7\times10^{-7})}{7\times10^{-6}} = 0.6$$

$$\Rightarrow \quad \mu' = 1.6$$

Since the shifting of 5 bright fringes was equal to 10^{-3} m

$$\Rightarrow \quad 5\beta_{red} = 10^{-3}\ \text{m}, \text{ where } \beta \text{ is the Fringe width}$$

$$\Rightarrow \quad \beta_{red} = \frac{10^{-3}}{5}\ \text{m} = 0.2\times10^{-3}\ \text{m}$$

Now since $\beta = \frac{\lambda D}{d}$

$$\Rightarrow \quad \beta \propto \lambda$$

$$\Rightarrow \quad \frac{\beta_{green}}{\beta_{red}} = \frac{\lambda_{green}}{\lambda_{red}}$$

$$\Rightarrow \quad \beta_{green} = \beta_{red}\frac{\lambda_{green}}{\lambda_{red}} = (0.2\times10^{-3})\left(\frac{5\times10^{-7}}{7\times10^{-7}}\right)$$

$$\Rightarrow \quad \beta_{green} = 0.143\times10^{-3}\ \text{m}$$

$$\Rightarrow \quad \Delta\beta = \beta_{green} - \beta_{red} = (0.143 - 0.2)\times10^{-3}\ \text{m}$$

$$\Rightarrow \quad \Delta\beta = -5.71\times10^{-5}\ \text{m}$$

PROBLEM 17

A double slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å.

(i) Calculate the fringe width.

(ii) One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minimum as the axis.

SOLUTION

Given $\mu = 1.33$, $d = 1$ mm, $D = 1.33$ m and $\lambda = 6300$ Å

(i) Wavelength of light in the given liquid is

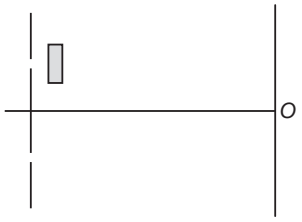
$$\lambda' = \frac{\lambda}{\mu} = \frac{6300}{1.33}\ \text{Å} \approx 4737\ \text{Å} = 4737\times10^{-10}\ \text{m}$$

$$\Rightarrow \text{ Fringe width, } \beta = \frac{\lambda' D}{d}$$

$$\Rightarrow \quad \beta = \frac{(4737\times10^{-10}\ \text{m})(1.33\ \text{m})}{(1\times10^{-3}\ \text{m})}$$

$$\Rightarrow \quad \beta = 6.3\times10^{-4}\ \text{m} = 0.63\ \text{mm}$$

(ii) Let t be the thickness of the glass slab.



Path difference due to glass slab at centre O is given by

$$\Delta x = \left(\frac{\mu_{glass}}{\mu_{liquid}} - 1\right)t = \left(\frac{1.53}{1.33} - 1\right)t$$

$$\Rightarrow \quad \Delta x = 0.15t$$

Now, for the intensity to be minimum at O, this path difference should be equal to $\frac{\lambda'}{2}$

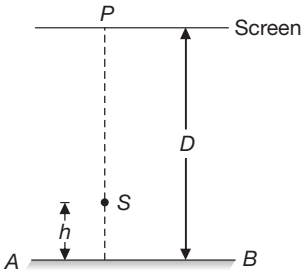
$$\Rightarrow \quad \Delta x = \frac{\lambda'}{2}$$

$$\Rightarrow \quad 0.15t = \frac{4737}{2} \text{ Å}$$

$$\Rightarrow \quad t = 15790 \text{ Å} = 1.579 \ \mu\text{m}$$

PROBLEM 18

A point source S emitting light of wavelength 600 nm is placed at a very small height h above a flat reflecting surface AB (shown in figure). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance D from it.



(a) What is the shape of the interference fringes on the screen?

(b) Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point P (shown in the figure).

(c) If the intensity at point P corresponds to a maximum, calculate the minimum distance through which the reflecting surface AB should be shifted so that the intensity at P again becomes maximum.

SOLUTION

(a) Since there is symmetry about the line SP, so the shape of the interference fringes will be circular.

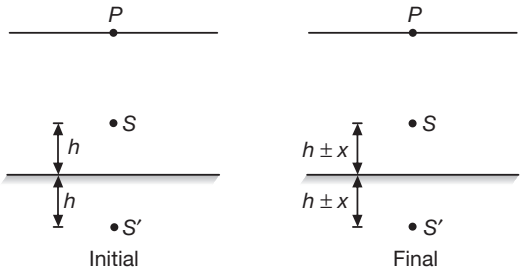
(b) Intensity of light reaching on the screen directly from the source $I_1 = I_0$ (say) and intensity of light reaching on the screen after reflecting from the mirror is $I_2 = 36\%$ of $I_0 = 0.36I_0$.

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{I_0}{0.36I_0} = \frac{1}{0.36}$$

$$\Rightarrow \quad \sqrt{\frac{I_1}{I_2}} = \frac{1}{0.6}$$

$$\Rightarrow \quad \frac{I_{\min}}{I_{\max}} = \frac{\left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2} = \frac{\left(\frac{1}{0.6} - 1\right)^2}{\left(\frac{1}{0.6} + 1\right)^2} = \frac{1}{16}$$

(c) Initially path difference at P between two waves reaching from S and S' is as shown.



Since the ray is reflected from the surface of a denser medium, so it suffers an additional path change of $\frac{\lambda}{2}$ or a phase change of π.

For maximum at P, path difference equals $n\lambda$.

If AB is shifted by x, then this will cause an additional path difference of $2\left(x - \frac{\lambda}{2}\right)$ (for object and its image taken as coherent sources). Since reflection takes place at surface of denser medium, so this will produce an additional phase change of π or a path change of $\frac{\lambda}{2}$. So, we get

$$2\left(x - \frac{\lambda}{2}\right) = n\lambda$$

$$2x - \lambda = n\lambda$$

$$\Rightarrow \quad 2x = (n+1)\lambda$$

$$\Rightarrow \quad x = (n+1)\frac{\lambda}{2} \text{ where } n = 0, 1, 2, 3, \ldots.$$

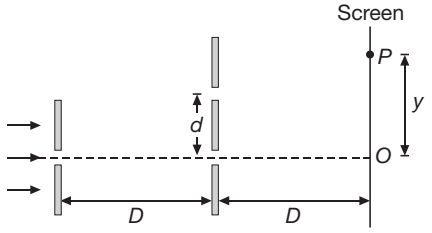
Now, to get minimum value of x, n must be minimum i.e., $n = 0$

$$\Rightarrow \quad x = \frac{\lambda}{2}$$

$$\Rightarrow \quad x = \frac{600}{2} = 300 \text{ nm}$$

PROBLEM 19

Consider the arrangement shown in figure. The distance D is large compared to the separation d between the slits.



(a) Find the minimum value of d so that there is a dark fringe at O.

(b) Suppose d has this value. Find the distance x at which the next bright fringe is formed.

(c) Find the fringe width.

SOLUTION

(a) The path difference at O is given as

$$\Delta x = 2\sqrt{D^2 + d^2} - 2D$$

For the dark fringe at O, this path difference should be

$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots$$

For minimum value of d, we have

$$\Delta x = 2\sqrt{D^2 + d^2} - 2D = \frac{\lambda}{2}$$

$$\Rightarrow \left(D^2 + d^2\right)^{\frac{1}{2}} - D = \frac{\lambda}{4}$$

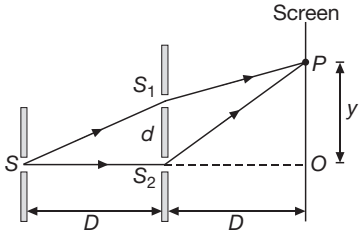
$$\Rightarrow D\left(1 + \frac{d^2}{D^2}\right)^{\frac{1}{2}} - D = \frac{\lambda}{4}$$

$$\Rightarrow D\left(1 + \frac{d^2}{2D^2}\right) - D = \frac{\lambda}{4}$$

$$\Rightarrow D + \frac{d^2}{2D} - D = \frac{\lambda}{4}$$

$$\Rightarrow d = \sqrt{\frac{D\lambda}{2}}$$

(b) At the above calculated value of d, first bright fringe will be obtained at a position where the path difference between the two waves will be λ.



For the situation shown in figure, the path difference in waves from S_1 and S_2 at point P is given as

$$\Delta x = (SS_1 + S_1P) - (SS_2 + S_2P)$$

$$\Rightarrow \Delta x = \left[\sqrt{D^2 + d^2} + \sqrt{(y-d)^2 + D^2}\right] - \left[D + \sqrt{D^2 + y^2}\right]$$

For the next bright fringe after first dark fringe,

$$\Delta x = \lambda$$

$$\Rightarrow \left[\sqrt{D^2 + d^2} + \sqrt{(y-d)^2 + D^2}\right] - \left[D + \sqrt{D^2 + y^2}\right] = \lambda$$

$$\Rightarrow D\left[\left(1 + \frac{d^2}{D^2}\right)^{\frac{1}{2}} + \left(1 + \frac{(y-d)^2}{D^2}\right)^{\frac{1}{2}}\right] - \left[D + D\left(1 + \frac{y^2}{D^2}\right)^{\frac{1}{2}}\right] = \lambda$$

$$\Rightarrow \left(D + \frac{d^2}{2D}\right) + D + \frac{(y-d)^2}{2D} - \left(D + D + \frac{y^2}{2D}\right) = \lambda$$

$$\Rightarrow \frac{d^2 + (y-d)^2 - y^2}{2D} = \lambda$$

$$\Rightarrow d^2 + y^2 + d^2 - 2yd - y^2 = 2\lambda D$$

$$\Rightarrow 2d^2 - 2yd = 2\lambda D$$

For $d = \sqrt{\frac{D\lambda}{2}}$, we get

$$2\left(\sqrt{\frac{D\lambda}{2}}\right)^2 - 2y\sqrt{\frac{D\lambda}{2}} = 2\lambda D$$

$$\Rightarrow 2\frac{D\lambda}{2} - 2y\sqrt{\frac{D\lambda}{2}} = 2\lambda D$$

$$\Rightarrow 2y\sqrt{\frac{D\lambda}{2}} = -D\lambda$$

Squaring both sides, we get

$$\left(2y\sqrt{\frac{D\lambda}{2}}\right)^2 = (-D\lambda)^2$$

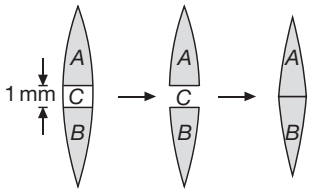
$$\Rightarrow y = \sqrt{\frac{D\lambda}{2}} = d$$

(c) Fringe width on screen can be given by the relation we studied in YDSE, given as

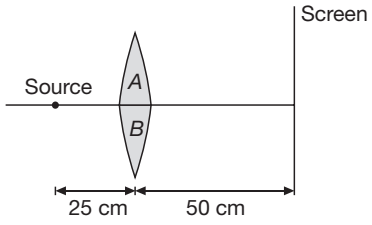
$$\beta = \frac{D\lambda}{d}$$

PROBLEM 20

In Billet's Lens Arrangement, a convex lens of focal length 50 cm is cut along the diameter into two identical halves A and B and in the process a layer C of the lens thickness 1 mm is lost. Then the two halves A and B are put together to form a composite lens as shown in figure.



Now, in front of this new composite lens a source of light emitting wavelength $\lambda = 6000$ Å is placed at a distance of 25 cm. Behind the lens there is a screen at a distance 50 cm from it as shown in the figure.



Calculate the fringe width of the interference pattern obtained on the screen.

SOLUTION

Applying lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ for $u = -25$ cm, we get

$$\frac{1}{v}-\frac{1}{(-25)}=\frac{1}{50}$$

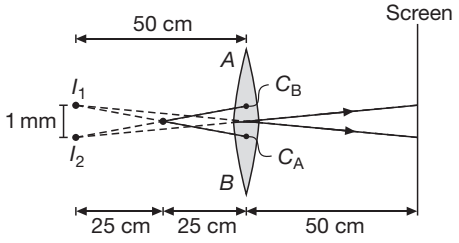
$$\Rightarrow \quad \frac{1}{v}=-\frac{1}{50}$$

$$\Rightarrow \quad v = -50 \text{ cm}$$

The two parts A and B of the lens produce two virtual images (of the source) at I_1 and I_2 at a distance 50 cm behind the lens. Figure shows the locations of I_1 and I_2 which are obtained by joining the source with the optic centres of the two lenses.

Please note that the optical centre of the original lens (from which the new composite lens is made) lies at the centre of the original lens.

On cutting the lens, the region C is lost and when the portions A and B are joined, then the optical centre for A will lie in the region of B and that for B will lie in the region of A.



Now the interference pattern is obtained on the screen due to the interference of light waves from the sources I_1 and I_2 which are separated by a distance 1 mm and are at a distance 1 m from the screen. So, fringe width of the fringes obtained on screen is given by

$$\beta = \frac{\lambda D}{d}$$

$$\Rightarrow \quad \beta = \frac{6\times10^{-7}\times1}{10^{-3}} = 6\times10^{-4} = 0.6 \text{ mm}$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. In an interference pattern produced by two identical slits, the intensity at the site of the central maximum is I. The intensity at the same spot when either of two slits is closed is

(A) $\frac{I}{2}$ (B) $\frac{I}{4}$

(C) $\frac{I}{2\sqrt{2}}$ (D) $\frac{I}{\sqrt{2}}$

2. In a $YDSE$ with two identical slits, when the upper slit is covered with a thin, perfectly transparent sheet of mica, the intensity at the centre of screen reduces to 75% of the initial value. Second minima is observed to be above this point and third maxima below it. Which of the following cannot be a possible value of phase difference caused by the mica sheet?

(A) $\frac{\pi}{3}$ (B) $\frac{13\pi}{3}$

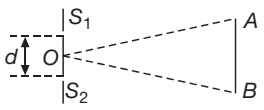
(C) $\frac{17\pi}{3}$ (D) $\frac{11\pi}{3}$

3. Specific rotation of sugar solution is 0.01 SI unit. If $200\ \text{kgm}^{-3}$ of impure sugar solution is taken in a polarimeter tube of length 0.25 m and an optical rotation of 0.4 rad is observed, then the percentage of purity of sugar is the sample is

(A) 80% (B) 89%

(C) 11% (D) 20%

4. The figure shows two coherent sources S_1, S_2 vibrating in same phase. AB is a screen lying at a far distance from the sources S_1 and S_2. Let $\frac{\lambda}{d}=10^{-3}$ and $\angle BOA = 0.12°$. The number of bright spots seen on the screen, including points A and B.



(A) 2
(B) 3
(C) 4
(D) more than 4

5. Huygens' conception of secondary waves
(A) helps us to find the focal length of a thick lens
(B) is a geometrical method to find the position of a wave-front at a later or an earlier instant
(C) is used to determine the velocity of light
(D) is used to explain polarization of light

6. Longitudinal waves do not exhibit
(A) refraction (B) reflection
(C) diffraction (D) polarization

7. The idea of the quantum nature of light has emerged in an attempt to explain
(A) interference
(B) diffraction
(C) polarization
(D) radiation spectrum of a black body

8. In the spectrum of light of a luminous heavenly body the wavelength of a spectral line is measured to be 4747 Å while actual wavelength of the line is 4700 Å. The relative velocity of the heavenly body with respect to earth will be (velocity of light is $3\times10^8\ \text{ms}^{-1}$)
(A) $3\times10^5\ \text{ms}^{-1}$ moving towards the earth
(B) $3\times10^5\ \text{ms}^{-1}$ moving away from the earth
(C) $3\times10^6\ \text{ms}^{-1}$ moving towards the earth
(D) $3\times10^6\ \text{ms}^{-1}$ moving away from the earth

9. A grating has $5000\ \text{lines cm}^{-1}$. The maximum order visible with wavelength 6000 Å
(A) 2 (B) 3
(C) 4 (D) 0

10. A beam of monochromatic light enters from vacuum into a medium of refractive index n. The ratio of the wavelengths of the incident and refracted waves is
(A) $n:1$ (B) $1:n$
(C) $n^2:1$ (D) $1:n^2$

11. In Young's double slit experiment, 62 fringes are seen in visible region for sodium light of wavelength 5893 Å. If violet light of wavelength 4358 Å is used in place of sodium light, then number of fringes seen will be
(A) 54 (B) 64
(C) 74 (D) 84

12. Monochromatic light of wavelength 5000 Å illuminates a pair of slits 1 mm apart. The separation of bright fringes in the interference pattern formed on a screen 2 m away is
(A) 0.25 mm (B) 0.1 mm
(C) 0.01 mm (D) 1.0 mm

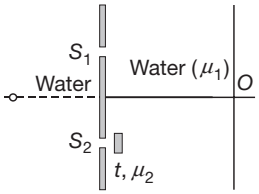
13. Air has refractive index 1.0003. The thickness of an air column, which will have one more wavelength of yellow light $(6000\ \text{Å})$ than in the same thickness of vacuum is
(A) 2 mm (B) 2 cm
(C) 2 m (D) 2 km

14. Two coherent monochromatic light beams of intensities I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are
(A) $4I$ and I (B) $5I$ and $3I$
(C) $9I$ and I (D) $9I$ and $3I$

15. There is a wavelength corresponding to each colour. How many colours are possible, then
(A) 3 (B) 1
(C) 7 (D) None of these

16. Though quantum theory of light can explain a number of phenomena observed with light, it is necessary to retain the wave nature of light to explain the phenomenon of
(A) photo-electric effect (B) diffraction
(C) compton effect (D) black body radiation

17. A Young's double slit experiment is carried out in water having refractive index μ_1 as shown in the figure. A glass plate of thickness t and refractive index μ_2 is placed in the path of S_2. The magnitude of the phase difference at O is
(Assume that λ is the wavelength of light in air, O is symmetrical w.r.t. S_1 and S_2).



(A) $\frac{2\pi}{\lambda}\left|\left(\frac{\mu_2}{\mu_1}-1\right)t\right|$ (B) $\frac{2\pi}{\lambda}\left|\left(\frac{\mu_1}{\mu_2}-1\right)t\right|$
(C) $\frac{2\pi}{\lambda}\left|(\mu_2-\mu_1)t\right|$ (D) $\frac{2\pi}{\lambda}\left|(\mu_2-1)t\right|$

18. Which of the following cannot be polarized?
(A) Radio wave (B) X-rays
(C) Infrared radiation (D) Sound waves in air

19. A blue object on a white background when seen through a blue filter will appear
(A) blue on a white background
(B) black on a blue background
(C) blue on red background
(D) invisible

20. Illumination of the sun at noon is maximum because
(A) the sun is nearer to the earth at noon
(B) rays are incident almost normally
(C) refraction of light is minimum at noon
(D) scattering is reduced at noon

21. In Young's double slit experiment, carried out with light of wavelength $\lambda = 5000$ Å, the distance between the slits is 0.2 mm and the screen is at 200 cm from the slits. The central maximum is at $x = 0$. The third maximum (taking the central maximum as zeroth maximum) will be at x equal to
(A) 1.67 cm (B) 1.5 cm
(C) 0.5 cm (D) 5.0 cm

22. Ray optics is valid when characteristic dimensions are
(A) of the same order as the wavelength of light
(B) much smaller than the wavelength of light
(C) much larger than the wavelength of light
(D) of the order of 1 mm

23. In order that a thin film of oil floating on the surface of water shows colours due to interference, the thickness of the oil film should be of the order of
(A) 1 cm (B) 10 Å
(C) 5000 Å (D) 10000 Å

24. The blue cross on a white background illuminated with white light is observed through a red filter. The pattern seen is
(A) a red cross on a black background
(B) a blue cross on a red background
(C) a red cross on a blue background
(D) a black cross on a blue background

25. In a single slit diffraction of light of wavelength λ by a slit of width a, the size of the central maximum on a screen at a distance b is
(A) $2b\lambda + a$ (B) $\frac{2b\lambda}{a}$
(C) $\frac{2b\lambda}{a} + a$ (D) $\frac{2b\lambda}{a} - a$

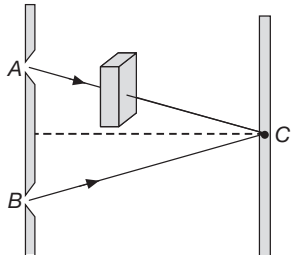
26. The deflection of light in a gravitational field was predicted first by
(A) Einstein (B) Newton
(C) Max Planck (D) Maxwell

27. Both the particle and wave aspects of the wave aspects of light appear to be used in
(A) photoelectric effect (B) gamma emission
(C) interference (D) classical mechanics

28. At sunset, the sun seems to be
(A) higher than it really is
(B) lower than it really is
(C) exactly where it really is
(D) lower than it would be at sunrise

29. In Huygens' wave theory, the locus of all the points in the same state of vibration is called a
(A) half period zone (B) vibrator
(C) wavefront (D) ray

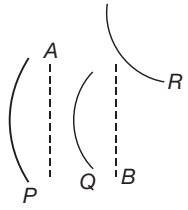
30. In Young's experiment, monochromatic light is used to illuminate the two slits A and B. Interference fringes are observed on a screen placed in front of the slits. Now if a thin glass plate is placed normally in the path of the beam coming from the slit



(A) The fringes will disappear
(B) The fringe width will increase
(C) The fringe width will increase
(D) There will be no change in the fringe width but the pattern shifts

31. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is
(A) zero (B) I_0
(C) $\frac{1}{2}I_0$ (D) $\frac{1}{4}I_0$

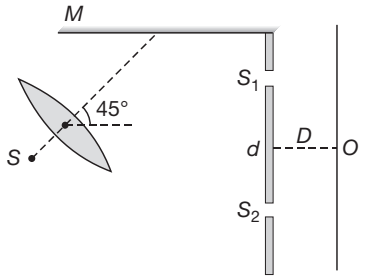
32. The figure shows a wave front P passing through two systems A and B, and emerging as Q and then as R. The systems A and B could, respectively, be



(A) a prism and a convergent lens
(B) a convergent lens and a prism
(C) a divergent lens and a prism
(D) a convergent lens and a divergent lens

33. In Young's double-slit experiment the separation between the slits is doubled and the distance between the slit and the screen is halved. The fringe-width becomes
(A) one-fourth (B) half
(C) double (D) quadruple

34. In Young's double slit experiment, distance between the slits is d and that between the slits and screen is D. Angle between principle axis of lens and perpendicular bisector of S_1 and S_2 is $45°$. The point source S is placed at the focus of lens. The lens has a focal length of 10 cm and its aperture is much larger than d. Assuming only the reflected light from plane mirror M is incident on slits, distance of central maxima from O will be



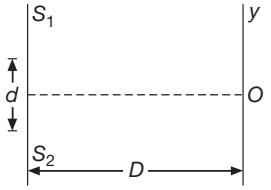
(A) $\frac{D}{\sqrt{2}}$ (B) $\frac{D}{\sqrt{3}}$
(C) $D\sqrt{3}$ (D) $\frac{D}{\sqrt{4}}$

35. Two coherent point sources s_1 and s_2 vibrating in phase emit light of wavelength λ. The separation between the sources is 2λ. The smallest distance from s_2 on a line passing through s_2 and perpendicular to s_1s_2, where a minimum of intensity occurs is
(A) $\frac{7\lambda}{12}$ (B) $\frac{15\lambda}{4}$
(C) $\frac{\lambda}{2}$ (D) $\frac{3\lambda}{4}$

36. In a Young's double slit experiment, the fringes are displaced by a distance x when a glass plate of refractive index 1.5 is introduced in the path of one of the beams. When this plate is replaced by another plate of same thickness, the shift of fringes is $\left(\frac{3}{2}\right)x$. The refractive index of second plate is

(A) 1.75 (B) 1.50
(C) 1.25 (D) 1.00

37. Consider a usual set-up of Young's double slit experiment with slits of equal intensity as shown in the figure. Take O as origin and the Y axis as indicated. If verage intensity between $y_1 = -\frac{\lambda D}{4d}$ and $y_2 = +\frac{\lambda D}{4d}$ equals n times the intensity of maxima, then n equals (take average over phase difference)



(A) $\frac{1}{2}\left(1+\frac{2}{\pi}\right)$ (B) $2\left(1+\frac{2}{\pi}\right)$
(C) $\left(1+\frac{2}{\pi}\right)$ (D) $\frac{1}{2}\left(1-\frac{2}{\pi}\right)$

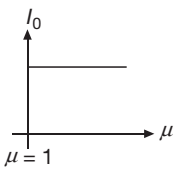
38. A plate of thickness t made of a material of refractive index μ is placed in front of one of the slits in a double slit experiment. What should be the minimum thickness t which will make the intensity at the centre of the fringe pattern zero?

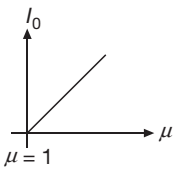
(A) $(\mu-1)\frac{\lambda}{2}$ (B) $(\mu-1)\lambda$
(C) $\frac{\lambda}{2(\mu-1)}$ (D) $\frac{\lambda}{(\mu-1)}$

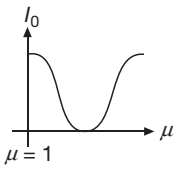
39. Two polaroids are placed in the path of unpolarized beam of intensity I_0 such that no light is emitted from the second polaroid. If a third polaroid whose polarization axis makes an angle θ with the polarization axis of first polaroid, is placed between these polaroids then the intensity of light emerging from the last polaroid will be

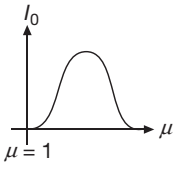
(A) $\left(\frac{I_0}{8}\right)\sin^2 2\theta$ (B) $\left(\frac{I_0}{4}\right)\sin^2 2\theta$
(C) $\left(\frac{I_0}{2}\right)\cos^4\theta$ (D) $I_0\cos^4\theta$

40. In a $YDSE$ experiment, if a slab whose refractive index can be varied is placed in front of one of the slits, then the variation of resultant intensity at mid-point of screen with $\mu(\geq 1)$ will be best represented by
(Assume slits are of equal width, there is no absorption by slab and midpoint of screen is the point where the waves interfere with zero phase difference, in absence of slab)

(A) 

(B) 

(C) 

(D) 

41. A plane wavefront $\left(\lambda = 6\times10^{-7}\text{ m}\right)$ falls on a slit 0.4 mm wide. A convex lens of focal length 0.8 m placed behind the slit focuses the light on a screen. The linear diameter of second maximum is

(A) 6 mm (B) 12 mm
(C) 3 mm (D) 9 mm

42. If two slightly different wavelengths are present in the light used in Young's double-slit experiment, then
(A) the sharpness of fringes will be more than the case when only one wavelength is present
(B) the sharpness of fringes will decrease as we move away from the central fringe
(C) the central fringe will be white
(D) the central fringe will be dark

43. Two identical coherent sources placed on a diameter of a circle of radius R at separation $x(<<R)$ symmetrically about the centre of the circle. The sources emit identical wavelength λ each. The number of points on the circle with maximum intensity is $(x = 5\lambda)$

(A) 20 (B) 22
(C) 24 (D) 26

44. Laser is
(A) intense, coherent and monochromatic
(B) only intense and coherent
(C) only coherent and monochromatic
(D) only intense and monochromatic

45. Imagine a hypothetical convex lens material which can transmit all the following radiation. This lens will have minimum focal length for

(A) ultraviolet rays (B) infrared rays
(C) radio waves (D) X-rays

46. A star emitting yellow light starts accelerating towards earth, its colour as seen from the earth will

(A) turn gradually red (B) turn suddenly red
(C) remains same (D) turn gradually blue

47. The transverse nature of light is shown by
(A) interference of light (B) refraction of light
(C) polarization of light (D) dispersion of light

48. The wave front of a light beam is given by the equation $x+2y+3z=a$, where a is arbitrary constant. The angle made by the direction of light with the y-axis is
(A) $\cos^{-1}\left(\frac{1}{\sqrt{14}}\right)$ (B) $\sin^{-1}\left(\frac{2}{\sqrt{14}}\right)$
(C) $\cos^{-1}\left(\frac{2}{\sqrt{14}}\right)$ (D) $\sin^{-1}\left(\frac{3}{\sqrt{14}}\right)$

49. Laser light is considered to be coherent, because it consists of
(A) many wavelengths
(B) uncoordinated wavelengths
(C) coordinated waves of exactly the same wavelength
(D) divergent beams

50. The wavelength of light observed on the earth, from a moving star is found to decrease by 0.05%. Relative to the earth the star is
(A) Moving away with a velocity of 1.5×10^5 ms^{-1}
(B) Coming closer with a velocity of 1.5×10^5 ms^{-1}
(C) Moving away with a velocity of 1.5×10^4 ms^{-1}
(D) Coming closer with a velocity of 1.5×10^4 ms^{-1}

51. A beam of electron is used in an $YDSE$ experiment. The slit width is d. When the velocity of electron is increased, then
(A) No interference is observed
(B) Fringe width increases
(C) Fringe width decreases
(D) Fringe width remains same

52. The ratio of the intensity at the centre of a bright fringe to the intensity at a point one-quarter of the distance between two fringe from the centre is
(A) 2 (B) $\frac{1}{2}$
(C) 4 (D) 16

53. The ratio of intensities of consecutive maxima in the diffraction pattern due to a single slit is
(A) 1 : 4 : 9 (B) 1 : 2 : 3
(C) $1:\frac{4}{9\pi^2}:\frac{4}{25\pi^2}$ (D) $1:\frac{1}{\pi^2}:\frac{9}{\pi^2}$

54. Minimum thickness of a mica sheet having $\mu=\frac{3}{2}$ which should be placed in front of one of the slits in $YDSE$ is required to reduce the intensity at the centre of screen to half of maximum intensity is
(A) $\frac{\lambda}{4}$ (B) $\frac{\lambda}{8}$
(C) $\frac{\lambda}{2}$ (D) $\frac{\lambda}{3}$

55. A beam of natural light falls on a system of 6 polaroids, which are arranged in succession such that each polaroid is turned through $30°$ with respect to the preceding one. The percentage of incident intensity that passes through the system will be
(A) 100% (B) 50%
(C) 30% (D) 12%

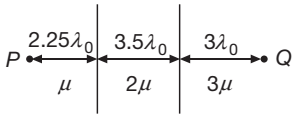
56. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double-slit experiment is
(A) Infinite (B) Five
(C) Three (D) Zero

57. A rocket is going towards moon with a speed v. The astronaut in the rocket sends signals of frequency ν towards the moon and receives them back on reflection from the moon. What will be the frequency of the signal received by the astronaut (Take $v \ll c$)
(A) $\frac{c}{c-v}\nu$ (B) $\frac{c}{c-2v}\nu$
(C) $\frac{2v}{c}\nu$ (D) $\frac{2c}{v}\nu$

58. In Young's double slit experiment the y-coordinates of central maxima and 10^{th} maxima are 2 cm and 5 cm respectively. When the YDSE apparatus is immersed in a liquid of refractive index 1.5 the corresponding y-coordinates will be
(A) 2 cm, 7.5 cm (B) 3 cm, 6 cm
(C) 2 cm, 4 cm (D) $\frac{4}{3}$ cm , $\frac{10}{3}$ cm

59. In Young's double slit experiment how many maximas can be obtained on a screen (including the central maximum) on both sides of the central fringe if $\lambda=2000$ Å and $d=7000$ Å
(A) 12 (B) 7
(C) 18 (D) 4

60. An electromagnetic wave of λ_0 (in vacuum) passes from P towards Q crossing three different media of refractive index μ, 2μ and 3μ respectively as shown in the figure.

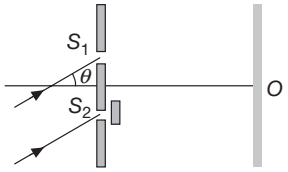


ϕ_P and ϕ_Q be the phase of the wave at points P and Q. The phase difference $\phi_Q - \phi_P$ for $\mu = 1$

(A) 0 (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) π

61. A monochromatic beam of light falls on $YDSE$ apparatus at some angle (say θ) as shown in figure. A thin sheet of glass is inserted in front of the lower slit S_2. The central bright fringe (path difference $= 0$) will be obtained



(A) At O
(B) Above O
(C) Below O
(D) Anywhere depending on angle θ, thickness of plate t and refractive index of glass μ

62. In Young's double slit experiment, the two slits act as coherent sources of equal amplitude A and wavelength λ. In another experiment with the same set up the two slits are of equal amplitude A and wavelength λ but are incoherent. The ratio of the intensity of light at the mid-point of the screen in the first case to that in the second case is

(A) 1 : 2 (B) 2 : 1
(C) 4 : 1 (D) 1 : 1

63. In the ideal double-slit experiment, when a glass-plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is

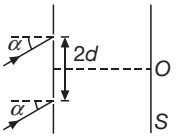
(A) 2λ (B) $\frac{2\lambda}{3}$

(C) $\frac{\lambda}{3}$ (D) λ

64. In a Young's double-slit experiment, the intensity ratio of maxima and minima is infinite. The ratio of the amplitudes of two sources

(A) is infinity (B) is unity
(C) is two (D) cannot be predicted

65. A parallel beam of monochromatic light of wavelength λ is used in Young's Double Slit Experiment. The beam makes an angle α with the normal to the plane of slits as shown in figure.

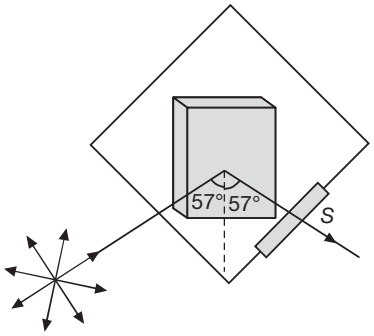


A screen is placed at a large distance $D(\gg 2d)$ from the slits. The value of α for which there is a dark fringe at O is

(A) $\cos^{-1}\left(\frac{\lambda}{4d}\right)$ (B) $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

(C) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$ (D) $\cos^{-1}\left(\frac{\lambda}{2d}\right)$

66. Figure represents a glass plate placed vertically on a horizontal table with a beam of unpolarised light falling on its surface at the polarising angle of 57° with the normal. The electric vector in the reflected light on screen S will vibrate with respect to the plane of incidence in a



(A) Vertical plane
(B) Horizontal plane
(C) Plane making an angle of 45° with the vertical
(D) Plane making an angle of 57° with the horizontal

67. A clear sheet of polaroid is placed on the top of similar sheet so that their axes make an angle $\sin^{-1}\left(\frac{3}{5}\right)$ with each other. The ratio of intensity of the emergent light to that of unpolarised incident light is

(A) 16 : 25 (B) 9 : 25
(C) 4 : 5 (D) 8 : 25

68. Optically active substances are those substances which

(A) produce polarized light
(B) rotate the plane of polarization of polarized light
(C) produce double refraction
(D) convert a plane polarized light into circularly polarized light

69. If I_0 is the intensity of the principal maximum in the single slit diffraction pattern, then what will be its intensity when the slit width is doubled

(A) I_0 (B) $\frac{I_0}{2}$

(C) $2I_0$ (D) $4I_0$

70. The critical angle of a certain medium is $\sin^{-1}\left(\frac{3}{5}\right)$. The polarising angle of the medium is

(A) $\sin^{-1}\left(\frac{4}{5}\right)$ (B) $\tan^{-1}\left(\frac{5}{3}\right)$

(C) $\tan^{-1}\left(\frac{3}{4}\right)$ (D) $\tan^{-1}\left(\frac{4}{3}\right)$

71. A 20 cm length of a certain solution causes right-handed rotation of $38°$. A 30 cm length of another solution causes left-handed rotation of $24°$. The optical rotation caused by 30 cm length of a mixture of the above solution in the volume ratio 1 : 2 is

(A) left handed rotation of $14°$
(B) right handed rotation of $14°$
(C) left handed rotation of $3°$
(D) right handed rotation of $3°$

72. The ray of light is incident on glass of refractive index 1.5 at polarising angle. The angle of deviation of the incident ray in glass is

(A) $57°$ (B) $33°$
(C) $24°$ (D) $114°$

73. Double refraction of light is shown by

(A) quartz and calcite only
(B) calcite only
(C) calcite and ice only
(D) calcite, ice and quartz

74. A slit of width a is illuminated by red light of wavelength 6500 Å. The first minimum will fall at $\theta = 30°$ if a is

(A) 3250 Å (B) 6.5×10^{-4} mm

(C) 1.3 μm (D) 2.6×10^{-4} cm

75. The resolution of the human eye is 1′. The resolving power of the human eye is nearly

(A) 360 (B) 3600
(C) 36000 (D) 360000

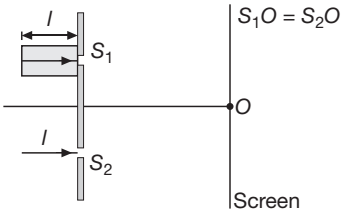
76. Colours of thin films are due to

(A) dispersion of light
(B) interference of light
(C) absorption of light
(D) scattering of light

77. A person standing at a distance of 3.6 km can just resolve two poles. The distance between the poles is

(A) 0.1 m (B) 100 m
(C) 1 m (D) 10 m

78. In the figure shown, a parallel beam of light is incident on the plane of the slits of a Young's double slit experiment. Light incident on the slit, S_1 passes through a medium of variable refractive index $\mu = 1 + ax$ (where x is the distance from the plane of slits as shown), up to a distance ℓ before falling on S_1. The entire remaining space is filled with air. If at O, a minima is formed, then the minimum value of the positive constant a (in terms of ℓ and wavelength λ in air) is



(A) $\frac{\lambda}{\ell}$ (B) $\frac{\lambda}{\ell^2}$

(C) $\frac{\ell^2}{\lambda}$ (D) $\frac{\lambda}{2\ell^2}$

79. A heavenly body is receding from earth such that the fractional change in λ is 1, then its velocity is

(A) c (B) $\frac{3c}{5}$

(C) $\frac{c}{5}$ (D) $\frac{2c}{5}$

80. A star is moving towards the earth with a speed of 4.5×10^6 ms^{-1}. If the true wavelength of a certain line in the spectrum received from the star is 5890 Å, its apparent wavelength will be about $(c = 3\times10^8$ ms$^{-1})$

(A) 5890 Å (B) 5978 Å
(C) 5802 Å (D) 5896 Å

81. Lights of wavelength $\lambda_1 = 4500$ Å, $\lambda_2 = 6000$ Å are sent through a double-slit arrangement simultaneously. Then

(A) no interference pattern will be formed
(B) the third bright fringe of λ_1 will coincide with the fourth bright fringe of λ_2
(C) the third bright fringe of λ_2 will coincide with fourth bright fringe of λ_1
(D) the fringes of wavelength λ_1 will be wider than the fringes of wavelength λ_2

82. Two slits separated by a distance of 1 mm are illuminated with light of wavelength 6×10^{-7} m. The interference fringes are observed on a screen placed 1 m from the slits. The distance between the second dark fringe and the fourth bright fringe is equal to
(A) 0.5 mm (B) 1.0 mm
(C) 1.5 mm (D) 2.0 mm

83. Interference fringes were produced in Young's double-slit experiment using light of wavelength 5000 Å. When a film of thickness 2.5×10^{-3} cm was placed in front of one of the slits, the fringe pattern shifted by a distance equal to 20 fringe-width. The refractive index of the material of the film is
(A) 1.25 (B) 1.35
(C) 1.4 (D) 1.5

84. The maximum intensity in Young's double slit experiment is I_0. Distance between the slits is $d=5\lambda$, where λ is the wavelength of monochromatic light used in the experiment. The intensity of light in front of one of the slits on a screen at a distance $D=10d$ is
(A) $\frac{I_0}{2}$ (B) $\frac{3}{4}I_0$
(C) I_0 (D) $\frac{I_0}{4}$

85. In Young's double-slit experiment, an interference pattern is obtained on a screen by a light of wavelength 6000 Å, coming from the coherent sources S_1 and S_2. At certain point P on the screen third dark fringe is formed. Then the path difference S_1P-S_2P in microns is
(A) 0.75 (B) 1.5
(C) 3.0 (D) 4.5

86. Young's double slit experiment is made in a liquid. The 10th bright fringe in liquid lies where 6th dark fringe lies in vacuum. The refractive index of the liquid is approximately
(A) 1.8 (B) 1.54
(C) 1.67 (D) 1.2

87. A point source emits light equally in all directions. Two points P and Q are at distances 9 m and 25 m respectively from the source. The ratio of the amplitudes of the waves P and Q is
(A) 9 : 25 (B) 25 : 9
(C) $9^2:25^2$ (D) $25^2:9^2$

88. In Young's double slit experiment the y-co-ordinates of central maxima and 10th maxima are 2 cm and 5 cm respectively. When the $YDSE$ apparatus is immersed in a liquid of refractive index 1.5 the corresponding y-co-ordinates will be
(A) 2 cm, 7.5 cm (B) 3 cm, 6 cm
(C) 2 cm, 4 cm (D) $\frac{4}{3}$ cm, $\frac{10}{3}$ cm

89. In Young's double slit experiment $\frac{d}{D}=10^{-4}$ ($d=$ distance between slits, $D=$ distance of screen from the slits). At a point P on the screen resulting intensity is equal to the intensity due to individual slit I_0. Then the distance of point P from the central maximum is ($\lambda=6000$ Å)
(A) 2 mm (B) 1 mm
(C) 0.5 mm (D) 4 mm

90. The Young's double-slit experiment is carried out with light of wavelength 5000 Å. The distance between the slits is 0.2 mm and the screen is at 200 cm from the slits. The central maximum is at $x=0$. The third maximum will be at x equal to
(A) 1.67 cm (B) 1.5 cm
(C) 0.5 cm (D) 5.0 cm

91. Light passes successively through two polarimeters tubes each of length 0.29 m. The first tube contains dextro rotatory solution of concentration 60 kgm^{-3} and specific rotation 0.01 $radm^2kg^{-1}$. The second tube contains laevo rotatory solution of concentration 30 kgm^{-3} and specific rotation 0.02 $radm^2kg^{-1}$. The net rotation produced is
(A) 15° (B) 0°
(C) 20° (D) 10°

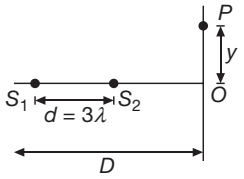
92. The blue colour of the sky is explained by
(A) refraction (B) reflection
(C) polarisation (D) scattering

93. A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen is
(A) Straight line (B) Parabola
(C) Hyperbola (D) Circle

94. Among the two interfering monochromatic sources A and B; A is ahead of B in phase by 66°. If the observation be taken from point P, such that $PB-PA=\frac{\lambda}{4}$. Then the phase difference between the waves from A and B reaching P is
(A) 156° (B) 140°
(C) 136° (D) 126°

95. Two coherent narrow slits emitting light of wavelength λ in the same phase are placed parallel to each other at a small separation of 3λ. The light is collected on a

screen S which is placed at a distance $D(\gg \lambda)$ from the slits. The smallest distance y for P to be a maxima is



(A) $\sqrt{3}D$ (B) $\sqrt{8}D$

(C) $\sqrt{5}D$ (D) $\sqrt{5}\dfrac{D}{2}$

96. Two coherent sources S_1 and S_2 are separated by a distance four times the wavelength λ of the source. The sources lie along y-axis whereas a detector moves along $+x$-axis. Leaving the origin and far off points the number of points where maxima are observed is

(A) 2 (B) 3

(C) 4 (D) 5

97. The first minimum due to a Fraunhofer diffraction using light of wavelength 500 nm and a slit of width 0.5 mm will be formed at an angle (in minutes)

(A) 2.42 (B) 3.43

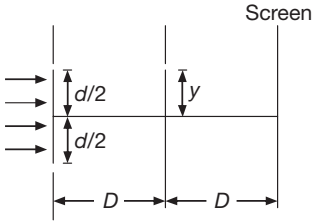
(C) 4.84 (D) 1.71

98. Aperture of the human eye is 2 mm. Assuming the mean wavelength of light to be 5000 Å, the angular resolution limit of the eye is nearly

(A) 2 minute (B) 1 minute

(C) 0.5 minute (D) 1.5 minute

99. In the Young's double slit experiment apparatus shown in figure, the ratio of maximum to minimum intensity on the screen is 9. The wavelength of light used is λ, then the value of y is



(A) $\dfrac{\lambda D}{d}$ (B) $\dfrac{\lambda D}{2d}$

(C) $\dfrac{\lambda D}{3d}$ (D) $\dfrac{\lambda D}{4d}$

100. In Young's experiment, using red and blue lights of wavelengths 7800 Å and 5200 Å respectively, the value of n for which nth red fringe coincides with $(n+1)$th blue fringe is

(A) 2 (B) 3

(C) 4 (D) 5

101. Angular width of central maxima in the Fraunhofer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, the angular width decreases by 30%. The wavelength of this light will be

(A) 6000 Å (B) 4200 Å

(C) 3000 Å (D) 1800 Å

102. In Young's experiment for interference of light with two slits, maxima occur at angles for which $\sin\theta = \dfrac{m\lambda}{d}$. Here d is

(A) distance of slits from the screen

(B) distance between dark and bright fringes

(C) distance between slits

(D) width of mth fringe

103. Interference is observed in a chamber with air present inside the chamber. The chamber is then evacuated and the same light is again used to produce interference. A careful observer will see

(A) no change in the pattern

(B) that the fringewidth slightly increases

(C) that the fringewidth slightly decreases

(D) no interference pattern

104. Finger prints of a piece of paper may be detected by sprinkling fluorescent powder on the paper and than looking into it under

(A) yellow light (B) brightness

(C) infrared light (D) ultraviolet light

105. Two nicol prisms (polariser and analyser) have their axes at angles of 30° in between. If I is the intensity of light falling on first nicol, then that of emerging light is

(A) $0.125I$ (B) $0.25I$

(C) $0.375I$ (D) $0.5I$

106. The Young's double-slit experiment is performed with blue light and green light of wavelengths 4360 Å and 5460 Å respectively. If X is the distance of 4th maximum from the central one, then

(A) $X(\text{Blue}) = X(\text{Green})$

(B) $X(\text{Blue}) < X(\text{Green})$

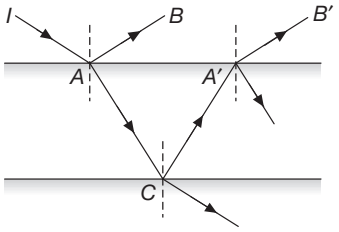
(C) $X(\text{Blue}) > (\text{Green})$

(D) $\dfrac{X(\text{Blue})}{X(\text{Green})} = \dfrac{5460}{4360}$

107. v_O and v_E represent the velocity, μ_O and μ_E the refractive indices of ordinary and extraordinary rays for a doubly refracting crystal, then

(A) $v_O \geq v_E$, $\mu_O \leq \mu_E$ if the crystal is calcite
(B) $v_O \leq v_E$, $\mu_O \leq \mu_E$ if the crystal is quartz
(C) $v_O \leq v_E$, $\mu_O \geq \mu_E$ if the crystal is calcite
(D) $v_O \geq v_E$, $\mu_O \geq \mu_E$ if the crystal is quartz

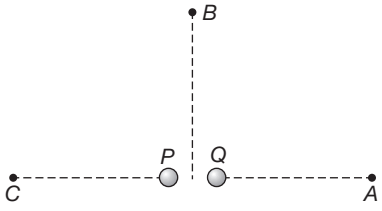
108. A ray of light of intensity I is incident on a parallel glass-slab at a point A as shown in figure. It undergoes partial reflection and refraction. At each reflection 25% of incident energy is reflected. The rays AB and $A'B'$ undergo interference. The ratio I_{max} / I_{min} is



(A) 4 : 1 (B) 8 : 1
(C) 7 : 1 (D) 49 : 1

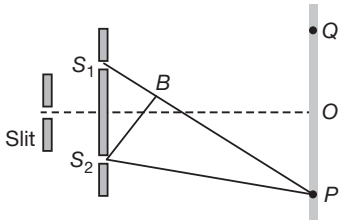
109. Plane polarised light is passed through a polaroid. On viewing through the polaroid we find that when the polaroid is given one complete rotation about the direction of light
(A) the intensity of light gradually decreases to zero and remains at zero
(B) the intensity of light gradually increases to a maximum and remains maximum
(C) there is no change in the intensity of light
(D) the intensity of light varies such that it is twice maximum and twice zero

110. Figure here shows P and Q as two equally intense coherent sources emitting radiations of wavelength 20 m. The separation PQ is 5 m and phase of P is ahead of the phase of Q by 90°. A, B and C are three distant points of observation equidistant from the mid-point of PQ. The intensity of radiations at A, B, C will bear the ratio



(A) 0 : 1 : 4 (B) 4 : 1 : 0
(C) 0 : 1 : 2 (D) 2 : 1 : 0

111. In the figure is shown Young's double slit experiment. Q is the position of the first bright fringe on the right side of O. P is the 11th fringe on the other side, as measured from Q. If the wavelength of the light used is 6000×10^{-10} m, then S_1B will be equal to



(A) 6×10^{-6} m (B) 6.6×10^{-6} m
(C) 3.138×10^{-7} m (D) 3.144×10^{-7} m

112. In $YDSE$, both slits produce equal intensities on the screen. A 100% transparent thin film is placed in front of one the slits. Now the intensity of the geometrical centre of system on the screen becomes 75% of the previous intensity. The wavelength of the light is 6000 Å and $\mu_{film} = 1.5$. The thickness of the film cannot be
(A) 0.2 μm (B) 1.0 μm
(C) 1.4 μm (D) 1.6 μm

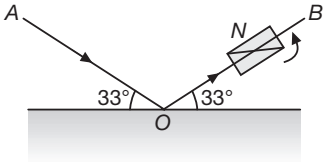
113. The maximum intensity in Young's double slit experiment is I_0. Distance between the slits is $d = 5\lambda$, where λ is the wavelength of monochromatic light used in the experiment. What will be the intensity of light in front of one of the slits on a screen at a distance $D = 10d$

(A) $\frac{I_0}{2}$ (B) $\frac{3}{4}I_0$
(C) I_0 (D) $\frac{I_0}{4}$

114. The polarising angle of diamond is 67°. The critical angle of diamond is nearest to
(A) 22° (B) 34°
(C) 45° (D) 60°

115. No longitudinal wave will show
(A) interference (B) diffraction
(C) T I R (D) polarisation

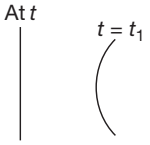
116. A beam of light AO is incident on a glass slab ($\mu = 1.54$) in a direction as shown in figure. The reflected ray OB is passed through a Nicol prism on viewing through a Nicol prism, we find on rotating the prism that



(A) The intensity is reduced down to zero and remains zero
(B) The intensity reduces down some what and rises again
(C) There is no change in intensity
(D) The intensity gradually reduces to zero and then again increases

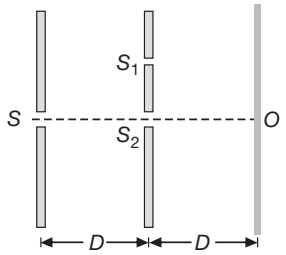
117. When one of the slits of Young's experiment is covered with a transparent sheet of thickness 4.8 mm, the central fringe shifts to a position originally occupied by the 30^{th} bright fringe. What should be the thickness of the sheet if the central fringe has to shift to the position occupied by 20^{th} bright fringe
(A) 3.8 mm (B) 1.6 mm
(C) 7.6 mm (D) 3.2 mm

118. The figure shows a plane wave front at a time t and a time t_1. In the time interval $(t_1 - t)$ the wave front must have passed through



(A) a prism
(B) a prism and a convex lens
(C) a convex lens
(D) a plane mirror and a concave lens

119. Two ideal slits S_1 and S_2 are at a distance d apart, and illuminated by light of wavelength λ passing through an ideal source slit S placed on the line through S_2 as shown. The distance between the planes of slits and the source slit is D. A screen is held at a distance D from the plane of the slits. The minimum value of d for which there is darkness at O is



(A) $\sqrt{\frac{3\lambda D}{2}}$ (B) $\sqrt{\lambda D}$
(C) $\sqrt{\frac{\lambda D}{2}}$ (D) $\sqrt{3\lambda D}$

120. Two waves originating from sources S_1 and S_2 having zero phase difference and common wavelength λ will show completely destructive interference at a point P if $S_1P - S_2P$ is
(A) 5λ (B) $\frac{3\lambda}{4}$
(C) 2λ (D) $\frac{11\lambda}{2}$

121. A thin air film between a plane glass plate and a convex lens is irradiated with parallel beam of monochromatic light and is observed under a microscope. We see
(A) uniform brightness
(B) complete darkness
(C) field crossed over by concentric bright and dark rings
(D) field crossed over by straight bright and dark fringes

122. In Young's double-slit experiment, the intensity of light at a point on the screen where the path difference is λ is I, λ being the wavelength of light used. The intensity at a point where the path difference is $\frac{\lambda}{4}$ will be
(A) $\frac{I}{4}$ (B) $\frac{I}{2}$
(C) I (D) zero

123. In a two slit experiment with monochromatic light fringes are obtained on a screen placed at some distance from the sits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5} m. If separation between the slits is 10^{-3} m, the wavelength of light used is
(A) 6000 Å (B) 5000 Å
(C) 3000 Å (D) 4500 Å

124. In the visible region of the spectrum the rotation of the place of polarization is given by $\theta = a + \frac{b}{\lambda^2}$. The optical rotation produced by a particular material is found to be $30°$ per mm at $\lambda = 5000$ Å and $50°$ per mm at $\lambda = 4000$ Å. The value of constant a will be

(A) $+\frac{50^\circ}{9}$ per mm (B) $-\frac{50^\circ}{9}$ per mm

(C) $+\frac{9^\circ}{50}$ per mm (D) $-\frac{9^\circ}{50}$ per mm

125. In Young's experiment using monochromatic light, the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 micron is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the slits and the screen is doubled. It is found that the distance between successive maxima now is the same as the observed fringe shift upon the introduction of the mica sheet. The wavelength of light is

(A) 5762 Å (B) 5825 Å

(C) 5892 Å (D) 6500 Å

126. The phenomenon of interference is shown by

(A) longitudinal mechanical waves only
(B) transverse mechanical waves only
(C) non-mechanical transverse waves only
(D) all the above types of waves

127. In Young's double-slit experiment, if L is the distance between the slits and the screen upon which the interference pattern is observed, x is the average distance between the adjacent fringes and d is the slit separation, then the wavelength of light is

(A) $\frac{xd}{L}$ (B) $\frac{xL}{d}$

(C) $\frac{Ld}{x}$ (D) $\frac{1}{Ldx}$

128. Interference can take place between

(A) transverse waves only, but not in longitudinal waves
(B) longitudinal waves only, but not in transverse waves
(C) both longitudinal and transverse waves
(D) light waves only, but not sound waves

129. Young's double-slit experiment is performed with light of wavelength $\lambda = 6000$ Å. A glass plate of thickness 0.01 mm and $\mu = 1.5$ is introduced. The number of fringes shifting in the system is

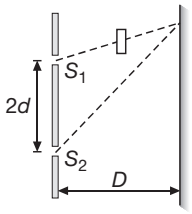
(A) 2000 (B) 8

(C) 120 (D) 4910

130. The contrast in the fringes in an interference pattern depends on

(A) fringe width
(B) wavelength
(C) intensity ratio of the sources
(D) distance between the slits

131. If a thin mica sheet of thickness t and refractive index $\mu = \frac{5}{3}$ is placed in the path of one of the interfering beams as shown in the figure, then the displacement of the fringe system is



(A) $\frac{Dt}{3d}$ (B) $\frac{Dt}{5d}$

(C) $\frac{Dt}{4d}$ (D) $\frac{2Dt}{5d}$

132. In a Young's double slit experiment the source S and the two slits A and B are vertical with slit A above slit B. The fringes are observed on a vertical screen K. The optical path length from S to B is increased very slightly (by introducing a transparent material of higher refractive index) and the optical path length from S to A is not changed, as a result the fringe system on K moves

(A) Vertically downwards slightly
(B) Vertically upwards slightly
(C) Horizontally, slightly to the left
(D) Horizontally, slightly to the right

133. In the Young's double slit experiment, if the phase difference between the two waves interfering at a point is ϕ, the intensity at that point can be expressed by the expression

(A) $I = \sqrt{A^2 + B^2\cos^2\phi}$ (B) $I = \frac{A}{B}\cos\phi$

(C) $I = A + B\cos\frac{\phi}{2}$ (D) $I = A + B\cos\phi$

Where A and B depend upon the amplitudes of the two waves.

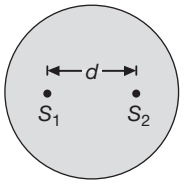
134. The time period of rotation of the sun is 25 days and its radius is 7×10^8 m. The Doppler shift for the light of wavelength 6000 Å emitted from the surface of the sun will be

(A) 0.04 Å
(B) 0.40 Å
(C) 4.00 Å
(D) 40.0 Å

135. A flake of glass (refractive index 1.5) is placed over one of the openings of a double slit apparatus. The interference pattern displaces itself through seven successive maxima towards the side where the flake is placed. if wavelength of the diffracted light is $\lambda = 600$ nm , then the thickness of the flake is

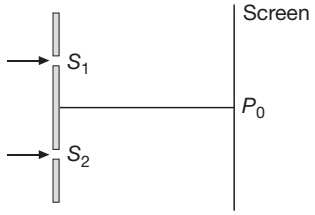
(A) 2100 nm (B) 4200 nm
(C) 8400 nm (D) None of these

136. Two coherent sources separated by distance d are radiating in phase having wavelength λ. A detector moves in a big circle around the two sources in the plane of the two sources. The angular position of $n = 4$ interference maxima is given as



(A) $\sin^{-1}\left(\frac{n\lambda}{d}\right)$ (B) $\cos^{-1}\left(\frac{4\lambda}{d}\right)$
(C) $\tan^{-1}\left(\frac{d}{4\lambda}\right)$ (D) $\cos^{-1}\left(\frac{\lambda}{4d}\right)$

137. In a standard Young's slit experiment with coherent light of wavelength 600 nm , the fringe width of the fringes in the central region (near the central fringe, P_0) is observed to be 3 mm . An extremely thin glass plate is introduced in front of the first slit, and the fringes are observed to be displaced by 11 mm. Another thin plate is placed before the second slit and it is observed that the fringes are now displaced by an additional 12 mm . If the additional optical path lengths introduced are Δ_1 and Δ_2 , then



(A) $11\Delta_1 = 12\Delta_2$ (B) $12\Delta_1 = 11\Delta_2$
(C) $11\Delta_1 > 12\Delta_2$ (D) None of these

138. White light may be considered to be a mixture of waves with λ ranging between 3900 Å and 7800 Å. An oil film of thickness 10,000 Å is examined normally by reflected light. If $\mu = 1.4$, then the film appears bright for

(A) 4308 Å, 5091 Å, 6222 Å
(B) 4000 Å, 5091 Å, 5600 Å
(C) 4667 Å, 6222 Å, 7000 Å
(D) 4000 Å, 4667 Å, 5600 Å, 7000 Å

139. The k line of singly ionised calcium has a wavelength of 393.3 nm as measured on earth. In the spectrum of one of the observed galaxies, this spectral line is located at 401.8 nm . The speed with which the galaxy is moving away from us, will be

(A) 6480 kms^{-1} (B) 3240 kms^{-1}
(C) 4240 kms^{-1} (D) None of these

140. The two coherent sources of equal intensity produce maximum intensity of 100 units at a point. If the intensity of one of the sources is reduced by 36% by reducing its width then the intensity of light at the same point will be

(A) 90 (B) 89
(C) 67 (D) 81

141. In a double slit arrangement fringes are produced using light of wavelength 4800 Å. One slit is covered by a thin plate of glass of refractive index 1.4 and the other with another glass plate of same thickness but of refractive index 1.7. By doing so the central bright shifts to original fifth bright fringe from centre. Thickness of glass plate is

(A) 8 μm (B) 6 μm
(C) 4 μm (D) 10 μm

142. Polarisation of light establishes

(A) corpuscular theory of light
(B) quantum nature of light
(C) transverse nature of light
(D) all the three

143. A ray of unpolarised light is incident on glass plate at the polarising angle, then

(A) the reflected and transmitted rays will be completely plane polarised
(B) the reflected ray is completely polarised and the transmitted ray is partially polarised
(C) the reflected ray is partially polarised and the transmitted ray is completely polarised
(D) the reflected ray and the transmitted ray will be partially polarised

144. To observe diffraction, the size of the obstacle

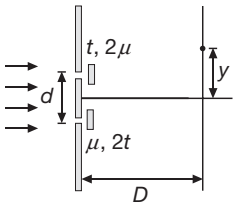
(A) should be of the same order as the wavelength
(B) should be much larger than the wavelength
(C) has no relation to wavelength
(D) should be exactly half the wavelength

145. A slit is illuminated by red light of wavelength 6500 Å. The first minimum is obtained at $\theta = 30°$. The width of the slit is

(A) 3200 Å (B) 1.24 micron

(C) 6.5×10^{-4} mm (D) 2.6×10^{-4} cm

146. In *YDSE* setup shown, the two slits are covered with thin sheets having thickness t and $2t$ and refractive index 2μ and μ. The position (y) of central maxima is



(A) zero (B) $\frac{tD}{d}$

(C) $-\frac{tD}{d}$ (D) None of these

147. A ray of light is incident on the surface of a glass plate at an angle of incidence equal to Brewster's angle ϕ. If μ represents the refractive index of glass with respect to air, then the angle between reflected and refracted rays is

(A) $90° + \phi$ (B) $\sin^{-1}(\mu \cos\phi)$

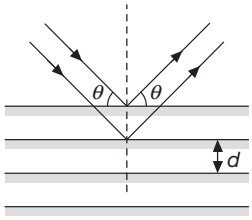
(C) $90°$ (D) $90° - \sin^{-1}\left(\frac{\sin\phi}{\mu}\right)$

148. A beam of plane polarized light falls normally on a polarizer of cross sectional area 3×10^{-4} m^2. Flux of energy of incident ray in 10^{-3} W. The polarizer rotates with an angular frequency of 31.4 rads^{-1}. The energy of light passing through the polarizer per revolution will be

(A) 10^{-4} Joule (B) 10^{-3} Joule

(C) 10^{-2} Joule (D) 10^{-1} Joule

149. A beam with wavelength λ falls on a stack of partially reflecting planes with separation d. The angle θ that the beam should make with the planes so that the beams reflected from successive planes may interfere constructively is (where $n = 1, 2, \ldots\ldots$)



(A) $\sin^{-1}\left(\frac{n\lambda}{d}\right)$ (B) $\tan^{-1}\left(\frac{n\lambda}{d}\right)$

(C) $\sin^{-1}\left(\frac{n\lambda}{2d}\right)$ (D) $\cos^{-1}\left(\frac{n\lambda}{2d}\right)$

150. A diffraction pattern is obtained using a beam of red light. If the red light is replaced by blue light, then

(A) the diffraction pattern remains unchanged

(B) diffraction bands become narrower and crowded together

(C) bands become broader and farther apart

(D) bands disappear

151. At sunrise or at sunset the sun appears to be reddish while at mid-day the sun looks white. The reason is that

(A) the sun is less hot at sunrise or at sunset than at noon

(B) diffraction sends red rays to the earth at these time

(C) refraction is responsible for this effect

(D) scattering due to dust particles and air molecules

152. A screen is placed at 50 cm from a single slit, which is illuminated with 600 nm light. If separation between the first and third minima in the diffraction pattern is 3.0 mm , then width of the slit is

(A) 4 mm (B) 0.1 mm

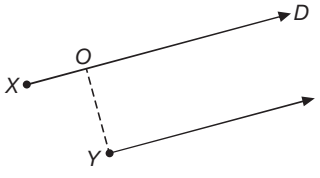
(C) 0.3 mm (D) 0.2 mm

153. Light of wavelength 6328 Å is incident normally on a slit having a width of 0.2 mm . The distance of the screen from the slit is 0.9 m. The angular width of the central maximum is

(A) 0.09 degree (B) 0.72 degree

(C) 0.18 degree (D) 0.36 degree

154. Two point sources X and Y emit waves of same frequency and speed but Y lags in phase behind X by $2\pi l$ radian. If there is a maximum in direction D the distance XO using n as an integer is given by



(A) $\frac{\lambda}{2}(n-l)$ (B) $\lambda(n+l)$

(C) $\frac{\lambda}{2}(n+l)$ (D) $\lambda(n-l)$

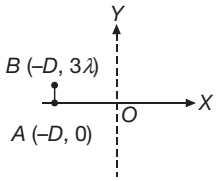
155. In a Young's double slit experiment, the slits are 2 mm apart and are illuminated with a mixture of two wavelength $\lambda_0 = 750$ nm and $\lambda = 900$ nm. The minimum distance from the common central bright fringe on a screen $2m$ from the slits where a bright fringe from one interference pattern coincides with a bright fringe from the other is
(A) 1.5 mm (B) 3 mm
(C) 4.5 mm (D) 6 mm

156. If sound waves can be assumed to be diffracted, which of the following objects will diffract sound waves in air from a 384 Hz tuning fork
(A) A sphere of radius 1 cm
(B) A sphere of radius 1 mm
(C) A sphere of radius 1 m
(D) A sphere of radius 10 m

157. A beam of light of wavelength 600 nm from a distant source falls on a single slit 1 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of the central bright fringe is
(A) 1.2 cm (B) 1.2 mm
(C) 2.4 cm (D) 2.4 mm

158. A nicol prism is based on the action of
(A) refraction (B) double refraction
(C) dichroism (D) both (B) and (C)

159. Two coherent points sources A and B emitting light of wavelength λ is are placed at position $(-D, 0)$ and $(-D, 3\lambda)$ respectively. Here, $D >> \lambda$. The shape of fringes on the screen placed along y-axis (in YZ-plane) is



(A) straight line (B) circular
(C) hyperbolic (D) parabolic

160. A thin sheet of glass (refractive index 1.5) of thickness 6 micron, introduced in the path of one of the interfering beams in a double-slit experiment, shifts the central fringe to a position earlier occupied by the fifth bright fringe. The wavelength of light used is
(A) 3000 Å (B) 6000 Å
(C) 4500 Å (D) 7000 Å

161. Which of the following cannot be polarised?
(A) Radio waves (B) β rays
(C) Infrared rays (D) γ rays

162. When light is incident on a transparent surface at the polarizing angle, which of the following is completely polarized?
(A) Reflected light
(B) Refracted light
(C) Both reflected as well as refracted light
(D) Neither reflected nor refracted light

163. An optically active substance
(A) produces polarized light
(B) rotates the plane of polarization of polarized light
(C) converts a plane polarized light into circularly polarized light
(D) converts a circularly polarized light into plane polarized light

164. Diffraction pattern of a single slit consists of a central bright band which is
(A) wide, and is flanked by alternate dark and bright bands of decreasing intensity
(B) narrow, and is flanked by alternate dark and bright bands of equal intensity
(C) wide, and is flanked by alternate dark and bright bands of equal intensity
(D) narrow, and is flanked by alternate dark and bright bands of decreasing intensity

165. In Young's experiment with one source and two slits, one of the slits is covered with black paper. Then
(A) the fringes will be darker
(B) the fringes will be narrower
(C) the fringes will be broader
(D) no fringes will be obtained and the screen will have uniform illumination

166. The distance between two coherent sources is 0.1 mm. The fringewidth on a screen 1.2 m away from the sources is 6.0 mm. The wavelength of light used is
(A) 4000 Å (B) 5000 Å
(C) 6000 Å (D) 7200 Å

167. If three slits are used in Young's experiment instead of two, we get
(A) no fringe pattern
(B) the same fringe pattern as that with two slits
(C) a pattern with fringe width reduced to half of that in the two slit pattern
(D) alternate bright and dim fringes

168. When a transparent parallel plate of uniform thickness t and refractive index n is interposed normally in the path of a beam of light, the optical path is
(A) increased by nt
(B) decreased by nt
(C) decreased by $(n-1)t$
(D) increased by $(n-1)t$

169. In Young's experiment, monochromatic light is used to illuminate the two slits and interference fringes are observed on a screen placed in front of the slits. Now if a thin glass plate is placed normally in the path of the beam coming from one of the slits, then
(A) the fringes will disappear
(B) the fringe-width will decrease
(C) the fringe-width will increase
(D) there will be no change in the fringe-width

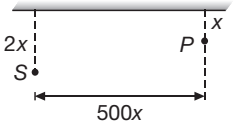
170. If the Young's double slit-experiment is performed with white light, then the colour which will have maximum fringe width is
(A) Blue (B) Green
(C) Yellow (D) Violet

171. In the interference pattern, energy is
(A) created at the positions of maxima
(B) destroyed at the positions of minima
(C) conserved but is redistributed
(D) not conserved

172. Fluorescent tubes give more light than a filament bulb of same power because
(A) the tube contains gas at low temperature
(B) ultraviolet light is converted into visible light by fluorescence
(C) light is diffused through the walls of the tube
(D) it produces more heat than bulb

173. Energies of photons of four different electromagnetic radiations are given below. The energy value corresponds to a visible photon is equal to
(A) 1 eV (B) 2 eV
(C) 5 eV (D) 1000 eV

174. The point S is a monochromatic source of light emitting light of wavelength λ. At the point P, at a distance x from the mirror as shown in the figure, interference takes place between two light rays, one directly coming from source S and another after reflection from the mirror such that a maxima is formed at P. The minimum value of x is



(A) 120λ (B) 125λ
(C) 62.5λ (D) None of these

175. If a torch is used in place of monochromatic light in Young's experiment
(A) fringes will appear as for monochromatic light
(B) fringes will appear for a moment and then they will disappear
(C) no fringes will appear
(D) only bright fringes will appear

176. beam of unpolarized light of intensity I is passed first through a tourmaline crystal A and then through another tourmaline crystal B oriented so that its principal plane is parallel to that of A. If A is now rotated by 45° in a plane perpendicular to the direction of the incident ray, the intensity of the emergent light will be
(A) $\frac{I}{2}$ (B) $\frac{I}{\sqrt{2}}$
(C) I (D) $\frac{I}{4}$

177. Interference pattern is obtained on a screen due to two identical coherent sources of monochromatic light. The intensity of the central bright fringe is I. When one of the sources is blocked, the intensity become I_0. The intensity in the two situations are related as
(A) $I = I_0$ (B) $I = 2I_0$
(C) $I = 3I_0$ (D) $I = 4I_0$

178. The phase difference between two wave trains giving rise to a dark fringe in Young's double-slit experiment is (where n is an integer)
(A) zero (B) $2\pi n + \frac{\pi}{2}$
(C) $2\pi n + \pi$ (D) $2\pi n + \frac{\pi}{4}$

179. A Young's double-slit set-up for interference is shifted from air to within water. Then the
(A) fringe pattern disappears
(B) fringewidth decreases
(C) fringewidth increases
(D) fringewidth remains unchanged

180. Two interfering beams have intensities in the ratio of $9:4$. Then the ratio of maximum to minimum intensity in the interference pattern is
(A) $25:1$ (B) $13:5$
(C) $5:1$ (D) $3:2$

181. In the far field diffraction pattern of a single slit under polychromatic illumination, the first minimum with the wavelength λ_1 is found to be coincident with the third maximum at λ_2. So
(A) $3\lambda_1 = 0.3\lambda_2$ (B) $3\lambda_1 = \lambda_2$
(C) $\lambda_1 = 3.5\lambda_2$ (D) $0.3\lambda_1 = 3\lambda_2$

182. In interference with two coherent beams of light, the fringe width is proportional
(A) to wavelength
(B) to inverse wavelength
(C) to square of wavelength
(D) to inverse square of wavelength

183. The fringe pattern observed in Young's double-slit experiment is
(A) a diffraction pattern
(B) an interference pattern
(C) a combination of diffraction and interference patterns
(D) neither a diffraction nor an interference pattern

184. In Young's interference experiment with one source and two slits, one slit is covered with a cellophane sheet which absorbs half the intensity. Then
(A) no fringes are obtained
(B) bright fringes will be brighter and dark fringes will be darker
(C) all fringes will be dark
(D) bright fringes will be less bright and dark fringes will be less dark

185. The distance between sources in a biprism of angle α and refractive index μ, if the source is placed at a distance a from it is
(A) $2(\mu-1)\alpha$ (B) $2(\mu-1)\alpha a$
(C) $(\mu-1)\alpha$ (D) $(\mu-1)\alpha a$

186. To obtain a sustained interference pattern, we require two sources which emit radiation of
(A) the same frequency.
(B) nearly the same frequency.
(C) the same frequency having a definite phase relationship.
(D) different wavelengths.

187. A thin mica sheet of thickness 2×10^{-6} m and refractive index $(\mu = 1.5)$ is introduced in the path of the first wave. The wavelength of the wave used is 5000 Å. The central bright maximum will shift
(A) 2 fringes upward
(B) 2 fringes downward
(C) 10 fringes upward
(D) None of these

188. In Young's double slit experiment, the slits are 0.5 mm apart and interference pattern is observed on a screen placed at a distance of 1 m from the plane containing the slits. If wavelength of the incident light is 6000 Å, then the separation between the third bright fringe and the central maxima is
(A) 4 mm (B) 3.5 mm
(C) 3 mm (D) 2.5 mm

189. Interference fringes are obtained due to the interference of waves from two coherent sources of light with amplitudes a_1 and $a_2 (a_1 = 2a_2)$. The ratio of the maximum and minimum intensities of light in the interference pattern is
(A) 2 (B) 4
(C) 9 (D) ∞

190. In the double-slit experiment, the distance of the second dark fringe from the central line is 3 mm. The distance of the fourth bright fringe from the central line is
(A) 6 mm (B) 8 mm
(C) 12 mm (D) 16 mm

191. In Young's double-slit experiment, we get 60 fringes in the field of view if we use light of wavelength 4000 Å. The number of fringes we will get in the same field of view if we use light of wavelength 6000 Å is
(A) 40 (B) 90
(C) 60 (D) 50

192. With a monochromatic light, the fringe-width obtained in a double-slit experiment is 1.33 mm. If the whole set-up is immersed in water of refractive index 1.33, the new fringe-width will be
(A) 1.33 mm (B) 1 mm
(C) 1.33×1.33 mm (D) $\frac{1.33}{2}$ mm

193. Two waves having amplitudes in the ratio 5:1 produce interference. The ratio of the maximum to the minimum intensity is
(A) 25:1 (B) 6:4
(C) 9:4 (D) 3:2

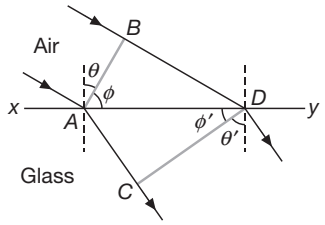
194. If the intensities of the two interfering beams in Young's double-slit experiment are I_1 and I_2, then the contrast between the maximum and minimum intensities is good when
(A) $|I_1 - I_2|$ is large
(B) $|I_1 - I_2|$ is small
(C) either I_1 or I_2 is zero
(D) $I_1 = I_2$

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

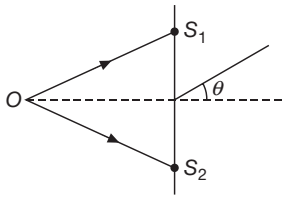
1. In Young's double slit experiment the two slits are covered by slabs of same thickness but refractive index 1.4 and 1.7. If slit to screen separation is 1 m and slits are at 1 mm separation using a coherent source of wavelength 4000 Å and the central fringe shifts to the 3rd bright fringe position, then
(A) shift will be towards slab of index 1.7 by 1.2 mm
(B) shift will be towards slab of index 1.4 by 1.2 mm
(C) slabs are of thickness 4 μm
(D) slabs are of thickness 4 Å

2. In the given diagram a wavefront AB moving in air is incident on a plane glass surface xy. Its position CD after refraction through the glass slab is shown also along with normals drawn at A and D. The refractive index of glass will be equal to $(\mu_{air} = 1)$



(A) $\left(\frac{BD}{AC}\right)$ (B) $\left(\frac{AB}{CD}\right)$
(C) $\left(\frac{\sin\phi}{\sin\phi'}\right)$ (D) $\left(\frac{\cos\theta}{\cos\theta'}\right)$

3. In a modified $YDSE$ experiment if point source of monochromatic light O is placed in such a manner that $OS_1 - OS_2 = \frac{\lambda}{4}$, where λ is the wavelength of light and S_1, S_2 are the slits separated by a distance 2λ. Then value(s) of θ for which a maxima is obtained is/are



(A) $\sin^{-1}\left(\frac{1}{8}\right)$ (B) $\sin^{-1}\left(-\frac{1}{4}\right)$
(C) $\sin^{-1}\left(\frac{5}{6}\right)$ (D) $\sin^{-1}\left(-\frac{7}{8}\right)$

4. Two coherent waves represented by

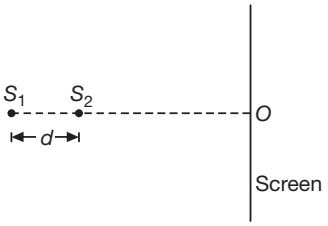
$$y_1 = A\sin\left(\frac{2\pi x_1}{\lambda} - \omega t + \frac{\pi}{4}\right) \text{ and}$$

$$y_2 = A\sin\left(\frac{2\pi x_2}{\lambda} - \omega t + \frac{\pi}{6}\right)$$

are superposed. The two waves will produce
(A) constructive interference at $x_1 - x_2 = \frac{11\lambda}{24}$
(B) constructive interference at $x_1 - x_2 = \frac{23\lambda}{24}$
(C) destructive interference at $x_1 - x_2 = \frac{23\lambda}{24}$
(D) destructive interference at $x_1 - x_2 = \frac{11\lambda}{24}$

5. To observe a stationary interference pattern formed by two light waves, it is not necessary that they must have
(A) the same frequency
(B) the same amplitude
(C) a constant phase difference
(D) the same intensity

6. Two point monochromatic and coherent sources of light of wavelength λ are placed on the dotted line in front of a large screen. The source emit waves in phase with each other. The distance between S_1 and S_2 is d while their distance from the screen is much larger. Then for

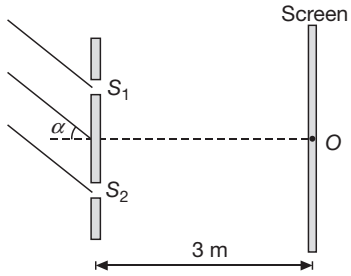


(A) $d = \frac{3\lambda}{2}$, O will be a minima
(B) $d = 3\lambda$, there will be a total of 6 minima on screen
(C) $d = \lambda$, there will be one maxima on the screen
(D) $d = 2\lambda$, there will be two maxima on the screen

7. If white light is used in a Young's double-slit experiment, then
(A) bright white fringe is formed at the centre of the screen

(B) fringes of different colours are observed clearly only in the first order
(C) the first-order violet fringes are closer to the centre of the screen than the first order red fringes
(D) the first-order red fringes are closer to the centre of the screen than the first order violet fringes

8. A parallel beam of light $(\lambda = 5000\ \text{Å})$ is incident at an angle $\alpha = 30°$ with the normal to the slit plane in a young's double slit experiment. Assume that the intensity due to each slit at any point on the screen is I_0. Point O is equidistant from S_1 and S_2. The distance between slits is 1 mm.



(A) the intensity at O is $4I_0$
(B) the intensity at O is zero
(C) the intensity at a point on the screen 4 m from O is $4I_0$
(D) the intensity at a point on the screen 4 m from O is zero

9. Four coherent light waves are represented by

(i) $y_1 = a_1 \sin(\omega t)$
(ii) $y_2 = a_2 \sin(\omega t + \phi)$
(iii) $y_3 = a_1 \sin(2\omega t)$
(iv) $y_4 = a_2 \sin(2\omega t + \phi)$

Interference fringes may be observed due to superposition of

(A) (i) and (ii) (B) (i) and (iii)
(C) (ii) and (iv) (D) (iii) and (iv)

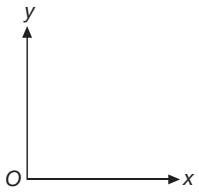
10. If one of the slits of a standard Young's double slit experiment is covered by a thin parallel slit glass so that it transmits only one half the light intensity of the other, then

(A) The fringe pattern will get shifted towards the covered slit
(B) The fringe pattern will get shifted away from the covered slit
(C) The bright fringes will become less bright and the dark ones will become more bright
(D) The fringe width will remain unchanged

11. If the first minima in a Young's slit experiment occurs directly in front of one of the slits, (distance between slits and screen $d = 5$ cm) then the wavelength of the radiation used can be

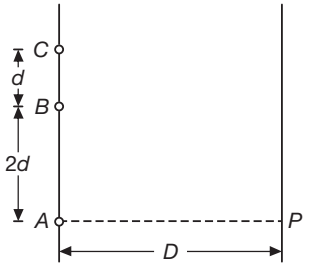
(A) 2 cm (B) 4 cm
(C) $\frac{2}{3}$ cm (D) $\frac{4}{3}$ cm

12. Two coherent sources A and B emitting light of wavelength λ are placed at positions $(-D, O)$ and $(-D, 3\lambda)$ respectively $D \gg \lambda$



(A) number of minima on y-axis is 6
(B) number of minima is more than number of maxima on y-axis
(C) number of maxima on x-axis is 3
(D) number of maxima on x-axis is more than number of minima on x-axis

13. In the figure A, B and C are three slits each of them individually producing the same intensity I_0 at P when they are illuminated by parallel beam of light of wavelength λ. It is given that $BP - AP = \frac{\lambda}{2}$. Also given that $d \ll D$, then wavelength λ and resultant intensity I at P will be



(A) $\lambda = \frac{2d^2}{D}$ (B) $\lambda = \frac{4d^2}{D}$
(C) $I = 2I_0$ (D) $I = I_0$

14. Light waves travel in vacuum along the x-axis. Which of the following may represent the wavefronts

(A) $x = c$ (B) $y = c$
(C) $z = c$ (D) $x + y + z = c$

15. A Young's double-slit apparatus is immersed in oil of refractive index $\frac{5}{3}$. The wavelength of light used in 500 nm (in oil), slit separation 2 mm, and distance to screen is 3 m. A glass slab of thickness 10 μm and refractive index $\frac{3}{2}$ is placed before one slit. The fringe pattern will shift
(A) 2 mm towards the other slit
(B) 2 mm away from the other slit
(C) 2.5 mm towards the other slit
(D) 2.5 mm away from the other slit

16. The fringe width in Young's double-slit experiment can be increased by decreasing
(A) separation of the slits
(B) frequency of the source of light
(C) distance between slit and screen
(D) wavelength of the source of light

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** If the phase difference between the light waves emerging from the slits of the Young's experiment is π-radian, the central fringe will be dark.
Statement-2: Phase difference is equal to $\frac{2\pi}{\lambda}$ times the path difference.

2. **Statement-1:** When a thin transparent sheet is placed in front of both the slits of Young's experiment, the fringe width will increase.
Statement-2: In Young's experiment the fringe width is proportional to wavelength of the source used.

3. **Statement-1:** In Young's double slit experiment, we observe an interference pattern on the screen if both the slits are illuminated by two bulbs of same power.
Statement-2: The interference pattern is observed when source is monochromatic and coherent.

4. **Statement-1:** No interference pattern is detected when two coherent sources are infinitely close to each other in simple $YDSE$.
Statement-2: The fringe width is inversely proportional to the distance between the two slits in simple $YDSE$.

5. **Statement-1:** The minimum slit separation d for interference to produce at least one maxima other than central maxima is 3λ.
Statement-2: For a maxima, path difference equals $n\lambda$. The maximum value of path difference is d.

6. **Statement-1:** Two slits in $YDSE$ are illuminated by two different sodium lamps emitting light of same wavelength. No interference pattern is observed.
Statement-2: To obtain interference pattern, source must be coherent. Two different light sources can never be coherent.

7. **Statement-1:** In Young's double slit experiment interference pattern disappears when one of the slits is covered by transparent slab.
Statement-2: Interference occurs due to superimposition of light wave from two coherent sources.

8. **Statement-1:** When a thin transparent sheet is placed in front of both the slits of Young's experiment, the fringe width will increase.
Statement-2: In Young's experiment the fringe width is proportional to wavelength of the source used.

9. **Statement-1:** Total number of maxima obtained over screen remains same whether Young's Double slit experiment is performed in air or in water with same setup.
Statement-2: $\beta_{water} = \frac{\beta_{air}}{\mu_{water}}$ (in Young's double slit experiment).

10. **Statement-1:** Interference obeys the Law of Conservation of Energy.
Statement-2: The energy is redistributed in case of interference.

11. **Statement-1:** Geometrical optics can be regarded as the limiting case of wave optics.
Statement-2: When size of obstacle or opening is very large compared to the wavelength of light then wave nature can be ignored and light can be assumed to be travelling in straight line.

12. **Statement-1:** Light from two coherent sources is reaching the screen. If the path difference at a point on the screen for yellow light is $\frac{3\lambda}{2}$, then the fringe at the point will be coloured.
Statement-2: Two coherent sources always have constant phase relationship.

13. **Statement-1:** The maximum intensity in interference pattern is four times the intensity due to each slit.
Statement-2: Intensity is directly proportional to square of amplitude.

14. **Statement-1:** Interference can be obtained by using two different lamps.
Statement-2: Two different lamps are incoherent sources as constant phase difference cannot be maintained between them.

15. **Statement-1:** Interference pattern is made by using blue light instead of red light, the fringes becomes narrower.
Statement-2: In Young's double slit experiment, fringe width is given by relation $\beta = \frac{\lambda D}{d}$.

16. **Statement-1:** Interference pattern is obtained on a screen due to two identical coherent sources of monochromatic light. The intensity at the central part of the screen becomes one-fourth if one of the source is blocked.
Statement-2: The resultant intensity is the sum of the intensities due to two sources.

17. **Statement-1:** Thin films such a soap bubble or a thin layer of oil on water show beautiful colours when illuminated by monochromatic light.
Statement-2: Colour in film are obtained due to interference between reflected light from the upper & lower layer of film.

18. **Statement-1:** The fringe obtained at the centre of the screen is known as zeroth order fringe, or the central fringe.
Statement-2: Path difference between the wave from S_1 and S_2, reaching the central fringe (or zero order fringe) is zero.

19. **Statement-1:** The phase difference between any two points on a wavefront is zero.
Statement-2: Light from the source reaches every point of the wavefront at the same time.

20. **Statement-1:** In Young's double slit experiment interference pattern disappears when one of the slits is closed.
Statement-2: Interference occurs due to superimposition of light wave from two coherent sources.

21. **Statement-1:** If a clean glass slide is observed under white light, one does not observe any colours. However, if this slide is touches with oily hands, coloured fringes appear on the slide.
Statement-2: These fringes are due to interference of reflected light, reflected from the upper and lower surfaces of the thin oil film.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension I

In a Young's experiment the upper slit is covered by a thin glass plate of refractive index $\mu = 1.4$. The interference pattern is observed using light of interference 5000 Å. Based on above information, answer the following questions.

1. It is observed that
   (A) the central maxima shifts upwards.
   (B) the central maxima shifts downwards.
   (C) fringe pattern will change after introduction of thin plate.
   (D) none of the above phenomenon is observed.

2. Now a thin plate of refractive index 1.7 is placed in front of lower slit then
   (A) central maxima will be obtained above centre.
   (B) central maxima will be obtain at centre.
   (C) there will not be any change in central maxima after introduction of thin plate.
   (D) no conclusion can be withdrawn without knowing the thickness of the plate.

3. Assume the thickness of both thin glass plate is t, the path difference between waves incident at center is
   (A) $0.5t$ (B) $0.3t$
   (C) $0.8t$ (D) $\frac{1.7}{1.5}t$

4. If minima is to be obtained at center then minimum value of t is (source wavelength is λ)

(A) $t = \frac{\lambda}{0.6}$ (B) $t = \frac{\lambda}{0.3}$

(C) $t = \lambda$ (D) $t = \frac{\lambda}{0.15}$

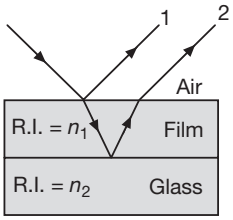
5. If intensity at center is $\frac{3}{4}$ of maximum intensity then minimum value of t is

(A) $t = 2777.7$ Å (B) $t = 3188$ Å

(C) $t = 4188.8$ Å (D) $t = 2122.9$ Å

Comprehension 2

A thin film of a specific material can be used to decrease the intensity of reflected light. There is destructive interference of waves reflected from upper and lower surfaces of the film. These films are called non–reflecting or antireflection coatings. The process of coating the lens or surface with non–reflecting film is called blooming as shown in the figure. The refracting index of coating (n_1) is less than that of the glass (n_2). Based on above information, answer the following questions.



6. If the light of wavelength λ is incident normally and the thickness of film is t then optical path difference between waves reflected from upper and lower surface of film is

(A) $2n_1t$ (B) $2n_1t - \frac{\lambda}{2}$

(C) $2n_1t + \frac{\lambda}{2}$ (D) $2t$

7. Magnesium fluoride (MgF_2) is generally used as anti–reflection coating. If refractive index of MgF_2 is 1.38 then minimum thickness of film required for $\lambda = 550$ nm is

(A) 112.4 nm (B) 78.2 nm

(C) 99.64 nm (D) 225 nm.

8. Assuming that the thickness of film in above problem is not technically possible to manufacture, then next thickness of film required is (approximately)

(A) 298.9 nm

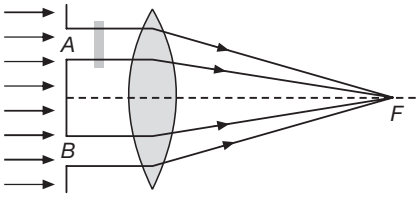
(B) 271.7 nm

(C) 304.7 nm

(D) 550 nm

Comprehension 3

In a modified Young's double slit experiment, a monochromatic uniform and parallel beam of light of wavelength 6000 Å and intensity $\frac{10}{\pi}$ Wm^{-2} is incident normally on two circular apertures A and B of radii 0.001 m and 0.002 m respectively. A perfect transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength of 6000 Å is placed in front of aperture A. The lens is symmetrically placed with respect to the apertures. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot.



Based on the above facts, answer the following questions.

9. The power received at A is

(A) 10^{-5} W (B) 4×10^{-5} W

(C) 10^{-6} W (D) 4×10^{-6} W

10. The power received at B is

(A) 10^{-5} W (B) 4×10^{-5} W

(C) 10^{-6} W (D) 4×10^{-6} W

11. The power transmitted through A is

(A) 10^{-5} W (B) 4×10^{-5} W

(C) 10^{-6} W (D) 4×10^{-6} W

12. The power transmitted through B is

(A) 10^{-5} W (B) 4×10^{-5} W

(C) 10^{-6} W (D) 4×10^{-6} W

13. The path difference introduced by the film is

(A) 10^{-3} m (B) 10^{-5} m

(C) 10^{-7} m (D) 10^{-9} m

14. The phase difference introduced by the film is

(A) π radian (B) $\frac{\pi}{2}$ radian

(C) $\frac{\pi}{3}$ radian (D) $\frac{\pi}{4}$ radian

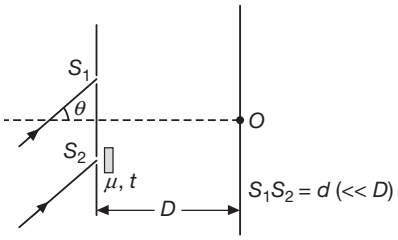
15. The power in watt received at the focal point F of the lens is

(A) 2 μW (B) 5 μW

(C) 6 μW (D) 7 μW

Comprehension 4

A monochromatic beam of light of wavelength $\lambda = 600$ nm falls on Young's double slit experiment apparatus as shown in figure. A thin sheet of glass is inserted in front of lower slit S_2. Based on above information, answer the following questions.



16. The central bright fringe can be obtained
(A) at O
(B) at O or below O
(C) at O or above O
(D) anywhere on the screen

17. If central bright fringe is obtained on screen at O, then we have
(A) $(\mu-1)t = d\sin\theta$ (B) $(\mu-1)t = d\cos\theta$
(C) $\mu t = d\theta$ (D) $\dfrac{t}{\mu-1} = \dfrac{d}{\sin\theta}$

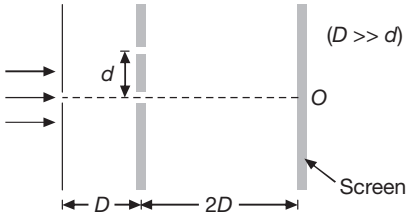
18. The phase difference between central maxima and fifth minima is
(A) $\dfrac{\pi}{6}$ (B) 9π
(C) $\dfrac{3\pi}{2}$ (D) $8\pi \pm \dfrac{\pi}{6}$

19. Fringe width for the pattern obtained on screen, if $\lambda = 600$ nm, $\mu = 1.5$, $d = 3$ mm, $D = 2$ m and $\theta = 30°$ is
(A) 2×10^5 nm (B) 4×10^5 nm
(C) 10^4 nm (D) 3×10^6 nm

20. Assume if $\theta\left(<\dfrac{\pi}{2}\right)$ is increased then for a given value of μ
(A) central maxima will move downwards.
(B) central maxima will move upward.
(C) fringe width will increase.
(D) fringe width will decrease.

Comprehension 5

In the arrangement shown in the figure, the distance D is large compared to the separation d between the slits. Monochromatic light of wavelength λ is incident on the slit, based on the information provided answer the following questions.



21. The minimum value of d for which there is a dark fringe at the point O is
(A) $\sqrt{\dfrac{\lambda D}{2}}$
(B) $\sqrt{\dfrac{D\lambda}{3}}$
(C) $\sqrt{\lambda D}$
(D) not possible to be calculated

22. The position of first bright fringe for the minimum value of d is
(A) $\dfrac{d}{2}$ below (B) d above
(C) $\dfrac{3d}{2}$ below (D) $\dfrac{3d}{2}$ above

23. The fringe width is
(A) $\dfrac{3D\lambda}{4d}$ (B) $\dfrac{3D\lambda}{2d}$
(C) $\dfrac{D\lambda}{d}$ (D) $\dfrac{2D\lambda}{d}$

Comprehension 6

If light incident on a thin film has wavelength as 900 nm and refractive index of film is 1.5. Based on above information, answer the following questions.

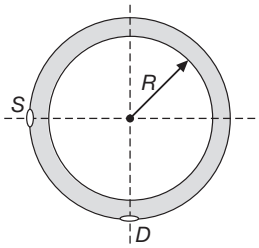
24. Minimum thickness of film needed for constructive interference in reflected light system is
(A) 100 nm (B) 150 nm
(C) 200 nm (D) 250 nm

25. Minimum thickness of film for destructive interference in transmitted light system is
(A) 150 nm (B) 200 nm
(C) 250 nm (D) 100 nm

Comprehension 7

A narrow tube is bent in the form of circle of radius R as shown. Two small holes S and D are made in the tube at the positions right angles to each other. A source placed at S generates a wave of intensity I_0 which is equally divided into two parts. One part travels along the longer path,

while the other travels along the shorter path. Both the part waves meet at point D where a detector is placed. Based on above information, answer the following questions.



26. Maximum intensity produced at D is given by

(A) $4I_0$ (B) $3I_0$

(C) $2I_0$ (D) I_0

27. The maximum value of wavelength λ to produce a maximum at D is given by

(A) πR (B) $2\pi R$

(C) $\frac{\pi R}{2}$ (D) $\frac{3\pi R}{2}$

28. The maximum value of wavelength λ to produce a minimum at D is given by

(A) πR (B) $2\pi R$

(C) $\frac{\pi R}{2}$ (D) $\frac{3\pi R}{2}$

Comprehension 8

When Fresnel's biprism experiment is performed in air then distance between coherent sources is 0.5 mm and distance between source and screen is 1 m. Fringe width obtained in air is 1 mm. Refractive index of biprism is 1.5. Now the experiment is performed in water having refractive index $\mu_w = \frac{4}{3}$. If the refractive index of the biprism is $\mu = \frac{3}{2}$. Based on above information, answer the following questions.

29. The distance between coherent sources in water is

(A) 1 mm (B) $\frac{1}{2}$ mm

(C) $\frac{1}{4}$ mm (D) $\frac{1}{8}$ mm

30. The fringe width in water is

(A) 1 mm (B) 2 mm

(C) 3 mm (D) 4 mm

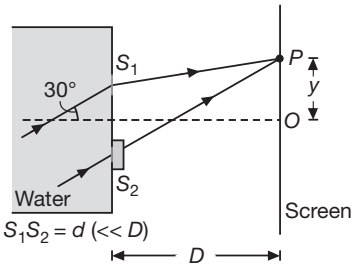
31. The wavelength of light in air is

(A) 4000 Å (B) 4500 Å

(C) 5000 Å (D) 6000 Å

Comprehension 9

In a $YDSE$ experiment, the two slits are covered with transparent membranes of negligible thickness which allow light to pass through it but does not allow water. A glass slab of thickness $t = 0.41$ mm and refractive index $\mu_g = 1.5$ is placed in front of one of the slits as shown in figure. The separation between the slits is $d = 0.30$ mm. The entire space to the left of the slits is filled with water of refractive index $\mu_w = \frac{4}{3}$. A coherent light of intensity I and absolute wavelength $\lambda = 5000$ Å is being incident on the slits making an angle of $30°$ with horizontal. Screen is placed at a distance $D = 1$ m from the slits. Based on above information, answer the following questions.



32. At point O, equidistant from slits we get

(A) 9^{th} dark fringe (B) 10^{th} dark fringe

(C) 11^{th} bright fringe (D) 10^{th} bright fringe

33. Central maxima is located at

(A) $y = +\frac{5}{6}$ cm (B) $y = -\frac{5}{6}$ cm

(C) $y = +\frac{5}{3}$ cm (D) $y = -\frac{5}{3}$ cm

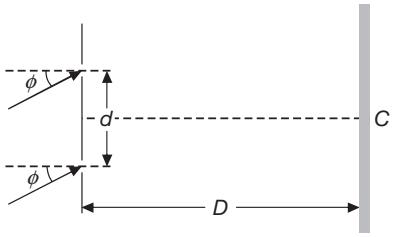
34. The ratio of intensity at point P at $y = \frac{1}{8}$ cm on screen and maximum intensity is

(A) 0 (B) 1

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

Comprehension 10

Light of wavelength $\lambda = 500$ nm falls on two narrow slits placed a distance $d = 50 \times 10^{-4}$ cm apart, at an angle $\phi = 30°$ relative to the slits shown in figure. On the lower slit a transparent slab of thickness 0.1 nm and refractive index $\frac{3}{2}$ is placed. The interference pattern is observed on a screen at a distance $D = 2$ m from the slits. Based on above information, answer the following questions.



35. The angular position of the central maxima w.r.t. central line is

(A) 60° (B) 45°

(C) 30° (D) 15°

36. The order of minima closest to centre C of screen is

(A) 50 (B) 49

(C) 48 (D) 47

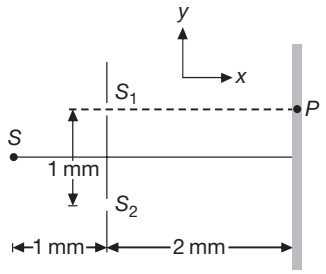
37. The number of fringes that will pass over C, when the transparent slab from the lower slit is removed is

(A) 100 (B) 98

(C) 96 (D) 94

Comprehension 11

In a Young's double slit experiment set-up source S of wavelength 5000 Å illuminates two slits S_1 and S_2, which act as two coherent sources. The source S oscillates about its shown position according to the equation $y = 0.5\sin(\pi t)$, where y is in millimetres and t in seconds. Based on above information, answer the following questions.



38. The position of the central maxima as a function of time is best represented by the equation

(A) $-\cos(\pi t)$ (B) $\cos(\pi t)$

(C) $-\sin[(\pi+1)t]$ (D) $-\sin[\pi(1+t)]$

39. The minimum time t at which the intensity at point P on the screen exactly in front of the upper slit becomes maximum is

(A) $\frac{1}{2}$ s (B) $\frac{1}{3}$ s

(C) $\frac{1}{6}$ s (D) 1 s

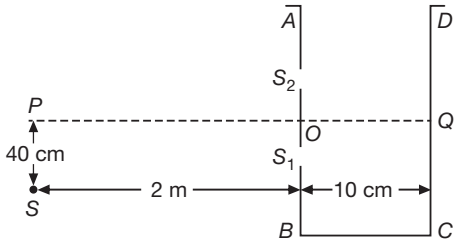
40. The minimum time t at which the intensity at point P on the screen exactly in front of the upper slit becomes minimum is

(A) 1 s (B) $\frac{1}{2}$ s

(C) $\frac{1}{3}$ s (D) $\frac{1}{6}$ s

Comprehension 12

A vessel $ABCD$ of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of S_1 and S_2. A monochromatic light source is kept at S, 40 cm below P and 2 m from the vessel, to illuminate the slits as shown in the figure. Based on above information, answer the following questions.



41. The position of the central bright fringe on the other wall CD with respect to the line OQ is

(A) 2 cm below Q (B) 2 cm above Q

(C) 4 cm below Q (D) 4 cm above Q

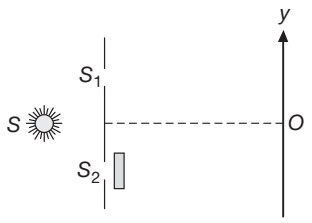
42. It is observed that when a liquid is poured into the vessel and filled upto OQ, the central bright fringe is found to be at Q. The refractive index of the liquid is

(A) 1.0008 (B) 1.0004

(C) 1.0016 (D) 1.0012

Comprehension 13

The Young's double slit experiment is done in a medium of refractive index $\frac{4}{3}$. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10.4 μm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown in the figure. Assume that all wavelengths in the problem are for the given medium of refractive index $\frac{4}{3}$ and ignore dispersion. Based on above information, answer the following questions.



43. The central maximum formed on the y-axis is located at

(A) $y = \frac{11}{3}$ mm above O

(B) $y = \frac{11}{3}$ mm below O

(C) $y = \frac{14}{3}$ mm above O

(D) $y = \frac{13}{3}$ mm below O

44. The ratio of light intensity of point O to the maximum fringe intensity is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{3}{4}$ (D) 1

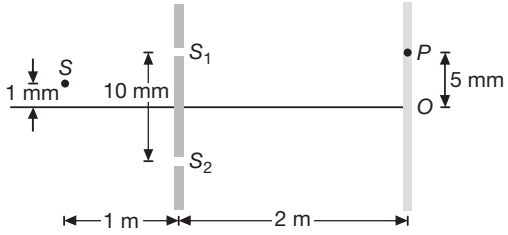
45. Assuming the 600 nm light to be replaced by white light of range 400 to 700 nm, the wavelengths of the light that form maxima exactly at point O are

(A) $\frac{1300}{3}$ nm, 500 nm (B) $\frac{1400}{3}$ nm, 600 nm

(C) $\frac{1300}{3}$ nm, 650 nm (D) $\frac{1400}{3}$ nm, 650 nm

Comprehension 14

In the Young's double slit experiment a point source of $\lambda = 5000$ Å is placed slightly off the central axis as shown in the figure. Based on the information provided, answer the following questions.



46. The order and nature of the interference at the point P is

(A) 50^{th} order, maxima (B) 50^{th} order, minima

(C) 70^{th} order, maxima (D) 70^{th} order, minima

47. The order and the nature of the interference at O is

(A) 50^{th} order, minima

(B) 50^{th} order, maxima

(C) 20^{th} order, minima

(D) 20^{th} order, maxima

48. If the zero order maxima is formed at O, then

(A) we should place a film of refractive index $\mu = 1.5$, thickness 10 μm in front of S_2.

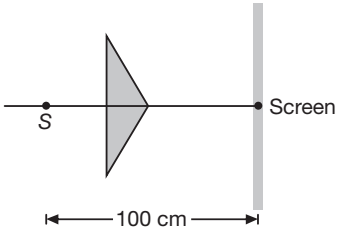
(B) we should place a film of refractive index $\mu = 1.5$, thickness 20 μm in front of S_2.

(C) we should place a film of refractive index $\mu = 1.5$, thickness 10 μm in front of S_1.

(D) we should place a film of refractive index $\mu = 1.5$, thickness 20 μm in front of S_1.

Comprehension 15

A narrow slit S allows monochromatic light of wavelength $\lambda = 6000$ Å to fall on a prism of very small angle as shown in figure. A screen is placed at a distance $l = 100$ cm from the source to obtain an interference pattern. To determine the distance between the virtual images formed by the prism an experiment is done. The prism and screen are kept fixed and a convex lens is moved between the prism and the screen. For two positions of the lens (between the prism and the screen) we get two sharp point images on the screen in each case. The images are separated from each other by a distance 6 mm and 1.5 mm in the other. Now lens is removed and interference pattern is obtained on the screen. Based on the information provided, answer the following questions.



49. Focal length of the lens is

(A) 16 cm (B) 20 cm

(C) 36 cm (D) 40 cm

50. Fringe width of the pattern on the screen is

(A) 0.1 mm (B) 0.2 mm

(C) 0.3 mm (D) 0.4 mm

51. If screen is displaced slightly away from prism, then

(A) No interference pattern is observed

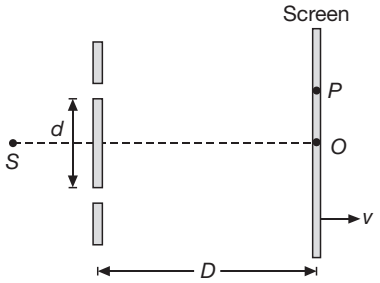
(B) fringe width remains same.

(C) fringe width decreases.

(D) fringe width increases.

Comprehension 16

In YDSE apparatus shown in figure, wavelength of light used is λ. The screen is moved away from the source with a constant speed v. Initial distance between screen and plane of slits was D. Based on the information provided answer the following questions.



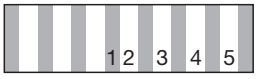
Based on the information provided, answer the following questions.

52. At a point P on the screen, the order of fringe will
(A) increase
(B) decrease
(C) remain constant
(D) first increase then decrease

53. Suppose P is the point where 5^{th} order maxima was lying at $t = 0$. Then after how much time third order minima will lie at this point
(A) $\frac{2D}{v}$ (B) $\frac{D}{v}$
(C) $\frac{3D}{2v}$ (D) $\frac{3D}{v}$

Comprehension 17

The figure shows the interference pattern obtained in a double-slit experiment using light of wavelength 600 nm with fringes 1, 2, 3, 4 and 5 marked on it. If fringe 2 represents the central bright fringe, then based on the information provided answer the following questions.



54. The third order bright fringe is
(A) 2 (B) 3
(C) 4 (D) 5

55. Which fringe results from a phase difference of 4π between the light waves incidenting from two slits
(A) 2 (B) 3
(C) 4 (D) 5

56. Let Δx_1 and Δx_3 represent path differences between waves interfering at 1 and 3 respectively then $\left(|\Delta x_3| - |\Delta x_1|\right)$ is equal to
(A) 0 (B) 300 nm
(C) 600 nm (D) 900 nm

Comprehension 18

If light of wavelength 900 nm is incident on a thin film of refractive index 1.5, then answer the following questions.

57. Minimum thickness of film required for constructive interference in reflected light is
(A) 100 nm (B) 150 nm
(C) 200 nm (D) 250 nm

58. Minimum thickness of film for destructive interference in transmitted light is
(A) 150 nm (B) 200 nm
(C) 250 nm (D) 100 nm

Comprehension 19

When Fresnel's biprism experiment is performed in air, then distance between coherent sources is 0.5 mm and distance between source and screen is 1 m. The fringe width obtained in air is 1 mm. Refractive index of biprism is 1.5. Now the experiment is performed in water $\left(\mu = \frac{4}{3}\right)$.

Based on the information provided answer the following questions.

59. Distance between coherent sources in water is
(A) $\frac{1}{2}$ mm (B) $\frac{1}{4}$ mm
(C) 1 mm (D) None of these

60. Fringe width in water is
(A) 3 mm (B) 2 mm
(C) 1 mm (D) None of these

61. Wavelength of light in air is
(A) 5000 Å (B) 4000 Å
(C) 6000 Å (D) 4500 Å

Comprehension 20

A Young's double slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and interference pattern is observed on the screen at a distance 1.33 m from plane of slits. The wavelength in air 6300 Å. Based on the information provided answer the following questions.

62. The distance of seventh bright fringe from third bright fringe lying on the same side of central bright fringe is

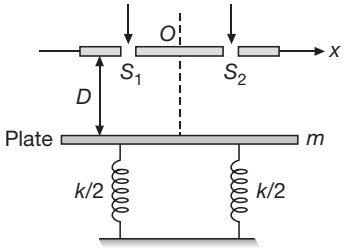
(A) 2.52 mm (B) 4.41 mm
(C) 1.89 mm (D 1.26 mm

63. One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. The smallest thickness of the sheet to interchange the position of minima and maxima assuming that the apparatus is still in same liquid is

(A) 2.575 μm (B) 1.575 μm
(C) 2.095 μm (D) None of these

Comprehension 21

In an arrangement, two slits S_1 and S_2 (lie on the x-axis and symmetric with respect to y-axis) are illuminated by a parallel monochromatic light beam of wavelength λ as shown.



The distance between slits is $d(\gg\lambda)$. Point O is the midpoint of the line S_1S_2 and this point is considered as origin. The slits are in horizontal plane. The interference pattern is observed on a horizontal plate (acting as screen) of mass m to which two vertical massless springs each of spring constant $\frac{k}{2}$ are connected. The other ends of springs are fixed to the ground. At $t=0$, plate is at C, a distance $D(\gg d)$ below the plane of slits and springs are in their natural lengths. The plate is released from rest from its initial position. Based on the information provided answer the following questions.

64. The rate by which fringe width will increase when the acceleration of the plate is zero is

(A) $\frac{\lambda g}{d}\sqrt{\frac{m}{k}}$ (B) $\frac{\lambda g}{3d}\sqrt{\frac{m}{k}}$
(C) $\frac{\lambda g}{4d}\sqrt{\frac{m}{k}}$ (D) $\frac{\lambda g}{2d}\sqrt{\frac{m}{k}}$

65. The difference between two fringe widths when the plate is at rest for a moment is

(A) $\frac{2\lambda}{d}$ (B) $\frac{\lambda mg}{kd}$
(C) $\frac{2\lambda mg}{kd}$ (D) $\frac{mgd}{k\lambda}$

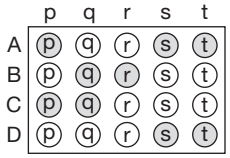
66. A thin slab of refractive index μ is kept in front of one of slits such that position of first maxima shifts to the position of central maxima (at the instant when the plate has been held at rest initially). The thickness of slab is

(A) $\frac{d}{\mu-1}$ (B) $\frac{\lambda d}{D(\mu-1)}$
(C) $\frac{\lambda D}{d(\mu-1)}$ (D) $\frac{\lambda}{(\mu-1)}$

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

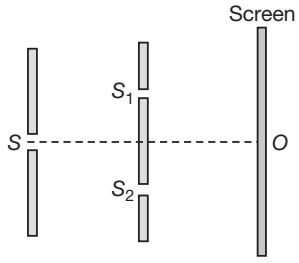
If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following :



1. **COLUMN-I** shows the effect on the fringe pattern in *YDSE* corresponding to the changes mentioned in **COLUMN-II**. Match the effects in **COLUMN-I** with the corresponding causes in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Angular fringe width remains same	(p) Screen is moved away from the plane of the slits
(B) Angular fringe width changes	(q) Wavelength of light used is decreased
(C) Fringe width (linear separation between two consecutive fringes) changes	(r) The separation between the slits is increased
(D) The fringe pattern may disappear	(s) The width of the source slit is increased

2. Figure shows a set-up to perform Young's double slit experiment. A monochromatic source of light is placed at S. S_1 and S_2 act as coherent sources and interference pattern is obtained on the screen. Match **COLUMN-I** with **COLUMN-II** keeping in mind the Young's double slit experiment.



COLUMN-I	COLUMN-II
(A) S is removed and two real sources emitting light of same wavelength are placed at S_1 and S_2.	(p) Interference fringes disappear.
(B) Width of S_1 is two times the width of S_2.	(q) There is uniform illumination on a large part of the screen.
(C) S_1 is closed.	(r) The zero order fringe will not form at O.
(D) A thin transparent plate is placed in front of S_1. Assuming negligible absorption by the plate.	(s) Intensity of a dark fringe will be non-zero, but less than the intensity of bright fringe.

(*Continued*)

3. Match the contents of **COLUMN-I** with the respective phenomenon in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Shining of diamonds.	(p) interference
(B) Light waves projected on oil surface shows seven colours.	(q) total internal reflection
(C) Huygen's wave theory of light cannot explain.	(r) origin of spectra
(D) Phenomena which is not Explained by Huygen's construction of wavelength.	(s) photoelectric effect

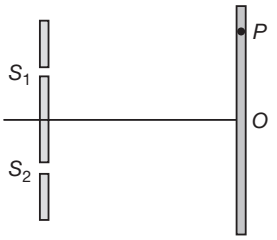
4. In Young's Double Slit Experiment, if distance between slits is d, distance between slit and screen is D, wavelength of light used is λ. Then match **COLUMN-I** with **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) For bright fringe, path difference.	(p) $\frac{D\lambda}{2d}$
(B) For dark fringe, path difference.	(q) $\frac{D}{d}(\mu-1)t$
(C) Displacement of fringe when glass plate of thickness t is placed.	(r) $n\lambda$
(D) Distance between central maxima and first dark fringe when glass plate of thickness t is used.	(s) $(2n-1)\frac{\lambda}{2}$

5. Match the contents of **COLUMN-I** with the respective contents of **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Sources of variable phase difference.	(p) Incoherent sources
(B) Point on a wavefront behaves as a light source.	(q) Coherent sources
(C) Net displacement is the vector sum of individual displacement.	(r) Superposition principle
(D) Young's double slit experiment uses.	(s) Huygen's principle

6. In the *YDSE* appratus shown in figure, Δx is the path difference between S_2P and S_1P. If, now a glass slab is introduced in front of S_2, then match the contents of **COLUMN-I** with the respective matches in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) Fringe width will	(p) increase
(B) Fringe pattern will	(q) decrease
(C) Number of fringes between O and P will	(r) remain same
(D) Δx at P will	(s) shift upward
	(t) shift downward

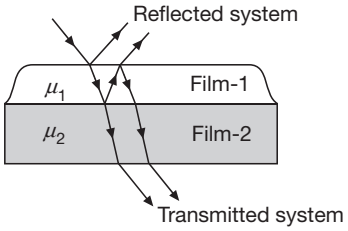
7. Match the quantities in **COLUMN-I** with their respective matches in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Point source of light	(p) Spherical wavefront
(B) Limit of resolution of telescope	(q) Amplitude division
(C) Interference	(r) Superposition of waves
(D) Coherent sources	(s) Radius of lens

8. In the light of possibility of occurrence of phenomena listed in **COLUMN-I** match the listings in **COLUMN-I** to the corresponding waves in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Reflection	(p) Non-mechanical waves
(B) Interference	(q) Electromagnetic waves
(C) Diffraction	(r) Visible light waves
(D) Polarisation	(s) Sound waves

9. For the situation shown in the figure below, match the entries of **COLUMN-I** with **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) $\mu_1 = \mu_2$	(p) Film 1 appears shiny from the reflected system
(B) $\mu_1 > \mu_2$	(q) Film 1 appears dark from the reflected system
(C) $\mu_1 < \mu_2$	(r) Film 1 appears shiny from the transmitted system
(D) $\mu_1 \neq \mu_2$	(s) Film 1 appears dark from the transmitted system

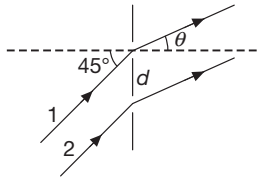
INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. Interference pattern with Young's double slit 1.5 mm apart are formed on a screen at a distance 1.5 m from the plane of slits. In the path of the beam from one of the slits, a transparent film of 10 micron thickness and the refractive index 1.6 is interposed while in the path of the beam from the other slit a transparent film of 15 micron thickness and a refractive index 1.2 is interposed. Find the displacement of the fringe pattern, in mm.

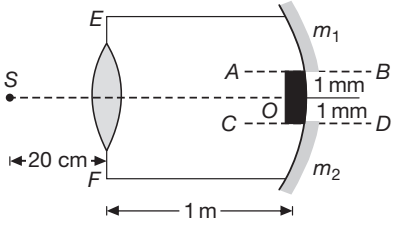
2. Two coherent radio point sources that are separated by 2 m are radiating in phase with a wavelength of 0.25 m. If a detector moves in a large circle around their midpoint, at how many points will the detector show a maximum signal?

3. In the figure shown the distance between the slits is $d = 20\lambda$, where λ is the wavelength of light used. Find the angle θ, in degree, where



(a) central maxima (where path difference is zero) is obtained.
(b) third order maxima is obtained.

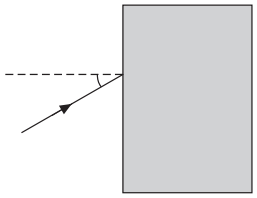
4. An equiconvex lens of focal length 10 cm (in air) and refractive index $\frac{3}{2}$ is put at a small opening on a tube of length 1 m fully filled with liquid of refractive index $\frac{4}{3}$. A concave mirror of radius of curvature 20 cm is cut into two halves m_1 and m_2 and placed at the end of the tube. m_1 and m_2 are placed such that their principal axes AB and CD respectively are separated by 1 mm each from the principle axes of the lens.



A slit S placed in air illuminates the lens with light of frequency 7.5×10^{14} Hz. The light reflected from m_1 and m_2 forms interference pattern on the left and EF of the tube. O is an opaque substance to cover the hole created by the placement of m_1 and m_2. Find :

(a) The position of the image, in cm, formed by lens-water combination.
(b) The distance, in mm, between the images formed by m_1 and m_2.
(c) Width of the fringes on EF, in μm.

5. A ray of light is incident on the left vertical face of the glass slab. If the incident light has an intensity I and on each reflection the intensity decreases by 90% and on each refraction the intensity decreases by 10%, find the ratio of the intensities of maximum to minimum in the reflected pattern.



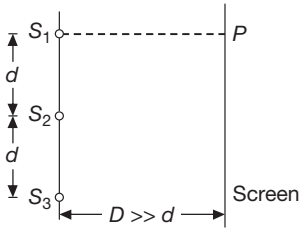
6. Two slits are separated by 0.32 mm. A beam of 500 nm light strikes the slits producing an interference pattern. Determine the number of maxima observed in the angular range $-30° < \theta < 30°$.

7. Interference fringes were produced by Young's double slit method, the wavelength of light used being 6000 Å. The separation between the two slits is 2 mm. The distance between the slits and screen is 10 cm. When a transparent plate of thickness 0.5 mm is placed over one of the slits, the fringe pattern is displaced by 5 mm. If μ be the refractive index of the material of the plate, then find 5μ.

8. In young's double slit experiment mixture of two light wave having wavelengths $\lambda_1 = 500$ nm and $\lambda_2 = 700$ nm are being used. Find the position next to central maxima, where maximas due to both waves coincides. $\left(\text{Given } \frac{D}{d} = 1000\right)$

9. Consider the interference at P between waves emanating from three coherent sources in same phase located at S_1, S_2 and S_3. If intensity due to each

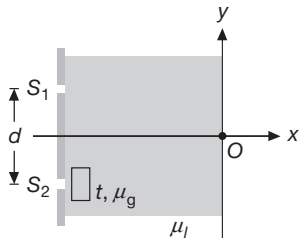
source is $I_0 = 12\ \text{Wm}^{-2}$ at P and $\frac{d^2}{2D} = \frac{\lambda}{3}$ then find the resultant intensity at P, in Wm^{-2}.



10. In a *YDSE* (young double slit experiment) screen is placed 1 m from the slits wavelength of light used is 6000 Å. The fringes formed on the screen are observed by a student sitting close to the slits. The student's eye can distinguish two neighboring fringes, if they subtend an angle more than 1 minute of the arc. Calculate the maximum distance between the slits, in mm, so that fringes are clearly visible. Give your answer to the nearest integer.

11. A parallel beam of white light falls from air on a thin film in air whose refractive index is $\sqrt{3}$. The angle of incidence is $i = 60°$. Find the minimum film thickness (in nanometer), if the reflected light is most intense for $\lambda = 6000$ Å.

12. In a modified *YDSE* the region between screen and slits is immersed in a liquid whose refractive index varies with time as $\mu_l = \frac{5}{2} - \frac{t}{4}$, until it reaches a steady state value $\frac{5}{4}$. A glass plate of thickness $T = 36\ \mu\text{m}$ and refractive index $\mu = \frac{3}{2}$ is introduced in front of one of the slits. If the separation between the sources and the screen is 1 m and the separation between the sources is 2 mm, then calculate the speed of the central maxima, in mms^{-1}, when it is at O,



13. In a Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is now changed to 400 nm, what will be the new number of fringes observed in the same segment of screen.

14. A glass wedge of angle 0.01 radian is illuminated by monochromatic light of wavelength 6000 Å falling normally on it. Find the distance from the edge of the wedge, in mm where the 10^{th} fringe will be observed due to the reflected light.

15. In Young's double slit experiment the two slits act as coherent sources of equal amplitude A and wavelength λ. In another experiment with the same set up the two slits are source of equal amplitude A and wavelength λ but are incoherent. Find the ratio of intensity of light at the mid-point of the screen in the first case to that in second case.

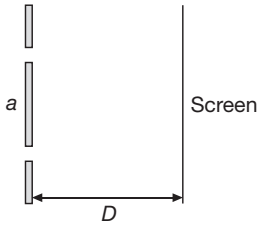
16. In *YDSE* setup, a light of wavelength 6000 Å is used. Calculate the separation between the slits (in mm rounded off to nearest integer) so that at a point (on the screen 1 m from the sources) in front of one of the slits, a third bright fringe is obtained.

ARCHIVE: JEE MAIN

1. **[Online April 2019]**
In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be

(A) 4 (B) 18
(C) 9 (D) 2

2. **[Online April 2019]**
The figure shows a young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gets shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be



(A) $\frac{2nD\lambda}{a(\mu-1)}$ (B) $\frac{n\lambda}{\mu-1}$

(C) $\frac{na\lambda}{\mu-1}$ (D) $\frac{nD\lambda}{\mu-1}$

3. [Online April 2019]

In a Young's double slit experiment, the ratio of the slit's width is $4:1$. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be

(A) $\left(\sqrt{3}+1\right)^4:16$ (B) $4:1$

(C) $25:9$ (D) $9:1$

4. [Online April 2019]

In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used)

(A) $\frac{2\lambda}{(\mu-1)}$ (B) $\frac{\lambda}{2(\mu-1)}$

(C) $\frac{\lambda}{(2\mu-1)}$ (D) $\frac{\lambda}{(\mu-1)}$

5. [Online April 2019]

A system of three polarizers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at $60°$ to the pass axis of P_3. When a beam of unpolarized light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio $\left(\frac{I_0}{I}\right)$ equals (nearly)

(A) 1.80 (B) 5.33

(C) 10.67 (D) 16.00

6. [Online January 2019]

Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio

(A) $25:9$ (B) $4:1$

(C) $16:9$ (D) $5:3$

7. [Online January 2019]

In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda = 500$ nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30° \le \theta \le 30°$ is

(A) 640 (B) 320

(C) 321 (D) 641

8. [Online January 2019]

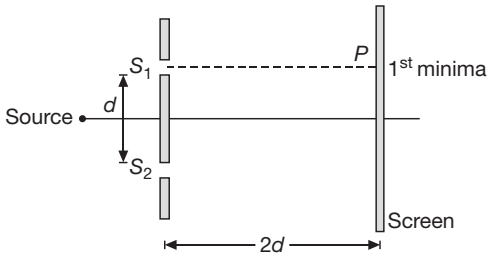
In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ rad by using light of wavelength λ_1. When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are

(A) 380 nm, 500 nm (B) 625 nm, 500 nm

(C) 380 nm, 525 nm (D) 400 nm, 500 nm

9. [Online January 2019]

Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms wavelength λ such that the first minima occurs directly in front of the slit (S_1)?



(A) $\frac{\lambda}{2(\sqrt{5}-2)}$ (B) $\frac{\lambda}{2(5-\sqrt{2})}$

(C) $\frac{\lambda}{(5-\sqrt{2})}$ (D) $\frac{\lambda}{(\sqrt{5}-2)}$

10. [Online January 2019]

In a Young's double slit experiment, the path difference, at a certain point on the screen, between two interfering waves is $\left(\frac{1}{8}\right)^{th}$ of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to

(A) 0.74 (B) 0.94

(C) 0.80 (D) 0.85

11. [Online January 2019]

In a double-slit experiment, green light (5303 Å) falls on a double slit having a separation of 19.44 μm and a width of 4.05 μm. The number of bright fringes between the first and the second diffraction minima is

(A) 05 (B) 09

(C) 10 (D) 04

12. [2018]

Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is

found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is

(A) $0°$ (B) $30°$

(C) $45°$ (D) $60°$

13. [2018]

The angular width of the central maximum in a single slit diffraction pattern is $60°$. The width of the slit is $1\ \mu m$. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance? (i.e. distance between the centres of each slit.)

(A) $25\ \mu m$ (B) $50\ \mu m$

(C) $75\ \mu m$ (D) $100\ \mu m$

14. [Online 2018]

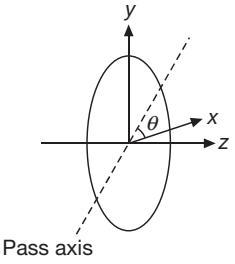
Light of wavelength 550 nm falls normally on a slit of width 22.0×10^{-5} cm. The angular position of the second minima from the central maximum will be (in radians)

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{12}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

15. [Online 2018]

A plane polarized light is incident on a polariser with its pass axis making angle θ with x-axis, as shown in the figure. At four different values of θ, $\theta = 8°$, $38°$, $188°$ and $218°$, the observed intensities are same. What is the angle between the direction of polarization and x-axis ?



(A) $203°$ (B) $128°$

(C) $98°$ (D) $45°$

16. [Online 2018]

Unpolarized light of intensity I is incident on a system of two polarizers, A followed by B. The intensity of emergent light is $\frac{I}{2}$. If a third polarizer C is placed between A and B, the intensity of emergent light is reduced to $\frac{I}{3}$. The angle between the polarizers A and C is θ. Then

(A) $\cos\theta = \left(\frac{1}{3}\right)^{\frac{1}{2}}$ (B) $\cos\theta = \left(\frac{2}{3}\right)^{\frac{1}{4}}$

(C) $\cos\theta = \left(\frac{2}{3}\right)^{\frac{1}{2}}$ (D) $\cos\theta = \left(\frac{1}{3}\right)^{\frac{1}{4}}$

17. [2017]

An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light $= 3\times10^8\ ms^{-1}$)

(A) 10.1 GHz (B) 12.1 GHz

(C) 17.3 GHz (D) 15.3 GHz

18. [2017]

In a Young's double slit experiment, slits are separated by 0.5 mm and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is

(A) 1.56 mm (B) 7.8 mm

(C) 9.75 mm (D) 15.6 mm

19. [Online 2017]

A single slit of width b is illuminated by a coherent monochromatic light of wavelength λ. If the second and fourth minima in the diffraction pattern at a distance 1 m from the slit are at 3 cm and 6 cm respectively from the central maximum, what is the width of the central maximum? (i.e. distance between first minimum on either side of the central maximum)

(A) 6.0 cm (B) 1.5 cm

(C) 4.5 cm (D) 3.0 cm

20. [Online 2017]

A single slit of width 0.1 mm is illuminated by a parallel beam of light of wavelength 6000 Å and diffraction bands are observed on a screen 0.5 m from the slit. The distance of the third dark band from the central bright band is

(A) 3 mm (B) 1.5 mm

(C) 9 mm (D) 4.5 mm

21. [2016]
The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say $b_{\min}$) when

(A) $a = \frac{\lambda^2}{L}$ and $b_{\min} = \left(\frac{2\lambda^2}{L}\right)$

(B) $a = \sqrt{\lambda L}$ and $b_{\min} = \left(\frac{2\lambda^2}{L}\right)$

(C) $a = \sqrt{\lambda L}$ and $b_{\min} = \sqrt{4\lambda L}$

(D) $a = \frac{\lambda^2}{L}$ and $b_{\min} = \sqrt{4\lambda L}$

22. [Online 2016]
In Young's double slit experiment, the distance between slits and the screen is 1.0 m and monochromatic light of 600 nm is being used. A person standing near the slits is looking at the fringe pattern. When the separation between the slits is varied, the interference pattern disappears for a particular distance d_0 between the slits. If the angular resolution of the eye is $\frac{1^\circ}{60}$, the value of d_0 is close to

(A) 1 mm (B) 3 mm
(C) 2 mm (D) 4 mm

23. [2015]
On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam

(A) bends downwards
(B) bends upwards
(C) becomes narrower
(D) goes horizontally without any deflection

24. [Online 2015]
In a Young's double slit experiment with light of wavelength λ the separation of slits is d and distance of screen is D such that $D \gg d \gg \lambda$. If the fringe width is β, the distance from point of maximum intensity to the point where intensity falls to half of maximum intensity on either side is

(A) $\frac{\beta}{2}$ (B) $\frac{\beta}{4}$
(C) $\frac{\beta}{3}$ (D) $\frac{\beta}{6}$

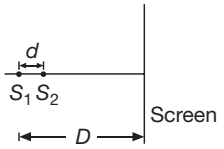
25. [Online 2015]
Unpolarized light of intensity I_0 is incident on surface of a block of glass at Brewster's angle. In that case, which one of the following statements is true?

(A) Transmitted light is partially polarized with intensity $\frac{I_0}{2}$.
(B) Transmitted light is completely polarized with intensity less than $\frac{I_0}{2}$.
(C) Reflected light is completely polarized with intensity less than $\frac{I_0}{2}$.
(D) Reflected light is partially polarized with intensity $\frac{I_0}{2}$.

26. [2014]
Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then $\frac{I_A}{I_B}$ equals

(A) $\frac{1}{3}$ (B) 3
(C) $\frac{3}{2}$ (D) 1

27. [2013]
Two coherent point sources S_1 and S_2 are separated by a small distance d as shown. The fringes obtained on the screen will be



(A) concentric circles (B) points
(C) straight lines (D) semi-circles

28. [2013]
A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is

(A) $\frac{I_0}{8}$ (B) I_0
(C) $\frac{I_0}{2}$ (D) $\frac{I_0}{4}$

29. [2012]

In Young's double slit experiment, one of the slit is wider than other, so that the amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by

(A) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$ (B) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$

(C) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$ (D) $\frac{I_m}{9}(4+5\cos\phi)$

30. [2011]

Direction: The question has a paragraph followed by two statements, Statement-1 and Statement-2. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement-1: When light reflects form the air-glass plate interface, the reflected wave suffers a phase change of π.

Statement-2: The centre of the interference pattern is dark.

(A) Statement-I is true, Statement-2 is false.

(B) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.

(C) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.

(D) Statement-1 is false, Statement-2 is true.

31. [2009]

A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4^{th} bright fringe of the unknown light. From this data, the wavelength of the unknown light is

(A) 393.4 nm (B) 885.0 nm

(C) 442.5 nm (D) 776.8 nm

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct)

1. [IIT-JEE 2013]

In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is

(A) $(2n+1)\frac{\lambda}{2}$ (B) $(2n+1)\frac{\lambda}{4}$

(C) $(2n+1)\frac{\lambda}{8}$ (D) $(2n+1)\frac{\lambda}{16}$

2. [IIT-JEE 2012]

Young's double slit experiment is carried out by using green, red and blue light, one color at a time. The fringe widths recorded are β_G, β_R and β_B respectively. Then,

(A) $\beta_G > \beta_B > \beta_R$ (B) $\beta_B > \beta_B > \beta_R$

(C) $\beta_R > \beta_B > \beta_G$ (D) $\beta_R > \beta_G > \beta_B$

3. [IIT-JEE 2005]

In Young's double slit experiment intensity at a point is $\left(\frac{1}{4}\right)$ of the maximum intensity. Angular position of this point is

(A) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ (B) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$

(C) $\sin^{-1}\left(\frac{\lambda}{3d}\right)$ (D) $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

4. [IIT-JEE 2004]

In $YDSE$ bi-chromatic light of wavelengths 400 nm and 560 nm are used. The distance between the slits is 0.1 mm and the distance between the plane of the slits and the screen is 1 m. The minimum distance between two successive regions of complete darkness is

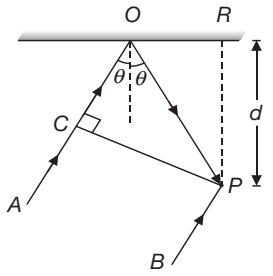
(A) 4 mm (B) 5.6 mm

(C) 14 mm (D) 28 mm

5. [IIT-JEE 2003]

In the diagram, CP represent a wavefront and AO and BP, the corresponding two rays. Find the condition

on θ for constructive interference at P between the ray BP and reflected ray OP



(A) $\cos\theta = \frac{3\lambda}{2d}$ (B) $\cos\theta = \frac{\lambda}{4d}$

(C) $\sec\theta - \cos\theta = \frac{\lambda}{d}$ (D) $\sec\theta - \cos\theta = \frac{4\lambda}{d}$

6. [IIT-JEE 2002]

In the ideal double-slit experiment, when a glass-plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is

(A) 2λ (B) $\frac{2\lambda}{3}$

(C) $\frac{\lambda}{3}$ (D) λ

7. [IIT-JEE 2001]

In a Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by

(A) 12 (B) 18

(C) 24 (D) 30

8. [IIT-JEE 2001]

Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\frac{\pi}{2}$ at point A and π at point B. Then the difference between resultant intensities at A and B is

(A) $2I$ (B) $4I$

(C) $5I$ (D) $7I$

9. [IIT-JEE 2000]

In a double slit experiment instead of taking slits of equal widths, one slit is made twice as wide as the other, then in the interference pattern

(A) the intensities of both the maxima and the minima increases

(B) the intensity of the maxima increases and the minima has zero intensity

(C) the intensity of maxima decreases and that of minima increases

(D) the intensity of maxima decreases and the minima has zero intensity

10. [IIT-JEE 1999]

Yellow light is used in a single slit diffraction experiment with slit width of 0.6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal

(A) that the central maximum is narrower

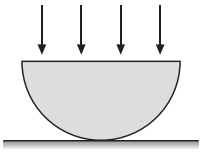
(B) more number of fringes

(C) less number of fringes

(D) no diffraction pattern

11. [IIT-JEE 1999]

A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat plate as shown. The observed interference fringes from this combination shall be



(A) straight

(B) circular

(C) equally spaced

(D) having fringe spacing which increases as we go outwards

12. [IIT-JEE 1998]

A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is

(A) ZERO (B) $\frac{\pi}{2}$

(C) π (D) 2π

13. [IIT-JEE 1995]

Consider Fraunhoffer diffraction pattern obtained with a single slit illuminated at normal incidence. At the angular position of the first diffraction minimum the phase difference (in radian) between the wavelets from the opposite edges of the slit is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) 2π (D) π

14. [IIT-JEE 1994]

A narrow slit of width 1 mm is illuminated by monochromatic light of wavelength 600 nm. The distance between the first minima on either side of a screen at a distance of 2 m is

(A) 1.2 cm (B) 1.2 mm

(C) 2.4 cm (D) 2.4 mm

15. [IIT-JEE 1988]

Two coherent monochromatic light beams of intensities I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are

(A) $5I$ and I (B) $5I$ and $3I$

(C) $9I$ and I (D) $9I$ and $3I$

16. [IIT-JEE 1981]

In Young's double slit experiment, the separation between the slits is halved and the distance between the slits and the screen is doubled. The fringe width is

(A) unchanged (B) halved

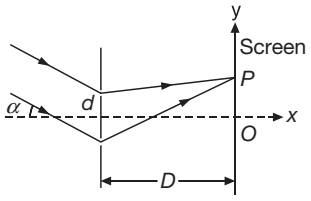
(C) doubled (D) quadrupled

Multiple Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct)

1. [JEE (Advanced) 2019]

In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1 m. A parallel beam of light of wavelength 600 nm is incident on the slits at angle α as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11 mm. Which of the following statement(s) is/are correct?



(A) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P.

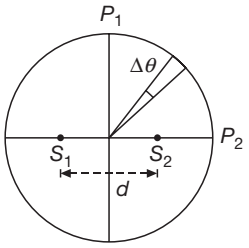
(B) Fringe spacing depends on.

(C) For $\alpha = 0$, there will be constructive interference at point P.

(D) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O.

2. [JEE (Advanced) 2017]

Two coherent monochromatic point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the centre of the circle as shown. The sources are separated by a distance $d = 1.8$ mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct?



(A) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant

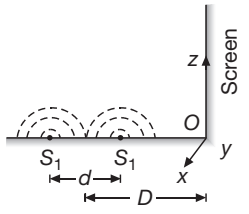
(B) A dark spot will be formed at the point P_2

(C) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000

(D) At P_2 the order of the fringe will be maximum

3. [JEE (Advanced) 2016]

While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting light of wavelength 600 mm. The student mistakenly placed the screen parallel to the x-z plane (for $z > 0$) at a distance $D = 3$ m from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the source $d = 0.6003$ mm. The origin O is at the intersection of the screen and the line joining S_1S_2.



Which of the following is (are) true of the intensity pattern on the screen?

(A) Semi circular bright and dark bands centered at point O

(B) The region very close to the point O will be dark

(C) Straight bright and dark bands parallel to the X-axis

(D) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

4. [JEE (Advanced) 2014]

A light source, which emits two wavelengths $\lambda_1 = 400$ nm and $\lambda_2 = 600$ nm, is used in a Young's double-slit experiment. If recorded fringe width for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then

(A) $\beta_2 > \beta_1$

(B) $m_1 > m_2$

(C) from the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1

(D) the angular separation of fringes of λ_1 is greater than λ_2

5. [JEE (Advanced) 2013]

Using the expression $2d\sin\theta = \lambda$, one calculates the values of d by measuring the corresponding angles θ in the range 0 to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from 0°

(A) the absolute error in d remains constant

(B) the absolute error in d increases

(C) the fractional error in d remains constant

(D) the fractional error in d decreases

6. [IIT-JEE 2008]

In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s).

(A) If $d = \lambda$, the screen will contain only one maximum.

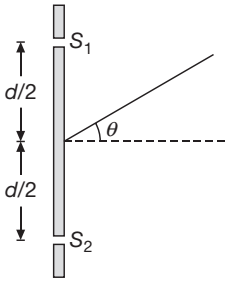
(B) If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen.

(C) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase.

(D) If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase.

7. [IIT-JEE 1995]

In an interference arrangement similar to Young's double-slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by a distance $d = 150.0$ m. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown. If I_0 is the maximum intensity, then $I(\theta)$ for $0 \le \theta \le 90°$ is given by



(A) $I(\theta) = \frac{I_0}{2}$ for $\theta = 30°$

(B) $I(\theta) = \frac{I_0}{4}$ for $\theta = 90°$

(C) $I(\theta) = I_0$ for $\theta = 0°$

(D) $I(\theta)$ is constant for all values of θ

8. [IIT-JEE 1984]

White light is used to illuminate the two slits in a Young's double-slit experiment. The separation between the slits is b and the screen is at a distance $d(\gg b)$ from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths are

(A) $\lambda = \frac{b^2}{d}$ (B) $\lambda = \frac{2b^2}{d}$

(C) $\lambda = \frac{b^2}{3d}$ (D) $\lambda = \frac{2b^2}{3d}$

9. [IIT-JEE 1982]

In the Young's double slit experiment, the interference pattern is found to have an intensity ratio between bright and dark fringes as 9. This implies that -

(A) the intensities at the screen due to the two slits are 5 units and 4 units respectively

(B) the intensities at the screen due to the two slits can be 4 units and 1 unit respectively

(C) the amplitude ratio is 3

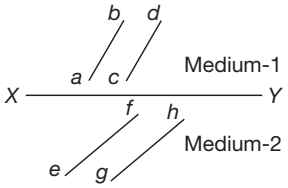
(D) the amplitude ratio is 2

Comprehension Type Questions

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension I

The figure shows a surface XY separating two transparent media, Medium-1 and Medium-2. The lines ab and cd represent wavefronts of a light wave travelling in Medium-1 and incident on XY. The lines ef and gh represent wavefronts of the light wave in Medium-2 after refraction.



Based on the above facts, answer the following questions.

1. [IIT-JEE 2007]

Light travels as a

(A) parallel beam in each medium
(B) convergent beam in each medium
(C) divergent beam in each medium
(D) divergent beam in one medium and convergent beam in the other medium

2. [IIT-JEE 2007]

The phases of the light wave at c, d, e and f are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c \neq \phi_f$

(A) ϕ_c cannot be equal to ϕ_d
(B) ϕ_d can be equal to ϕ_e
(C) $(\phi_d - \phi_f)$ is equal to $(\phi_c - \phi_e)$
(D) $(\phi_d - \phi_c)$ is not equal to $(\phi_f - \phi_e)$

3. [IIT-JEE 2007]

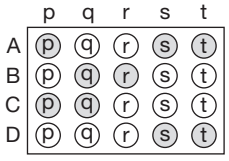
Speed of light is:

(A) the same in medium-1 and medium-2
(B) larger in medium-1 than in medium-2
(C) larger in medium-2 than in medium-1
(D) different at b and d

Matrix Match/Column Match Type Questions

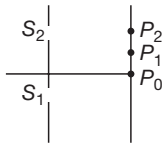
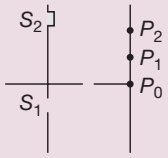
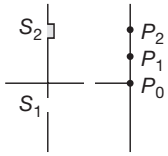
Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

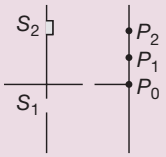


1. [IIT-JEE 2009]

COLUMN I shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \frac{\lambda}{4}$ and $S_1P_2 - S_2P_2 = \frac{\lambda}{2}$ when λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thickness of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match each situation given in **COLUMN-I** with the statement(s) in **COLUMN-II** valid for that situation.

COLUMN-I	COLUMN-II
A. 	p. $\delta(P_0) = 0$
B. $(\mu - 1)t = \frac{\lambda}{4}$ 	q. $\delta(P_1) = 0$
C. $(\mu - 1)t = \frac{\lambda}{2}$ 	r. $I(P_1) = 0$

(*Continued*)

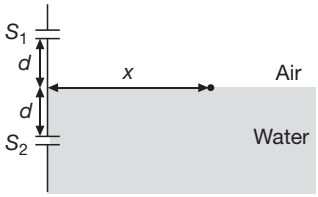
COLUMN-I	COLUMN-II
D. $(\mu-1)t=\frac{3\lambda}{4}$ 	s. $I(P_0)>I(P_1)$
	t. $I(P_2)>I(P_1)$

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

1. [JEE (Advanced) 2015]

A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index $\frac{4}{3}$) as shown in the figure. The positions of maxima on the surface of water are given by $x^2=p^2m^2\lambda^2-d^2$, where λ is the wavelength of light in air (refractive index $=1$), $2d$ is the separation between the slits and m is an integer. The value of p is



ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Interference)

1. (a) 7.8 μm (b) 0.6 mm

2. 5 μm

3. (a) $\frac{2\pi}{\lambda}\left[\left(\sqrt{d^2+\ell^2}-d\right)+\frac{\mu d^2}{2D}\right]$

(b) $\frac{2\pi}{\lambda}\left(\mu\left(\sqrt{d^2+\ell^2}-\ell\right)+\frac{d^2}{2D}\right)$

4. 94.8 nm

5. 0.546 mm

6. 112.78 nm

7. (a) 0.5 mm (b) 2.25 mm

(c) $3I_0$ (d) $\frac{1}{\sqrt{3}}$ meter

(e) $n = 5000$ is not possible

8. (a) $\frac{5\pi}{2}$

(b) $\frac{25\pi}{4}$

9. 5890 Å

10. 1.2 μm

11. Zero order maxima will remain unchanged. Tenth order will now be at 4.55 mm.

12. 8.75 mm

14. 4.5 mm

15. 7

16. 0.21

17. (a) $3\times10^{-4}I_0$ (b) 5.49

18. 3.5 mm

19. 1.5

Test Your Concepts-II (Based on Diffraction)

1. 11.8 mm

2. 9×10^{-4} m

3. 0.2 mm

4. 40 m

5. 12.5 cm

6. 1.3 μm

7. 0.2 mm

8. 3.534×10^{-3} rad

9. 10^{-6} m, 5×10^{-7} m

10. (a) 8.54×10^{-4} radian (b) 1.823×10^{-15} m^2

(c) 10.97×10^{12} Wm^{-2}

Test Your Concepts-III (Based on Polarisation)

1. $\pm45°$, $\pm135°$

2. (a) 0.75 (b) 0.25

3. 37.5%

4. $\frac{I_0}{4}$

5. 1:3

7. 30°, 45°

8. 59°, 31°

9. (a) 2.136×10^{-5} m (b) 2.762×10^{-5} m

10. 60° cm^{3}g^{-1}(dm)$^{-1}$

11. 7.5 cm

Single Correct Choice Type Questions

1. B	**2.** A	**3.** A	**4.** B	**5.** B	**6.** D	**7.** D	**8.** D	**9.** B	**10.** A
11. D	**12.** D	**13.** A	**14.** C	**15.** D	**16.** B	**17.** C	**18.** D	**19.** D	**20.** B
21. B	**22.** C	**23.** C	**24.** D	**25.** C	**26.** B	**27.** D	**28.** A	**29.** C	**30.** D
31. C	**32.** D	**33.** A	**34.** A	**35.** A	**36.** A	**37.** A	**38.** C	**39.** A	**40.** C
41. A	**42.** B	**43.** A	**44.** A	**45.** D	**46.** D	**47.** C	**48.** C	**49.** C	**50.** B
51. B	**52.** A	**53.** C	**54.** C	**55.** D	**56.** B	**57.** B	**58.** C	**59.** B	**60.** C

61. D	**62.** B	**63.** A	**64.** B	**65.** B	**66.** A	**67.** D	**68.** B	**69.** D	**70.** B
71. D	**72.** C	**73.** D	**74.** C	**75.** B	**76.** B	**77.** C	**78.** B	**79.** A	**80.** C
81. C	**82.** C	**83.** C	**84.** A	**85.** B	**86.** A	**87.** B	**88.** C	**89.** A	**90.** B
91. B	**92.** D	**93.** C	**94.** A	**95.** B	**96.** B	**97.** B	**98.** B	**99.** C	**100.** A
101. B	**102.** C	**103.** B	**104.** D	**105.** C	**106.** B	**107.** C	**108.** D	**109.** D	**110.** D
111. A	**112.** D	**113.** A	**114.** A	**115.** D	**116.** D	**117.** D	**118.** B	**119.** C	**120.** D
121. C	**122.** B	**123.** A	**124.** B	**125.** C	**126.** D	**127.** A	**128.** C	**129.** B	**130.** C
131. A	**132.** A	**133.** D	**134.** A	**135.** C	**136.** B	**137.** B	**138.** A	**139.** A	**140.** D
141. A	**142.** C	**143.** B	**144.** A	**145.** B	**146.** B	**147.** C	**148.** A	**149.** C	**150.** B
151. D	**152.** D	**153.** D	**154.** A	**155.** C	**156.** C	**157.** D	**158.** D	**159.** B	**160.** B
161. B	**162.** A	**163.** B	**164.** A	**165.** D	**166.** B	**167.** D	**168.** D	**169.** D	**170.** C
171. C	**172.** B	**173.** B	**174.** C	**175.** C	**176.** D	**177.** D	**178.** C	**179.** B	**180.** A
181. C	**182.** A	**183.** C	**184.** D	**185.** B	**186.** C	**187.** A	**188.** B	**189.** C	**190.** B
191. A	**192.** B	**193.** C	**194.** D						

Multiple Correct Choice Type Questions

1. A, C	**2.** A, C, D	**3.** A, D	**4.** B, D	**5.** B, D
6. A, B, C	**7.** A, B, C	**8.** A, C	**9.** A, B	**10.** A, C, D
11. A, C	**12.** A, B, C	**13.** B, D	**14.** A	**15.** C
16. A, B				

Reasoning Based Questions

1. B	**2.** D	**3.** D	**4.** B	**5.** D	**6.** A	**7.** D	**8.** D	**9.** D	**10.** A
11. A	**12.** D	**13.** B	**14.** D	**15.** A	**16.** C	**17.** D	**18.** D	**19.** A	**20.** A
21. A									

Linked Comprehension Type Questions

1. A	**2.** D	**3.** B	**4.** A	**5.** A	**6.** A	**7.** C	**8.** A	**9.** A	**10.** B
11. C	**12.** D	**13.** C	**14.** C	**15.** D	**16.** D	**17.** A	**18.** B	**19.** B	**20.** B
21. B	**22.** C	**23.** D	**24.** B	**25.** A	**26.** C	**27.** A	**28.** B	**29.** D	**30.** C
31. C	**32.** D	**33.** D	**34.** C	**35.** C	**36.** B	**37.** A	**38.** D	**39.** C	**40.** A
41. B	**42.** C	**43.** D	**44.** C	**45.** C	**46.** C	**47.** D	**48.** D	**49.** A	**50.** B
51. D	**52.** B	**53.** B	**54.** D	**55.** C	**56.** B	**57.** B	**58.** A	**59.** D	**60.** A
61. A	**62.** A	**63.** B	**64.** A	**65.** C	**66.** D				

Matrix Match/Column Match Type Questions

1. A → (p, s)	B → (q, r)	C → (p, q, r)	D → (q, r, s)
2. A → (p, q)	B → (s)	C → (p, q)	D → (r)
3. A → (q)	B → (p)	C → (s)	D → (r)
4. A → (r)	B → (s)	C → (q)	D → (p)
5. A → (p)	B → (s)	C → (r)	D → (q, r)

6. A → (r)	B → (t)	C → (r)	D → (p)
7. A → (p)	B → (s)	C → (r)	D → (q)
8. A → (p, q, r, s)	B → (p, q, r, s)	C → (p, q, r, s)	D → (p, q, r)
9. A → (q, r)	B → (q, s)	C → (p, s)	D → (p, q, r, s)

Integer/Numerical Answer Type Questions

1. 3	**2.** 32	**3.** (a) 45, (b) 59	**4.** (a) 80, (b) 4, (c) 60	**5.** 361
6. 739	**7.** 6	**8.** 3	**9.** 36	**10.** 2
11. 100	**12.** 3	**13.** 18	**14.** 3	**15.** 2
16. 2				

ARCHIVE: JEE MAIN

1. A	**2.** B	**3.** D	**4.** D	**5.** C	**6.** A	**7.** D	**8.** B	**9.** A	**10.** D
11. D	**12.** C	**13.** A	**14.** C	**15.** A	**16.** B	**17.** C	**18.** B	**19.** D	**20.** C
21. C	**22.** C	**23.** B	**24.** B	**25.** C	**26.** A	**27.** A	**28.** D	**29.** C	**30.** C
31. C									

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. B	**2.** D	**3.** C	**4.** D	**5.** B	**6.** A	**7.** B	**8.** B	**9.** A	**10.** D
11. A	**12.** D	**13.** C	**14.** D	**15.** C	**16.** D				

Multiple Correct Choice Type Problems

1. A	**2.** C, D	**3.** A, B	**4.** A, C	**5.** D
6. A, B	**7.** A, C	**8.** A, C	**9.** B, D	

Comprehension Type Questions

1. A	**2.** C	**3.** B

Matrix Match/Column Match Type Questions

1. A → (p, s)	B → (q)	C → (t)	D → (r, s, t)

Integer/Numerical Answer Type Questions

1. 3

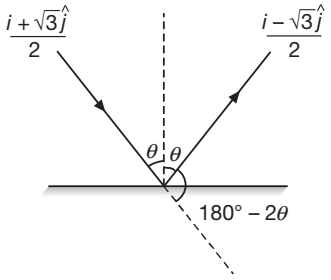
HINTS AND EXPLANATIONS

This page is intentionally left blank

CHAPTER 1: RAY OPTICS

Test Your Concepts-I
(Based on Reflection at Plane Surfaces)

1. The angle between the incident ray and the reflected ray is $180-2\theta$, so, we have



$$\cos(180°-2\theta)=\frac{\left(\frac{i+\sqrt{3}\hat{j}}{2}\right)\cdot\left(\frac{i-\sqrt{3}\hat{j}}{2}\right)}{\left|\frac{i+\sqrt{3}\hat{j}}{2}\right|\left|\frac{i-\sqrt{3}\hat{j}}{2}\right|}$$

$$\Rightarrow \quad -\cos(2\theta)=\frac{\frac{(1-3)}{4}}{1}$$

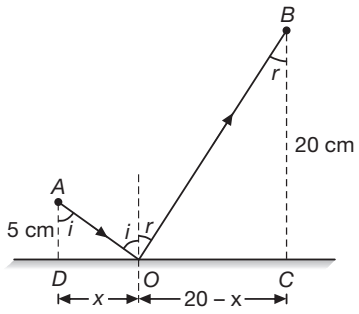
$$\Rightarrow \quad -\cos(2\theta)=-\frac{1}{2}$$

$$\Rightarrow \quad \cos(2\theta)=\frac{1}{2}$$

$$\Rightarrow \quad 2\theta=60°$$

$$\Rightarrow \quad \theta=30°$$

2. Drawing the ray diagram and using the Law of Reflection, we get



$$i=r \qquad \ldots(1)$$

$$\Rightarrow \quad \sin i=\sin r$$

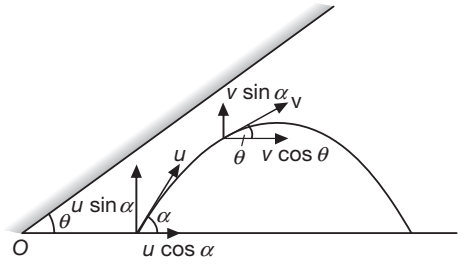
So, we can say that ΔADO and ΔOBC are similar

$$\Rightarrow \quad \frac{x}{5}=\frac{20-x}{20}$$

$$\Rightarrow \quad x=4 \text{ cm}$$

So, point of incidence of light from A should be at 4 cm from D on mirror.

3. The image will be momentarily at rest when the particle moves parallel to the mirror. Let at the time t the particle has a velocity v parallel to the mirror.



$$v\sin\theta=u\sin\alpha-gt \qquad \ldots(1)$$

and $v\cos\theta=u\cos\alpha$

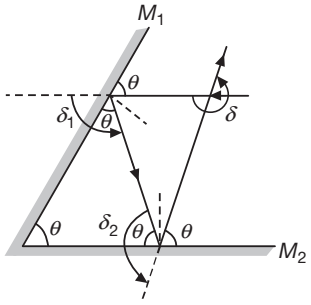
$$\Rightarrow \quad v=\frac{u\cos\alpha}{\cos\theta} \qquad \ldots(2)$$

From (1) and (2)

$$\left(\frac{u\cos\alpha}{\cos\theta}\right)\sin\theta=u\sin\alpha-gt$$

$$\Rightarrow \quad t=\frac{u\cos\alpha(\tan\alpha-\tan\theta)}{g}$$

4.



From the figure, we observe that

$$3\theta=180°$$

$$\Rightarrow \quad \theta=60°$$

So, $\delta_1=180°-2(30°)=120°$ (CCW)

and $\delta_2=180°-2(30°)=120°$ (CCW)

So, total deviation $\delta=\delta_1+\delta_2$

$$\Rightarrow \quad \delta=240° \text{ (CCW)}$$

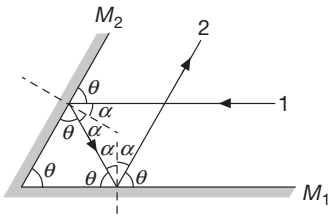
Alternatively from the figure, we observe that

$$\delta=180°+\theta=240° \text{ (CCW) or } 120° \text{ (CW)}$$

5. Various angles made are as shown in figure. In triangle ABC, we observe that

$$\theta+\theta+\theta=180^\circ$$

$$\Rightarrow \quad \theta=60^\circ$$

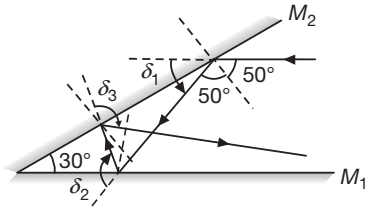


6. From the figure, we observe that

$$\delta_1=180^\circ-2(50^\circ)=100^\circ \text{ (CCW)}$$

$$\delta_2=180^\circ-2(20^\circ)=140^\circ \text{ (CW)}$$

$$\delta_3=180^\circ-2(10^\circ)=160^\circ \text{ (CW)}$$

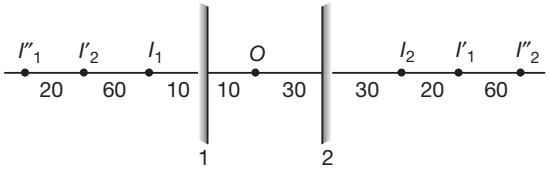


So, total deviation

$$\delta=100^\circ(\text{CCW})+140^\circ(\text{CW})+160^\circ(\text{CW})$$

$$\Rightarrow \quad \delta=100^\circ \text{ (CW) or } 260^\circ \text{ (CCW)}$$

7. The image is formed as far behind the mirror as the object is in front of it. Also image formed by mirror 1 i.e., I_1, acts as object for mirror 2, so I_1' is formed 50 cm behind the mirror 2 as shown.



Taking all distances to be in cm and plotting them as shown (but not to scale), we get

$OI_1=20$ cm $\qquad OI_2=60$ cm

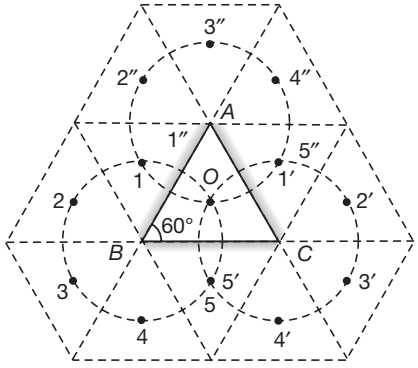
$OI_1'=80$ cm $\qquad OI_2'=80$ cm

$OI_1''=100$ cm $\qquad OI_2''=140$ cm

So, the respective distances are 20 cm, 60 cm, 80 cm, 100 cm and 140 cm

8. Let us first consider the mirrors AB and BC, for which we have

$$\frac{360^\circ}{60^\circ}=6$$



So, the number of images formed by the combination is given by

$$N=6-1=5$$

Combination of Mirrors	Images Formed
AB & BC	1, 2, 3, 4, 5
AC & BC	1′, 2′, 3′, 4′, 5′
AB & AC	1″, 2″, 3″, 4″, 5″

These images along with the object must lie on a circle as shown in figure with an angular separation of

$$\frac{360^\circ}{N}=\frac{360^\circ}{5}=72^\circ$$

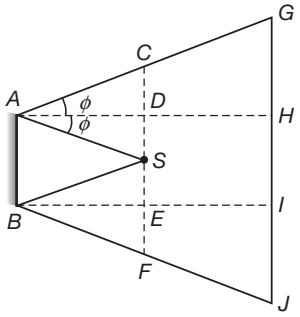
Similarly, the other two combination of mirrors also form 5 images each but we find from symmetry that 5 and 5′, 1 and 5″, 1 and 1″ coincide. So the total number of images formed by three mirrors AB, BC and AC is

$$N'=(5)(3)-3=12$$

9. The ray diagram is shown in figure. We observe that

$$HI=AB=d$$

$$DS=CD=\frac{d}{2}$$



Also, $AH=2AD$

$\Rightarrow \quad GH = 2CD = 2\left(\frac{d}{2}\right) = d$

Similarly, $IJ = d$

$\Rightarrow \quad GJ = GH + HI + IJ = d + d + d = 3d$

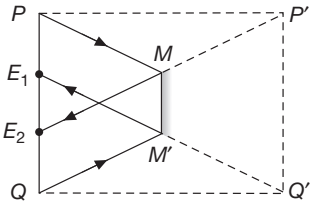
10. (a) For a one eyed man, the required size will be half the each dimension of the face i.e., $12\text{ cm} \times 8\text{ cm}$

(b) For a two eyed man, the

Smallest length of the mirror = Half the length of face

$$\Rightarrow \quad \left(\begin{matrix}\text{Smallest length}\\ \text{of the Mirror}\end{matrix}\right) = \frac{1}{2} \times 24 = 12\text{ cm}$$

The smallest breadth of the mirror is calculated by using the fact that the rays from extreme part of face should reach one of the eyes after reflection from the mirror. The common overlapping portion is then the required breadth of the mirror. The ray diagram is shown in figure.



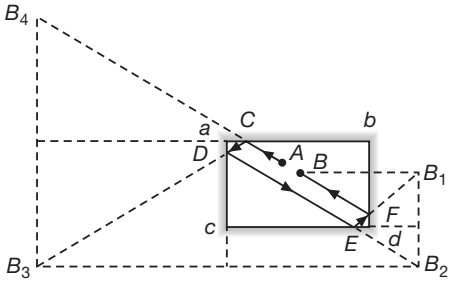
From figure, we get

$$MM' = \frac{1}{2}PQ - \frac{1}{2}E_1E_2$$

$$\Rightarrow \quad \left(\begin{matrix}\text{Smallest Breadth}\\ \text{of the Mirror}\end{matrix}\right) = \frac{1}{2}(16 - 8) = 4\text{ cm}$$

So, the shortest size of mirror is 12 cm by 4 cm.

11. Let us first find the image of point B in mirror bd (shown in figure). Let us then construct image B_1 in mirror cd. Also, B_3 is the image of B_2 in mirror ac and B_4 is the image of B_3 in mirror ab. Let us connect points A and B_4. Point C is the point of intersection of ab with line AB_4. Let us now draw line B_3C from B_3 and connect point D at which this line intersects ac with B_2, E with B_1 and F with B.



It can be stated that the line $ACDEFB$ is the sought path of the beam. Further, we observe that since, B_3CB_4 is an isosceles triangle, CD is the reflection of beam AC. Similarly, we can show that DE is the reflection of CD and so on. This solution of the problem is not unique, as the beam will not necessarily always be sent initially to mirror ab.

12. Along x-direction i.e., perpendicular to the mirror, we have

$$\left(\begin{matrix}\text{Relative Velocity of}\\ \text{Image w.r.t. mirror}\end{matrix}\right) = -\left(\begin{matrix}\text{Relative Velocity of}\\ \text{Object w.r.t. mirror}\end{matrix}\right)$$

$\Rightarrow \quad v_I - v_m = -(v_0 - v_m)$

$\Rightarrow \quad v_I - (-5\cos 30°) = -(10\cos 60° - (-5\cos 30°))$

$\Rightarrow \quad v_I = -5\left(1 + \sqrt{3}\right)\text{ ms}^{-1}$

In the direction parallel to the surface of mirror, i.e., along y-direction we have

$v_I = v_0$

$\Rightarrow \quad v_I = 10\sin(30°) = 5\text{ ms}^{-1}$

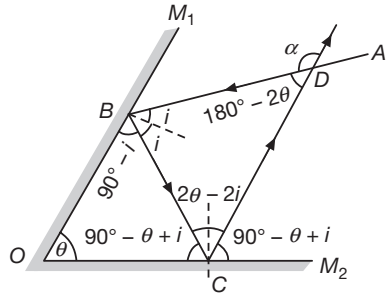
Since $(v_{Im})_{\|} = (v_{Om})_{\|}$

So, velocity of the image

$\vec{v}_I = (v_I)_x\,\hat{i} + (v_I)_y\,\hat{j}$

$\Rightarrow \quad \vec{v}_I = -5\left(1 + \sqrt{3}\right)\hat{i} + 5\hat{j}$

13. Let AB be the incident ray and angle of incidence at the mirror M_1 be i, then



$\angle CBO = 90° - i$

$\Rightarrow \quad \angle BCO = 180° - \theta - (90° - i)$

$\Rightarrow \quad \angle BCO = 90° - \theta + i$

Using the Laws of Reflection, we get

$\angle DCB = 2\theta - 2i$

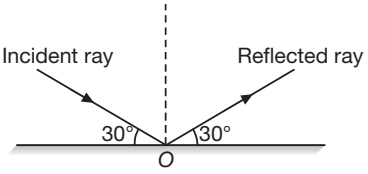
$\Rightarrow \quad \angle CDB = 180° - 2\theta$

$\Rightarrow \quad \alpha = 2\theta$

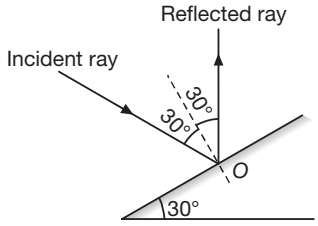
The angle between incident and emergent ray is 2θ and it is independent of the angle of incidence i.

CHAPTER 1

14. Suppose that a plane mirror is kept horizontal as shown in figure. The reflected ray will make an angle of $30°$ with horizontal, or an angle of $60°$ with the vertical.



To make the reflected ray to go vertically upwards, the mirror is required to be rotated about O counterclockwise by $60°$. To achieve this, therefore, the plane mirror is required to rotate about O by half the angle, i.e., by $30°$, as shown in figure.

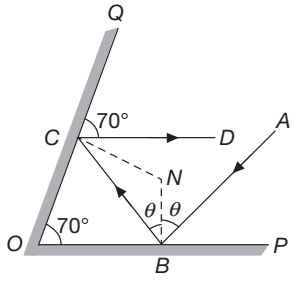


15. Ray AB is incident on mirror OP at an angle θ. The reflected ray BC is incident on second mirror OQ. Finally, the reflected ray CD is parallel to OP. Since CD and OP are parallel, and CO cuts them,

$$\angle QCD = \angle COP = 70°$$

$$\Rightarrow \quad \angle DCN = 90° - \angle QCD = 90° - 70° = 20°$$

$$\Rightarrow \quad \angle NCB = \angle DCN = 20°$$



Further, $\angle OCB = 90° - \angle NCB = 90° - 20° = 70°$

Now, in ΔCOB, we have

$$\angle CBO = 180° - (\angle COB + \angle OCB)$$

$$\Rightarrow \quad \angle CBO = 180° - (70° + 70°) = 40°$$

$$\Rightarrow \quad \theta = \angle NBC = 90° - \angle CBO = 90° - 40° = 50°$$

Test Your Concepts-II (Based on Reflection at Curved Surfaces)

1. Since, $m = \dfrac{f}{f-u}$

Now, $f = -f$ and $u = -1.5\ \text{f}$, so

$$m = \frac{-f}{-f + 1.5\ \text{f}} \quad \Rightarrow \quad m = \frac{-f}{0.5\ \text{f}}$$

$$\Rightarrow \quad m = -2$$

Since $m = \dfrac{h_i}{h_0} = -2$

$$\Rightarrow \quad h_i = -2h_0 = -5\ \text{cm}$$

The image is 5 cm long. The minus sign shows that it is inverted.

2. Given: $u = -25$ m, $m = +4$ (since the image is erect).
Now, the magnification is given by

$$m = -\frac{v}{u}$$

$$\Rightarrow \quad v = -mu = -(+4) \times (-2.5) = 10\ \text{m}$$

Using the mirror formula, we get

$$\frac{1}{10} + \frac{1}{-2.5} = \frac{1}{f}$$

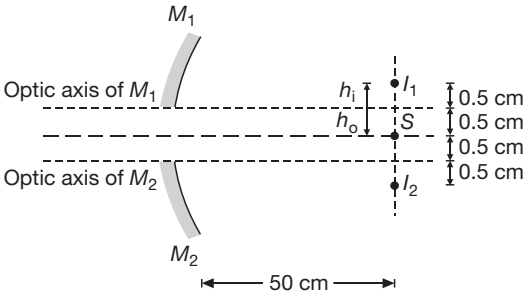
$$\Rightarrow \quad \frac{1}{f} = 0.1 - 0.4 = -0.3$$

$$\Rightarrow \quad f = -\frac{1}{0.3} = -\frac{10}{3}\ \text{m}$$

Since f is negative so, the mirror is concave.
The radius of curvature of the mirror is given by

$$R = 2f = 2\left(-\frac{10}{3}\right) = -\frac{20}{3}\ \text{m} = -6.67\ \text{m}$$

3. According to the mirror formula, we have $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$



$$\Rightarrow \quad \frac{1}{v} - \frac{1}{50} = \frac{1}{-25}$$

$\Rightarrow \quad v = -50$ cm

So, $m = -\dfrac{v}{u} = -1$

$\Rightarrow \quad \left|\dfrac{h_i}{h_o}\right| = +1$

$\Rightarrow \quad h_i = h_o = 0.5$ cm

4. Given, $f = -10$ cm . Since a concave mirror can form real as well as virtual image and since the nature of image is not given in the question. So we will consider two possible cases.

CASE-1 (when image is real):

So, $m = -4$

Since $m = \dfrac{f}{f-u}$

$\Rightarrow \quad -4 = \dfrac{-10}{-10-u}$

$\Rightarrow \quad u = -12.5$ cm

Please note that here,
$|u| > |f|$ and we know that in case of a concave mirror, image is real when object lies beyond F.

CASE-2 (When image is virtual):

So, $m = +4$

Since $m = \dfrac{f}{f-u}$

$\Rightarrow \quad 4 = \dfrac{-10}{-10-u}$

$\Rightarrow \quad u = -7.5$ cm

Please, note that here, $|u| < |f|$, as we know that image is virtual when the object lies between F and P.

5. Since $f = \dfrac{R}{2} = -12$ cm

Let the object be placed at a distance u from the pole. Since, we know that magnification m is given by

$$m = \frac{f}{f-u}$$

So, now we apply this formula to these situations one by one.

(a) Here, we have $m = +3$

$\Rightarrow \quad 3 = \dfrac{-12}{-12-u}$

$\Rightarrow \quad -4 = -12 - u$

$\Rightarrow \quad 12 + u = 4$

$\Rightarrow \quad u = -8$ cm

(b) Here, $m = -3$

$\Rightarrow \quad -3 = \dfrac{-12}{-12-u}$

$\Rightarrow \quad 4 = -12 - u \quad \Rightarrow \quad u = -16$ cm

(c) Here, $m = -\dfrac{1}{3}$

$\Rightarrow \quad -\dfrac{1}{3} = \dfrac{-12}{-12-u}$

$\Rightarrow \quad 36 = -12 - u \quad \Rightarrow \quad u = -48$ cm

6. Distance of image formed by the plane mirror is $(b-a)$ i.e., $(b-5)$ cm and distance of object from mirror is $(b+a)$ i.e., $(b+5)$ cm . Using mirror formula, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$ we get

$$\frac{1}{(b-5)} - \frac{1}{(b+5)} = \frac{1}{20}$$

Solving this equation, we get

$b = 15$ cm

The coincidence of the images can be established by observing the changes in the relative position of the images when the eye is moved away from the optical axis of the mirror.
When the images are at various distance from the eye the images will be displaced with respect to each other. When the images are at the same distance, they will coincide irrespective of the placement of the eye.

7. (a) At any instant t, we have

$u = -(2f + x) = -(2f + f\cos\omega t)$

Using, the mirror formula, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$, we get

$$\frac{1}{v} - \frac{1}{2f + f\cos\omega t} = \frac{-1}{f}$$

$\Rightarrow \quad v = -\left(\dfrac{2 + \cos\omega t}{1 + \cos\omega t}\right) f$

i.e., distance of image from mirror at time any instant t is

$$\left(\frac{2 + \cos\omega t}{1 + \cos\omega t}\right) f$$

(b) Ball coincides with its image at centre of curvature, i.e., at $x = 0$

(c) At $t = \dfrac{T}{2}$, we have

$\omega t = \pi$

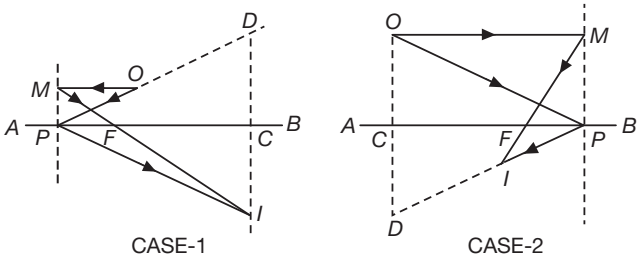
CHAPTER 1

$\Rightarrow \quad x = f\cos(\pi) = -f$

So, $u = -f$ i.e., ball is at focus. So, its image is formed at infinity, so $m \to \infty$

8. (a) Since the image is on the opposite side of the principal axis, the mirror is concave. Because convex mirror always forms a virtual and erect image.

(b) The ray diagrams for two different cases are shown in figure.



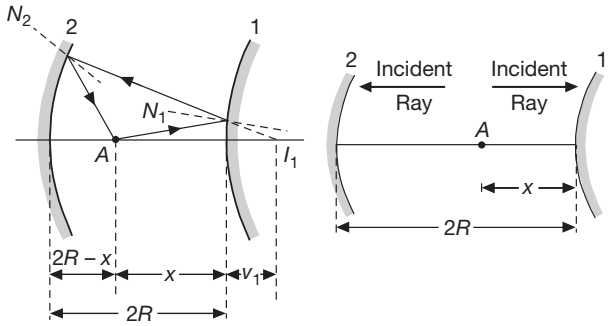
The following steps of construction for drawing the ray diagrams are used.

(i) From I or O drop a perpendicular on principal axis, such that $CI = CD$ or $OC = CD$.

(ii) Draw a line joining D and O or D and I so that it meets the principal axis at P. The point P will be the pole of the mirror as a ray reflected from the pole is always symmetrical about principal axis.

(iii) From O draw a line parallel to principal axis towards the mirror so that it meets the mirror at M. Join M to I, so that it intersects the principal axis at F. F is the focus of the mirror as any ray parallel to principal axis after reflection from the mirror intersects the principal axis at the focus.

9. Let the point A be at a distance x from the convex mirror as shown in mirror, then assuming the origin to be placed at the pole of convex mirror, we get



For convex mirror, we have

$$\frac{1}{v_1} - \frac{1}{x} = \frac{2}{R}$$

$\Rightarrow \quad v_1 = \dfrac{xR}{2x+R}$

For concave mirror, we have

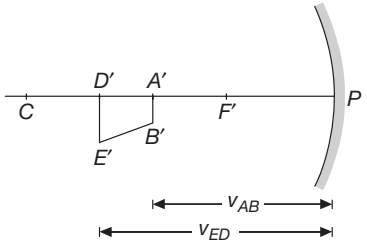
$$\frac{1}{-(2R-x)} - \frac{1}{\left(2R + \dfrac{xR}{2x+R}\right)} = -\frac{2}{R}$$

Solving this equation, we get

$$x = \left(\frac{1+\sqrt{3}}{2}\right)R \text{ and } x = -\left(\frac{\sqrt{3}-1}{2}\right)R$$

Ignoring the negative value, as we have already used a negative sign with x, so the object should be placed at a distance $x = \left(\dfrac{\sqrt{3}+1}{2}\right)R$ from the convex mirror.

10. Object is placed beyond C. Hence, the image will be real and it will lie between C and F. Further u, v and f all are negative, hence the mirror formula becomes



$$-\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}$$

$\Rightarrow \quad \dfrac{1}{v} = \dfrac{1}{f} - \dfrac{1}{u} = \dfrac{u-f}{uf}$

$\Rightarrow \quad v = \dfrac{f}{1 - \dfrac{f}{u}}$

Now since, $u_{AB} > u_{ED}$

$\Rightarrow \quad v_{AB} < v_{ED}$

Hence, $|m_{AB}| < |m_{ED}|$ $\qquad \left\{\because m = -\dfrac{v}{u}\right\}$

Therefore, shape of the image will be as shown in figure.

Also note that $v_{AB} < u_{AB}$ and $v_{ED} < u_{ED}$, $|m_{AB}| < 1$ and $|m_{ED}| < 1$

11. Since the image is inverted, so the mirror is concave.

Now, $u = -30$ cm

$$m = -\frac{1}{2} = -\frac{v}{u}$$

$\Rightarrow \quad v = \dfrac{u}{2} = -15$ cm

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\Rightarrow \quad \frac{1}{(-15)}+\frac{1}{(-30)}=\frac{1}{f}$

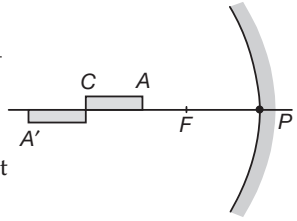
$\Rightarrow \quad f=-10 \text{ cm}$

12. Since image touches the rod, the rod must be placed with one end at centre of curvature. However, two cases arise here.

CASE-1: When the other end lies between C and F
For A, we have

$$u=-\left(2f-\frac{f}{3}\right)=-\frac{5f}{3}$$

$$f=-f$$



Since, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, so we get

$$\frac{1}{v}+\frac{1}{\left(\frac{-5f}{3}\right)}=\frac{1}{(-f)}$$

$\Rightarrow \quad v=\frac{5f}{2}$

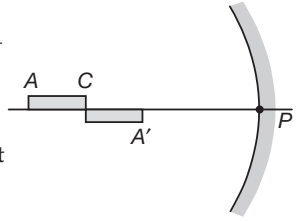
So, magnification, $m=\frac{\text{Length of Image}}{\text{Length of Object}}$

$\Rightarrow \quad m=\frac{v_A-v_C}{u_A-u_C}=\frac{-\frac{5f}{2}-(-2f)}{-\frac{5f}{3}-(-2f)}=-\frac{3}{2}$

CASE-2: When the other end lies beyond C
For A, we have

$$u=-\left(2f+\frac{f}{3}\right)=-\frac{7f}{3}$$

$$f=-f$$



Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, so we get

$$\frac{1}{v}+\frac{1}{\left(-\frac{7f}{3}\right)}=\frac{1}{-f}$$

$\Rightarrow \quad v=-\frac{7f}{4}$

So, magnification, $m=\frac{v_A-v_C}{u_A-u_C}$

$\Rightarrow \quad m=\frac{-\frac{7f}{4}-(-2f)}{-\frac{7f}{3}-(-2f)}=-\frac{3}{4}$

13. Using coordinate convention for mirror formula, we have

$u=-25 \text{ cm}$, $f=-20 \text{ cm}$, $v=?$

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\Rightarrow \quad v=\frac{uf}{u-f}=\frac{-25\times-20}{-5}=-100 \text{ cm}$

Thus magnification in this case is

$$m=-\frac{v}{u}=-\left(\frac{-100}{-25}\right)=-4$$

Image velocity along the principal axis is given by

$$v_{i(\text{along } PA)}=m^2\times v_{o(\text{along } PA)}$$

$\Rightarrow \quad v_{i(\text{along } PA)}=16\times\frac{5}{\sqrt{2}}=40\sqrt{2} \text{ cm}$

Image velocity normal to principal axis is given by

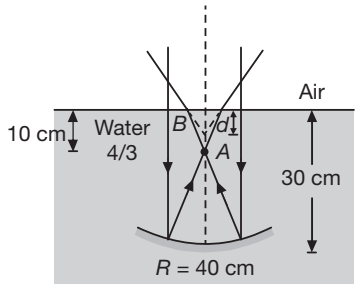
$$v_{i(\text{normal to } PA)}=m\times v_{o(\text{normal to } PA)}$$

$\Rightarrow \quad v_{i(\text{normal to } PA)}=4\times\frac{5}{\sqrt{2}}=10\sqrt{2} \text{ cm}$

Test Your Concepts-III (Based on General Refraction)

1. $d'=\frac{d}{n_{\text{relative}}}=\frac{100}{\frac{4/3}{1}}=75 \text{ cm}$

2. The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore for the mirror, object is at ∞. Its image A (in figure) will be formed at focus which is 20 cm from the mirror. Now for the interface between water and air, $d=10$ cm.



$\Rightarrow \quad d'=\frac{d}{\left(\frac{n_w}{n_a}\right)}=\frac{10}{\left(\frac{4/3}{1}\right)}=7.5 \text{ cm}$

CHAPTER 1

3. **CASE-1: When No Slab is Inserted**

According to mirror formula, we have

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v_1}+\frac{1}{-30}=\frac{1}{-10}$$

$$\Rightarrow \quad v_1=-15 \text{ cm}$$

Magnification, $m_1=-\frac{v}{u}=-\frac{1}{2}=-0.5$

CASE-2: When Slab is Inserted

$$\text{Shift}=\left(1-\frac{1}{\mu}\right)t=\left(1-\frac{1}{1.5}\right)6=2 \text{ cm}$$

Again, applying the mirror formula, we get

$$\frac{1}{v_2}\frac{1}{-(30-2)}=\frac{1}{-10}$$

$$\Rightarrow \quad v_2=-15.55 \text{ cm}$$

Magnification, $m_2=-\frac{v}{u}=-0.55$

So, $\Delta v=0.55$ cm

$$\frac{m_2}{m_1}\approx 1.1$$

4. The refractive index of glass,

$$\mu=\frac{v_{\text{air}}}{v_{\text{glass}}}=\frac{c}{v}$$

$$\Rightarrow \quad v=\frac{c}{\mu}=\frac{3\times 10^8}{1.5}=2\times 10^8 \text{ ms}^{-1}$$

Since the frequency of light remains the same when it passes from one medium to another, so we have

$c=f\lambda_0$ and $v=f\lambda$

Since, $\mu=\frac{c}{v}=\frac{\lambda_0}{\lambda}$

$$\Rightarrow \quad \lambda=\frac{\lambda_0}{\mu}=\frac{6000}{1.5}=4000 \text{ Å}$$

The colour remains yellow, as the colour depends on the frequency and not on the wavelength.

5. Using equation, the total apparent shift is

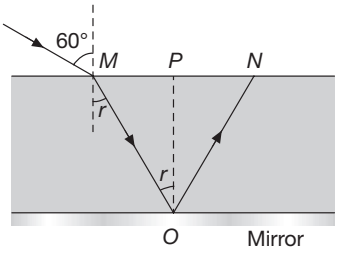
$$\Delta x=h_1\left(1-\frac{1}{\mu_1}\right)+h_2\left(1-\frac{1}{\mu_2}\right)$$

$$\Rightarrow \quad \Delta x=2\left(1-\frac{1}{4/3}\right)+3\left(1-\frac{1}{3/2}\right)=1.5 \text{ cm}$$

Thus, $h=h_1+h_2-\Delta x=2+3-1.5=3.5$ cm

6. $1.8=\frac{\sin(60°)}{\sin r}$

$$\Rightarrow \quad r=28.76°$$



Since, $MP=PO\tan r$

$$\Rightarrow \quad MP=(6)\tan(28.76°)$$

$$\Rightarrow \quad MP=3.3 \text{ cm}$$

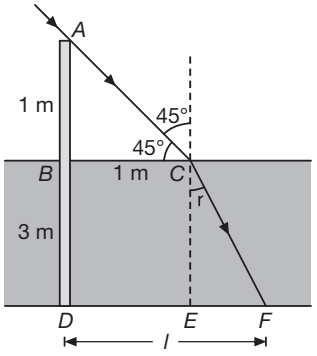
So, $MN=2MP=6.6$ cm

7. From Snell's Law, we have

$$\frac{4}{3}=\frac{\sin(45°)}{\sin r}$$

$$\Rightarrow \quad \sin r=\frac{3}{4}\sin(45°)$$

$$\Rightarrow \quad r=32°$$



Since, $EF=EC\tan r$

$$\Rightarrow \quad EF=(3)\tan 32°=1.88 \text{ m}$$

Length of shadow at the bottom of the lake is

$$\ell=DF=DE+EF=2.88 \text{ m}$$

8. Total deviation suffered by the ray is given by

$$\delta_{\text{Total}}=\delta_P+\delta_Q$$

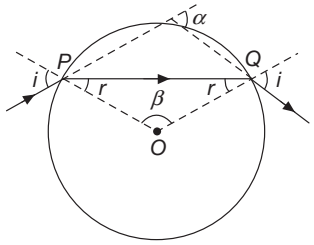
$$\Rightarrow \quad \alpha=(i-r)+(i-r)$$

$$\Rightarrow \quad i-r=\frac{\alpha}{2} \qquad \ldots(1)$$

Further, in ΔOPQ, we have

$$r+r+\beta=180°$$

$$\Rightarrow \quad r = 90° - \frac{\beta}{2} \qquad \ldots(2)$$



From equation (1), we get

$$i = r + \frac{\alpha}{2} = 90° + \left(\frac{\alpha - \beta}{2}\right) \qquad \ldots(3)$$

According to Snell's Law, we have

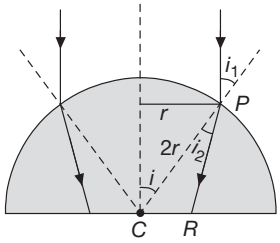
$$\mu = \frac{\sin i}{\sin r}$$

$$\Rightarrow \quad \mu = \frac{\sin\left[90° + \left(\frac{\alpha-\beta}{2}\right)\right]}{\sin\left(90° - \frac{\beta}{2}\right)} = \frac{\cos\left(\frac{\beta-\alpha}{2}\right)}{\cos\left(\frac{\beta}{2}\right)}$$

$$\Rightarrow \quad \cos\left(\frac{\beta-\alpha}{2}\right) = \mu\cos\frac{\beta}{2}$$

9. $\sin i_1 = \frac{r}{2r} = \frac{1}{2}$

$$\Rightarrow \quad i_1 = 30°$$



Applying Snell's Law at P, we get

$$\frac{3}{2} = \frac{\sin i_1}{\sin i_2}$$

$$\Rightarrow \quad i_2 = 19.5°$$

Now, Applying Sine Law (Lami's Theorem), on ΔCPR, we get

$$\frac{2r}{\sin(180° - 60° - 19.5°)} = \frac{CR}{\sin(19.5°)}$$

$$\{\because \angle PCR = 60°\}$$

$$\Rightarrow \quad CR = 0.7r$$

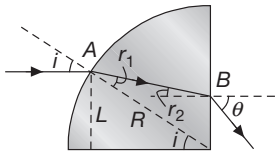
10. Since, $\sin i = \frac{L}{R}$

$$\Rightarrow \quad i = \sin^{-1}\left(\frac{L}{R}\right)$$

According to Snell's Law applied at A, we have

$$\mu = \frac{\sin i}{\sin r_1}$$

$$\Rightarrow \quad r_1 = \sin^{-1}\left(\frac{\sin i}{\mu}\right) = \sin^{-1}\left(\frac{L}{\mu R}\right)$$



Deviation suffered by the ray is

$$\delta = i - r_1 = \sin^{-1}\left(\frac{L}{R}\right) - \sin^{-1}\left(\frac{L}{\mu R}\right)$$

This is also the angle r_2, so we have

$$r_2 = \sin^{-1}\left(\frac{L}{R}\right) - \sin^{-1}\left(\frac{L}{\mu R}\right)$$

From the knowledge of inverse trigonometry, we have

$$\sin^{-1}(C) - \sin^{-1}(D) = \sin^{-1}\left(C\sqrt{1-D^2} - D\sqrt{1-C^2}\right)$$

$$\Rightarrow \quad r_2 = \sin^{-1}\left[\frac{L}{R}\sqrt{1-\frac{L^2}{\mu^2R^2}} - \frac{L}{\mu R}\sqrt{1-\frac{L^2}{R^2}}\right]$$

Now, again applying Snell's Law at B, we get

$$\mu = \frac{\sin\theta}{\sin r_2}$$

$$\Rightarrow \quad \theta = \sin^{-1}(\mu \sin r_2)$$

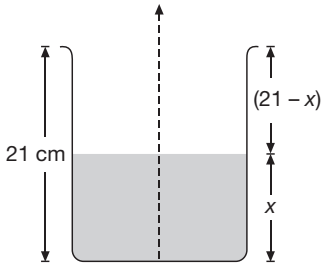
$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{\mu L}{R}\sqrt{1-\frac{L^2}{\mu^2R^2}} - \frac{L}{R}\sqrt{1-\frac{L^2}{R^2}}\right)$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{L}{R^2}\sqrt{\mu^2R^2 - L^2} - \frac{L}{R^2}\sqrt{R^2 - L^2}\right)$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{L}{R^2}\left(\sqrt{\mu^2R^2 - L^2} - \sqrt{R^2 - L^2}\right)\right)$$

11. Figure shows the container filled with water upto a height x so that when observed from top, it appears to be half filled.

CHAPTER 1



The apparent depth of container is such that it should be equal to the empty length of container for it to appear half filled, so we use

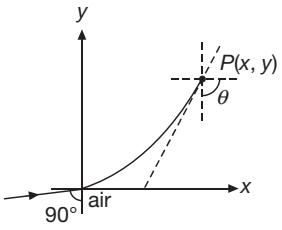
$$\frac{x}{\mu} = 21 - x$$

$$\Rightarrow \quad \frac{3x}{4} + x = 21$$

$$\Rightarrow \quad \frac{7x}{4} = 21$$

$$\Rightarrow \quad x = 12 \text{ cm}$$

12. We draw a tangent at any point (x, y) on the trajectory which makes an angle θ with optical normal parallel to y-axis as shown in figure.



By using Snell's law at the initial point and at the general point of the trajectory of light, we have

$$1\sin 90° = \mu \sin\theta$$

$$\Rightarrow \quad \sin\theta = \frac{1}{\mu} = \frac{1}{\sqrt{1+y}} \qquad \ldots(1)$$

Slope of tangent is

$$\frac{dy}{dx} = \tan(90 - \theta)$$

$$\Rightarrow \quad \frac{dy}{dx} = \cot\theta \qquad \ldots(2)$$

From Equation (1) and (2) we get,

$$\frac{dy}{dx} = y^{\frac{1}{2}}$$

$$\Rightarrow \quad \int_0^y \frac{dy}{y^{\frac{1}{2}}} = \int_0^x dx$$

$$\Rightarrow \quad 2\sqrt{y} = x$$

$$\Rightarrow \quad y = \frac{x^2}{4}$$

Test Your Concepts-IV (Based on Total Internal Reflection (TIR))

1. (a) Critical angle between 2 and 3, is given by

$$\sin C = \frac{1.3}{1.8}$$

Now, applying Snell's Law, we get

$$1.6\sin\theta = 1.8\sin C = (1.8)\left(\frac{1.3}{1.8}\right) = 1.3$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{13}{16}\right) \approx 54.34°$$

(b) If θ is decreased, the angle of incidence at the interface between 2 and 3 gets decreased or $i < C$, so the light will refract into medium 3.

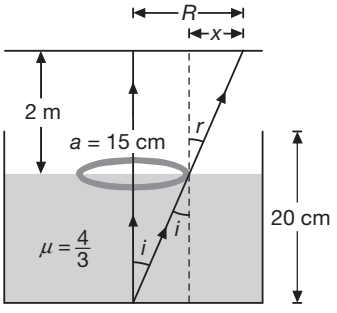
2. (a) Using Snell's Law, we get

$$\mu \sin i = \sin r$$

$$\Rightarrow \quad \frac{4}{3}\left(\frac{15}{\sqrt{15^2 + 20^2}}\right) = 1\left(\frac{x}{\sqrt{x^2 + 200^2}}\right)$$

Solving, we get

$$x = \frac{800}{3} \text{ cm}$$



So, the radius of shadow is $R = \left(15 + \frac{800}{3}\right)$ cm

$$\Rightarrow \quad R = \frac{845}{3} \text{ cm} = 2.81 \text{ m}$$

(b) For shadow to be formed, angle of incidence must be less than critical angle.
Using Snell's Law, we get

$$\frac{4}{3}\left(\frac{a_{max}}{\sqrt{a_{max}^2 + 20^2}}\right) = 1\sin(90°)$$

$$\Rightarrow \quad 16a_{max}^2 = 9a_{max}^2 + 9(20^2)$$

$$\Rightarrow \quad 7a_{max}^2 = 9(20^2)$$

$$\Rightarrow \quad a_{max} = \left(\sqrt{\frac{9}{7}}\right)(20\text{ cm}) = 0.23\text{ m}$$

3. (a) At interface AB, applying Snell's Law, we get

$$1\sin\left(\frac{\pi}{2}\right) = \mu\sin\alpha$$

$$\Rightarrow \quad \sin\alpha = \frac{1}{\mu} \qquad \ldots(1)$$

At interface BC, applying Snell's Law again, we get

$$\mu\sin(90-\alpha) = 1(\sin\theta)$$

$$\Rightarrow \quad \sin\theta = \mu\cos\alpha \qquad \ldots(2)$$

From equation (1) and (2), we get $\sin\theta = \cot\alpha$

(b) For emergence from BC, we must have

$$90-\alpha \le C$$

At grazing incidence we have

$$\alpha = C$$

$$\Rightarrow \quad 90-\alpha \le C$$

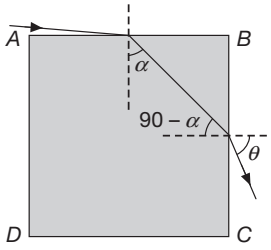
$$\Rightarrow \quad 2C \ge 90°$$

$$\Rightarrow \quad C \ge 45°$$

$$\Rightarrow \quad \sin^{-1}\left(\frac{1}{\mu}\right) \ge 45°$$

$$\Rightarrow \quad \frac{1}{\mu} \ge \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \mu \le \sqrt{2}$$

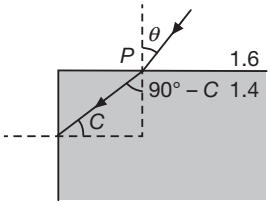


So, the greatest value of refractive index is

$$\mu_{max} = \sqrt{2}$$

4. (a) Critical angle between 2 and 3

$$\sin C = \frac{1.2}{1.4} = \frac{6}{7}$$



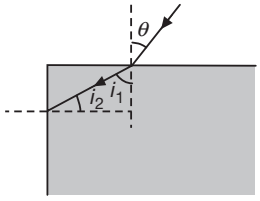
Applying, Snell's Law, at P, we get

$$1.6\sin\theta = 1.4\sin(90°-C) = 1.4\cos C$$

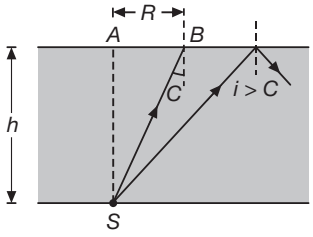
$$\Rightarrow \quad \sin\theta = \frac{1.4\sqrt{1-\frac{36}{49}}}{1.6} = 0.45$$

$$\Rightarrow \quad \theta \approx 26.8°$$

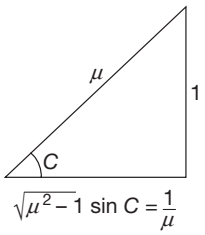
(b) As θ is increased, i_1 will increase or i_2 will decrease or $i_2 < C$ and hence the light will refract in medium 3.



5. As shown in figure, the light from the source will not emerge out of water if $i = C$.



Therefore, minimum radius R corresponds to the situation when $i = C$



In ΔSAB,

$$\frac{R}{h} = \tan C$$

$$\Rightarrow \quad R = h\tan C$$

$$\Rightarrow \quad R = \frac{h}{\sqrt{\mu^2-1}}$$

6. (a) Applying Snell's Law, we get

$$1.3\sin\theta_1 = (1)\sin\theta_5$$

$$\Rightarrow \quad \sin\theta_5 = (1.3)\sin(30°) = \frac{1.3}{2} = 0.65$$

$$\Rightarrow \quad \theta_5 \approx 40.54°$$

CHAPTER 1

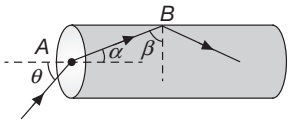
(b) Applying Snell's Law, we get

$$1.3\sin\theta_1 = 1.45\sin\theta_4$$

$$\Rightarrow \quad \sin\theta_4 = \frac{1.3\left(\frac{1}{2}\right)}{1.45} = 0.49$$

$$\Rightarrow \quad \theta_4 \approx 26.6^\circ$$

7. Critical angle, $C = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1}{1.36}\right) = 47.3^\circ$



For TIR to take place at B, we have

$$\beta > C$$

$$\Rightarrow \quad \beta > 47.3^\circ$$

For this to happen, we have

$$\alpha < 90^\circ - 47.3^\circ$$

$$\Rightarrow \quad \alpha < 42.7^\circ$$

Applying Snell's Law at A, we get

$$\theta < \sin^{-1}(\mu\sin 42.7^\circ)$$

$$\Rightarrow \quad \theta < \sin^{-1}[1.36\sin(42.7^\circ)]$$

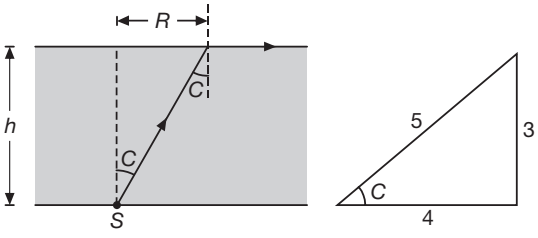
$$\Rightarrow \quad \theta < 67.3^\circ$$

So, the maximum value of θ for TIR to take place at B is 67.3°.

8. $\sin C = \frac{1}{\mu} = \frac{3}{5}$

$$\frac{R}{h} = \tan C = \frac{3}{4}$$

$$\therefore \quad h = \frac{4}{3}R = \frac{4}{3}\text{ cm}$$



9. At the maximum, the ray can enter the glass at the grazing angle, so $(i)_{max} = 90^\circ$

According to Snell's Law

$$\mu_g \sin r_1 = \mu\sin i$$

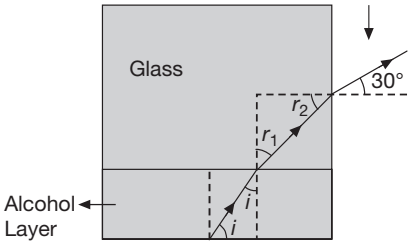
$$\Rightarrow \quad \sin r_1 = \frac{\mu}{1.5}\sin(90^\circ) = \frac{2\mu}{3}$$

$$\Rightarrow \quad (r_1)_{max} = \sin^{-1}\left(\frac{2\mu}{3}\right)$$

Since $r_1 + r_2 = 90^\circ$, so

$$\therefore \quad (r_2)_{min} = \frac{\pi}{2} - \sin^{-1}\left(\frac{2\mu}{3}\right)$$

Again, using Snell's Law, we get $\mu_g = \frac{\sin(30^\circ)}{\sin r_2}$



$$\Rightarrow \quad \frac{3}{2} = \frac{\frac{1}{2}}{\sin\left(\frac{\pi}{2} - r_1\right)} = \frac{1}{2\cos r_1}$$

$$\Rightarrow \quad 3\cos r_1 = 1$$

$$\Rightarrow \quad 3\sqrt{1 - \frac{4\mu^2}{9}} = 1$$

$$\Rightarrow \quad 1 - \frac{4\mu^2}{9} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{4\mu^2}{9} = \frac{8}{9}$$

$$\Rightarrow \quad \mu = \sqrt{2}$$

Now, when the paper is dry then $\mu = 1$

$$\Rightarrow \quad r_1 = \sin^{-1}\left(\frac{2}{3}\right) \approx 42^\circ$$

Since $r_1 + r_2 = 90^\circ$, so we get

$$r_2 = 48^\circ$$

Critical angle at glass air interface, is given by

$$C = \sin^{-1}\left(\frac{1}{1.5}\right) \approx 42^\circ$$

So, we observe that as $r_2 > C$, so it can't be seen.

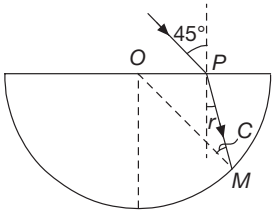
10. According to Snell's Law, we have

$$\sqrt{2} = \frac{\sin(45^\circ)}{\sin r}$$

$$\Rightarrow \quad r = 30^\circ$$

The critical angle C is given by

$$C = \sin^{-1}\left(\frac{1}{\mu}\right) = 45°$$



Applying, Sine Law (i.e., Snell's Law) in ΔOPM, we get

$$\frac{OP}{\sin C} = \frac{OM}{\sin(90° + r)}$$

$$\Rightarrow \quad \frac{OP}{\left(\frac{1}{\sqrt{2}}\right)} = \frac{R}{\cos r} \qquad \{R = \text{radius}\}$$

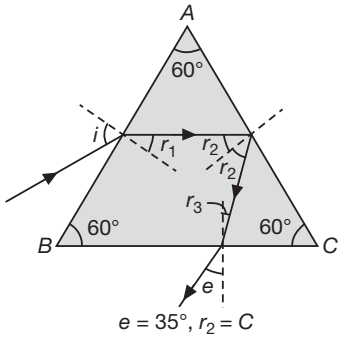
$$\Rightarrow \quad OP = \sqrt{\frac{2}{3}}R$$

As we move away from O, angle PMO will increase.

Therefore, $OP \gtrless \sqrt{\frac{2}{3}}R$. Same is the case on left side of O.

Test Your Concepts-V (Based on Prism)

1. The ray diagram for the situation discussed is shown in figure.



(a) From the figure, we observe that

$$r_1 + r_2 = r_2 + r_3 = 60°$$

$$\Rightarrow \quad r_1 = r_3$$

Applying Snell's Law at the faces AB and BC, we get

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin(30°)}{\sin r_3}$$

Since $r_1 = r_3$

$$\Rightarrow \quad i = 30°$$

(b) Since $r_1 + r_2 = A = 60°$

$$\Rightarrow \quad r_1 = 60° - r_2 \cong 60 - C \qquad \{\because\ r_2 \approx C\}$$

Further, $\mu = \frac{\sin i}{\sin r_1} = \frac{\sin(30°)}{\sin(60° - C)}$

$$\mu = \frac{0.5}{\sin(60°)\cos C - \cos(60°)\sin C}$$

Since $\sin C = \frac{1}{\mu}$

$$\Rightarrow \quad \cos C = \sqrt{1 - \frac{1}{\mu^2}}$$

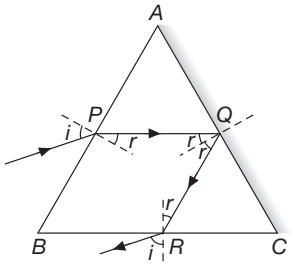
$$\Rightarrow \quad \mu\left(\frac{\sqrt{3}}{2}\right)\sqrt{1 - \frac{1}{\mu^2}} - \frac{1}{2} = 0.5$$

$$\Rightarrow \quad \left(\frac{\sqrt{3}}{2}\right)\sqrt{\mu^2 - 1} = 1$$

$$\Rightarrow \quad \mu = \sqrt{\frac{7}{3}}$$

2. At minimum deviation, we have

$$r = \frac{A}{2} = 30°$$



(a) Applying Snell's Law at AB, we get

$$1.5 = \frac{\sin i}{\sin(30°)}$$

$$\Rightarrow \quad i = 48.6°$$

Since, $\delta_{\text{Total}} = \delta_P + \delta_Q + \delta_R$

$$\Rightarrow \quad \delta_{\text{Total}} = (i - r) + (180° - 2r) + (i - r)$$

$$\Rightarrow \quad \delta_{\text{Total}} = 180° + 2i - 4r$$

$$\Rightarrow \quad \delta_{\text{Total}} = 157.2°$$

(b) Again applying Snell's Law for water-glass interface, we get

$$\frac{4}{3}\sin i' = \frac{3}{2}\sin(30°)$$

CHAPTER 1

$\Rightarrow \quad i' = 34.2°$

$\Rightarrow \quad \delta_{\text{Total}} = 180° + 2i' - 4r$

$\Rightarrow \quad \delta_{\text{Total}} = 128.4°$

3. Since, the condition for no emergence is

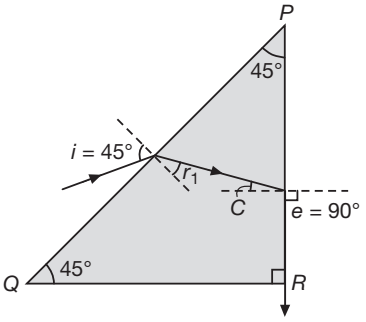
$A > 2C$

$\Rightarrow \quad A > 2\sin^{-1}\left(\frac{1}{\mu}\right)$

$\Rightarrow \quad A > 2\sin^{-1}\left(\frac{1}{1.5}\right) > 83.62°$

Therefore, $A_{\max} = 83.62°$, for escaping of the ray through the adjacent face.

4. The situation is shown in figure



(a) Since, $e = 90°$

Also, $r_2 = C = \sin^{-1}\left(\frac{1}{\mu}\right)$

Now, $i = A = 45°$ and $r_1 = A - r_2 = 45° - C$

Applying Snell's Law at AB, we get

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin(45°)}{\sin(45° - C)}$$

$$\Rightarrow \quad \mu = \frac{\sin(45°)}{\sin(45°)\cos C - \cos(45°)\sin C}$$

Since $\sin C = \frac{1}{\mu}$

$\Rightarrow \quad \cos C = \sqrt{1 - \frac{1}{\mu^2}}$

$\Rightarrow \quad \mu\sqrt{1 - \frac{1}{\mu^2}} - 1 = 1 \qquad (\sin 45° = \cos 45°)$

$\Rightarrow \quad \mu^2 - 1 = 4$

$\Rightarrow \quad \mu = \sqrt{5}$

(b) At minimum deviation, we have

$$r_1 = r_2 = \frac{A}{2} = 22.5°$$

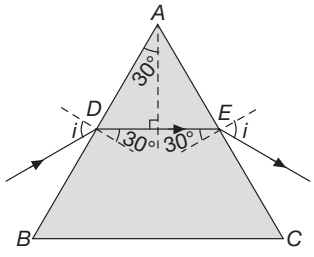
Since, $\sqrt{5} = \frac{\sin i}{\sin(22.5°)}$

$\Rightarrow \quad i = 58.8°$

5. (a) Applying Snell's law at D, we get

$$\left(\frac{4}{3}\right)\sin i = \left(\frac{3}{2}\right)\sin 30°$$

$\Rightarrow \quad i = 34.2°$



(b) Total deviation suffered by the ray is

$$\delta = \delta_D + \delta_E = 2\delta_D$$

$\Rightarrow \quad \delta = 2(i - 30°) = 8.4°$

6. At near normal incidence, $i \approx r_1 = 0°$

Since $r_1 + r_2 = A$

$\Rightarrow \quad r_2 = \alpha$

From Snell's Law applied at the face from where the refracted ray emerges, we get

$$\mu = \frac{\sin e}{\sin r_2}$$

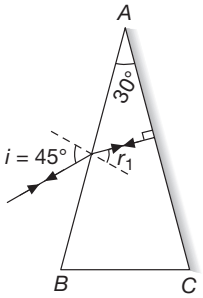
$\Rightarrow \quad e = \sin^{-1}(\mu \sin\alpha)$

Now, deviation $\delta = i + e - A = \sin^{-1}(\mu\sin\alpha) - \alpha$

$\Rightarrow \quad \delta = \sin^{-1}(\mu\sin\alpha) - \alpha \qquad \{\because i = 0°\}$

7. For the ray to retrace its path, it must be incident normally to the face AC. So, we have

$r_2 = 0°$



Since $r_1 + r_2 = A$

$\Rightarrow \quad r_1 = A = 30°$

From Snell's Law, we have

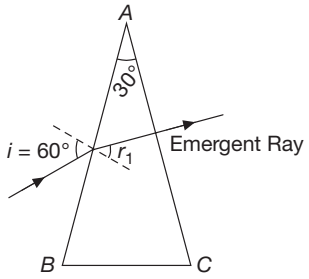
$$\mu = \frac{\sin i}{\sin r_1} = \sqrt{2}$$

8. From the statement of the problem, we gather the information that

$$i = 60°,\ A = 30°,\ \delta = 30°$$

Since, $\delta = i + e - A$

$\Rightarrow \quad e = \delta - i + A = 30° - 60° + 30° = 0°$



i.e., the emergent ray is perpendicular to the face through which it emerges.

Further, $r_2 = 0$ and $r_1 + r_2 = A$

$\Rightarrow \quad r_1 = A = 30°$ $\{\text{as } e = 0\}$

From Snell's Law applied at face AC, we get

$$\mu = \frac{\sin i}{\sin r_1} = \sqrt{3}$$

9. **For Violet Light:** According to Snell's Law, applied at the plane of incidence, we get

$$\mu = \frac{\sin i}{\sin r_1}$$

$$\Rightarrow \quad 1.66 = \frac{\sin(50°)}{\sin r_1}$$

$\Rightarrow \quad r_1 = 27.5°$

Since, $r_1 + r_2 = A$

$\Rightarrow \quad r_2 = A - r_1 = 32.5°$

Applying Snell's Law at the plane of emergence, we get

$$\mu = \frac{\sin e}{\sin r_2}$$

$$\Rightarrow \quad 1.66 = \frac{\sin e}{\sin(32.5°)}$$

$\Rightarrow \quad e = 63.1°$

Since, $\delta_V = (i + e) - A$

$\Rightarrow \quad \delta_V = 53.1°$

For Red Light: According to Snell's Law, applied at the plane of incidence, we get

$$1.62 = \frac{\sin(50°)}{\sin r_1}$$

$\Rightarrow \quad r_1 = 28.2°$

Since, $r_1 + r_2 = A$

$\Rightarrow \quad r_2 = A - r_1 = 31.8°$

Applying Snell's Law at the plane of emergence, we get

$$1.62 = \frac{\sin e}{\sin(31.8°)}$$

$\Rightarrow \quad e = 58.6°$

Since, $\delta_R = (i + e) - A = 48.6°$

So, angular dispersion is given by

$$\delta_V - \delta_R = 4.5°$$

10. For minimum deviation, we have

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \quad 1.3\sin(45°) = \sin(45° + \delta_m)$$

Solving, this we get

$$\delta_m = 22°$$

For maximum deviation, we have the emergent ray to be grazing on the surface of emergence. So,

$$e = 90°,\ r_2 = C \text{ and } r_1 = 90° - r_2 = 90° - C$$

$$\Rightarrow \quad \delta_{\max} = i + e - A$$

We can find i by using $\mu = \frac{\sin i}{\sin r_1}$ and $\sin C = \frac{1}{\mu}$.

Substituting the values, we get

$$\delta_{\max} = 56°$$

11. Given that, $i = 60°$, $A = 30°$ and $\delta = 30°$

Since, $\delta = i + e - A$

Substituting the values we get, $e = 0°$

Now, $e = 0°$, means that the emergent ray is normal to the face through which it emerges.

12. Given that $A = 30°$ and $i = 0°$, so $r_1 = 0°$

Since, $r_1 + r_2 = A$

$\Rightarrow \quad r_2 = A = 30°$

Further applying Snell's Law at the plane of emergence, we get

$$1.5 = \frac{\sin e}{\sin r_2}$$

CHAPTER 1

Substituting the values, we get

$e = 49°$

$\Rightarrow \quad \delta = i + e - A = 19°$

13. For no total internal reflection, when the ray leaves the prism,

$r_2 = C$

But $\sin C = \frac{1}{\mu} = \frac{1}{1.6}$

$r_2 = C = \sin^{-1}\left(\frac{1}{1.6}\right) = 38.7°$

Further $r_1 + r_2 = A = 45°$

$\Rightarrow \quad r_1 = 45° - 38.7° = 6.3°$

Now, $i = \sin^{-1}(\mu \sin r_1) = \sin^{-1}[1.6 \times \sin(6.3°)] = 10.1°$

14. For red ray, we have

$e = 0° = r_2$

Since, $r_1 + r_2 = A$

$\Rightarrow \quad r_1 = A = 45°$

Now, according to Snell's Law, we have

$$1.37 = \frac{\sin i}{\sin(45°)}$$

$\Rightarrow \quad i = 75.6°$

$\Rightarrow \quad \delta_{\text{red}} = i + e - A = 30.6°$

For violet ray, we have

$$1.42 = \frac{\sin(75.6°)}{\sin r_1}$$

$\Rightarrow \quad r_1 = 43°$

$\Rightarrow \quad r_2 = A - r_1 = 2° \qquad \{\because r_1 + r_2 = A\}$

Again, applying Snell's Law at the emerging face, for violet rays, we get

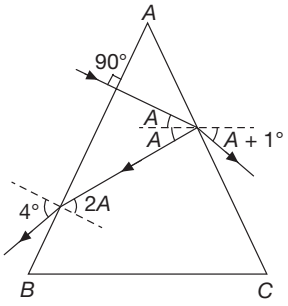
$$1.42 = \frac{\sin e}{\sin r_2}$$

$\Rightarrow \quad e = 2.84°$

$\Rightarrow \quad \delta_{\text{violet}} = i + e - A = 33.4°$

15. As the angles are small, we can take,

$\sin\theta \approx \theta$



Applying Snell's Law for the two emerging rays at AC and AB, we get

$$\mu = \frac{\sin(A + 1°)}{\sin A} = \frac{\sin(4°)}{\sin(2A)}$$

$\Rightarrow \quad \mu \approx \frac{A + 1°}{A} = \frac{4°}{2A}$

$\Rightarrow \quad A = 1°$ and $\mu = 2$

Test Your Concepts-VI (Based on Refraction at Curved Surfaces)

1. Let us see where do the parallel rays converge (or diverge) on the principal axis. Let us call it the focus and the corresponding length the focal length f. Using $\frac{\mu_2}{v} = \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ with appropriate values and signs, we get

$$\frac{\frac{4}{3}}{f} - \frac{1}{\infty} = \frac{\frac{4}{3} - 1}{+10}$$

$\Rightarrow \quad f = 40 \text{ cm} = 0.4 \text{ m}$

Since, the rays are converging, its power should be positive. Hence,

$$P \text{ (in dioptre)} = \frac{1}{f(\text{metre})} = \frac{1}{0.4}$$

$\Rightarrow \quad P = 2.5 \text{ dioptre} = 2.5 \text{ D}$

2. For first refraction at the unsilvered surface, we have

$$\frac{1.5}{v_1} - \frac{1}{(-2r)} = \frac{1.5 - 1}{r}$$

$\Rightarrow \quad v_1 \to \infty$

i.e., rays become parallel to the principal axis.
Hence the image formed by the curved mirror will lie at the focus of the mirror i.e., a distance $\frac{r}{2}$ from pole of mirror.

So, $v_2 = \frac{r}{2}$ {from pole of the mirror}

For second refraction at the unsilvered surface, we have

$$\frac{1}{v_3} - \frac{1.5}{\left(-\frac{3r}{2}\right)} = \frac{1 - 1.5}{(-r)}$$

$\Rightarrow \quad v_3 = -2R$

i.e., final image is formed at pole of the mirror.

3. For the image of object O to be formed at O, the light should fall normally on mirror. First image I_1 (after refraction from the plane surface) will be formed

at a distance of $\frac{2R+x}{\mu}$ from plane surface, because $d_{app.} = \frac{d_{actual}}{\mu}$.

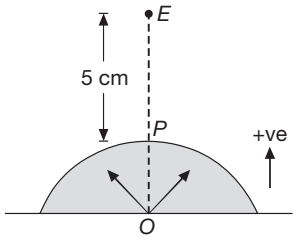
Now applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ with the idea that I_2 is formed at C, because light falls normally on the mirror.

$$\Rightarrow \quad \frac{\mu}{-(R+x)} - \frac{1}{-\left(\frac{2R+x}{\mu} + R\right)} = \frac{\mu - 1}{-R}$$

Solving this equation for $\mu = 1.5$, we get

$$x \approx 0.75R$$

4. Applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, we get



$$\frac{1}{PI} - \frac{1.6}{(-3)} = \frac{1-1.6}{-5}$$

$$\Rightarrow \quad PI = -2.42 \text{ cm}$$

$$\Rightarrow \quad EI = (5 + 2.42) \text{ cm}$$

$$\Rightarrow \quad EI = 7.42 \text{ cm}$$

5. (a) Applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, we get

$$\frac{1.5}{v} - \frac{1}{-1} = \frac{1.5-1}{6}$$

$$\Rightarrow \quad v = -90 \text{ cm}$$

So, the distance between object and its image is 80 cm

(b) Again applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \quad \frac{1.5}{v} - \frac{1}{(-u)} = \frac{1.5-1}{6}$$

$$\Rightarrow \quad v = \frac{18}{\left(1 - \frac{12}{u}\right)}$$

v is negative when $\frac{12}{u} > 1$

$$\Rightarrow \quad u < 12 \text{ cm}$$

6. Applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ twice, we get

$$\frac{1.5}{v_1} - \frac{1}{(-2.5)} = \frac{1.5-1}{+10}$$

$$\Rightarrow \quad v_1 = -\frac{30}{7} \text{ cm}$$

Further, $\frac{1}{v_2} - \frac{1.5}{-\left(20 + \frac{30}{7}\right)} = \frac{1-1.5}{-10}$

$$\Rightarrow \quad v_2 = -85 \text{ cm}$$

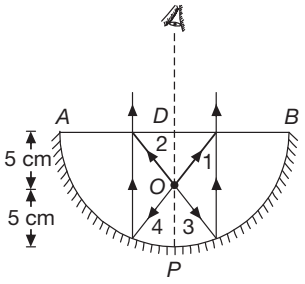
i.e., final image is formed at 65 cm from first face on the same side of the object.

7. First image will be formed by direct rays 1 and 2, etc.

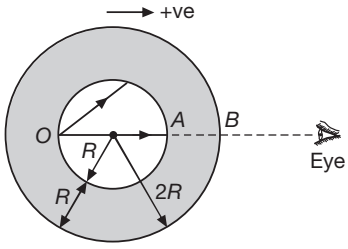
$$DI_1 = \frac{DO}{\mu} = \frac{5}{1.5} = 3.33 \text{ cm}$$

Second image will be formed by reflected rays 3 and 4, etc.

Object is placed at the focus of the mirror. Hence, I_2 is formed at infinity.



8. We have to see the image of O from the other side



Applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ twice, we get

$$\frac{\mu}{AI_1} - \frac{1}{(-2R)} = \frac{\mu - 1}{-R}$$

$$\Rightarrow \quad AI_1 = \frac{2\mu R}{1 - 2\mu}$$

Further, $\frac{1}{BI_2} - \frac{\mu}{(AI_1 - R)} = \frac{1-\mu}{-R}$

CHAPTER 1

Solving this equation, we get

$$BI_2 = -\frac{2R(4\mu-1)}{3\mu-1}$$

So, the distance between the final image and the object is

$$d = 3R - \frac{2R(4\mu-1)}{3\mu-1} = \frac{(\mu-1)R}{(3\mu-1)}$$

9. For refraction formula, we use

$$u = +15 \text{ cm}\ ;\ \mu_1 = \frac{4}{3}\ ;\ \mu_2 = 1 \text{ and } R = +10 \text{ cm}$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Substituting the values, we get

$$\frac{1}{v} - \frac{4}{3\times 15} = \frac{1-\frac{4}{3}}{10} = -\frac{1}{30}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{4}{45} - \frac{1}{30} = \frac{1}{18}$$

$$\Rightarrow \quad v = +18 \text{ cm}$$

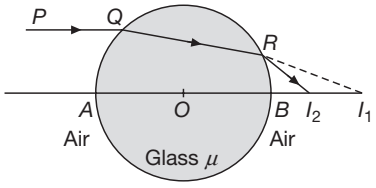
Velocity magnification along optic axis is given as

$$m = \frac{\mu_1 v^2}{\mu_2 u^2} = \frac{\frac{4}{3}\times(18)^2}{1\times(15)^2} = 1.92$$

Velocity of image of fish is

$$\Rightarrow \quad V_{\text{fish}} = 1.92\times 2 = 3.84 \text{ mms}^{-1}$$

10. Figure shows the refraction of a parallel beam on a solid glass sphere at near normal incidence. At first refraction image I_1 is produced which acts as an object for second refraction at other surface of sphere at which after refraction final image I_2 is produced.



Using refraction formula at first surface we have

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u_1} = \frac{\mu_1 - \mu_2}{R} \qquad \ldots(1)$$

Here we use $\mu_1 = 1$, $\mu_2 = \mu$, $u_1 = \infty$ and $R = +R$

Substituting the values in refraction formula we get

$$v_1 = \frac{\mu R}{(\mu-1)} \qquad \ldots(2)$$

For the refraction at the second surface, we use

$$\mu_1 = \mu\ ,\ \mu_2 = 1\ ,\ R = -R \text{ and}$$

$$u_2 = +\frac{\mu R}{(\mu-1)} - 2R = \frac{2R-\mu R}{(\mu-1)} = \frac{R(2-\mu)}{(\mu-1)}$$

For the second surface,

$$\frac{1}{v_2} - \frac{\mu}{u_2} = \frac{(1-\mu)}{-R}$$

$$\Rightarrow \quad \frac{1}{v_2} - \frac{\mu(\mu-1)}{R(2-\mu)} = \frac{(1-\mu)}{-R}$$

$$\frac{1}{v_2} = \frac{\mu(\mu-1)}{R(2-\mu)} - \frac{(1-\mu)}{R}$$

Simplifying, we get

$$v_2 = \frac{R(2-\mu)}{2(\mu-1)}$$

11. For refraction formula, we use

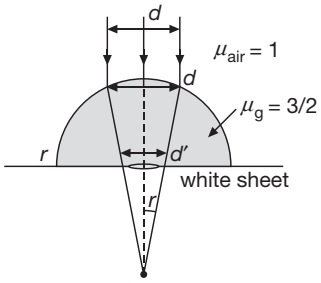
$$u = \infty\ ;\ \mu_1 = 1\ ;\ \mu_2 = \frac{3}{2} \text{ and } R = +r$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Substituting the values, we have

$$\frac{3}{2v} = \frac{\frac{3}{2}-1}{r} = \frac{1}{2r}$$

$$\Rightarrow \quad v = +3r$$



by similarity, we can write

$$\frac{d}{d'} = \frac{3r}{2r}$$

$$\Rightarrow \quad d' = \frac{2d}{3}$$

12. We consider refractive index of glass to be μ and after first refraction, image is produced at v then by refraction formula, we use

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u_1} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \quad \frac{\mu}{v}-\frac{1}{\infty}=\frac{\mu-1}{R}$$

$$\Rightarrow \quad v=\frac{\mu R}{\mu-1}$$

For second refraction using refraction formula, we have

$$u_1=\frac{\mu R}{\mu-1}-R=+\frac{R}{\mu-1};\ v_1=+R;\ R=+\frac{R}{2};\ \mu_1=\mu$$

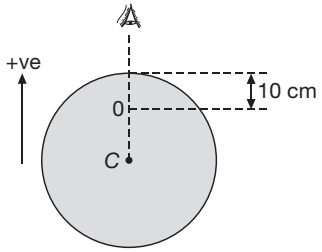
and $\mu_2=1$

$$\Rightarrow \quad \frac{1}{R}-\frac{\mu(\mu-1)}{R}=\frac{2(1-\mu)}{R}$$

$$\Rightarrow \quad \mu^2-3\mu+1=0$$

Solving we get $\mu=\dfrac{3+\sqrt{5}}{2}$

13. Figure below shows the situation described in the question.



We apply refraction formula

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

Here we use $u=-10$ cm ; $R=-15$ cm ; $\mu_1=1.5$ and $\mu_2=1$

Substituting the values, we have

$$\frac{1}{v}-\frac{1.5}{-10}=\frac{1-1.5}{-15}$$

Solving we get $v=-8.57$ cm

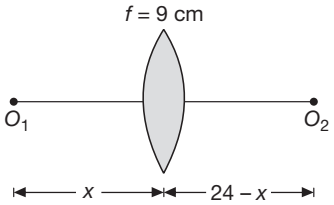
Thus, the apparent depth of bubble will be 8.57 cm from the top of the sphere.

Test Your Concepts-VII (Based on Lens Formula)

1. When the images of both the sources are formed at the same point, then v will be same for both (in value). However, for one case the image will be real and for the other case it will be virtual. So,

For O_1, we have $\dfrac{1}{v}+\dfrac{1}{x}=\dfrac{1}{9}$ …(1)

For O_2, we have $-\dfrac{1}{v}+\dfrac{1}{24-x}=\dfrac{1}{9}$ …(2)



Adding equations (1) and (2), we get

$$\frac{1}{x}+\frac{1}{24-x}=\frac{2}{9}$$

Solving we get $x=6$ cm

Hence, the lens should be kept at a distance of 6 cm from either of the object.

2. Here the image formed can be virtual as well as real, so the value of m should be $+3$ in one case (virtual image) and -3 in the other (real image). Magnification of $+3$ can be obtained only when the object is placed within F (i.e., for smaller value of the object distance). The magnification of -3 is obtained when the object is kept between F and $2F$ (i.e., for greater value of object distance). So,

$m=+3$, for $u=-8$ cm

Therefore, from the definition of magnification, we have

$$m=\frac{v}{u}$$

$$\Rightarrow \quad +3=\frac{v}{-8}$$

$$\Rightarrow \quad v=3\times(-8)=-24 \text{ cm}$$

Using lens formula, we get

$$\frac{1}{-24}-\frac{1}{-8}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{-1+3}{24}=\frac{1}{f}$$

$$\Rightarrow \quad f=12 \text{ cm}$$

Note that you would get the same answer by considering the other case ($m=-3$, for $u=-16$ cm)

3. The Lens Maker's Formula is given by

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Here, $f=30$ cm , $R_1=10$ cm , $R_2=\infty$

$$\Rightarrow \quad \frac{1}{30}=(\mu-1)\left(\frac{1}{10}-\frac{1}{\infty}\right)=\frac{\mu-1}{10}$$

CHAPTER 1

$\Rightarrow \quad 30\mu - 30 = 10$

$\Rightarrow \quad \mu = \frac{4}{3}$

4. Applying Lens Maker's Formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $\mu = 1.5$, $f = 60$ cm, $R_1 = +R$, $R_2 = -R$

$\Rightarrow \quad \frac{1}{60} = (1.5 - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$

$\Rightarrow \quad R = 60$ cm

Therefore, the focal length of the spherical silvered surface, is given by

$$f_m = \frac{R}{2} = \frac{60}{2} = +30 \text{ cm}$$

(Positive, because it is a converging mirror)

The equivalent focal length of the lens-mirror combination is then given by

$$-\frac{1}{F} = \frac{2}{f_l} - \frac{1}{f_m} = \frac{2}{60} - \frac{1}{(-30)}$$

$\Rightarrow \quad F = -15$ cm

The negative sign indicates that the combination behaves as a concave mirror.

5. According to the Lens Maker's Formula, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \text{ where } f = 10 \text{ cm} \qquad \ldots(1)$$

When placed in medium 1, then

$$\frac{1}{f_1} = \left(\frac{\mu}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots(2)$$

When placed in medium 2, then

$$\frac{1}{f_2} = \left(\frac{\mu}{\mu_2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots(3)$$

From equations (1) and (2), we get

$$\frac{f_1}{f} = \frac{(\mu - 1)}{\left(\frac{\mu}{\mu_1} - 1\right)} = \frac{1.6 - 1}{\left(\frac{1.6}{1.5} - 1\right)} = 9$$

$\Rightarrow \quad f_1 = 9f = 90$ cm

i.e., still it behaves as a converging lens

From equations (1) and (3), we get

$$\frac{f_2}{f} = \frac{(\mu - 1)}{\left(\frac{\mu}{\mu_2} - 1\right)} = \frac{1.6 - 1}{\left(\frac{1.6}{1.7} - 1\right)} = -10.2$$

$\Rightarrow \quad f_2 = -10.2f = -102$ cm

i.e., it now behaves like a diverging lens.

6. Here $R_1 = +25$ cm, $R_2 = -25$ cm, $\mu_1 = 1$ and $\mu_2 = \frac{3}{2}$

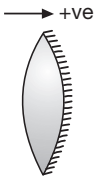


Image coincides with object, hence, $u = v = -x$ (say)

$$\frac{1}{-x} - \frac{1}{x} = \frac{2(3/2)}{-25} - \frac{2(3/2 - 1)}{25}$$

$\Rightarrow \quad \frac{2}{x} = \frac{3}{25} + \frac{1}{25} = \frac{4}{25}$

$\Rightarrow \quad x = 12.5$ cm

Hence, the object should be placed at a distance 12.5 cm in front of the silvered lens.

7. Given, $f_1 = +5$ cm and $f_2 = -10$ cm

The combined focal length F is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{5} - \frac{1}{10} = +\frac{1}{10}$$

$\Rightarrow \quad F = +10$ cm

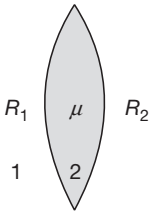
i.e., the combination behaves as a converging lens of focal length 10 cm.

8. According to Lens Maker's Formula, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\Rightarrow \quad \frac{1}{10} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{-10}\right)$

$\Rightarrow \quad R_1 = +10$ cm



Now, using, $\frac{1}{v} + \frac{1}{u} = \frac{2(\mu_2/\mu_1)}{R_2} - \frac{2(\mu_2/\mu_1 - 1)}{R_1}$

Substituting the values, we get

$$\frac{1}{v} - \frac{1}{-15} = \frac{2(1.5)}{-10} - \frac{2(1.5 - 1)}{+10}$$

$\Rightarrow \quad v = -2.14$ cm

9. Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{36} - \frac{1}{-45} = \frac{1}{f}$$

$\Rightarrow \quad f = 20$ cm

In the second case, let μ be the refractive index of the liquid, then

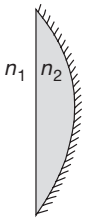
$$\frac{1}{48}-\frac{1}{-\left(5+\frac{40}{\mu}\right)}=\frac{1}{20}$$

Solving, we get

$$\mu = 1.37$$

10. The system behaves like a mirror of focal length given by,

$$\frac{1}{F}=\frac{2(\mu_2/\mu_1)}{R_2}-\frac{2(\mu_2/\mu_1-1)}{R_1}$$



Substituting the values with appropriate signs, we get

$$\frac{1}{F}=\frac{2\left(\frac{4}{3}\right)}{-20} \qquad \{\because R_1 \to \infty\}$$

$$\Rightarrow \quad F = -7.5 \text{ cm}$$

So, the system behaves as a concave mirror of focal length 7.5 cm .

11. For a convex lens, the distance between an object and its real image is minimum when $u = 2f_1$ and $v = 2f_1$ When concave lens is placed in contact, then we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_{\text{comb}}}$$

$$\frac{1}{v'}+\frac{1}{2f_1}=\frac{1}{f_1}-\frac{1}{f_2}=\frac{f_2-f_1}{f_1f_2}$$

$$\Rightarrow \quad v'=\frac{2f_1f_2}{f_2-2f_1}$$

Shift of image is

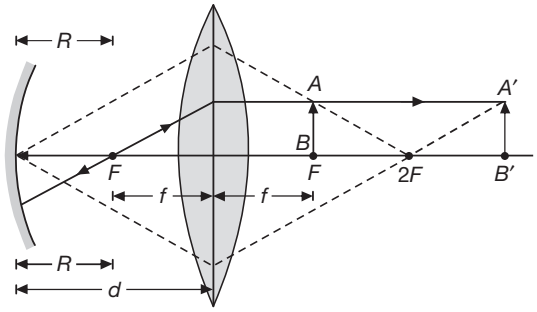
$$\Delta v = v'-v=\frac{2f_1f_2}{f_2-2f_1}-2f_1=\frac{4f_1^2}{f_2-2f_1}$$

Since, $f_2 \gg f_1$, so the shift of image is

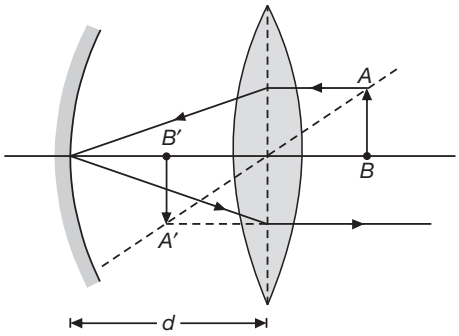
$$\Delta v \approx \frac{4f_1^2}{f_2}$$

12. The following two cases are possible.

CASE-1: The mirror is at a distance of $d = f + R = 2$ m from the lens. The path of the beam parallel to the optical axis of the system and the image of object AB are shown in figure. Image $A'B'$ (direct and real) is obtained to full scale with the object in any position.



CASE-2: The mirror is at a distance of $d = f = R = 1$ m from the lens. The image of object $A'B'$, also full scale, will be inverted and virtual with the object in any position.



13. For a convex lens, distance between an object and its real image is minimum when $u = 2f_1$. Hence,

(a) $$3f_1-\left(1-\frac{1}{\mu}\right)t=2f_1$$

$$\Rightarrow \quad \left(1-\frac{1}{\mu}\right)t=f_1$$

$$\Rightarrow \quad \frac{1}{\mu}=1-\frac{f_1}{t}=\frac{t-f_1}{t}$$

$$\Rightarrow \quad \mu=\frac{t}{t-f_1}$$

(b) If a concave lens of very large focal length f_2 (i.e., very small power) is placed in contact with the convex lens, then its power and hence the focal length are almost unaltered. Therefore, there will be no shifting of the image.

14. Using the lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get for the first lens

$$\frac{1}{v_1}+\frac{1}{30}=\frac{1}{20}$$

$$\Rightarrow \quad v_1 = 60 \text{ cm}$$

CHAPTER 1

For the second lens

$$\frac{1}{v_2}-\frac{1}{30}=\frac{1}{10}$$

$$\Rightarrow \quad v_2=7.5 \text{ cm}$$

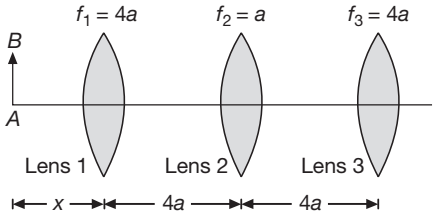
15. Since, $\frac{1}{F}=\frac{2\left(\frac{\mu_2}{\mu_1}\right)}{R_2}-\frac{2\left(\frac{\mu_2}{\mu_1}-1\right)}{R_1}=\frac{2\mu}{-r}-\frac{2(\mu-1)}{r}$

$$\Rightarrow \quad \frac{1}{F}=-\left(\frac{4\mu-2}{r}\right)$$

$$\Rightarrow \quad F=-\left(\frac{r}{4\mu-2}\right)$$

i.e., the lens is equivalent to a concave mirror of focal length $\frac{r}{4\mu-2}$

16. Let x be the distance between first lens and the object AB. Applying the lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ thrice, we get for lens 1,



$$\frac{1}{v_1}+\frac{1}{x}=\frac{1}{4a} \qquad \ldots(1)$$

$$\Rightarrow \quad v_1=\frac{4ax}{x-4a}$$

$$\Rightarrow \quad m_1=\frac{v}{u}=\frac{v_1}{-x}=\frac{4a}{4a-x} \qquad \ldots(2)$$

Similarly, for Lens 2, we get

$$\frac{1}{v_2}-\frac{1}{v_1-4a}=\frac{1}{a}$$

$$\Rightarrow \quad \frac{1}{v_2}-\frac{x-4a}{16a^2}=\frac{1}{a} \qquad \ldots(3)$$

$$\Rightarrow \quad v_2=\frac{16a^2}{x+12a}$$

$$\Rightarrow \quad m_2=\frac{\left(\frac{16a^2}{x+12a}\right)}{\left(\frac{16a^2}{x-4a}\right)}=\frac{x-4a}{x+12a} \qquad \ldots(4)$$

Similarly, for lens 3, we get

$$\frac{1}{v_3}-\frac{1}{v_2-4a}=\frac{1}{4a} \qquad \ldots(5)$$

$$\Rightarrow \quad v_3=-(x+8a)$$

$$\Rightarrow \quad m_3=\frac{x+12a}{4a} \qquad \ldots(6)$$

So, we observe from (2), (4) and (6) that

$$m_1m_2m_3=-1$$

Also, $v_3=-(x+8a)$ means that the object and its image lie at the same place and $m_1m_2m_3=-1$ means final image is of the same size but inverted.

17. According to the lens formula, we have, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

Since $u=-mf$, so we get

$$\frac{1}{v}+\frac{1}{mf}=\frac{1}{(-f)}$$

$$\Rightarrow \quad v=-\left(\frac{m}{m+1}\right)f$$

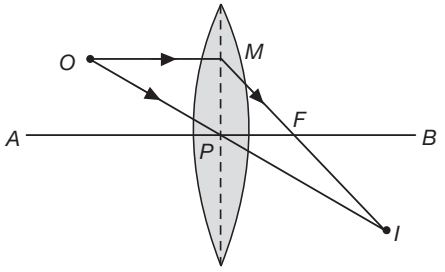
So, lateral magnification is given by

$$m_L=\frac{v}{u}=\frac{1}{m+1}$$

Hence, the image will be $(m+1)$ times smaller than the object.

18. (a) Since a concave lens always forms an erect image whereas the given image I is on the other size of the optic axis, so the lens is convex.

(b) Join O with I. Line OI cuts the optic axis AB at pole (P) of the lens. The dotted line shows the position of lens.



From point O, draw a line parallel to AB which after refraction must pass through the focus F of the lens.

19. Let parallel rays be incident on first lens, then

$$v_1=f_1=7 \text{ cm}$$

For the second lens, we have

$$\frac{1}{v_2}-\frac{1}{(7-3)}=\frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{v_2}-\frac{1}{4}=\frac{1}{6}$$

$$\Rightarrow \quad v_2 = 2.4 \text{ cm}$$

20. (a) From Lens Maker's Formula, we get

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{f_1}{f_2}=\frac{\mu_2-1}{\mu_1-1}=\frac{1.7-1}{1.5-1}=\frac{7}{5}=1.4$$

(b) In this liquid the first lens will be a diverging (as refractive index of liquid >1.5) and the second a converging one (as refractive index of liquid <1.7).

21. Since the incident beam is parallel, so we have

$$v_1 = f_1 = 10 \text{ cm}$$

For the second lens, we have

$$\frac{1}{v_2}-\frac{1}{-5}=\frac{1}{-20}$$

$$\Rightarrow \quad v_2 = -4 \text{ cm}$$

For the third lens, we have

$$\frac{1}{v_3}-\frac{1}{-9}=\frac{1}{9}$$

$$\Rightarrow \quad v_3 \to \infty$$

i.e., rays will become parallel to the optic axis.

22. Since the focal length is equal to two times the radius of curvature, so $f=-2r$

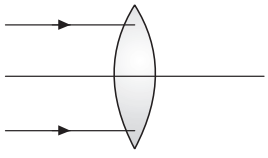
$$\frac{1}{(-2r)}=(\mu_0+a\text{I}-1)\left(\frac{1}{\infty}-\frac{1}{r}\right)$$

$$\Rightarrow \quad \text{I}=\frac{3-2\mu_0}{2a}$$

23. The rays will first get refracted, then n -times reflected and finally again refracted. So, using $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ for first refraction, we get

$$\frac{\mu}{v_i}-\frac{1}{\infty}=\frac{\mu-1}{R}$$

$$\Rightarrow \quad v_i=\left(\frac{\mu}{\mu-1}\right)R$$



For first reflection, let us use the mirror formula, i.e.,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{R}$$

$$\Rightarrow \quad \frac{1}{v_1}+\left(\frac{\mu-1}{\mu R}\right)=\frac{-2}{R}$$

$$\Rightarrow \quad \frac{1}{v_1}=-\left(\frac{3\mu-1}{\mu R}\right)$$

For second reflection, similarly we get

$$\frac{1}{v_2}+\frac{3\mu-1}{\mu R}=\frac{-2}{R}$$

$$\Rightarrow \quad \frac{1}{v_2}=-\left(\frac{5\mu-1}{\mu R}\right)$$

Similarly, after nth reflections, we get

$$\frac{1}{v_n}=-\left[\frac{(2n+1)\mu-1}{\mu R}\right]$$

Finally, again using the refraction formula, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$, applied at the second curved surface, we get

$$\frac{1}{v_f}-\left\{\frac{(2n+1)\mu-1}{R}\right\}=\frac{1-\mu}{-R}$$

$$\Rightarrow \quad v_f=\frac{R}{2(\mu n+\mu-1)}$$

24.

$$\frac{1}{f_1}=\frac{2\left(\frac{\mu_2}{\mu_1}\right)}{R_2}-\frac{2\left(\frac{\mu_2}{\mu_1}-1\right)}{R_1}$$

$$\Rightarrow \quad \frac{1}{f_1}=\frac{2\mu}{\infty}-\frac{2(\mu-1)}{R}$$

$$\Rightarrow \quad f_1=\frac{-R}{2(\mu-1)}$$

So, the system behaves as a concave mirror of focal length $\frac{R}{2(\mu-1)}$. The object will coincide with image when the object is placed at centre of curvature. So, we get

$$x_1 = 2|f_1| = \frac{R}{\mu-1} \qquad \ldots(1)$$

In the second case, $\frac{1}{f_2}=\frac{-2\mu}{R}$

$$\Rightarrow \quad f_2=-\frac{R}{2\mu}$$

CHAPTER 1

$$\Rightarrow \quad x_2 = 2|f_2| = \frac{R}{\mu} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$\mu = \frac{x_1}{x_1 - x_2}$$

and $R = \dfrac{x_1 x_2}{x_1 - x_2}$

Now, according to Lens Maker's Formula, we have

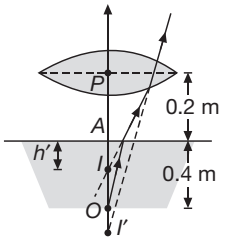
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R}\right) = \frac{\mu - 1}{R} = \frac{1}{x_1} \quad \{\text{from equation (1)}\}$$

$$\Rightarrow \quad f = x_1$$

25. The apparent position of the object O from the surface of water is

$$h' = \frac{h}{\mu} = \frac{0.4}{\frac{4}{3}}$$

$$\Rightarrow \quad h' = 0.3$$



The distance $PI = 0.3 + 0.2 = 0.5$ m
For convex lens, we use

$u = -0.5$ m, $f = +3$ m

By lens formula,

$\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$, we have

$$\frac{1}{v} - \frac{1}{-0.5} = \frac{1}{3}$$

$$\Rightarrow \quad v = -0.6 \text{ m}$$

26. Focal length of an equiconvex lens is given as

$$f = \frac{R}{2(\mu - 1)}$$

For lenses A and B, we use

$$\frac{R_A}{2(\mu_A - 1)} = \frac{R_B}{2(\mu_B - 1)}$$

$$\Rightarrow \quad \frac{\mu_A - 1}{0.9} = \mu_B - 1$$

$$\Rightarrow \quad \frac{0.63}{0.9} = \mu_B - 1$$

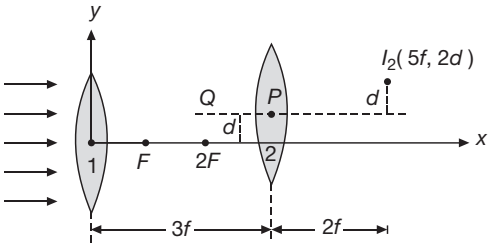
$$\Rightarrow \quad \mu_B = 1 + 0.7 = 1.7$$

27. The x-axis is the principal axis for first lens and PQ is the principal axis for second lens. First lens produces image at its focal point at a distance f on x-axis. Now for second lens, we use lens formula as

$$\frac{1}{v} - \frac{1}{-2f} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{2f}$$

$$\Rightarrow \quad v = 2f$$



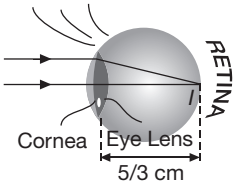
Magnification for the second lens is given as

$$m = \frac{h_i}{h_0} = +\frac{v}{u} = \frac{+2f}{-2f} = -1$$

Here negative sign indicates that the image will be formed above the principal axis PQ for the second lens. Thus, final coordinates of image are $(5f, 2d)$

Test Your Concepts-VIII (Based on Aberrations, Human Eye and Optical Instruments)

1. The human eye has a cornea, an eye lens and retina as shown in figure.



The cornea has fixed focal length whereas the eye lens has variable focal length. Since

$$P_{\text{eye}} = P_{\text{cornea}} + P_{\text{eye lens}} = 40 + 20 = 60 \text{ D}$$

If f_{eye} be the focal length of human eye (cornea + eye lens), then

$$f_{eye} = \frac{1}{P} = \frac{1}{60} \text{ m} = \frac{100}{60} \text{ cm} = \frac{5}{3} \text{ cm}$$

Therefore, the distance between eye lens and retina is $\frac{5}{3}$ cm.

So, for rays coming from infinity, the eye will make the image at retina i.e. at $\frac{5}{3}$ cm from the lens.

For object lying at distance of distinct vision i.e. $D = 25$ cm, we have $u = -25$ cm, $v = \frac{5}{3}$ cm

Applying lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{f'} = \frac{1}{5/3} - \frac{1}{(-25)} = \frac{3}{5} + \frac{1}{25} = \frac{16}{25}$$

$$\Rightarrow \quad f' = \frac{25}{16} \text{ cm}$$

$$\Rightarrow \quad P' = \frac{16}{25} \times 100 = 64 \text{ D}$$

Since $P' = P_{\text{cornea}} + P'_{\text{eye lens}}$

$$\Rightarrow \quad 64 = 40 + P'_{\text{eye lens}}$$

$$\Rightarrow \quad P'_{\text{eye lens}} = 24 \text{ D}$$

Therefore, power of accommodation of eye lens is between 20 D and 24 D.

Hence the power of accommodation of human eye (cornea + eye lens) is between 60 D (when object is at infinity) and 64 D (when object is at 25 cm)

2. Chromatic aberration is given by

$$\Delta f = f_r - f_b = (1.0 - 0.968) \text{ m} = 0.032 \text{ m}$$

Since dispersive power of material of lens is given by

$$\omega = \frac{f_r - f_b}{f}, \text{ where } f = \sqrt{f_r f_b}$$

$$\Rightarrow \quad \omega = \frac{f_r - f_b}{\sqrt{f_r f_b}} = \frac{0.032}{\sqrt{1 \times 0.968}} = 0.0325$$

3. If f_e be the accommodate focal length of the eye-lenses for 2 m, then by lens formula, we have

$$\frac{1}{v} - \frac{1}{(-2)} = \frac{1}{f_e} \qquad \ldots(1)$$

where v is the distance between eye lens and retina.

If f be the focal length of the correcting lens for seeing distant objects then by lens formula for combination, we have

$$\frac{1}{v} - \frac{1}{\infty} = \frac{1}{f_e} + \frac{1}{f} \qquad \ldots(2)$$

Subtracting Equation (1) from Equation (2), we get

$$f = -2 \text{ m}$$

Thus, power of the lens required is

$$P = \frac{1}{f} = \frac{1}{(-2)} = -0.5 \text{ dipotre}$$

4. (i) We are given that the focal length of the lens is $f = 10$ cm and distance between the screen and lens is $v = 500$ cm.

Since, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{u} = \frac{1}{500} - \frac{1}{10} = -\frac{49}{500}$$

$$\Rightarrow \quad u = -\frac{500}{49} \text{ cm}$$

The linear magnification is given by

$$m = \frac{\text{Size of Image}}{\text{Size of Object}} = \frac{v}{u}$$

$$\Rightarrow \quad \text{Image size} = \frac{v}{u} \times \text{Object size}$$

$$\Rightarrow \quad \text{Image size} = \frac{500 \times 49}{500} \times 2 = 98 \text{ cm}$$

$$\Rightarrow \quad \text{Size of the slide image} = 98 \text{ cm} \times 98 \text{ cm}$$

(ii) If the illuminating power of the source is P, then Illumination of slide is $\frac{P}{2 \times 2}$

So, Illumination of slide image picture is $\frac{P}{98 \times 98}$

$$\Rightarrow \quad \frac{\text{Illumination of slide}}{\text{Illumination of picture}} = \frac{P}{2 \times 2} \times \frac{98 \times 98}{P}$$

$$\Rightarrow \quad \frac{\text{Illumination of slide}}{\text{Illumination of picture}} = 2401$$

5. Since, $M = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$

In first case, $M_1 = \frac{(v_o)_1}{u_o}\left(1 + \frac{D}{f_e}\right)$

where $f_e = 0.03$ m $= 3$ cm and $D = 25$ cm

$$\Rightarrow \quad M_1 = \frac{(v_o)_1}{u_o}\left(1 + \frac{25}{3}\right)$$

$$\Rightarrow \quad M_1 = \frac{(v_o)_1}{u_o} \times \frac{28}{3}$$

CHAPTER 1

Similarly in second case

$$M_2 = \frac{(v_o)_2}{u_o}\left(1+\frac{25}{4}\right)$$

$$\Rightarrow \quad M_2 = \frac{(v_o)_2}{u_o}\times\frac{29}{4}$$

According to the problem $M_1 = M_2$

$$\Rightarrow \quad \frac{(v_o)_1}{u_o}\times\frac{28}{3} = \frac{(v_o)_2}{u_o}\times\frac{29}{4}$$

$$\Rightarrow \quad (v_o)_2 = \frac{112}{87}(v_o)_1$$

Let the distance of the object from eyepiece be x in the first arrangement. Then by lens formula for eyepiece, we get

$$-\frac{1}{25}+\frac{1}{x}=\frac{1}{3}$$

$$\Rightarrow \quad x = \frac{75}{28}\text{ cm}$$

$$\Rightarrow \quad (v_o)_1 = 20 - x = 20 - \frac{75}{28} = \frac{485}{28}\text{ cm}$$

and $(v_o)_2 = \frac{112}{87}\times\frac{485}{28} = \frac{1940}{87}$ cm

Let the distance of the object from eyepiece by y in the second arrangement. Then by lens formula, we get

$$-\frac{1}{25}+\frac{1}{y}=\frac{1}{4}$$

$$\Rightarrow \quad y = \frac{100}{29}$$

So, the distance between lenses in the second case is

$$L = (v_o)_2 + y$$

$$\Rightarrow \quad L = \frac{1940}{87}+\frac{100}{29} = \frac{2240}{87} = 25.75\text{ cm} = 0.2575\text{ m}$$

6. Let v_0 be the distance of the image formed by the objective alone so by lens formula for objective, we use

$$\frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0}$$

$$\Rightarrow \quad \frac{1}{v_0}=\frac{1}{u_0}+\frac{1}{f_0}$$

$$\Rightarrow \quad \frac{1}{v_0}=-\frac{1}{3}+\frac{1}{2}=\frac{1}{6}$$

$$\Rightarrow \quad v_0 = +6\text{ cm}$$

The image is formed at 6 cm behind the objective. If f_0' be the new focal length of the objective when a lens of focal length $0.1\text{ m}(=10\text{ cm})$ is removed from it, then for combination, we can use

$$\frac{1}{f_0'}=\frac{1}{f_0}-\frac{1}{10}=\frac{1}{2}-\frac{1}{10}=\frac{2}{5}$$

$$\Rightarrow \quad f_0' = +\left(\frac{5}{2}\right)\text{ cm}$$

Let v_0' be the new distance of the image formed by the objective then by using lens formula again, we have

$$\frac{1}{v_0'}=\frac{1}{u_0}+\frac{1}{f_0'}=-\frac{1}{3}+\frac{2}{5}=\frac{1}{15}$$

$$v_0' = +15\text{ cm}$$

Thus, the image is shifted from the objective through a distance $15\text{ cm} - 6\text{ cm} = 9\text{ cm}$. So, the eyepiece should be moved away from the objective by 9 cm to refocus the image at same position.

7. Magnification of microscope is given as

$$M = \frac{v_0}{u_0}\left(\frac{D}{f_e}\right)$$

Here $v_0 = 16 - 2.5 = 13.5$ cm

and $f_0 = +4\text{ mm} = +0.4\text{ cm}$

Using the lens formula, we have

$$\frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0}$$

$$\Rightarrow \quad u_0 = -\left(\frac{54}{131}\right)\text{ cm}$$

$$\Rightarrow \quad M = -\frac{13.5}{\left(\frac{54}{131}\right)}\times\frac{25}{2.5} = -327.5$$

8. For objective, by lens formula, we have

$$\frac{1}{v_0}-\frac{1}{-1.5}=\frac{1}{+0.25}$$

$$\Rightarrow \quad \frac{1}{v_0}=\frac{1}{0.25}-\frac{1}{1.5}=4-0.6667=3.3333$$

$$\Rightarrow \quad v_0 = +0.3\text{ m}$$

For eyepiece, by lens formula we have

$$\frac{1}{-0.25}-\frac{1}{-u_e}=\frac{1}{+0.02}$$

$$\Rightarrow \quad \frac{1}{u_e}=\frac{1}{0.25}+\frac{1}{0.02}=4+50=54$$

$$\Rightarrow \quad u_e = +0.01852\text{ m}$$

The tube length of the telescope is given as

$$L = 0.3 + 0.01852 = 0.31852 \text{ m}$$

Magnification by objective is

$$m_o = \frac{v_o}{u_o} = \frac{0.3}{1.5} = 0.2$$

Magnification by eyepiece is

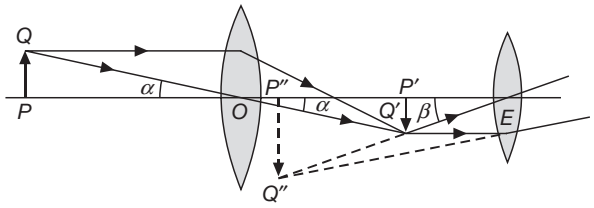
$$m_e = \frac{v_e}{u_e} = \frac{0.25}{0.001852} = 136.98$$

Total Magnification $m_T = m_1 \times m_2 = 27.39$

9. When the telescope is focussed on a distant object and the final image is also at infinity, the distance between lenses

$$L = f_0 + f_e = 50 + 5 = 55 \text{ cm}$$

When it is turned on a nearer object, the image formed by the objective will not be at its focus, but a little away from it. The eye-piece has to be shifted exactly by the same distance through which the image is shifted from focus of objective because the final image is at the same distance without change in accommodation of eye. Figure below shows the ray diagram of this situation.



Using lens formula for the objective, we have

$$\frac{1}{v_0} - \frac{1}{-10} = \frac{1}{+0.5}$$

$$\Rightarrow \quad v_0 = +\frac{50}{95} \text{ m} = +52.63 \text{ cm}$$

The distance by which the eye-piece is to be shifted is

$$s = 52.63 - 50$$

$$\Rightarrow \quad s = 2.63 \text{ cm}$$

When the object is at a finite but large distance then the magnifying power of telescope is given as

$$M = \frac{\beta}{\alpha} = \frac{v_0}{u_e}$$

For the case of focussing the telescope in both cases, using lens formula for eye-piece, we have

$$\frac{1}{\infty} - \frac{1}{-u_e} = \frac{1}{+5}$$

$$\Rightarrow \quad u_e = 5 \text{ cm}$$

Thus, magnification of telescope for the case of focussing on object located at a distance of 10 m is

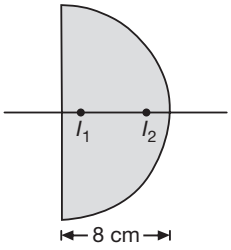
$$M = \frac{52.63}{5} = 10.52$$

Previous magnification when telescope is focussed on Moon, we know magnification is given as

$$M' = \frac{f_0}{f_e} = \frac{50}{5} = 10$$

Single Correct Choice Type Questions

1.



Distance of image from the plane surface is

$$x_1 = \frac{4}{1.6} = 2.5 \text{ cm} \qquad \left\{\because d_{\text{app}} = \frac{d_{\text{actual}}}{\mu}\right\}$$

For the curved surface, we have

$$\frac{1.6}{4} + \frac{1}{x_2} = \frac{1 - 1.6}{-8}$$

$$\Rightarrow \quad x_2 \approx -3 \text{ cm}$$

The minus sign means the image is on the side where the object lies. So,

$$I_1I_2 = (8 - 2.5 - 3) \text{ cm} = 2.5 \text{ cm}$$

Hence, the correct answer is (C).

2. Area of object $= 9 \text{ cm}^2$

Also, we know that

$$\text{Areal Magnification} = m_{ar} = \frac{A_I}{A_0} = \frac{v^2}{u^2} = \left(\frac{f}{f-u}\right)^2$$

$$\Rightarrow \quad \frac{A_I}{9} = \left[\frac{-10}{-25-(-10)}\right]^2$$

$$\Rightarrow \quad A_I = \left(\frac{2}{3}\right)^2 \times 9$$

$$\Rightarrow \quad A_I = 4 \text{ cm}^2$$

Hence, the correct answer is (C).

3. The similar thing is extended and applied here too. Here the answer fabricated by the MISCONCEPTION is 1 (but we must know this is the answer only for a Concave Mirror (or Convex Lens). For Convex Mirror we have

CHAPTER 1

$$m = \frac{f}{f-u}$$

$$\Rightarrow \quad m = \frac{f}{f-(-2f)}$$

$$\Rightarrow \quad m = \frac{1}{3}$$

Hence, the correct answer is (A).

4. Only one image will be formed by this lens system, because the optic axis of both the parts coincide. Two images would have been formed if their optic axis would had been different.
Hence, the correct answer is (D).

5. **MISCONCEPTION**
Maximum students are gripped by the misconception that when object lies at focus then image is formed at infinity. This is true but only for a Concave Mirror (Convex Lens) and is absolutely wrong when applied to Convex Mirror (Concave Lens)
How To Proceed Then?
If object placed at focus then just check out the MIRROR. If CONCAVE then image is formed at infinity and if Convex then apply $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{20}=\frac{1}{+20}$$

$$\Rightarrow \quad v = 10 \text{ cm}$$

Hence, the correct answer is (B).

6. Image will be formed at infinity, when the object is placed at focus of the lens i.e., at 20 cm from the lens. So, we have

$$\text{Shift } \Delta x = 25-20 = \left(1-\frac{1}{\mu}\right)t$$

$$\Rightarrow \quad 5 = \left(1-\frac{1}{1.5}\right)t$$

$$\Rightarrow \quad t = \frac{5\times 1.5}{0.5} = 15 \text{ cm}$$

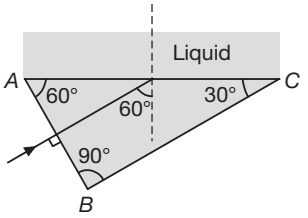
Hence, the correct answer is (C).

7. Critical angle between glass and liquid interface is

$$\sin C = \frac{\mu}{3/2} = \frac{2\mu}{3}$$

Angle of incidence at face AC is $60°$
For TIR to take place, we have

$$i > C$$



$$\Rightarrow \quad \sin(60°) > \frac{2\mu}{3}$$

$$\Rightarrow \quad \mu < \frac{3\sqrt{3}}{4}$$

Hence, the correct answer is (C).

8. Since no parallax exists between the images formed by two mirrors (convex and plane) hence the images for both coincide. But for a plane mirror an image is as far behind the mirror as the object is in front of it. Hence the image for plane mirror should be 30 cm behind it or 10 cm behind the convex mirror. So for convex mirror.

$$u = -50 \text{ cm}, \; v = +10 \text{ cm}$$

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{10}+\frac{1}{-50}=\frac{1}{f}$$

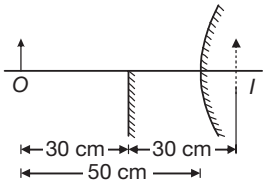
$$\Rightarrow \quad \frac{1}{f}=\frac{5-1}{50}$$

$$\Rightarrow \quad f = \frac{50}{4}$$

$$\Rightarrow \quad f = 12.5 \text{ cm}$$

Since $R = 2f$

$$\Rightarrow \quad R = 25 \text{ cm}$$



Hence, the correct answer is (D).

9. This case is the refering to the setup of displacement method experiment as product of the two magnifications is unity.

$$\text{Since } f = \frac{x}{m_1-m_2} = \frac{x}{\eta-\frac{1}{\eta}} = \frac{\eta x}{\eta^2-1}$$

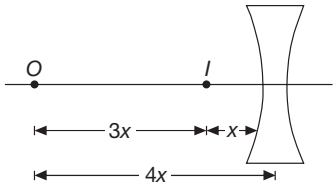
$$\Rightarrow \quad \frac{x}{f} = \frac{\eta^2-1}{\eta} \text{ for } \eta > 1$$

Hence, the correct answer is (A).

10. Concave lens forms the virtual image of a real object. So, we have

$$m = -\frac{v}{u} = \frac{1}{4}. \text{ Now, if}$$

$u = -4x$, then $v = -x$ then $3x = 10$ cm



$\Rightarrow \quad x = \dfrac{10}{3}$ cm

$\Rightarrow \quad u = -\dfrac{40}{3}$ cm

and $v = -\dfrac{10}{3}$ cm

Substituting in $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}$, we get

$$\frac{1}{f} = \frac{-3}{10} + \frac{3}{40}$$

$\Rightarrow \quad f = -\dfrac{40}{9}$

$\Rightarrow \quad f = -4.4$ cm

Hence, the correct answer is (D).

11. $|m_{\text{real}}| = 2|m_{\text{virtual}}|$

$\Rightarrow \quad m_{\text{real}} = -2m_{\text{virtual}}$

$\Rightarrow \quad \dfrac{f}{f-(-15)} = -2\dfrac{f}{f-(-20)}$

$\Rightarrow \quad f + 20 = 2f + 30$

$\Rightarrow \quad f = -10$ cm

Hence, the correct answer is (C).

12. $m = +\dfrac{1}{2}$

$\Rightarrow \quad \dfrac{1}{2} = \dfrac{20}{20-u}$

$\Rightarrow \quad 40 = 20 - u$

$\Rightarrow \quad u = -20$ cm

Hence, the correct answer is (B).

13. Focal length of mirror is independent of the refractive index of medium in which it is placed.
Hence, the correct answer is (B).

14. Since image formed is erect, hence it must be virtual. So,

$$m = -\frac{v}{u} = 3$$

Also $|u| + |v| = D$

$|u| + 3|u| = 80$

$|u| = 20$

Since object always lies on negative side. So,

$u = -20$ cm

$\Rightarrow \quad 3 = \dfrac{f}{f-(-20)}$

$\Rightarrow \quad 3f + 60 = f$

$\Rightarrow \quad f = -30$ cm

Negative sign indicates the mirror is concave.

An Advice

I would always advice you to write $|u| + |v| = D$ wherever you are given the distance between object and image as no error will creep in these because the MOD signs prevent the errors.

Hence, the correct answer is (A).

16. Since, $\delta = (\mu - 1)A = (1.5 - 1)(4) = 2°$

$\Rightarrow \quad i = \delta = 2°$

Let the mirror be rotated by an angle θ, then

$i' = (2° + \theta)$

Since, $\delta_{\text{total}} = 180°$

$\Rightarrow \quad \delta + 180° - 2i' = 180°$

$\Rightarrow \quad \delta = 2i'$

$\Rightarrow \quad 2° = 2(2 + \theta)$

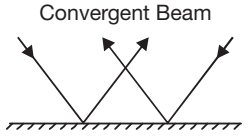
$\theta = -2°$

Here, negative sign implies that i gets decreased or $i' = 0$. i.e., light should fall normally on mirror.
Hence, the correct answer is (B).

17. The minimum length of the mirror required for the purpose is half the height of the person.
Hence, the correct answer is (B).

18. When mirror is turned, about an axis perpendicular to plane of mirror, then there will be no change in incident angle and reflected angle so angle between incident and reflected rays after rotation will be same as before.
Hence, the correct answer is (D).

19. A divergent beam appears to converge behind the mirror thus giving a virtual image. So, a convergent beam will give a real image.



Hence, the correct answer is (B).

CHAPTER 1

21. Applying $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{b}+\frac{1}{a}=\frac{1}{f}$$

$$\Rightarrow \quad f=\frac{ab}{a+b} \qquad \ldots(1)$$

Further in right triangle ACB, we have

$$AC^2+BC^2=AB^2$$

$$\Rightarrow \quad (a^2+c^2)+(b^2+c^2)=(a+b)^2$$

$$\Rightarrow \quad a^2+b^2+2c^2=a^2+b^2+2ab$$

$$\Rightarrow \quad ab=c^2$$

Substituting this in equation (1), we get

$$f=\frac{c^2}{a+b}$$

Hence, the correct answer is (C).

22. If the mirror approaches the object or the object approaches the stationary mirror with speed v then image approaches object with speed 2v.
Hence, the correct answer is (C).

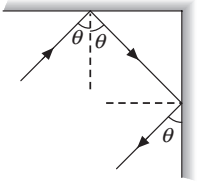
23. $m_{\text{real}}=-n=\frac{f}{f-u}$

$$\Rightarrow \quad -nf+nu=f$$

$$\Rightarrow \quad u=\left(\frac{n+1}{n}\right)f$$

Hence, the correct answer is (C).

24. The incident and the second reflected ray make the same angle θ with vertical. Hence, they are parallel for any value of θ.



Hence, the correct answer is (D).

25. $\frac{1}{n}=\frac{f}{f-u}$

$$f-u=nf$$

$$\Rightarrow \quad u=(1-n)f=-(n-1)f$$

According to the sign convention used u must always be negative.
Hence, the correct answer is (D).

26. $m_{\text{real}}=-4.5$

$$m=\frac{f}{f-u}$$

$$-4.5=\frac{f}{f-(-20)}$$

$$\Rightarrow \quad -4.5f-90=f$$

$$\Rightarrow \quad 90=-5.5f$$

$$\Rightarrow \quad f=-\frac{90}{5.5}$$

$$\Rightarrow \quad f=-\frac{900}{55}=-\frac{180}{11}\ \text{cm}$$

Negative sign with focal length implies that the mirror is a concave mirror.
Hence, the correct answer is (D).

27. $f=+20$ cm

$$u=-10\ \text{cm}$$

Since, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{10}=\frac{1}{20}$$

$$\Rightarrow \quad v=\frac{20}{3}\ \text{cm}$$

Hence, the correct answer is (B).

28. Taking leftwards as negative, we have acceleration of block AB is $a_1=-\frac{3mg}{3m+m}=-\frac{3}{4}g$

Acceleration of block CD is $a_2=\frac{2mg}{2m+m}=\frac{2g}{3}$

Acceleration of image in mirror AB is twice acceleration of the mirror

$$(a_1)_I=2\left(-\frac{3g}{4}\right)=-\frac{3}{2}g$$

Acceleration of image in mirror CD is

$$(a_2)_I=2\left(\frac{2g}{3}\right)=\frac{4g}{3}$$

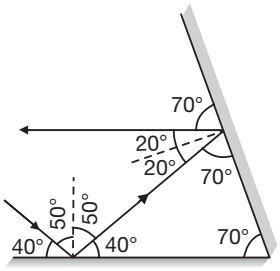
$\Rightarrow$ Acceleration of the two images w.r.t. each other is

$$a_{rel}=\frac{4g}{3}-\left(-\frac{3g}{2}\right)=\frac{17g}{6}$$

Hence, the correct answer is (D).

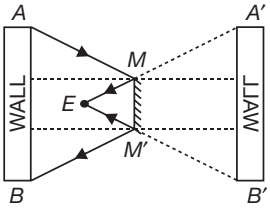
29. Covering the lower half will just make the image less bright (not blurred) as less number of rays will be reflected as compared to the previous case.
Hence, the correct answer is (A).

30.



Hence, the correct answer is (B).

31.



From symmetry we observe that length of mirror is one third of height of wall.

$$\Rightarrow \quad \ell = \frac{H}{3}$$

Hence, the correct answer is (C).

32. $n = \dfrac{360}{0} \to \infty$

Hence, the correct answer is (D).

33. $\dfrac{360}{\theta} = 6$ which is Even

$$\Rightarrow \quad n = \frac{360}{\theta} - 1 = 5$$

Hence, the correct answer is (A).

34. Ray's after reflections from two perpendicular mirrors are always parallel to incident ray irrespective of angle of incidence.
Hence, the correct answer is (B).

35. Power of concave lens must be less than that of the convex lens to form a real image. So, net power will decrease or focal length will increase. For real image v is positive, u is negative and f is positive. Applying lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$ (substituting all values with sign), we get

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

u is constant, f is increasing. So v will also increase.

Hence, the correct answer is (C).

37. Using lens formula we can see that first image after refraction through lens is obtained 10 cm behind the lens from which when light rays are reflected from plane mirror next image will be obtained at a distance 20 cm behind the plane mirror which will act as an object for the next refraction at lens so we use

$u = +30$ cm and $f = +10$ cm

Using lens formula, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

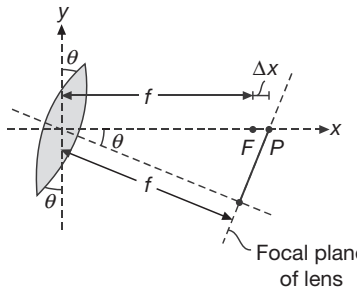
$$\Rightarrow \quad \frac{1}{v} - \frac{1}{30} = \frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{10} + \frac{1}{30} = \frac{4}{30}$$

$$\Rightarrow \quad v = +7.5 \text{ cm}$$

Thus final image is obtained 7.5 cm to the right of lens.
Hence, the correct answer is (D).

38. When the lens is tilted by θ, the image is formed at the intersection point P of focal plane of lens in tilted position and x-axis.

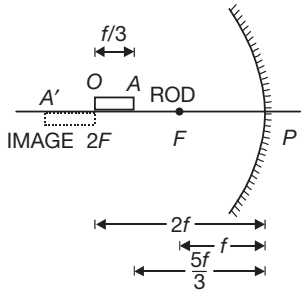


As the lens oscillates, the image shifts on x-axis in between F and P. Thus distance between two extreme positions of the oscillating image is

$$\Delta x = PF = \frac{f}{\cos\theta} - f = f(\sec\theta - 1)$$

Hence, the correct answer is (C).

39. Since an elongated image is formed and it touches one end of the rod, so the rod must lie with one end at $2F$ and other end between $2F$ and F (shown in figure).



CHAPTER 1

For end A, $u = -\frac{5f}{3}$

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\Rightarrow \quad \frac{1}{v}-\frac{3}{5f}=\frac{1}{-f}$

$\Rightarrow \quad \frac{1}{v}=\frac{3}{5f}-\frac{5}{5f}$

$\Rightarrow \quad \frac{1}{v}=-\frac{2}{5f}$

$\Rightarrow \quad v=-\frac{5f}{2}$

$\Rightarrow \quad PA'=\frac{5f}{2}$

$\Rightarrow \quad OA'=\frac{5f}{2}-2f$

$\Rightarrow \quad OA'=\frac{f}{2}$

Hence, the correct answer is (B).

40. First consider two adjacent walls (not the ceiling). If n_1 is the number of images formed due to these perpendicular walls, then

$$n_1=\frac{360}{90}-1$$

$\Rightarrow \quad n_1=3$

Now, when we consider the mirror on the ceiling then it will make a total of 4 images (one of the original object and three images of the previous arrangement) so, total images formed equals 7.
However, if the object were the observer himself then total number of images is $7-1=6$
Hence, the correct answer is (B).

41. Velocity of light is always normal to the wavefront.
Hence, the correct answer is (C).

42. Both concave and convex mirror give virtual image but a concave mirror gives a magnified virtual image (when object placed between F and P). Since the boy sees his image of diminished size, so the mirror must be convex.
Hence, the correct answer is (B).

43. Let object be placed at a distance x from mirror.

$u=-x$, $v=-(x+10)$, $f=-12$ cm

Since, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\Rightarrow \quad \frac{1}{-(x+10)}+\frac{1}{-x}=\frac{1}{-12}$

$\Rightarrow \quad x=20$ cm

$\Rightarrow \quad u=-20$ cm,

$v=-30$ cm

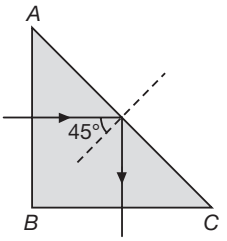
So, $m=-\frac{v}{u}=\frac{-(-30)}{-20}=-1.5$

Negative sign with magnification indicates image is real.
Hence, the correct answer is (A).

44. At face AB, the ray of light suffers no deviation, so applying Snell's Law at face AC, we get

$$\mu=\frac{1}{\sin C}=\frac{1}{\sin(45°)}$$

$\Rightarrow \quad \mu_{\min}=\sqrt{2}$



Hence, the correct answer is (B).

45. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\Rightarrow \quad v^{-1}+u^{-1}=f^{-1}=$ constant

Take derivative w.r.t. time on both sides

$$\frac{d}{dt}(v^{-1})+\frac{d}{dt}(u^{-1})=0$$

$\Rightarrow \quad (-1)v^{-2}\frac{dv}{dt}+(-1)u^{-2}\frac{du}{dt}=0$

$\Rightarrow \quad \frac{dv}{dt}=-\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$

$\Rightarrow \quad$ Image speed $=-\frac{v^2}{u^2}$(object speed)

when object moves towards mirror u decreases with passage of time and hence $\frac{du}{dt}=-9$ cms^{-1}

$\Rightarrow \quad \frac{dv}{dt}=-\left(\frac{f}{f-u}\right)^2\frac{du}{dt}$

$\Rightarrow \quad \frac{dv}{dt}=-\left(\frac{-24}{-24+60}\right)^2\frac{du}{dt}$

$\Rightarrow \quad \frac{dv}{dt}=-\left(\frac{2}{3}\right)^2\frac{du}{dt}$

$$\Rightarrow \quad \frac{dv}{dt} = -\frac{4}{9}(-9)$$

$$\Rightarrow \quad \frac{dv}{dt} = +4 \text{ cms}^{-1}$$

Positive value of $\frac{dv}{dt}$ indicates that v increases with the passage of time i.e. image must be going away from mirror.

Hence, the correct answer is (C).

46. $A = 60°$ for equilateral prism.

$$i = e = \frac{3}{4}A$$

Since, $i + e = A + D$

$$\Rightarrow \quad 2i = A + D$$

$$\Rightarrow \quad 2\left(\frac{3}{4}A\right) = A + D$$

$$\Rightarrow \quad D = \frac{A}{2} = 30°$$

Hence, the correct answer is (D).

47. As the object moves from infinity to centre of curvature, the distance between object and image reduces from infinity to zero.
When the object moves from centre of curvature to focus, the distance between object and image increases from zero to infinity.
When the object moves from focus to pole, the distance between object and its image reduces from infinity to zero. Hence the distance between object and its image shall be 40 cm three times.
Hence, the correct answer is (C).

48. Since, $u_{ab} > u_{cd}$

$$\Rightarrow \quad v_{ab} < v_{cd} \qquad \left\{\because \frac{1}{v} + \frac{1}{u} = \frac{1}{f}\right\}$$

Since, $m = -\frac{v}{u}$

$$\Rightarrow \quad |m_{ab}| < |m_{cd}|$$

Hence, the correct answer is (B).

49. $\mu = \frac{c}{v}$

$$\Rightarrow \quad 2 = \frac{3 \times 10^8}{v}$$

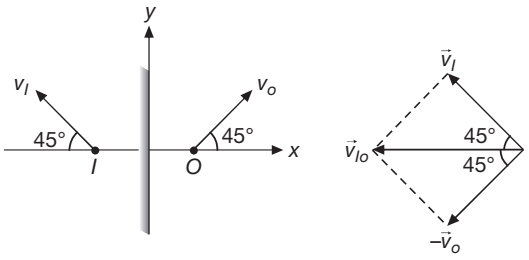
$$\Rightarrow \quad v = 1.5 \times 10^8 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = 1.5 \times 10^{10} \text{ cms}^{-1}$$

Hence, the correct answer is (D).

50. $|\vec{v}_O| = |\vec{v}_I| = \sqrt{(2)^2 + (2)^2} = 2\sqrt{2} \text{ ms}^{-1}$

Relative velocity of image with respect to object is in negative x-direction as shown in figure.



Hence, the correct answer is (C).

51. $i = 2r$

Since, $n = \frac{\sin i}{\sin r}$

$$\Rightarrow \quad n = \frac{\sin i}{\sin\left(\frac{i}{2}\right)} = \frac{2\sin\left(\frac{i}{2}\right)\cos\left(\frac{i}{2}\right)}{\sin\left(\frac{i}{2}\right)}$$

$$\left\{\because 2\sin\left(\frac{i}{2}\right)\cos\left(\frac{i}{2}\right) = \sin i\right\}$$

$$\Rightarrow \quad n = 2\cos\left(\frac{i}{2}\right)$$

$$\Rightarrow \quad \cos\left(\frac{i}{2}\right) = \frac{n}{2}$$

$$\Rightarrow \quad i = 2\cos^{-1}\left(\frac{n}{2}\right)$$

Hence, the correct answer is (C).

52. The source cannot be seen if angle of incidence in the denser medium is greater than the critical angle.
Since,

$$r = \frac{d}{\sqrt{\mu^2 - 1}} = \frac{h}{\sqrt{\mu^2 - 1}}$$

$$\Rightarrow \quad h = (1)\sqrt{\left(\frac{5}{3}\right)^2 - 1}$$

$$\Rightarrow \quad h = \frac{4}{3} \text{ cm}$$

Hence, the correct answer is (B).

53. $\sin C = \frac{1}{\mu}$

TIR will take place at AC if $i > C$ i.e. $45^o > C$ for

CHAPTER 1

Red Colour

$\mu_R = 1.39$

$\Rightarrow \quad \sin C_R = \frac{1}{1.39} > \frac{1}{1.41} = \sin 45$

$\Rightarrow \quad \sin C_R > \sin 45$

$\Rightarrow \quad C_R > 45^o$ (No TIR will take place)

Green Colour

$\mu_G = 1.44$

$\Rightarrow \quad \sin C_G = \frac{1}{1.44} < \frac{1}{1.41} = \sin 45$

$\Rightarrow \quad \sin C_G < \sin 45$

$\Rightarrow \quad C_G < 45°$ (TIR will take place for Green Colour)

Blue Colour

$\mu_B = 1.47$

$\Rightarrow \quad \sin C_B = \frac{1}{1.47} < \frac{1}{1.41} = \sin 45$

$\Rightarrow \quad \sin C_B < \sin 45$

$\Rightarrow \quad C_B < 45°$ (TIR will take place for Blue Colour)

So, Red is separated from Green and Blue.
Hence, the correct answer is (A).

54. Equivalent focal length of the system of three lenses in contact is given as

$$\frac{1}{f_{eq}} = \frac{1}{f_G} + \frac{1}{f_W} + \frac{1}{f_G} \qquad \ldots(1)$$

where focal length of glass lenses is given as

$$f_G = \frac{R}{\mu - 1} = 2R$$

$\Rightarrow \quad R = 5$ cm

Focal length of water lens is given as

$$f_W = \frac{R}{2(\mu - 1)} = \frac{3R}{2} = 7.5 \text{ cm}$$

Now from Equation (1), we have

$$\frac{1}{f_{eq}} = 2\left(\frac{1}{10}\right) + \left(-\frac{2}{15}\right) = \frac{1}{15}$$

Equivalent optical power of the given system is given as

$$P_e = \frac{1}{15} \times 100 = 6.67 \text{ D}$$

Hence, the correct answer is (A).

56. $\sin C = \frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}} = \frac{v_{\text{denser}}}{v_{\text{rarer}}}$

$\Rightarrow \quad \sin\theta = \frac{v_{\text{denser}}}{v_{\text{rarer}}} = \frac{v_A}{v_B}$

$\Rightarrow \quad \sin\theta = \frac{v}{v_B}$

$\Rightarrow \quad v_B = \frac{v}{\sin\theta}$

Hence, the correct answer is (A).

57. Since, $\mu v = \text{constant}$

$\Rightarrow \quad \mu_g v_g = \mu_w v_w$

$\Rightarrow \quad \frac{3}{2}(2 \times 10^8) = \frac{4}{3} v_w$

$\Rightarrow \quad v_w = 2.25 \times 10^8 \text{ ms}^{-1}$

Hence, the correct answer is (C).

58. After two reflections from two mirrors placed at right angles, the emergent ray will always be parallel to the incident ray for any value of i.
Hence, the correct answer is (D).

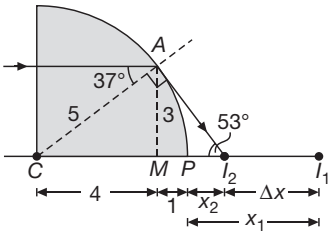
59. The rays just above x-axis (i.e. close to principal axis) are paraxial and after refraction at curved surface intersect the x axis at point I_1 which is at a distance x_1 from P.

Since $\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$

where $u \to \infty$, $v = +x_1$, $R = 5$ cm

$\Rightarrow \quad \frac{1}{x_1} = \frac{\mu - 1}{R}$

$\Rightarrow \quad x_1 = \frac{R}{\mu - 1} = \frac{15}{2}$ cm



The critical angle for air-glass interface is

$$C = \sin^{-1}\left(\frac{3}{5}\right) = 37° .$$

The rays above the ray incident on curved surface at $i = 37°$ will suffer total internal reflection and will not be considered.

The light ray which incident on curved surface at $C = 37°$, after refraction, intersects curved surface at point I_2, a distance x_2 from P, the location of which can be obtained from ΔI_2AM. Since

$$\frac{3}{1+x_2} = \tan(53°) = \frac{4}{3}$$

$$\Rightarrow \quad x_2 = \frac{5}{4}$$

So, required width of region I_1I_2 is

$$\Delta x = x_1 - x_2 = \frac{15}{2} - \frac{5}{4} = \frac{25}{4} \text{ cm}$$

Hence, the correct answer is (D).

60. Since light has to travel from denser to raser medium so, it must be made incident in the denser medium at an angle less than critical angle.
Hence, the correct answer is (C).

61. $C \propto \lambda$

Since $\lambda_{Red} > \lambda_{Violet}$

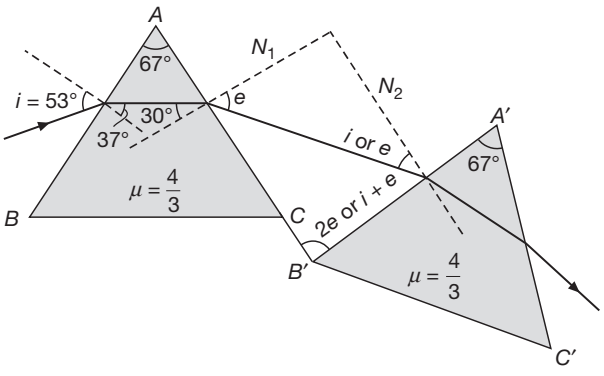
$$\Rightarrow \quad C_{Red} > C_{Violet}$$

Hence, the correct answer is (D).

63. If the angle of emergence for the first prism is e, then applying Snell's law, we get

$$e = \sin^{-1}\left(\frac{2}{3}\right)$$

For net deviation to be double, the incident ray on side $A'B'$ of second prism should make angles i or e with normal.



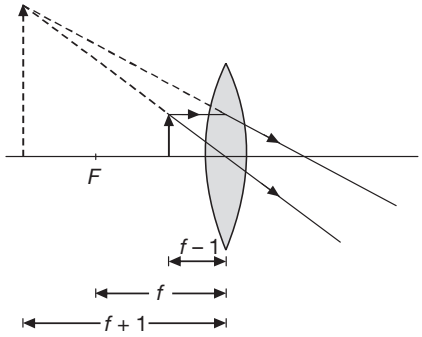
Thus the angle between CA and $A'B'$ will be $2e$ or $(i+e)$.

Hence, the correct answer is (A).

64. According to the problem, we have, $u = -(f-1)$

$$v = -(f+1)$$

$$f = +f$$



Applying $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{-(f+1)} + \frac{1}{(f-1)} = \frac{1}{f}$$

$$\Rightarrow \quad f^2 - 2f - 1 = 0$$

$$\Rightarrow \quad f = (\sqrt{2}+1) \text{ cm}$$

Hence, the correct answer is (B).

66. Apparent shift in the object O due to three slabs S_1, S_2 and S_3 with respect to the medium of $\mu = \frac{4}{3}$ is given by

$$\text{Shift} = 45\left(1 - \frac{1}{3/2 \,/\, 4/3}\right) + 24\left(1 - \frac{1}{1 \,/\, 4/3}\right) + 54\left(1 - \frac{1}{3/2 \,/\, 4/3}\right)$$

$$\Rightarrow \quad \text{Shift} = 45\left(1 - \frac{8}{9}\right) + 24\left(1 - \frac{4}{3}\right) + 54\left(1 - \frac{8}{9}\right)$$

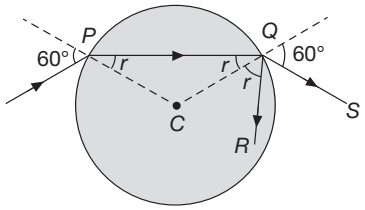
$$\Rightarrow \quad \text{Shift} = 5 + (-8) + 6 = 3 \text{ cm}$$

$\Rightarrow$ Object distance $u = 150$ cm $= R.O.C.$ of mirror

So, image will be formed on the object itself because light rays fall on mirror normally (as the object appears to be located at its centre of curvature).
Hence, the correct answer is (B).

67. Applying Snell's Law $\mu = \frac{\sin i}{\sin r}$, we get

$$\Rightarrow \quad \sin r = \frac{\sin i}{\mu}$$



$$\Rightarrow \quad \sin r = \frac{\sin 60°}{\sqrt{3}} = \frac{1}{2}$$

CHAPTER 1

$\Rightarrow \quad r = 30°$

Since, $PC = QC$

$\Rightarrow \quad \angle CPQ = \angle PQC = \angle r = 30°$

Angle between reflected ray QR and refracted ray QS at the other face is $180° - r - 60° = 90°$ $\quad \{\because r = 30°\}$

Hence, the correct answer is (D).

68. According to Snell's Law

$$\frac{\sin i}{\sin r} = \frac{\mu_w}{\mu_a}$$

$$\Rightarrow \quad \sin r = \frac{1}{\mu_w}\sin i$$

$$\Rightarrow \quad \sin r = \frac{1}{4/3}\sin 60$$

$$\Rightarrow \quad r = \sin^{-1}\left(\frac{3\sqrt{3}}{8}\right)$$

Hence, the correct answer is (A).

69. $\angle DBM = 45°$

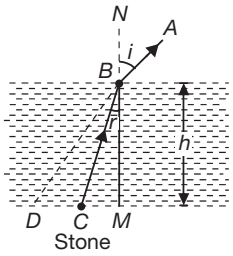
$\Rightarrow \quad DM = h\tan 45° = h = 32$ cm

$$\mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 45°}{4/3} = \frac{3}{4\sqrt{2}}$$

$$\Rightarrow \quad \tan r = \frac{\sin r}{\cos r} = \frac{\sin r}{\sqrt{1-\sin^2 r}}$$

$$\Rightarrow \quad \tan r = \frac{\frac{3}{4\sqrt{2}}}{\sqrt{1-\frac{9}{32}}} = \frac{3}{\sqrt{23}}$$



$$\Rightarrow \quad CM = h\tan r = 32 \times \frac{3}{\sqrt{23}} \simeq 20 \text{ cm}$$

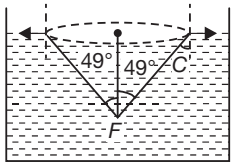
$\Rightarrow \quad CD = DM - CM = 32 - 20 = 12$ cm

Hence, the correct answer is (B).

70. $\sin C = \dfrac{1}{\mu_w} = \dfrac{3}{4}$

$\Rightarrow \quad C = 49°$

Apex angle of cone is $2C = 98°$

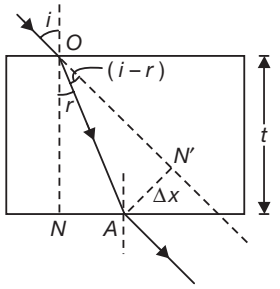


Hence, the correct answer is (C).

71. In $\Delta OAN'$

$$\sin(i-r) = \frac{\Delta x}{OA}$$

$$\Rightarrow \quad \Delta x = OA\sin(i-r)$$



In ΔOAN

$$\cos r = \frac{ON}{OA}$$

$$\Rightarrow \quad OA = \frac{t}{\cos r}$$

$$\Rightarrow \quad \Delta x = \frac{t\sin(i-r)}{\cos r}$$

Hence, the correct answer is (D).

74. For grazing incidence and emergence, we have

$$i = e = 90°, \; r_1 = r_2 = \frac{A}{2} = 30°$$

According to Snell's Law, we get

$$\mu = \frac{\sin i}{\sin r_1} = 2$$

Hence, the correct answer is (C).

75. $\sin C = \dfrac{\mu_r}{\mu_d} = \dfrac{4/3}{3/2}$

$$\Rightarrow \quad C = \sin^{-1}\left(\frac{8}{9}\right)$$

and light must go from denser to rarer medium.

Hence, the correct answer is (A).

76. According to Snell's Law

$\mu\lambda = \text{constant}$

Since $\lambda_{\text{Red}} > \lambda_{\text{Violet}}$

$\Rightarrow \quad \mu_{\text{Red}} < \mu_{\text{Violet}}$

Hence, the correct answer is (B).

78. Critical angle $C = \sin^{-1}\left(\dfrac{1}{\mu}\right)$

$$\Rightarrow \quad C = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

When $A > 2C$, the ray does not emerge from the prism. So, maximum refracting angle can be $60°$.
Hence, the correct answer is (C).

80. According to Snell's Law

$$\mu v = \text{constant}$$

Hence, the correct answer is (A).

81. This is a case of total internal reflection at B. For TIR at Q, we use

$$\theta > C, \text{ where } C = \sin^{-1}\left(\frac{1}{\mu}\right)$$

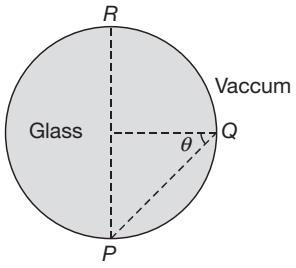
$$\Rightarrow \quad \frac{1}{\mu} < \sin\theta$$

As at point Q angle of incidence of light ray is maximum, we use

$$\frac{1}{\mu} < \sin(45°)$$

$$\Rightarrow \quad \mu > \frac{1}{\sin(45°)}$$

$$\Rightarrow \quad \mu > \sqrt{2}$$



Since $v = \frac{c}{\mu}$

$$\Rightarrow \quad \mu = \frac{c}{v}$$

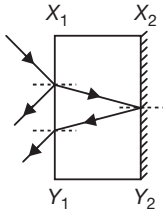
$$\Rightarrow \quad \frac{c}{v} > \sqrt{2}$$

$$\Rightarrow \quad v < \frac{c}{\sqrt{2}} = \frac{3\times10^8}{\sqrt{2}}$$

$$\Rightarrow \quad v < 2.1\times10^8 \text{ ms}^{-1}$$

So, only (B) is not possible.
Hence, the correct answer is (B).

82. This happens due to multiple refractions and reflections. The first image is formed due to reflection at X_1Y_1 and is fainter. The second image is formed due to reflection at X_2Y_2 and is brightest all other images formed further are faint.



Hence, the correct answer is (B).

83. In both A and B, the refracted ray is parallel to the base of prism.
Hence, the correct answer is (C).

85. By refraction formula, we use

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}$$

where $u = -x$, $R_1 = +10$ cm, $\mu_1 = 1$ and $\mu_2 = \frac{3}{2}$

$$\Rightarrow \quad \frac{\mu_2}{\infty} - \frac{1}{-x} = \frac{1.5-1}{10}$$

$$\Rightarrow \quad x = -20 \text{ cm}$$

Since the second surface is flat so parallel rays after first refraction will not suffer any deviation when falling normally on the flat surface.
Hence, the correct answer is (C).

87. $$\mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \quad \sqrt{2}\sin 30 = \sin\left(\frac{A+D_m}{2}\right)$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \sin\left(\frac{A+D_m}{2}\right)$$

$$\Rightarrow \quad \frac{60+D_m}{2} = 45$$

$$\Rightarrow \quad \frac{D_m}{2} = 15$$

$$\Rightarrow \quad D_m = 30°$$

Further we know that at minimum deviation $i = \frac{A+D_m}{2}$

$$\Rightarrow \quad i = 45°$$

Hence, the correct answer is (A).

88. Normal Emergence implies

$$e = 0$$

Since $i + e = A + D$

CHAPTER 1

For prism with small A,

$$D=(\mu-1)A$$

$$\Rightarrow \quad i=A+(\mu-1)A$$

$$\Rightarrow \quad i\cong \mu A$$

Hence, the correct answer is (C).

89. Since, ${}^{1}\mu_2\times{}^{2}\mu_3\times{}^{3}\mu_4\times{}^{4}\mu_1=1$

$$\Rightarrow \quad {}^{4}\mu_3\times{}^{3}\mu_2\times{}^{2}\mu_1\times{}^{1}\mu_4=1$$

$$\Rightarrow \quad {}^{4}\mu_3\times{}^{3}\mu_2\times{}^{2}\mu_1=\frac{1}{{}^{1}\mu_4}$$

Hence, the correct answer is (C).

90. Let thickness of slab be t and real depth from first side be x. Then

$$\mu=\frac{x}{6} \qquad \text{(when viewed from first side)}$$

$$\mu=\frac{t-x}{4} \qquad \text{(when viewed from second side)}$$

$$\Rightarrow \quad x=6\left(\frac{3}{2}\right) \qquad \left\{\because \mu=\frac{3}{2}\right\}$$

$$\Rightarrow \quad x=9 \text{ cm}$$

$$\Rightarrow \quad \frac{3}{2}=\frac{t-9}{4}$$

$$\Rightarrow \quad t=15 \text{ cm}$$

Hence, the correct answer is (C).

91. Since, $\mu=\dfrac{\sin\left(\dfrac{A+D_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

$$\Rightarrow \quad \cot\left(\frac{A}{2}\right)=\frac{\sin\left(\dfrac{A+D_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$$

$$\Rightarrow \quad \cos\left(\frac{A}{2}\right)=\sin\left(\frac{A+D_m}{2}\right) \qquad \left\{\because \cot\left(\frac{A}{2}\right)=\frac{\cos\left(\dfrac{A}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}\right\}$$

$$\Rightarrow \quad \sin\left(\frac{\pi}{2}-\frac{A}{2}\right)=\sin\left(\frac{A}{2}+\frac{D_m}{2}\right)$$

$$\Rightarrow \quad D_m=\pi-2A=180-2A$$

Hence, the correct answer is (D).

92. $i+e=A+D$

$$\Rightarrow \quad 60+e=30+30$$

$$\Rightarrow \quad e=0^\circ$$

Hence, the correct answer is (A).

93. $\sin(45)=\dfrac{1}{\mu}$

$$\Rightarrow \quad \mu=\sqrt{2}$$

Hence, the correct answer is (B).

94. $\sin C_1=\dfrac{\mu_r}{\mu_d}=\dfrac{\mu_a}{\mu_g}=\dfrac{1}{\mu_g}$

$$\sin C_2=\frac{\mu_w}{\mu_g}>\sin C_1$$

$$C_1<C_2$$

Hence, the correct answer is (B).

95. Since $r=\dfrac{d}{\sqrt{\mu^2-1}}$

$$\Rightarrow \quad r=\frac{4}{\sqrt{\left(\dfrac{5}{3}\right)^2-1}}$$

$$\Rightarrow \quad r=\frac{4\times 3}{4}=3 \text{ m}$$

Diameter $=2r=6$ m

Hence, the correct answer is (B).

96. Apparent Depth $=\dfrac{h}{n}$

Hence, the correct answer is (A).

98. From mirror formula, we get

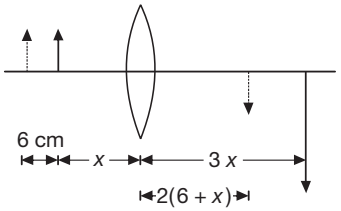
$$\frac{1}{3x}+\frac{1}{x}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{4}{3x}=\frac{1}{f} \qquad \ldots(1)$$

From mirror formula, we get

$$\frac{1}{2(6+x)}+\frac{1}{6+x}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{3}{2(6+x)}=\frac{1}{f} \qquad \ldots(2)$$



Equations (1) and (2) give

$$\frac{4}{3x}=\frac{3}{2(6+x)}$$

$\Rightarrow \quad 9x = 48 + 8x$

$\Rightarrow \quad x = 48 \text{ cm}$

Shift of screen $= 3x - 2(6 + x)$

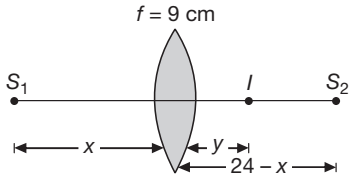
Shift of screen $= 3 \times 48 - 2(6 + 48) = 36 \text{ cm}$

Hence, the correct answer is (C).

99. Apparent Depth $= \dfrac{d}{2\mu_1} + \dfrac{d}{2\mu_2}$

Hence, the correct answer is (D).

100. In this case, one of the image will be real and the other will be virtual. Let us assume that image of S_1 is real and that of S_2 is virtual. Then, applying $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$



for S_1, we get

$$\frac{1}{y} + \frac{1}{x} = \frac{1}{9} \qquad \ldots(1)$$

for S_2, we get

$$-\frac{1}{y} + \frac{1}{24 - x} = \frac{1}{9} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$x = 6 \text{ cm}$

Hence, the correct answer is (A).

Conceptual Note(s)

This question may have following answer 6 cm from S_1 or 18 cm from S_2 and 18 cm from S_1 or 6 cm from S_2.

101. Optical path length $= nt$

So time taken $= \dfrac{nt}{c}$

Hence, the correct answer is (C).

102. To a fish the outer world is seen in a circle of radius $\dfrac{d}{\sqrt{\mu^2 - 1}}$, d is the depth at which the fish swims.

$$\Rightarrow \quad r = \frac{12}{\sqrt{\frac{16}{9} - 1}}$$

$$\Rightarrow \quad r = \frac{36}{\sqrt{7}} \text{ cm}$$

Hence, the correct answer is (C).

103. For the diver

$$\sin C = \frac{1}{\mu_w} = \frac{1}{(4/3)} \quad \left(\text{Remember that } \mu_{\text{water}} = \frac{4}{3}\right)$$

$\Rightarrow \quad C = 49^\circ$ with vertical

$\Rightarrow \quad \theta = 90 - 49^\circ = 41^\circ$ with the horizon.

Hence, the correct answer is (A).

104. Using refraction formula, we have

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

where, $v = 2R$, $u \to \infty$, $\mu_1 = 1$ and $\mu_2 = \mu$

$$\Rightarrow \quad \frac{\mu}{2R} - \frac{1}{\infty} = \frac{\mu - 1}{R}$$

$\Rightarrow \quad \mu = 2\mu - 2$

$\Rightarrow \quad \mu = 2$

Hence, the correct answer is (D).

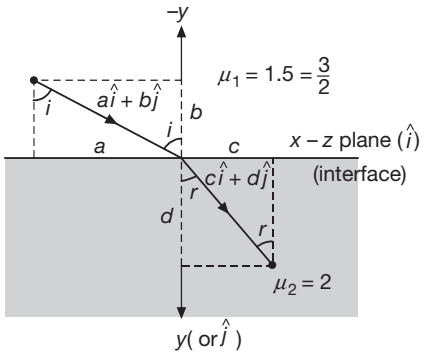
105. $\sin C = \dfrac{\mu_{\text{rarer}}}{\mu_{\text{denser}}} = \dfrac{\mu_w}{\mu_g}$

$$\Rightarrow \quad \sin C = \frac{4/3}{5/3} = \frac{4}{5}$$

Hence, the correct answer is (C).

106. Applying Snell's Law at the interface separating two media, we get

$\mu_1 \sin i = \mu_2 \sin r$



From the figure, we get

$$\left(\frac{3}{2}\right)\left(\frac{a}{\sqrt{a^2 + b^2}}\right) = 2\left(\frac{c}{\sqrt{c^2 + d^2}}\right) \qquad \ldots(1)$$

Since $a\hat{i} + b\hat{j}$ and $c\hat{i} + d\hat{j}$ are unit vectors, so we get

$$\sqrt{a^2 + b^2} = \sqrt{c^2 + d^2} = 1$$

CHAPTER 1

Substituting in equation (1), we get

$$\frac{a}{c}=\frac{4}{3}$$

Hence, the correct answer is (B).

107. $\sin C=\dfrac{\mu_{\text{rarer}}}{\mu_{\text{denser}}}=\dfrac{\mu_B}{\mu_A}$

Since, $\mu_A v_A=\mu_B v_B$

$$\Rightarrow \quad \sin C=\frac{v_A}{v_B}=\frac{2}{2.4}$$

$$\Rightarrow \quad \sin C=\frac{5}{6}$$

Hence, the correct answer is (B).

108. According to Snell's Law

$$\mu v=\text{constant}$$

$$\Rightarrow \quad \mu_g v_g=\mu_\ell v_\ell$$

$$\Rightarrow \quad (1.5)\times(2\times10^8)=\mu_\ell(2.5\times10^8)$$

$$\Rightarrow \quad \mu_\ell=\frac{3}{2.5}=\frac{30}{25}=1.2$$

Hence, the correct answer is (C).

109. $\sin C=\dfrac{1}{\sqrt{2}}$

$$\Rightarrow \quad C=45^\circ$$

For $i=C$ in denser medium angle of refraction in rarer medium is 90°.

Hence, the correct answer is (D).

110. $\text{Apparent Depth}=\dfrac{\text{Real Depth}}{\mu}$

$$\Rightarrow \quad \text{Apparent Depth}=\frac{24}{4/3}$$

$$\Rightarrow \quad \text{Apparent Depth}=18\text{ cm}$$

Hence, the correct answer is (C).

111. Given that $A=\delta_m=60^\circ$

At minimum deviation, we have

$$i=\left(\frac{A+\delta_m}{2}\right)=60^\circ$$

Hence, the correct answer is (C).

112. $r+i=90$

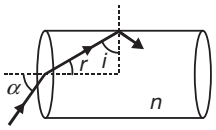
$$i=90-r$$

For ray not to emerge from curved surface

$$i>C$$

$$\Rightarrow \quad \sin i>\sin C$$

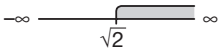
$$\Rightarrow \quad \sin(90-r)>\sin C$$



$$\Rightarrow \quad \cos r>\sin C$$

$$\Rightarrow \quad \sqrt{1-\sin^2 r}>\frac{1}{n} \qquad \left\{\because \sin C=\frac{1}{n}\right\}$$

$$\Rightarrow \quad 1-\frac{\sin^2 i}{n^2}>\frac{1}{n^2}$$



$$\Rightarrow \quad 1>\frac{1}{n^2}(1+\sin^2 i)$$

$$\Rightarrow \quad n^2>1+\sin^2 i$$

$$\Rightarrow \quad n>\sqrt{2} \qquad \{\sin i\to 1\}$$

$$\Rightarrow \quad \text{least value}=\sqrt{2}$$

Hence, the correct answer is (B).

113. $\text{Dispersive Power}=\dfrac{d\mu}{d\lambda}$

$$\Rightarrow \quad \frac{d\mu}{d\lambda}=-\frac{2B}{\lambda^3}$$

So, as B increases, dispersive power increases.

Hence, the correct answer is (B).

114. Maximum separation will be $4A$ when A is the amplitude which is given as

$$A\omega=v$$

$$\Rightarrow \quad A\sqrt{\frac{k}{m}}=v$$

$$\Rightarrow \quad A=v\sqrt{\frac{m}{k}}$$

Therefore, the maximum separation is $4v\sqrt{\dfrac{m}{k}}$

Hence, the correct answer is (D).

115. For the near end of the rod applying $\dfrac{1}{f}=\dfrac{1}{u}+\dfrac{1}{v}$

Since, here u and f are negative, so

$$|v|=\frac{uf}{u-f}$$

The farther end of the rod is at infinity, so its image will be formed at focus. Hence,

Length of the image is $\ell=|v|-f$

$$\Rightarrow \quad \ell=\frac{uf}{u-f}-f=\frac{f^2}{u-f}$$

Hence, the correct answer is (C).

116. **For A**

$$\text{Total number of waves}=\frac{(1.5)t}{\lambda} \qquad \ldots(1)$$

$$\because \quad \left(\begin{matrix}\text{Total number}\\ \text{of waves}\end{matrix}\right)=\left(\frac{\text{Optical path length}}{\text{wavelength}}\right)$$

For B and C

Total number of waves

$$= \frac{n_B\left(\frac{t}{3}\right)}{\lambda} + \frac{(1.6)\left(\frac{2t}{3}\right)}{\lambda} \quad \ldots(2)$$

Equating (1) and (2)

$\Rightarrow \quad n_B = 1.3$

Hence, the correct answer is (C).

117. Focal length of curved mirrors is independent of the refractive index of the medium in which the mirror is placed.

Hence, the correct answer is (C).

118. $n = \frac{c}{v}$

$$\Rightarrow \quad n = \frac{1/\sqrt{\mu_0\varepsilon_0}}{1/\sqrt{\mu\varepsilon}}$$

$$\Rightarrow \quad n = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$$

Hence, the correct answer is (B).

119. $\sin\theta_1 = \frac{1}{\mu_g}$ and $\sin\theta_2 = \frac{1}{\mu_w}$

Since, $\mu_g > \mu_w$, $\theta_1 < \theta_2$

The critical angle θ between glass-water interface is given by

$$\sin\theta = \frac{\mu_w}{\mu_g}$$

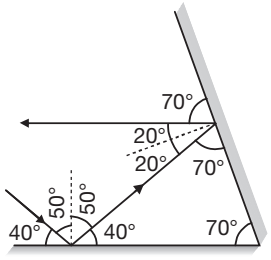
$\Rightarrow \quad \theta > \theta_2$

Hence, the correct answer is (D).

Conceptual Note(s)

Critical angle increases as the relative refractive index is decreased.

120.



Hence, the correct answer is (B).

121. $u = -20$ cm, $f = +20$ cm, $v = ?$

Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

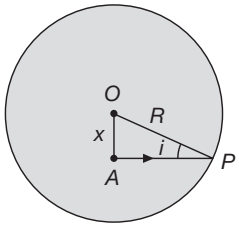
$$\Rightarrow \quad \frac{1}{v} + \frac{1}{-20} = \frac{1}{20}$$

$\Rightarrow \quad v = 10$ cm

Students generally give (A) as the answer to this problem because they have in their mind that if an object is placed at centre of curvature C, then image is also formed at C, but this is true only for the case of a convex lens or a concave mirror. So correct answer is **(B)**.

Hence, the correct answer is (B).

123. If O is the centre and A is the point source for the light ray incident at any point P on surface of sphere to angle have maximum angle of incidence i, then situation of maximum i is shown in figure.



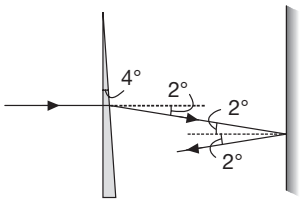
It happens when OA and AP are perpendicular, for which we have

$$i = \sin^{-1}\left(\frac{OA}{OP}\right) = \sin^{-1}\left(\frac{x}{R}\right)$$

Hence, the correct answer is (B).

124. Since, $\delta_{\text{prism}} = (\mu - 1)A = (1.5 - 1)4° = 2°$

$$\Rightarrow \quad \delta_{\text{total}} = \delta_{\text{prism}} + \delta_{\text{mirror}}$$



$$\Rightarrow \quad \delta_{\text{total}} = (\mu - 1)A + (180 - 2i)$$

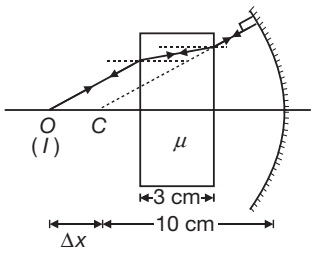
$$\Rightarrow \quad \delta_{\text{total}} = (1.5 - 1)4° + (180 - 2 \times 2°)$$

$$\Rightarrow \quad \delta_{\text{total}} = 2° + 176° = 178°$$

Hence, the correct answer is (C).

CHAPTER 1

125.



$$\Delta x = 3\left[1 - \frac{1}{\left(\frac{3}{2}\right)}\right]$$

$\Rightarrow \quad \Delta x = 1$ cm

$\Rightarrow$ Object distance $= 10 + 1 = 11$ cm

Hence, the correct answer is (C).

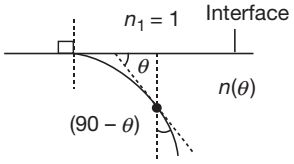
126. Apparent depth $= \frac{d}{n}$

Hence apparent separation $= \frac{2d}{n}$

Hence, the correct answer is (C).

127. According to Snell's Law for anisotropic medium

$$n(\theta)\sin(90 - \theta) = \text{constant}$$



or $\quad n_1 \sin i = n(\theta)\cos\theta$

$\Rightarrow \quad (1)\sin 90 = n(\theta)\cos\theta$

$\Rightarrow \quad n(\theta)\cos\theta = 1$

Hence, the correct answer is (B).

128. Velocity of approach of man towards the bicycle $= (u - v)$

Hence velocity of approach of image towards bicycle is $2(u - v)$.

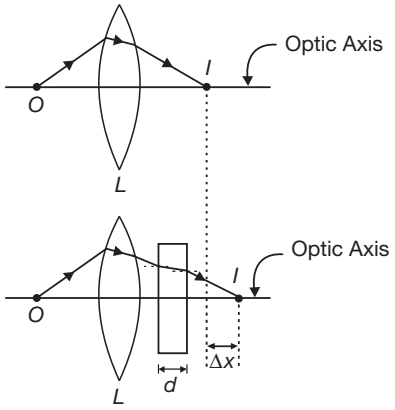
Hence, the correct answer is (D).

129. Optical path length $= \mu t$

Time taken $= \frac{\mu t}{c}$

Hence, the correct answer is (C).

130.



$$\Delta x = d\left(1 - \frac{1}{\mu}\right)$$

Hence, the correct answer is (C).

131. $\sin C = \frac{4000}{6000}$

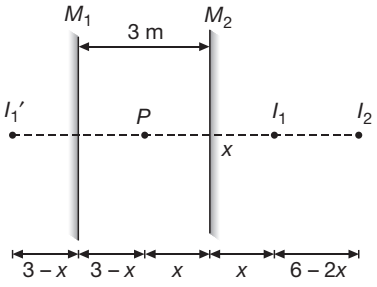
$\Rightarrow \quad C = \sin^{-1}\left(\frac{2}{3}\right)$

Hence, the correct answer is (C).

132. A virtual, erect image is obtained by using the mirror. The mirror can be both concave and convex. But a virtual image obtained by a concave mirror is always magnified and hence the mirror must be convex as we are getting diminished image.

Hence, the correct answer is (B).

133.



Distance between images I_1 and I_2 is given to be 4 m

$\Rightarrow \quad I_1 I_2 = 6 - 2x = 4$ m

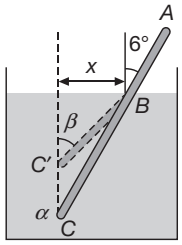
$\Rightarrow \quad x = 1$ m

Hence, the correct answer is (C).

134. Since rays after passing through the glass slab just suffer lateral displacement hence, we have angle between the emergent rays as α.

Hence, the correct answer is (B).

136. From figure, we get



$$\alpha = \frac{x}{OC} \text{ and } \beta = \frac{x}{OC'}$$

$$\Rightarrow \quad \frac{\alpha}{\beta} = \frac{OC'}{OC} = \frac{1}{\mu}$$

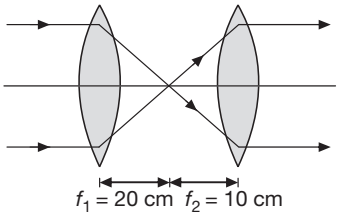
For small angles, we have

$$\beta = \mu\alpha = \left(\frac{4}{3}\right)(6°) = 8°$$

$$\Rightarrow \quad \text{Bending angle} = \beta - \alpha = 2°$$

Hence, the correct answer is (A).

137. As shown in figure the distance between the lenses should be 30 cm .



Hence, the correct answer is (A).

138. Two plano convex lens of focal length f, when combined using optical glue will give rise to a convex lens of focal length $\left(\frac{f}{2}\right)$.

i.e. size of image = size of object.

Object at 2F i.e. at a distance f from optical centre.

Hence, the correct answer is (C).

139. $f_{\text{combination}} \to \infty$

Since $\frac{1}{f_{\text{combination}}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$

$$\Rightarrow \quad 0 = \frac{1}{-10} + \frac{1}{f_2} - \frac{10}{(-10)f_2}$$

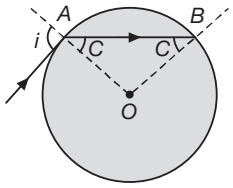
$$\Rightarrow \quad 0 = \frac{1}{-10} + \frac{1}{f_2} + \frac{1}{f_2}$$

$$\Rightarrow \quad \frac{1}{10} = \frac{2}{f_2}$$

$$\Rightarrow \quad f_2 = 20 \text{ cm}$$

Hence, the correct answer is (B).

140. Since, $\angle ABO = \angle OAB = C$



Further, by definition, we have

$$\sin C = \frac{1}{\mu} = \frac{2}{3}$$

Applying Snell's Law at A, we get

$$\frac{\sin i}{\sin C} = \frac{3}{2}$$

$$\Rightarrow \quad \sin i = \left(\frac{3}{2}\right)\sin C = \left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$$

$$\Rightarrow \quad i = 90°$$

Hence, the correct answer is (D).

141. For combination to produce dispersion without deviation.

$$(n_1 - 1)A_1 = (n_2 - 1)A_2$$

$$\Rightarrow \quad A_2 = \left(\frac{n_1 - 1}{n_2 - 1}\right)A_1$$

$$\Rightarrow \quad A_2 = \left(\frac{1.54 - 1}{1.72 - 1}\right)4°$$

$$\Rightarrow \quad A_2 = \frac{3}{4}(4°)$$

$$\Rightarrow \quad A_2 = 3°$$

Hence, the correct answer is (C).

142. $\frac{\mu}{\infty} + \frac{1}{-u} = \frac{\mu - 1}{R}$

$$\Rightarrow \quad u = -\frac{R}{\mu - 1} \text{ from the surface}$$

$$\Rightarrow \quad \text{Total distance from centre}$$

$$\Rightarrow \quad x = |u| + R = R + \frac{R}{\mu - 1}$$

CHAPTER 1

$x = R + 2R$

$x = 3R$

Hence, the correct answer is (C).

143. An air bubble in water always behaves as a concave lens and hence is always incapable to form a real image.
Hence, the correct answer is (D).

144. $m_{\text{virtual}} = m_{\text{real}}$

$\Rightarrow \quad \dfrac{f}{f+(-12)} = -\dfrac{f}{f+(-20)}$

$\Rightarrow \quad \dfrac{f}{f-12} = -\dfrac{f}{f-20}$

$\Rightarrow \quad f - 20 = -f + 12$

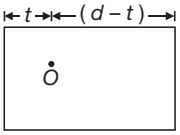
$\Rightarrow \quad 2f = 32$

$\Rightarrow \quad f = 16 \text{ cm}$

Hence, the correct answer is (B).

145. $4 = \dfrac{t}{\mu} \qquad \left\{\because \ \mu = \dfrac{\text{Real Depth}}{\text{Apparent Depth}}\right\}$

and $6 = \dfrac{d-t}{\mu}$



$\Rightarrow \quad 6\mu = d - 4\mu$

$\Rightarrow \quad d = 10\mu$

$\Rightarrow \quad d = 15 \text{ cm}$

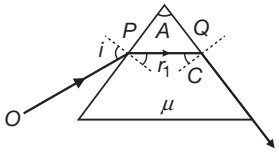
Hence, the correct answer is (C).

146. $\dfrac{P_A}{P_B} = \left(\dfrac{r_B}{r_A}\right)^2 = \dfrac{4}{9} = 0.44$

Hence, the correct answer is (A).

147. An 'achromatic combination' is made from a concave lens and a convex lens with greater power of convex lens so as to make the rays converge at a point.
Hence, the correct answer is (C).

148.



$\dfrac{\sin i}{\sin r_1} = \mu \ \text{(at } P\text{)} \qquad \ldots(1)$

and $r_1 + C = A \qquad \ldots(2)$

$\Rightarrow \quad \sin i = \mu \sin(A - C)$

$\Rightarrow \quad \sin i = \mu(\sin A \cos C - \cos A \sin C)$

$\Rightarrow \quad \sin i = \mu\left(\sin A \sqrt{1 - \dfrac{1}{\mu^2}} - \cos A \dfrac{1}{\mu}\right) \quad \left\{\because \sin C = \dfrac{1}{\mu}\right\}$

$\Rightarrow \quad i = \sin^{-1}\left\{\left(\sqrt{\mu^2 - 1}\right)\sin A - \cos A\right\}$

Hence, the correct answer is (C).

149. In first case, the apparent depth of the liquid is $(b-a)$

$\Rightarrow$ Real depth $= \mu(b-a)$

In second case, apparent depth $= (d-c)$

$\Rightarrow$ Real depth $= \mu(d-c)$

The difference in depth of liquid is

$\mu(d-c) - \mu(b-a)$

Here we use $\mu(d-c) > \mu(b-a)$

From experimental data, the difference is equal to $(d-b)$

$\Rightarrow \quad \mu(d-c) - \mu(b-a) = d - b$

$\Rightarrow \quad \mu = \dfrac{d-b}{(a+d-c-b)}$

Hence, the correct answer is (A).

150. At $u = f$ {focal length}

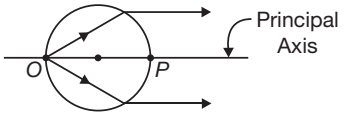
$v \to \infty$

At $u = 0$ {i.e., object is at pole}

$v = 0$ {image is also at pole}

Satisfying these two conditions, only OPTION (D) is correct.
Hence, the correct answer is (D).

151.



Considering pole at P, we have

$\dfrac{\mu}{-2R} + \dfrac{1}{\infty} = \dfrac{\mu - 1}{-R} \qquad \left\{\because \dfrac{\mu_1}{\mu} + \dfrac{\mu_2}{v} = \dfrac{\mu_2 - \mu_1}{R}\right\}$

$\Rightarrow \quad \dfrac{\mu}{2} = \mu - 1$

$\Rightarrow \quad \dfrac{\mu}{2} = 1$

$\Rightarrow \quad \mu = 2$

Hence, the correct answer is (C).

152. The first image is formed due to the reflection from concave mirror M_2.

$\Rightarrow \quad \dfrac{1}{v_1} + \dfrac{1}{(-2R)} = \dfrac{2}{-R}$

$\Rightarrow \quad \dfrac{1}{v_1} = \dfrac{1}{2R} - \dfrac{4}{2R}$

$$\Rightarrow \quad \frac{1}{v_1}=\frac{-3}{2R}$$

$$\Rightarrow \quad v_1=\frac{-2R}{3}$$

$$\Rightarrow \quad m_1=-\left(\frac{\frac{-2R}{3}}{2R}\right)=-\frac{1}{3}$$

$\Rightarrow$ Radius of image circle $=\frac{1}{3}a$, (in magnitude).

Now the second image is formed by convex mirror M_1. The second image will be formed because the image formed by the first acts as object for it.

$\Rightarrow$ Object distance $= 2R-\frac{2R}{3}=\frac{4R}{3}$.

$$\Rightarrow \quad \frac{1}{v_2}+\frac{1}{\left(-\frac{4R}{3}\right)}=\frac{1}{\left(\frac{R}{2}\right)}$$

$$\Rightarrow \quad \frac{1}{v_2}=\frac{2}{R}+\frac{3}{4R}$$

$$\Rightarrow \quad v_2=\frac{4R}{11}$$

$$\Rightarrow \quad m_2=\frac{-v_2}{u_2}=\frac{-\frac{4R}{11}}{-\frac{4R}{3}}=\frac{3}{11}$$

$$\Rightarrow \quad m_2=\frac{3}{11}$$

So, radius of second image $= a_2=\frac{3}{11}\cdot\frac{a}{3}=\frac{a}{11}$

Similarly, radius of third image is $a_3=\frac{a}{41}$

Hence, the correct answer is (C).

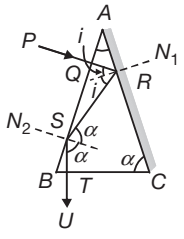
154. The two slabs will shift the image by a distance

$$\Delta x=2\left(1-\frac{1}{\mu}\right)t$$

$$\Rightarrow \quad \Delta x=2\left(1-\frac{1}{1.5}\right)(1.5)=1 \text{ cm}$$

Therefore, final image will be 1 cm above point P.
Hence, the correct answer is (D).

155.



Since

$PR \parallel N_2$

$\Rightarrow \quad \alpha=2i$

Also $A+2\alpha=180°$

$\Rightarrow \quad A+2(2i)=180$

$\Rightarrow \quad i+4i=180$

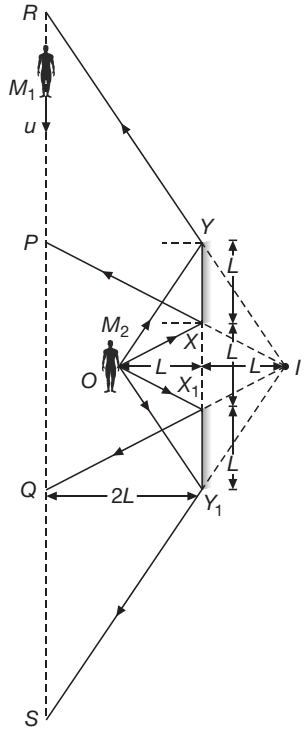
$\Rightarrow \quad 5i=180$

$\Rightarrow \quad i=36°$

$\Rightarrow \quad A=36°$

Hence, the correct answer is (C).

156. When the man M_1 is in the region RP and SQ, then man M_1 sees the image of M_2



From the similar triangle PIQ and XIX_1

$$\Rightarrow \quad \frac{PQ}{XX_1}=\frac{3L}{L}$$

$$\Rightarrow \quad PQ=3L$$

From the similar triangle, RIS and PIQ

$$\Rightarrow \quad \frac{RS}{YY_1}=\frac{3L}{L}$$

$$\Rightarrow \quad RS=9L$$

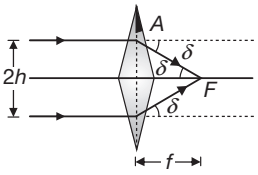
Then $RP+QS=RS-PQ=9L-3L=6L$

CHAPTER 1

$\Rightarrow$ Time $= \dfrac{6L}{u}$

Hence, the correct answer is (C).

157. Since $\tan\delta = \dfrac{h}{f}$



$\Rightarrow \quad f = \dfrac{h}{\tan\delta}$

Further

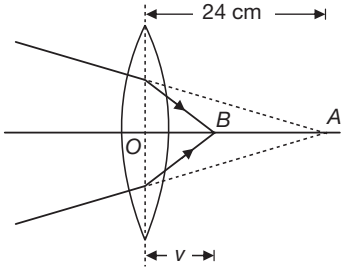
$\delta = (\mu - 1)A$

$\Rightarrow \quad f = \dfrac{h}{(\mu-1)A}$

Hence, the correct answer is (C).

158. If no lens had been there the rays would have met at A. On inserting the lens the rays meet at B. Thus A acts as an object for the lens.

Since $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$



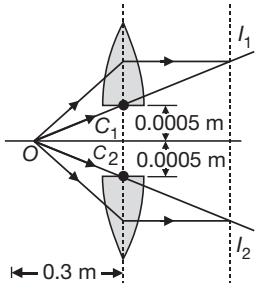
$u = +24$ cm, $v = ?$, $f = +24$ cm

$\Rightarrow \quad v = 12$ cm

$\Rightarrow$ Distance $AB = OA - OB = 24 - 12 = 12$ cm

Hence, the correct answer is (A).

159. $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$



$\Rightarrow \quad \dfrac{1}{v} - \dfrac{1}{(-0.3)} = \dfrac{1}{0.2}$

$\Rightarrow \quad v = 0.6$ m

$\Delta's \ OC_1C_2$ and OI_1I_2 are similar

Hence

$$\frac{0.001}{0.3} = \frac{I_1I_2}{0.3+0.6}$$

$\Rightarrow \quad I_1I_2 = 0.003$ m

Hence, the correct answer is (D).

160. Let the critical angle of interface between medium 1 and 2 is C_1 and between 1 and 3 is C_2

Then $\sin C_1 = \dfrac{\mu_2}{\mu_1}$ and $\sin C_2 = \dfrac{\mu_3}{\mu_1}$

For TIR to take place at second interface, we have

$90 - C_1 > C_2$

Taking sine of both sides, we get

$\sin(90 - C_1) > \sin C_2$

$\Rightarrow \quad \cos C_1 > \sin C_2$

$\Rightarrow \quad \sqrt{1 - \left(\dfrac{\mu_2}{\mu_1}\right)^2} > \dfrac{\mu_3}{\mu_1}$

$\Rightarrow \quad \mu_1^2 - \mu_3^2 > \mu_2^2$

Hence, the correct answer is (B).

161. A convex mirror can never form a real image.
Hence, the correct answer is (D).

162. Given, $\mu = \sqrt{2}$, $A = 60°$, $\delta = 30°$

For the minimum deviation, we have

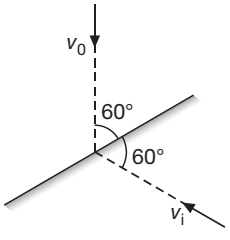
$$\mu = \frac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$$

Substituting the values of A and μ we get

$\delta_m = 30°$

i.e., the given deviation $\delta = 30°$ is actually the minimum deviation δ_m. At minimum deviation the ray inside the prism is parallel to the base of the prism (in case of an equilateral prism).
Hence, the correct answer is (A).

164.



$$v_0 = v_i = \omega A$$

$$\Rightarrow \quad v_0 = \left(\sqrt{\frac{k}{M}}\right)A$$

$$v_{rel} = \sqrt{v_0^2 + v_i^2 - 2v_0\cos(120°)}$$

$$\Rightarrow \quad v_{rel} = \sqrt{3}A\sqrt{\frac{k}{M}}$$

Hence, the correct answer is (C).

165. Here, PQ is the length of reflected light from the mirror. From the similar triangles, PMA and MXS'

$$\Rightarrow \quad \frac{PA}{XS'} = \frac{3H}{H}$$

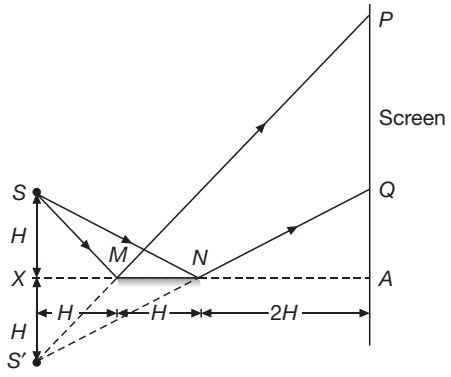
Then, $PA = 3(XS') = 3H$

From the similar triangles QNA and XNS'

$$\frac{QA}{XS'} = \frac{2H}{2H}$$

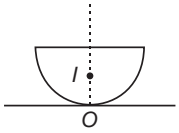
$$\Rightarrow \quad QA = H$$

$$\Rightarrow \quad PQ = PA - QA = 3H - H = 2H$$



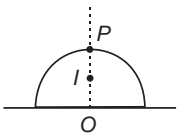
Hence, the correct answer is (A).

166. **CASE-1:**



$$n = \frac{\text{Real Depth}}{\text{Apparent Depth}} = \frac{4}{3}$$

CASE-2:



Consider origin to be at the pole P.

$$\frac{n_1}{-u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$$\Rightarrow \quad \frac{\frac{4}{3}}{-(-4)} + \frac{1}{\left(-\frac{25}{8}\right)} = \frac{1-\left(\frac{4}{3}\right)}{R}$$

$$\Rightarrow \quad R = -25 \text{ cm}$$

According to Lens Maker's Formula

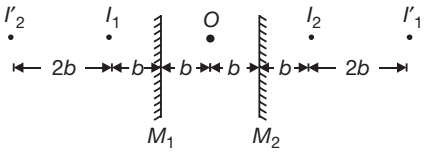
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{\infty} - \frac{1}{-25}\right)$$

$$\Rightarrow \quad f = 75 \text{ cm}$$

Hence, the correct answer is (B).

167.



I_1 is the direct image of O by mirror M_1

I_2 is the direct image of O by mirror M_2

I_1 acts as object for mirror M_2 and gives an image I_1' and similarly I_2 acts as object for mirror M_1 and gives image I_2'.

Since I_1 is located at a distance $3b$ from M_2.

So, I_1' must be at a distance $3b$ from M_2.

Similarly, I_2' must be at a distance $3b$ from M_1.

From above we observe that the first image is at a distance $2b$ from O.

Second image is at a distance $4b$ from O and so on.

So, nth image is at a distance $2nb$ from O.

Hence, the correct answer is (C).

168. $\theta = \frac{1}{2}(2\pi)\left(\frac{1}{4}\right)^2 \qquad \left\{\because \theta = \frac{1}{2}\alpha t^2\right\}$

$$\Rightarrow \quad \theta = \frac{\pi}{16}$$

Reflected ray turns through an angle $2\theta = 22.5°$

Hence, the correct answer is (C).

169. Distance of first image (I_1) formed due to refraction from the plane surface of water is

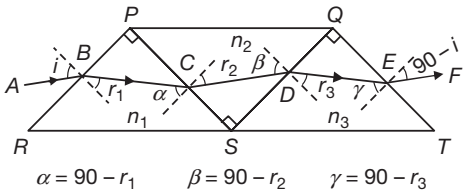
$$d_{app} = \frac{d_{real}}{\mu} = \frac{10}{4/3} = 7.5 \text{ cm}$$

CHAPTER 1

Now distance of this image from the plane mirror is $x = 5 + 7.5 = 12.5$ cm. Therefore, distance of second image (I_2) from the mirror will also be equal to 12.5 cm from it.

Hence, the correct answer is (C).

170.



$\alpha = 90 - r_1$ $\beta = 90 - r_2$ $\gamma = 90 - r_3$

At B

$$\sin i = n_1 \sin r_1$$

$$\Rightarrow \sin^2 i = n_1^2 \sin^2 r_1 \quad \ldots(1)$$

At C

$$n_1 \sin(90 - r_1) = n_2 \sin r_2$$

$$\Rightarrow n_2^2 \sin^2 r_2 = n_1^2 \cos^2 r_1 \quad \ldots(2)$$

At D

$$n_2 \sin(90 - r_2) = n_3 \sin r_3$$

$$\Rightarrow n_2^2 \cos^2 r_2 = n_3^2 \sin^2 r_3 \quad \ldots(3)$$

At E

$$n_3 \sin(90 - r_3) = (1)\sin(90 - i)$$

$$\Rightarrow \cos^2 i = n_3^2 \cos^2 r_3 \quad \ldots(4)$$

Adding (1), (2), (3) & (4)

$$\Rightarrow 1 + n_2^2 = n_1^2 + n_3^2$$

Hence, the correct answer is (D).

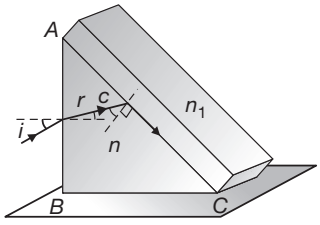
171. $P = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Hence, the correct answer is (A).

173. $(90 - r) + (90 - C) + 45 = 180$

$$\Rightarrow r + C = 45^\circ$$

$$\Rightarrow r = 45 - C$$



Since $\frac{\sin i}{\sin r} = n$

$$\Rightarrow \sin i = n \sin r$$

$$\Rightarrow \sin i = n \sin(45 - C)$$

$$\Rightarrow \sin i = n(\sin 45 \cos C - \cos 45 \sin C)$$

$$\Rightarrow \sin i = \frac{n}{\sqrt{2}}(\cos C - \sin C)$$

Further, $\sin C = \frac{n_1}{n}$

$$\Rightarrow \sin i = \frac{n}{\sqrt{2}}\left[\sqrt{1 - \frac{n_1^2}{n^2}} - \frac{n_1}{n}\right]$$

$$\Rightarrow \sin i = \frac{1}{\sqrt{2}}\left[\sqrt{n^2 - n_1^2} - n_1\right]$$

$$\Rightarrow i = \sin^{-1}\left[\frac{\sqrt{n^2 - n_1^2} - n_1}{\sqrt{2}}\right]$$

Hence, the correct answer is (D).

174. Here nose of the boy is the object and fish is observer. Using refraction formula, we have

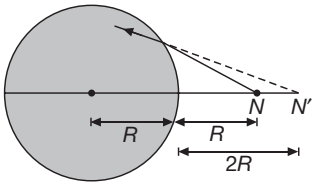
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

where $u = +R$, $\mu_1 = 1$, $\mu_2 = \frac{4}{3}$, $R = -R$

$$\Rightarrow \frac{4}{3v} - \frac{1}{R} = \frac{\frac{4}{3} - 1}{-R}$$

$$\Rightarrow v = +2R$$

Thus, the image of child's nose will appear at a distance $3R$ from the centre of the bowl. The ray diagram is shown in below figure.



Hence, the correct answer is (C).

175. For image of centre of mass of A and B, we have

$$(\vec{v}_{cm})_x = \frac{m_1 v_1}{m_1 + m_2}(-\hat{i})$$

$$(\vec{v}_{cm})_y = \frac{m_2 v_2}{m_1 + m_2}(+\hat{j})$$

Since, $\vec{v}_{cm} = (\vec{v}_{cm})_x + (\vec{v}_{cm})_y$

$$\Rightarrow \vec{v}_{cm} = \frac{1}{m_1 + m_2}\left(-m_1 v_1 \hat{i} + m_2 v_2 \hat{j}\right)$$

$$\Rightarrow \quad |\vec{v}_{cm}| = \sqrt{\frac{m_1^2 v_1^2}{(m_1+m_2)^2} + \frac{m_2^2 v_2^2}{(m_1+m_2)^2}}$$

$$\Rightarrow \quad |\vec{v}_{cm}| = \frac{1}{m_1+m_2}\sqrt{m_1^2 v_1^2 + m_2^2 v_2^2}$$

Hence, the correct answer is (D).

176. $\frac{n_1}{v} + \frac{n_2}{-u} = \frac{n_1 - n_2}{R}$

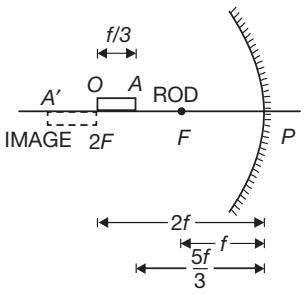
$$\Rightarrow \quad \frac{1}{v} - \frac{2}{(-15)} = \frac{1-2}{-10}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{10} - \frac{2}{15} = -\frac{1}{30}$$

$$\Rightarrow \quad v = -30 \text{ cm}$$

Hence, the correct answer is (B).

177. Since an elongated image is formed and it touches one end of the rod, so the rod must lie with one end at $2F$ and other end between $2F$ and F (shown in figure).



For end A, $u = -\frac{5f}{3}$

Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v} - \frac{3}{5f} = \frac{1}{-f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{3}{5f} - \frac{5}{5f}$$

$$\Rightarrow \quad \frac{1}{v} = -\frac{2}{5f}$$

$$\Rightarrow \quad v = -\frac{5f}{2}$$

$$\Rightarrow \quad PA' = \frac{5f}{2}$$

$$\Rightarrow \quad OA' = \frac{5f}{2} - 2f$$

$$\Rightarrow \quad OA' = \frac{f}{2}$$

$$\Rightarrow \quad m = \frac{OA'}{OA} = \frac{\frac{f}{2}}{\frac{f}{3}} = \frac{3}{2}$$

Hence, the correct answer is (C).

178. For a plane refracting surface, the lateral magnification is 1. So, the image of the coin will be of the same size as the coin itself.

Hence, the correct answer is (D).

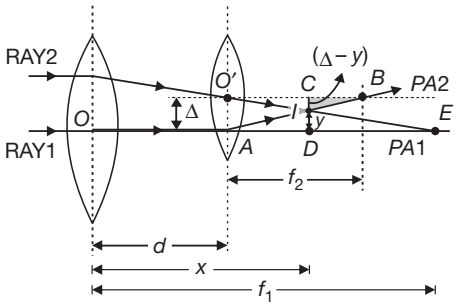
179. $\delta = \frac{A}{2} = \frac{60}{2} = 30$

$\Rightarrow$ The ray passes symmetrically through the prism parallel to its base.

Hence $\theta = 0°$

Hence, the correct answer is (D).

180.



For drawing the above ray diagram, we must consider two rays RAY 1 and RAY 2 in such a manner that RAY 1 passes through optical centre of first convex lens and is parallel to the Principal Axis for the second convex lens $(PA2)$.

So, wherever the RAY 1 cuts $PA2$ is the focal length f_2 of second convex lens. Similarly we have RAY 2 parallel to $PA1$ and let this pass through O' so as to cut $PA1$ at a distance equal to focal length (f_1) for the first convex lens.

ΔIDE and $\Delta O'AE$ are similar

$$\Rightarrow \quad \frac{y}{\Delta} = \frac{f_1 - x}{f_1 - d} \qquad \ldots(1)$$

ΔICB and $AO'B$ are similar

$$\Rightarrow \quad \frac{\Delta - y}{\Delta} = \frac{(f_2 + d) - x}{f_2}$$

$$\Rightarrow \quad -\frac{y}{\Delta} = \frac{d}{f_2} - \frac{x}{f_2}$$

$$\Rightarrow \quad \frac{y}{\Delta} = \frac{x}{f_2} - \frac{d}{f_2} \qquad \ldots(2)$$

CHAPTER 1

From (1) and (2)

$$\frac{x}{f_2}-\frac{d}{f_2}=\frac{f_1-x}{f_1-d}$$

$$\Rightarrow \quad \frac{x}{f_2}-\frac{d}{f_2}=\frac{f_1}{f_1-d}-\frac{x}{f_1-d}$$

$$\Rightarrow \quad x\left(\frac{1}{f_2}+\frac{1}{f_1-d}\right)=\frac{f_1}{f_1-d}+\frac{d}{f_2}$$

$$\Rightarrow \quad x=\frac{f_1f_2+d(f_1-d)}{f_1+f_2-d}$$

Similarly, we can calculate value of y.

OBJECTIVE TRICK

If $\Delta=0$, y must be zero (satisfied by A, B, C)

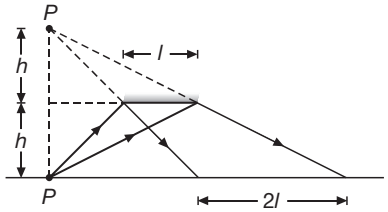
If $d\to f_1+f_2$, then $x\to\infty$ (satisfied by B, C, D)

If $d\to f_1$, then $x\to f_1$ (satisfied by C, D)

So, we observe that OPTION **(C)** is the only option satisfying both the x and y in special situations mentioned above.

Hence, the correct answer is (C).

181. As mirror is moving parallel to itself, it does not affect the position of image so length of spot on ground will remain same. This can also be seen by similar triangles in figure.



Hence, the correct answer is (B).

182. For the minimum angle of deviation produced by the prism we have $i=e$ and $r_1=r_2=\frac{A}{2}=\frac{90^\circ}{2}=45^\circ$

Applying Snell's Law at 1st interface, we get

$$1\sin i=\mu\sin r$$

$$\Rightarrow \quad \sin i=\sqrt{\frac{3}{2}}\times\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad i=60^\circ,$$

Then $D_{\min}=(i-r)+(e-r)=(60^\circ-45^\circ)+(60^\circ-45^\circ)$,

$$\Rightarrow \quad D_{\min}=30^\circ$$

Hence, the correct answer is (C).

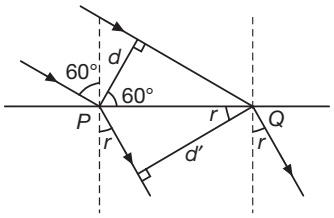
183. Let d' be the diameter of refracted beam. Then

$$d=PQ\cos(60^\circ)$$

and $d'=PQ\cos r$

$$\Rightarrow \quad \frac{d'}{d}=\frac{\cos r}{\cos 60^\circ}=2\cos r$$

$$\Rightarrow \quad d'=2d\cos r$$



Since, $\sin r=\frac{\sin i}{\mu}=\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}=\frac{1}{\sqrt{3}}$

Also, $\cos r=\sqrt{1-\sin^2 r}$

$$\Rightarrow \quad \cos r=\sqrt{\frac{2}{3}}$$

$$\Rightarrow \quad d'=(2)(2)\sqrt{\frac{2}{3}}=4\sqrt{\frac{2}{3}}\text{ cm}$$

$$\Rightarrow \quad d'\approx 3.26\text{ cm}$$

Hence, the correct answer is (C).

184. $$10=\frac{1}{f_1}+\frac{1}{f_2} \qquad \ldots(1)$$

Since $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1f_2}$

$$\Rightarrow \quad 6=\frac{1}{f_1}+\frac{1}{f_2}-\frac{0.25}{f_1f_2} \qquad \ldots(2)$$

$$\Rightarrow \quad 6=10-\frac{0.25}{f_1f_2}$$

$$\Rightarrow \quad 4=\frac{1}{4f_1f_2}$$

$$\Rightarrow \quad f_1f_2=\frac{1}{16} \qquad \ldots(3)$$

Put (3) in (1), we get

$$f_1+f_2=10\left(\frac{1}{16}\right)$$

$$\Rightarrow \quad f_1+f_2=\frac{5}{8} \qquad \ldots(4)$$

Since $(f_1 - f_2)^2 = (f_1 + f_2)^2 - 4f_1f_2$

$\Rightarrow \quad (f_1 - f_2)^2 = \frac{25}{64} - \frac{1}{4}$

$\Rightarrow \quad (f_1 - f_2)^2 = \frac{9}{64}$

$\Rightarrow \quad f_1 - f_2 = \frac{3}{8}$ …(5)

Adding (4) and (5), we get

$2f_1 = 1$

$f_1 = \frac{1}{2} = 0.5$ m

$\Rightarrow \quad f_2 = \frac{1}{8} = 0.125$ m

Hence, the correct answer is (A).

185. Since, $\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$

Using $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, we get

$-\frac{1}{12} = -\frac{1}{20} + \frac{1}{v}$

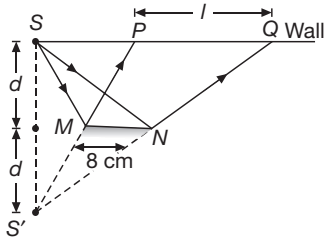
$\Rightarrow \quad v = -30$ cms^{-1}

$\Rightarrow \quad \frac{dv}{dt} = -\left(\frac{30}{20}\right)^2 \times 4 = -9$ cms^{-1}

i.e., 9 cms^{-1} away from the mirror.

Hence, the correct answer is (C).

186. Here, PQ is the size of spot formed on the wall. From the similar triangles $\Delta PS'Q$ and $\Delta MS'N$, we have



$\frac{PQ}{MN} = \frac{2d}{d}$

$\Rightarrow \quad PQ = 2 \times 8 = 16$ cm

Hence, the correct answer is (C).

187. After projection of the particle its velocity component in vertical direction is given as

$u_y = \sqrt{2}\sin(45°) = 1$ ms^{-1}

In vertical direction if we consider y_1 as the displacement of particle and y_2 the displacement of mirror in time t then we use

$y_1 = (1)(0.5) - \frac{1}{2}gt^2 = 0.5 - \frac{1}{2}gt^2$

and $y_2 = -\frac{1}{2}gt^2$

Vertical distance of particle from mirror is given by

$y = y_1 - y_2 = 0.5$ m

Thus, distance between particle and its image is

$2y = 2(0.5) = 1$ m

Hence, the correct answer is (B).

188. For the first medium, $\sin\theta_1 = \frac{a}{\sqrt{a^2 + b^2}}$

and for the second medium, $\sin\theta_2 = \frac{c}{\sqrt{c^2 + d^2}}$

By Snell's law, we have $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

$\Rightarrow \quad \frac{\mu_1 a}{\sqrt{a^2 + b^2}} = \frac{\mu_2 a}{\sqrt{c^2 + d^2}}$

Hence, the correct answer is (B).

189. For refraction at plane surface

$\frac{1}{-(-mR)} + \frac{1.5}{v} = \frac{1.5 - 1}{\infty}$

$\Rightarrow \quad v = -1.5\ mR$

For refraction at the curved surface

$u' = -(1.5\ mR + R)$, $v' \to \infty$

$\Rightarrow \quad \frac{1}{\infty} - \frac{1.5}{-(1.5\ mR + R)} = \frac{1 - 1.5}{-R}$

$\Rightarrow \quad \frac{3}{(1.5\ m + 1)} = 1$

$\Rightarrow \quad 3 = 1.5\ m + 1$

$\Rightarrow \quad 1.5\ m = 2$

$\Rightarrow \quad m = \frac{4}{3}$

Hence, the correct answer is (D).

190. Ray diagram of the grazing emergence is shown in figure.

Here we have $r + C = A$

$\Rightarrow \quad r + C = 60$

Using snell's law at the second surface, we have

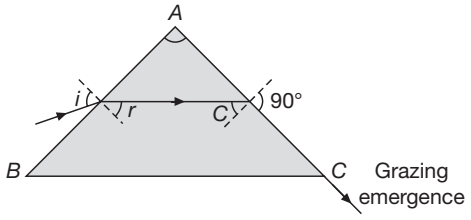
$\sqrt{2}\sin C = 1\sin(90)$

CHAPTER 1

$\Rightarrow \quad \sin C = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad C = 45°$

Then, we have $r = A - C = 15°$



Using Snell's law at surface AB, we have

$1 \sin i = \sqrt{2} \sin r$

and $\sin i = \sqrt{2} \sin(15°) = \sqrt{2} \sin(45° - 30°)$

$\Rightarrow \quad i = \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$

Hence, the correct answer is (A).

191. Deviation by a sphere is $2(i - r)$

Here, deviation $\delta = 60° = 2(i - r)$

$\Rightarrow \quad i - r = 30°$

$\Rightarrow \quad r = i - 30° = 60° - 30° = 30°$

According to Snell's Law, we have $\mu = \frac{\sin i}{\sin r}$

$\Rightarrow \quad \mu = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$

Hence, the correct answer is (C).

192. Incidence angle on face BC is

$i = 90 - \theta$

$i = A = 90 - \theta > C$ (for light not to cross BC)

$\Rightarrow \quad 90 - \theta > C$

$\Rightarrow \quad \sin(90 - \theta) > \sin C$

$\Rightarrow \quad \cos\theta > \sin C = \frac{\frac{6}{5}}{\frac{3}{2}} = \frac{4}{5}$

$\Rightarrow \quad \theta < \cos^{-1}\left(\frac{4}{5}\right) = 37°$

Hence, the correct answer is (B).

193. Equivalent power of the system is given as

$P_{eq} = 2(P_w + P_g) + P_m$

where P_w and P_g are the respective powers of the water and glass lenses and P_m is the power of the mirror. These are given by

$P_w = \frac{1}{f_w} = (\mu_w - 1)\left(\frac{1}{\infty} - \frac{1}{-60}\right)$

$\Rightarrow \quad P_w = \frac{1}{180}\ \text{cm}^{-1}$

Also, $P_g = \frac{1}{f_g} = (\mu_g - 1)\left(\frac{1}{-60} - \frac{1}{-20}\right)$

$\Rightarrow \quad P_g = \frac{1}{60}\ \text{cm}^{-1}$

and $P_m = \frac{1}{f_m} = \frac{2}{20}\ \text{cm}^{-1}$

Equivalent power of the system is given by

$P_{eq} = 2(P_w + P_g) + P_m,$

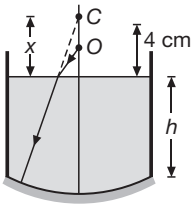
$\Rightarrow \quad P_{eq} = 2\left(\frac{1}{180} + \frac{1}{60}\right) + \frac{2}{20}$

$\Rightarrow \quad P_{eq} = \frac{13}{90}$

$\Rightarrow \quad f_e = \frac{90}{13}\ \text{cm}$

Hence, the correct answer is (A).

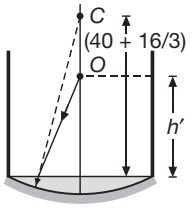
194. The image coincides with object O if the ray starting from O is incident normally on mirror as shown in figure.



Here the image is located at $x = \mu(4)$

$\Rightarrow \quad x = 4 \times \frac{4}{3} = \frac{16}{3}$

In second case, the image will coincides with object O if ray starting from O is incident normally on mirror as shown in figure.



Now, $40 + \frac{16}{3} = \frac{136}{3} = h' \times \frac{4}{3}$

$$\Rightarrow \quad h' = \frac{136}{3} \times \frac{3}{4} = 34 \text{ cm}$$

Hence, the correct answer is (A).

195. Let R be the radius of curvature of each surface. Then applying Lens Maker's Formula, we get

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\Rightarrow \quad R = f$$

For the water lens, again applying the Lens Maker's Formula, we get

$$\frac{1}{f'} = \left(\frac{4}{3} - 1\right)\left(-\frac{1}{R} - \frac{1}{R}\right) = \frac{1}{3}\left(-\frac{2}{f}\right)$$

$$\Rightarrow \quad \frac{1}{f'} = -\frac{2}{3f}$$

Since, we know that for lenses placed in contact, we have

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

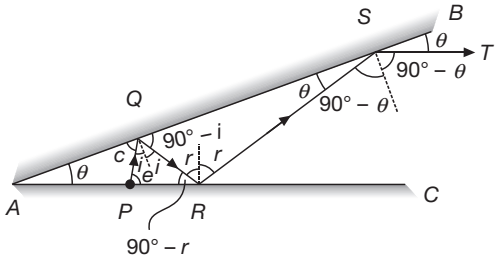
$$\Rightarrow \quad \frac{1}{F} = \frac{1}{f} + \frac{1}{f} + \frac{1}{f'}$$

$$\Rightarrow \quad \frac{1}{F} = \frac{2}{f} - \frac{2}{3f} = \frac{4}{3f}$$

$$\Rightarrow \quad F = \frac{3f}{4}$$

Hence, the correct answer is (B).

196. In triangle QRS, we have



$$90° - i + \theta + 2r = 180°$$

$$\Rightarrow \quad r = \left(\frac{90° + i - \theta}{2}\right)$$

$$\Rightarrow \quad \angle QPR = 180° - (90° - r) - 2i$$

$$\Rightarrow \quad \angle QPR = 180° - 90° + \left(\frac{90° + i - \theta}{2}\right) - 2i$$

$$\Rightarrow \quad \angle QPR = \frac{180° - 4i + 90° + i - \theta}{2}$$

$$\Rightarrow \quad \angle QPR = \left(\frac{270° - 3i - \theta}{2}\right)$$

Since $\angle QPR = \theta + \angle AQP$

$$\Rightarrow \quad \angle AQP = \angle QPR - \theta = 180° - (90° - i) - 2i$$

$$\Rightarrow \quad \left(\frac{270° - 3i - \theta}{2}\right) - \theta = 90 - i$$

Since, $\theta = 20°$, so we get

$$i = 30°$$

Hence, the correct answer is (B).

197. Focal length of lens does not depend on the aperture of lens. It depends on the refractive index of lens, surroundings and the radius of curvature of its surfaces. For intensity of image, we have

$$I \propto (\text{aperture diameter})^2$$

$$\Rightarrow \quad I_{\text{blocked}} \propto \left(\frac{d}{2}\right)^2 \text{ and } I_0 \propto d^2$$

Hence, $I_{\text{blocked}} = \frac{I_0}{4}$

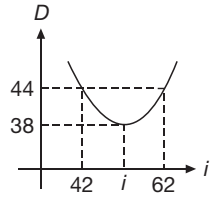
Final image intensity

$$I_f = I_{\text{original}} - I_{\text{blocked}}$$

$$\Rightarrow \quad I_f = I_0 - \frac{I_0}{4} = \frac{3I_0}{4}$$

Hence, the correct answer is (D).

198. D vs i Graph for the prism is shown in figure.



Since, $D = i + e - A$

$$\Rightarrow \quad 44° = 42° + 62° - A$$

$$\Rightarrow \quad A = 60°$$

For the condition of minimum deviation, we have

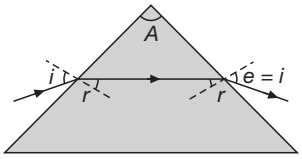
$$i = e \text{ and } r_1 = r_2 = \frac{A}{2}$$

$$\Rightarrow \quad r = 30°$$

$$\Rightarrow \quad D_{\min} = 2i - A$$

$$\Rightarrow \quad 38° = 2i - 60$$

$$\Rightarrow \quad i = 49°$$



Hence, the correct answer is (B).

CHAPTER 1

199. $A_{max} = 2C$

$\Rightarrow \quad A_{max} = 2(36^o)$

$\Rightarrow \quad A_{max} = 72^o$

Hence, the correct answer is (C).

200. $f = \dfrac{D^2 - x^2}{4D}$

$\Rightarrow \quad f = \dfrac{(90)^2 - (20)^2}{4(90)}$

$\Rightarrow \quad f = 21.4 \text{ cm}$

Hence, the correct answer is (C).

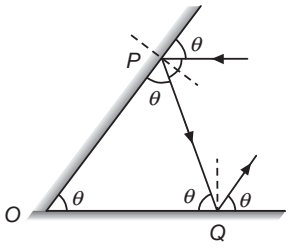
201. Since $\mu = \dfrac{\sin i}{\sin r}$

$\Rightarrow \quad \sqrt{2} = \dfrac{\sin i}{\sin 30}$

$\Rightarrow \quad i = 45°$

Hence, the correct answer is (C).

202.



$\angle OPQ = \angle OQP = \theta$

$\Rightarrow \quad 3\theta = 180°$

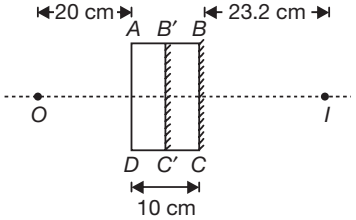
$\Rightarrow \quad \theta = 60°$

Hence, the correct answer is (C).

203. AB' = Apparent Depth = x

$AB = 10$ cm

The face BC appear's to be shifted to $B'C'$. If x be the apparent depth, then $AB' = x$ and $B'B = (10 - x)$.



Also, according to Laws of Reflection an image is formed as far behind the mirror as the object is in front of it

$\Rightarrow \quad 20 + x = 23.2 + (10 - x)$

$\Rightarrow \quad 2x = 13.2$

$\Rightarrow \quad x = 6.6 \text{ cm}$

Since $\mu = \dfrac{\text{Real Depth}}{\text{Apparent Depth}}$

$\Rightarrow \quad \mu = \dfrac{10}{6.6} = 1.51$

Hence, the correct answer is (C).

204. Since $\mu_2 t = \mu_1 t'$

$\Rightarrow \quad t' = \dfrac{\mu_2}{\mu_1} t$

$\Rightarrow \quad \Delta x = t' - t$

$\Rightarrow \quad \Delta x = \left(\dfrac{\mu_2}{\mu_1} - 1\right) t = \left(\dfrac{\frac{3}{2}}{\frac{4}{3}} - 1\right) t = \dfrac{t}{8}$

Hence, the correct answer is (A).

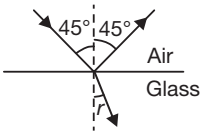
205. By Laws of Reflection

$i = r$

Also

$i + r = 90$

$\Rightarrow \quad i = 45$



According to Snell's Law

$\mu = \dfrac{\sin i}{\sin r}$

$\Rightarrow \quad 1.5 = \dfrac{3}{2} = \dfrac{\sin 45}{\sin r}$

$\Rightarrow \quad r = \sin^{-1}\left(\dfrac{\sqrt{2}}{3}\right)$

Hence, the correct answer is (B).

206. Shift $= t\left(1 - \dfrac{1}{\mu}\right)$ away

Hence, the correct answer is (A).

207. According to Newton's Formula, we have

$x_1 x_2 = f^2$

$\Rightarrow \quad (10)(40) = f^2$

$\Rightarrow \quad f^2 = 400$

$\Rightarrow \quad f = 20 \text{ cm}$

Hence, the correct answer is (B).

208. $A = 60°$

$i = 55°$, $e = 46°$

Since $i + e = A + D$

$\Rightarrow \quad 55 + 46 = 60 + D$

$\Rightarrow \quad D = 41°$

So, $D_m < D$

Hence, the correct answer is (B).

209. At $D = D_m$

$i = e$

Hence, the correct answer is (A).

210. Areal Magnification $= 9$

$$\Rightarrow \quad m_{ar} = 9 = \left(\frac{f}{f+u}\right)^2$$

$$\Rightarrow \quad \frac{f}{f+u} = -3 \qquad \text{(Since we get a real image)}$$

$$\Rightarrow \quad \frac{f}{f-40} = -3$$

$$\Rightarrow \quad -3f + 120 = f$$

$$\Rightarrow \quad 4f = 120$$

$$\Rightarrow \quad f = 30 \text{ cm}$$

Hence, the correct answer is (A).

211. Axial Magnification is given by

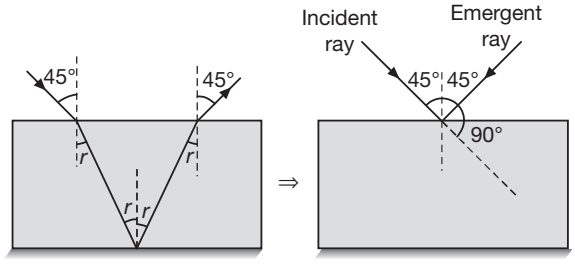
$$m_{ax} = \frac{I \text{ along PA}}{O \text{ along PA}} = \frac{v^2}{u^2}$$

$$\Rightarrow \quad m_{ax} = \frac{I}{b} = \left(\frac{f}{f-u}\right)^2$$

$$\Rightarrow \quad I = b\left(\frac{f}{f-u}\right)^2$$

Hence, the correct answer is (D).

212. From the figure it is clear that the angle between incident ray and the emergent ray is $90°$.



Hence, the correct answer is (B).

213. According to Lens Maker's Formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad f \propto \frac{1}{(\mu - 1)}$$

Since $\mu_{\text{Red}} < \mu_{\text{Violet}}$

$\Rightarrow \quad f_{\text{Red}} > f_{\text{Violet}}$

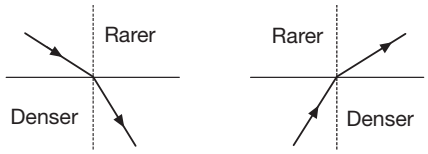
$\Rightarrow \quad f_{\text{Red}} > f_{\text{Blue}}$

Always keep in mind that whenever you are asked to compare (greater than or less than) u, v or f you must not apply sign conventions for comparison.

Hence, the correct answer is (D).

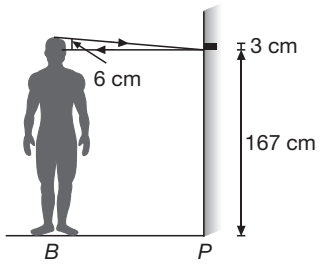
214. Since, the refractive index is increasing linearly from top to the bottom, so the light cannot travel in a straight line in the liquid as shown in options (A) and (B).

Initially it will bend towards normal and after reflecting from the bottom it will bend away from the normal as shown below.



Hence, the correct answer is (D).

215. To see the top of his head, the light ray from head after reflection should come to his eyes as shown in figure.



The person will not see the top of his head when the point from which these light rays are getting reflected, is replaced by a hole. Thus, this point is located 167 cm from P.

Hence, the correct answer is (A).

216. $u = -(f + x)$

$$m = \frac{f}{f+u}$$

CHAPTER 1

$$\Rightarrow \quad m = \frac{f}{f - f - x}$$

$$\Rightarrow \quad m = -\frac{f}{x}$$

Negative sign confirms that real magnification is negative.

Hence, the correct answer is (A).

217. For a convex mirror, both (A) and (B) are incorrect.

Hence, the correct answer is (D).

218. $u = -(f + x_1)$

$v = (f + x_2)$

Since $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{f + x_2} + \frac{1}{f + x_1} = \frac{1}{f}$$

$$\Rightarrow \quad f(2f + x_1 + x_2) = f^2 + (x_1 + x_2)f + x_1x_2$$

$$\Rightarrow \quad f^2 = x_1x_2$$

$$\Rightarrow \quad f = \sqrt{x_1x_2}$$

Hence, the correct answer is (D).

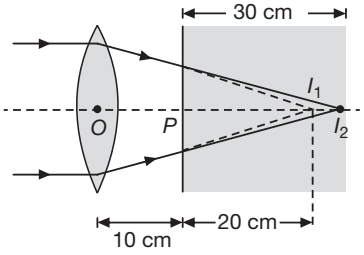
219. Two plano-convex lenses of focal length f on combining give a convex lens of focal length $\frac{f}{2}$. To obtain a real image of magnification unity the object must be at $2F$(or C) and so image is also formed at $2F$(or C). Hence $u = -2f = -40$ cm

Hence, the correct answer is (C).

220. The lens will converge the rays at its focus i.e., 30 cm from the lens or 20 cm from the refracting surface, so we have

$PI_1 = 20$ cm

Now, $PI_2 = \mu(PI_1) = \frac{3}{2} \times 20 = 30$ cm



Hence, the rays will converge at a distance of 40 cm from the lens.

Hence, the correct answer is (D).

221. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$

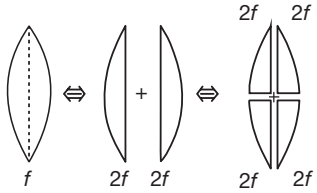
$$\Rightarrow \quad \frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right) \qquad \ldots(1)$$

On cutting $\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$

$$\Rightarrow \quad f' = 2f$$

Hence, the correct answer is (C).

222. When a lens is cut parallel to principal axis its focal length remains the same. Hence each part will have a focal length 2*f*.



Hence, the correct answer is (C).

223. According to Lens Maker's Formula

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The lens is made of two materials, so for a single object distance, two different image distances are obtained i.e. two images are formed.

Hence, the correct answer is (B).

225. Let f_1 and f_2 be the focal lengths of the lenses of refractive indices μ_1 and μ_2 in water, then

$$\frac{1}{f_1} = \left(\frac{\mu_1}{\mu_w} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\Rightarrow \quad \frac{1}{f_1} = \left(\frac{\mu_1}{\mu_w} - 1\right)\left(\frac{2}{R}\right) \qquad \ldots(1)$$

$$\frac{1}{f_2} = \left(\frac{\mu_2}{\mu_w} - 1\right)\left(-\frac{1}{R} - \frac{1}{R}\right)$$

$$\Rightarrow \quad \frac{1}{f_2} = \left(\frac{\mu_2}{\mu_w} - 1\right)\left(\frac{-2}{R}\right) \qquad \ldots(2)$$

Adding equations (1) and (2), we get

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{2(\mu_1 - \mu_2)}{\mu_w R}$$

$$\Rightarrow \quad \frac{1}{30} = \frac{2(\mu_1 - \mu_2)}{\mu_w R}$$

$$\Rightarrow \quad (\mu_1 - \mu_2) = \frac{\mu_w R}{60}$$

Substituting the values, we get

$$(\mu_1 - \mu_2) = \frac{4\times 15}{3\times 60} = \frac{1}{3}$$

Hence, the correct answer is (B).

226. Since image is formed on the screen, hence it must be real. So, Lens used must be convex (and cannot be concave)

$u = -45$ cm , $v = +90$ cm

Since, for a lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{90} + \frac{1}{45} = \frac{1}{f}$$

$$\Rightarrow \quad f = +30 \text{ cm}$$

Further again the positive sign with focal length indicates the lens is convex.

Hence, the correct answer is (A)

227. Size of Image $= \frac{v}{u}$(Size of Object)

$$\Rightarrow \quad I = \frac{90}{-45}(5) = -10 \text{ cm}$$

Here again the negative sign with I indicates that it is formed below origin i.e. inverted i.e. real.

Hence, the correct answer is (B).

228. $\frac{\mu_1}{-\mu} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \quad \frac{1}{-(-50)} + \frac{1.5}{v} = \frac{1.5-1}{20}$$

$$\Rightarrow \quad \frac{1}{50} + \frac{1.5}{v} = \frac{0.5}{20}$$

$$\Rightarrow \quad \frac{1.5}{v} = \frac{1}{40} - \frac{1}{50}$$

$$\Rightarrow \quad \frac{1.5}{v} = \frac{10}{2000}$$

$$\Rightarrow \quad v = \frac{15\times 200}{10} = +300 \text{ cm}$$

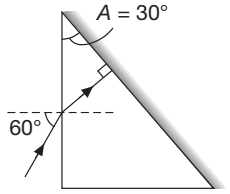
Positive side indicates that image is real.

Hence, the correct answer is (C).

229. $r_2 = 0°$

$$\Rightarrow \quad r_1 = A = 30°$$

and $i_1 = 60°$



So, from Snell's Law, we get

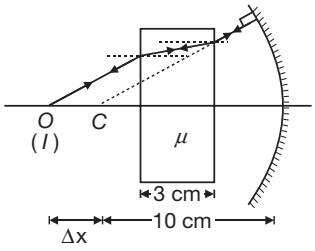
$$\mu = \frac{\sin i_1}{\sin r_i} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$$

Hence, the correct answer is (C).

230. $\Delta x = 3\left[1 - \frac{1}{\left(\frac{3}{2}\right)}\right]$

$$\Rightarrow \quad \Delta x = 1 \text{ cm}$$

$\Rightarrow$ Object distance $= 10 + 1 = 11$ cm



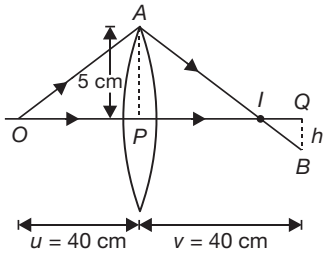
Hence, the correct answer is (C).

231. $f_\ell = \frac{\mu_g - 1}{\frac{\mu_g}{\mu_\ell} - 1} \times f_a = \frac{1.5-1}{\frac{1.5}{1.6} - 1} \times 20$

$$\Rightarrow \quad f_\ell = -\frac{0.5\times 1.6}{0.1} \times 20 = -160 \text{ cm}$$

Hence, the correct answer is (C).

232.



$f = 20$ cm , $u = -40$ cm

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ gives

CHAPTER 1

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{20}-\frac{1}{40}$$

$$\Rightarrow \quad v=40 \text{ cm}$$

The situation is shown in figure.

From similar triangles PAI and IQB

$$\frac{5}{h}=\frac{40}{20}$$

$$\Rightarrow \quad h=\frac{5}{2}=2.5 \text{ cm}$$

Hence, the correct answer is (C).

233. Since $O^2=I_1I_2$

$$\Rightarrow \quad 4=(0.5)I_2$$

$$\Rightarrow \quad I_2=8 \text{ cm}$$

Hence, the correct answer is (C).

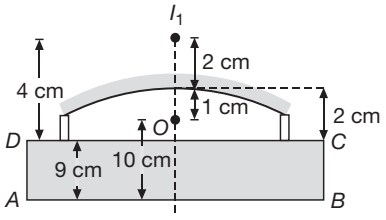
234. According to Newton's Formula

$$x_1x_2=f^2$$

$$\Rightarrow \quad f=\sqrt{16\times 25}=20 \text{ cm}$$

Hence, the correct answer is (C).

235. For reflection from mirror, we have



$$u=-1 \text{ cm}$$

$$f=-2 \text{ cm}$$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\left(-\frac{1}{2}\right)-\left(-\frac{1}{1}\right)=1-\frac{1}{2}$$

$$\Rightarrow \quad v=2 \text{ cm}$$

For the slab, image I_1 can be taken as the object. Here we use

$$h_{\text{app}}=\frac{d_1}{\left(\frac{\mu_1}{\mu_3}\right)}+\frac{d_2}{\left(\frac{\mu_2}{\mu_3}\right)}$$

$$\Rightarrow \quad h_{\text{app}}=\left(\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}\right)\mu_3$$

where $d_1=9$ cm, $d_2=4$ cm, $\mu_1=\frac{3}{2}$, $\mu_2=1$, $\mu_3=1$

$$\Rightarrow \quad h_{\text{app}}=\left(\frac{9}{\frac{3}{2}}+\frac{4}{1}\right)(1)=10 \text{ cm}$$

i.e. the final image formed at 10 cm from side AB of the slab i.e. at the object itself as shown in figure. Here the image formed is virtual.

Hence, the correct answer is (D).

236. Since we know that, along the normal, the velocity of image is double the velocity of mirror and opposite to the velocity of image and along the surface of mirror, image velocity components are same as that of object, so velocity of image is given as $\left(3\hat{i}+4\hat{j}+11\hat{k}\right)$.

Hence, the correct answer is (B).

237. For the mango located at a distance x from the water surface, apparent distance of the image of mango appears to tortoise as

$$x_{\text{app}}=\mu x$$

$$\Rightarrow \quad \frac{d^2x_{\text{app}}}{dt^2}=\mu\frac{d^2x}{dt^2}$$

$$\Rightarrow \quad a_{\text{app}}=\mu g$$

So, the acceleration of falling mango with respect to tortoise is given as

$$a_{\text{relative}}=a+a_{\text{app}}=g+\mu g=g(1+\mu)=\frac{7g}{3}$$

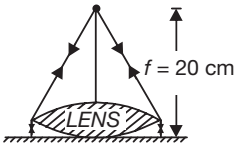
Hence, the correct answer is (D).

238. Focal Length of Lens is $f_\ell=20$ cm.

Focal length of combination is

$$f_c=20+5=25 \text{ cm}$$

when the space between lens and mirror is filled with a liquid then



$$\frac{1}{f_c}=\frac{1}{f_\ell}+\frac{1}{f}$$

$$\frac{1}{25}=\frac{1}{f_\ell}+\frac{1}{20}$$

$$\Rightarrow \quad \frac{1}{f_\ell}=\frac{1}{25}-\frac{1}{20}$$

$\Rightarrow \quad f_\ell = -100$ cm

For this liquid lens

$$\frac{1}{f_\ell} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad -\frac{1}{100} = (\mu - 1)\left(\frac{1}{-33} - \frac{1}{\infty}\right)$$

$$\Rightarrow \quad \mu - 1 = 0.33$$

$$\Rightarrow \quad \mu = 1.33 = \frac{4}{3}$$

Hence, the correct answer is (A).

240. For real image

$$u = -u_1,\ v = -2u_1,\ f = -20 \text{ cm}$$

Substituting in $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{-2u_1} - \frac{1}{u_1} = -\frac{1}{20}$$

$$\Rightarrow \quad u_1 = 30 \text{ cm}$$

For virtual image $u = -u_2,\ v = 2u_2,\ f = -20$ cm

Again applying $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{2u_2} - \frac{1}{u_2} = -\frac{1}{20}$$

$$\Rightarrow \quad u_2 = 10 \text{ cm}$$

So, the distance between two positions of the object is $u_1 - u_2$

$$\Rightarrow \quad u_1 - u_2 = 30 \text{ cm} - 10 \text{ cm} = 20 \text{ cm}$$

Hence, the correct answer is (D).

241. Since we know that a light ray after successive reflections from two mutually perpendicular mirrors reverses its direction. So the final reflected ray must have same slope as that of the given incident light ray. Hence only OPTION (D) can be the correct answer for a given set of mirrors.

Hence, the correct answer is (D).

242. $P = P_1 + P_2$

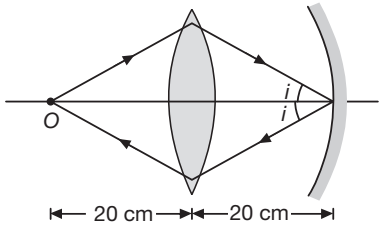
$$\Rightarrow \quad P = 10\ D$$

$$f = \frac{1}{P} = 0.1 \text{ m}$$

$$\Rightarrow \quad f = 10 \text{ cm}$$

Hence, the correct answer is (A).

243. Object is placed at a distance of $2f$ from the lens of focal length f i.e., the image formed by the lens will be at a distance of $2f$ or 20 cm from the lens. So, if the concave mirror is placed in this position, the first image will be formed at its pole and it will reflect all the rays symmetrically to other side as shown below and the final image will coincide with the object.



Hence, the correct answer is (B).

244. Shift $= d = t\left(1 - \frac{1}{n}\right)$

$$\Rightarrow \quad t = \frac{nd}{n-1}$$

Hence, the correct answer is (D).

245. Focal length of plano convex lens using Lens Maker's Formula is

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} - \frac{1}{\infty}\right)$$

$$\Rightarrow \quad f = 20 \text{ cm}$$

If point object O is placed at a distance of 20 cm from the plano convex lens rays become parallel and final image is formed at second focus or 20 cm from concave lens which is independent of y.

Hence, the correct answer is (A).

246. Focal length remains unchanged.

$I \propto$ Area of Aperture

$$I = kD^2$$

$$I' = k\left[D^2 - \left(\frac{D}{2}\right)^2\right]$$

$$\Rightarrow \quad I' = \frac{3I}{4}$$

Hence, the correct answer is (D).

247. $\lambda = \frac{v}{f}$

In moving from air to glass, f remains unchanged while v decreases. Hence, λ should decrease.

Hence, the correct answer is (B).

CHAPTER 1

248. $f = \dfrac{x}{m_1 - m_2}$

(See 'Displacement Method' in SYNOPSIS)

Hence, the correct answer is (B).

249. $f = \dfrac{D^2 - x^2}{4D}$

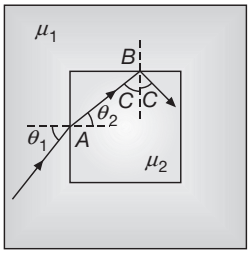
$\Rightarrow \quad f = \dfrac{(0.9)^2 - (0.2)^2}{4(0.9)}$

$\Rightarrow \quad f = 0.214 \text{ m}$

$\Rightarrow \quad f = 21.4 \text{ cm}$

Hence, the correct answer is (B).

250.



Applying Snell's Law at A, we get

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 \qquad \ldots(1)$$

and angle of incidence at B is

$$\theta_2 = 90° - C$$

for total internal reflection at top face we use

$$\theta_2 > C$$

$$\Rightarrow \quad \cos\theta_2 = \sin C = \frac{\mu_1}{\mu_2} \qquad \ldots(2)$$

Eliminating θ_2 between (1) and (2), we get

$$\sin\theta_1 = \sqrt{\left(\frac{\mu_2}{\mu_1}\right)^2 - 1}$$

Hence, the correct answer is (A).

251. When plane surface is silvered, we use

$$P_{eq} = 2P_L = 2\left(\frac{\mu - 1}{R}\right)$$

Then in case when curved surface is silvered, we use

$$P_{eq}' = 2P_L + P_m$$

$$\Rightarrow \quad P_{eq}' = 2\left(\frac{\mu - 1}{R}\right) + \frac{2}{R} = \frac{2\mu}{R}$$

Since, $f_{eq} = 28$ cm

$$\Rightarrow \quad 28 = \frac{R}{2(\mu - 1)} \qquad \ldots(1)$$

When curved surface is silvered, we have

$$f_{eq}' = 10 = \frac{R}{2\mu} \qquad \ldots(2)$$

From Equations (1) and (2) we get,

$$\mu = \frac{14}{9}$$

Hence, the correct answer is (B).

253. $\dfrac{1}{F} = \dfrac{2}{f_\ell} + \dfrac{1}{f_m}$

$$\Rightarrow \quad \frac{1}{F} = \frac{2}{f_\ell} + \frac{1}{\infty}$$

$$\Rightarrow \quad F = \frac{f_\ell}{2}$$

Also

$$\frac{1}{f_\ell} = (\mu - 1)\left(\frac{1}{-r} - \frac{1}{\infty}\right)$$

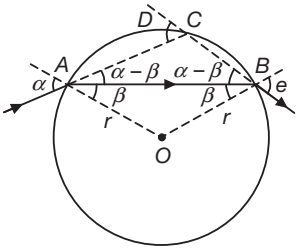
$$\Rightarrow \quad f_\ell = \frac{-r}{\mu - 1}$$

$$\Rightarrow \quad F = -\frac{r}{2(\mu - 1)}$$

Negative sign indicates that mirror is concave.

Hence, the correct answer is (B).

254.



Since angles opposite to equal sides are equal. So, in ΔAOB we have $\angle ABO = \beta$. In ΔCAB, external angle equals the sum of internal opposite angles. So,

$$D = (\alpha - \beta) + (\alpha - \beta)$$

$$D = 2(\alpha - \beta)$$

Hence, the correct answer is (B).

255. When magnification is $m = +2$, then we have

$$u = -x$$

$$v = -2x$$

$$f = +20$$

Applying Mirror Formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$, we get

$$\frac{1}{-2x}+\frac{1}{x}=\frac{1}{20}$$

$\Rightarrow \quad x = 10$ cm

To have a magnification of $m' = -2$, we have

$u = -y$

$v = +2y$

and $f = +20$

$$\Rightarrow \quad \frac{1}{2y}+\frac{1}{y}=\frac{1}{20}$$

$\Rightarrow \quad y = 30$ cm

So, the object has to be moved by

$y - x = 20$ cm

Hence, the correct answer is (C).

256. When water is filled in the mirror, a plano convex lens is formed, so now combination contains a plano-convex lens and a mirror. The effective focal length of combination is less than the focal length of above mirror, so image is shifted downwards.
Hence, the correct answer is (D).

258. The given lens is a convex lens. Let the magnification be m, then for real image, we have

$$\frac{1}{mx}+\frac{1}{x}=\frac{1}{f} \qquad \ldots(1)$$

and for virtual image, we have

$$\frac{1}{-my}+\frac{1}{y}=\frac{1}{f} \qquad \ldots(2)$$

From equation (1) and (2), we get

$$f=\frac{x+y}{2}$$

Hence, the correct answer is (D).

259. Using refraction formula for spherical surface (taking B as object, if its image is formed ∞), we have

$$\frac{\mu_2}{\infty}-\frac{\mu_1}{(-2R)}=\frac{\mu_2-\mu_1}{-R}$$

$$\Rightarrow \quad \frac{\mu_1}{\mu_2}=2$$

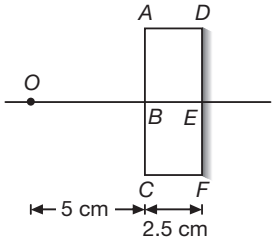
Hence, the correct answer is (A).

261. Let I_1, I_2 and I_3 be the images formed due to

(i) refraction from ABC

(ii) reflection from DEF and

(iii) again refraction from ABC



Then $BI_1 = (5)\mu_g = (5)(1.5) = 7.5$ cm

Now $EI_1 = (7.5 + 2.5) = 10$ cm

$\Rightarrow \quad EI_2 = 10$ cm behind the mirror

Now $BI_2 = (10 + 2.5) = 12.5$ cm

$$\Rightarrow \quad BI_3=\frac{12.5}{\mu_g}=\frac{12.5}{1.5}=\frac{25}{3} \text{ cm}$$

Hence, the correct answer is (B).

262. $f=\dfrac{D^2-x^2}{4D}$ (By Displacement Method)

$$\Rightarrow \quad P=\frac{4D}{D^2-x^2}$$

$$\Rightarrow \quad P=\frac{4(1)}{1-0.16}$$

$$\Rightarrow \quad P=\frac{4}{0.84}$$

$\Rightarrow \quad P = 4.76$ D

Hence, the correct answer is (C).

263. $\text{Shift} = d\left(1-\dfrac{1}{\mu}\right)=7\left(1-\dfrac{1}{1.4}\right)$

$\Rightarrow \quad \text{Shift} = 2$ cm (downwards)

Hence, the correct answer is (A).

264. $\dfrac{1}{F}=\dfrac{2}{f_\ell}+\dfrac{1}{f_m}$

$$\frac{1}{f_\ell}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f_\ell}=(1.5-1)\left(\frac{1}{20}-\frac{1}{60}\right)$$

$\Rightarrow \quad f_\ell = 60$ cm and

$$f_m=\frac{20}{2}=10 \text{ cm}$$

$$\Rightarrow \quad \frac{1}{F}=\frac{2}{60}+\frac{1}{10}$$

$\Rightarrow \quad F = 7.5$ cm

CHAPTER 1

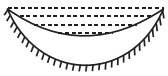
For image to be formed at the same place where object is situated we have $u = 2F = 15$ cm

Hence, the correct answer is (A).

265. $\frac{1}{F} = \frac{2}{f_\ell} + \frac{2}{f_w} + \frac{1}{f_m}$

(Because here we have two refractions at the concave surface of lens and two refraction for water lens). Since

$$\frac{1}{f_w} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{60} - \frac{1}{\infty}\right)$$



$$\Rightarrow \quad f_w = 180 \text{ cm}$$

$$\Rightarrow \quad \frac{1}{F} = \frac{2}{60} + \frac{2}{180} + \frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{F} = \frac{13}{90} \text{ cm}$$

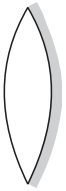
$$\Rightarrow \quad F = \frac{90}{13} \text{ cm}$$

So, required answer to get asked condition is 2F. Hence

$$u = \frac{180}{13} \text{ cm}$$

Hence, the correct answer is (D).

266. For equiconvex lens, we have



$$|R_1| = |R_2| = f = 10 \text{ cm}$$

Now $P = 2P_L + P_M$

$$\Rightarrow \quad -\frac{1}{F} = 2(\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - \frac{2}{R_2}$$

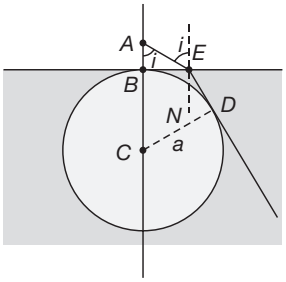
$$\Rightarrow \quad -\frac{1}{F} = 2(1.5 - 1)\left(\frac{1}{10} - \frac{1}{-10}\right) - \frac{2}{-10}$$

$$\Rightarrow \quad F = -2.5 \text{ cm}$$

Therefore, the system will behave like a concave mirror of focal length 2.5 cm.

Hence, the correct answer is (C).

267. The situation described in the problem is shown in the ray diagram.



Since, we are given that

$$\angle NED = 30°$$

$$\Rightarrow \quad \angle BED = 120°$$

Also, $BCDE$ is a cyclic quadrilateral

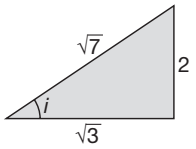
$$\Rightarrow \quad \angle BCD = 60°$$

The line CE will be angle bisector of $\angle BCD$

$$\Rightarrow \quad BE = a\tan(30°) = \frac{a}{\sqrt{3}}$$

Since $\tan i = \frac{BE}{AB} = \frac{a\sqrt{3}}{\frac{a}{2}} = \frac{2}{\sqrt{3}}$

$$\Rightarrow \quad \sin i = \frac{2}{\sqrt{7}}$$



Using Snell's law, we get

$$1\sin i = n\sin r$$

$$\Rightarrow \quad \frac{2}{\sqrt{7}} = n \times \frac{1}{2}$$

$$\Rightarrow \quad n = \frac{4}{\sqrt{7}}$$

Hence, the correct answer is (D).

268. $\frac{1}{F} = \frac{2}{f_\ell} + \frac{1}{f_m}$

Since $f_m \to \infty$

$$\Rightarrow \quad F = \frac{f_\ell}{2} = 15 \text{ cm}$$

Further

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{(-40)}=\frac{1}{15}$$

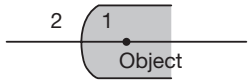
$$\Rightarrow \quad \frac{1}{v}=\frac{1}{15}-\frac{1}{40}$$

$$\Rightarrow \quad v=24 \text{ cm}$$

Hence, the correct answer is (B).

269. Applying $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$, we get

$$\frac{1}{v}-\frac{1.5}{(-u)}=\frac{1-(1.5)}{-R}$$



$$\Rightarrow \quad \frac{1}{v}+\frac{3}{2u}=\frac{1}{2R}$$

For v to be positive, we have

$$\frac{1}{2R}>\frac{3}{2u}$$

$$\Rightarrow \quad u>3R$$

Hence, the correct answer is (D).

270. $f_{\text{combination}} \to \infty$

Since $\frac{1}{f_{\text{combination}}}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1 f_2}$

$$\Rightarrow \quad 0=\frac{1}{-10}+\frac{1}{f_2}-\frac{10}{(-10)f_2}$$

$$\Rightarrow \quad 0=\frac{1}{-10}+\frac{1}{f_2}+\frac{1}{f_2}$$

$$\Rightarrow \quad \frac{1}{10}=\frac{2}{f_2}$$

$$\Rightarrow \quad f_2=20 \text{ cm}$$

Hence, the correct answer is (B).

271. $\frac{f}{f+(-12)}=-\frac{f}{f+(-20)}$

$$f-12=-(f-20)$$

$$f-12=-f+20$$

$$2f=32$$

$$f=16 \text{ cm}$$

272. $5=(0.5)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\Rightarrow \quad \frac{1}{R_1}-\frac{1}{R_2}=10$$

$$-1=\left({}^{\ell}\mu_g-1\right)10$$

$$\Rightarrow \quad \left(\frac{\mu_g}{\mu_\ell}-1\right)=-0.1$$

$$\Rightarrow \quad \frac{1.5}{\mu_\ell}=0.9$$

$$\Rightarrow \quad \mu_\ell=\frac{1.5}{0.9}=\frac{5}{3}$$

Hence, the correct answer is (C).

273. $|u|+|v|=D$

$$m=\frac{v}{u}$$

$$\Rightarrow \quad v=mu$$

$$|u|+m|u|=D$$

$$\Rightarrow \quad |u|=\frac{D}{m+1}$$

$$\Rightarrow \quad u=-\left(\frac{D}{m+1}\right)$$

Since image formed is real, so it must be on positive side. Hence

$$|v|=+v=\frac{mD}{m+1}$$

For a lens

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{m+1}{mD}+\frac{m+1}{D}=\frac{1}{f}$$

$$\Rightarrow \quad f=\frac{mD}{(m+1)^2}$$

Hence, the correct answer is (D).

274. $\frac{1}{f}=(1.5-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)$

$$\Rightarrow \quad \frac{1}{16}=0.5\ \frac{1}{R}$$

$$\Rightarrow \quad R=8 \text{ cm}$$

Hence, the correct answer is (A).

275. Since $m=\frac{f}{f+u}$

$$\Rightarrow \quad -1=\frac{f}{f+u}$$

$$\Rightarrow \quad \frac{1}{f}=-\frac{1}{10}$$

$$\Rightarrow \quad f=-10 \text{ cm}$$

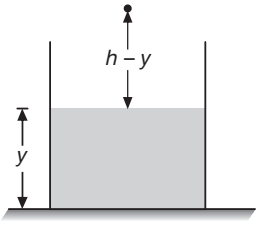
CHAPTER 1

Negative sign indicates diverging nature of lens in the liquid.

Hence, the correct answer is (B).

278. Since, we have, at time t

$$d = y + \mu(h - y) = \mu h - (\mu - 1)y$$



If A is the area of the tank, then we have

$$y = \frac{\alpha t}{A} \qquad \{\because Ay = \alpha t\}$$

$$\Rightarrow \quad d = \mu h - \frac{(\mu - 1)\alpha t}{A}$$

i.e., $d - t$ graph is a straight line with negative slope and positive intercept. But d becomes constant once $y = H$.

Hence, the correct answer is (C).

279. On immersing in water f increases and hence P decreases.

Hence, the correct answer is (B).

280. $$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{0.5} - \frac{1}{-0.5}\right)$$

$$\Rightarrow \quad \frac{1}{f} = 0.5\left(\frac{2}{0.5}\right)$$

$$\Rightarrow \quad P = 2\ D$$

Hence, the correct answer is (D).

281. $O = \sqrt{I_1 I_2}$

$O = \sqrt{4 \times 16} = 8$ cm

Hence, the correct answer is (A).

282. For a lens making real image

$$-m = \frac{f}{f + u}$$

$$\Rightarrow \quad -mf - mu = f$$

$$\Rightarrow \quad -mu = f(1 + m)$$

$$\Rightarrow \quad u = -f\left(1 + \frac{1}{m}\right)$$

$$\Rightarrow \quad |u| = f\left(1 + \frac{1}{m}\right)$$

For $|u|$ to be MINIMUM m must be MAXIMUM i.e. $m \to \infty$

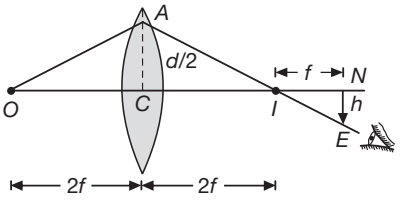
$$\Rightarrow \quad |u|_{\min} = f$$

For $|u|$ to be MAXIMUM m must be MINIMUM i.e. $m = 1$

$$\Rightarrow \quad |u|_{\max} = 2f$$

Hence, the correct answer is (A).

283. The ray diagram is as shown in figure



Since triangles CAI and NEI are similar, so we have

$$\frac{h}{d/2} = \frac{f}{2f}$$

$$\Rightarrow \quad \frac{2h}{d} = \frac{1}{2}$$

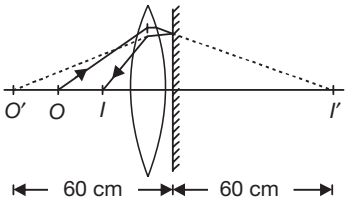
$$\Rightarrow \quad h = \frac{d}{4}$$

Hence, the correct answer is (A).

287. If mirror would have been absent then image is formed on the other side of lens (at I').
So

$$\frac{1}{v} - \frac{1}{-20} = \frac{1}{15}$$

$$\Rightarrow \quad v = 60 \text{ cm}$$



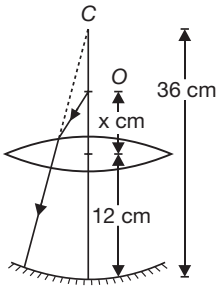
Since the mirror reflects the ray back, so O' serves as a virtual object and forms a real image I in front of mirror.

$$\frac{1}{v} + \frac{1}{-60} = \frac{1}{15}$$

$\Rightarrow \quad v = +12$ cm

Hence, the correct answer is (A).

288. Rays from O must fall normally on the mirror, only then the lens forms a virtual image at C (the centre of curvature).



$$\Rightarrow \quad \frac{1}{x} + \frac{1}{-(36-12)} = \frac{1}{40}$$

$\Rightarrow \quad x = 15$ cm

Hence, the correct answer is (B).

289. Let the length of rod be a.

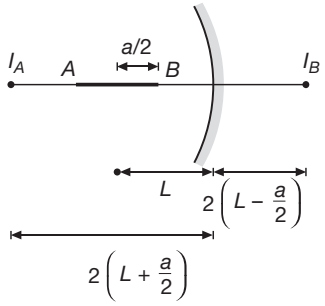
The object has a velocity V but the velocity of image of the end A and B is $4V$. Since

$$(\text{Image Velocity}) = m^2 (\text{Object Velocity})$$

$\Rightarrow \quad 4V = m^2 V$

$\Rightarrow \quad m = \pm 2$

This simply means that the magnitude of transverse magnification of images of ends A and B is 2 each. However, the image of B is virtual and that of A is real.



Applying mirror formula to end A, we get

$$-\frac{1}{2\left(L-\frac{a}{2}\right)} - \frac{1}{\left(L-\frac{a}{2}\right)} = -\frac{1}{f} \qquad \ldots(1)$$

Applying mirror formula to B, we get

$$-\frac{1}{2\left(L+\frac{a}{2}\right)} - \frac{1}{\left(L+\frac{a}{2}\right)} = -\frac{1}{f} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$a = L$

Hence, the correct answer is (C).

290. $$\frac{1}{F} = \frac{1}{(1/2)} + \frac{1}{(1/3)} = 5$$

$\Rightarrow \quad F = 0.2$ m $= 20$ cm

Since

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{(-30)} = \frac{1}{20}$$

$\Rightarrow \quad v = 60$ cm

Hence, the correct answer is (D).

291. $P_{\text{comb}} = P_1 + P_2 - xP_1P_2$

$$0 = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

$\Rightarrow \quad x = f_1 + f_2$

Hence, the correct answer is (D).

293. Apparent Separation $= 2(\text{Apparent Depth})$

$$\Rightarrow \quad \text{Apparent Separation} = \frac{2h}{\mu}$$

Hence, the correct answer is (D).

294. For distant vision

$u = -\infty$

$$\Rightarrow \quad \frac{1}{-2} - \frac{1}{-\infty} = \frac{1}{f}$$

$\Rightarrow \quad f = -2$ m

$\Rightarrow \quad P = -0.5\ D$

For near vision

$u = -D = -25$ cm

$$\Rightarrow \quad \frac{1}{-1} - \frac{1}{-0.25} = \frac{1}{f}$$

$\Rightarrow \quad P = +3\ D$

Hence he must use bifocal lenses with

$P = -0.5\ D$ and additional + 3.5 D

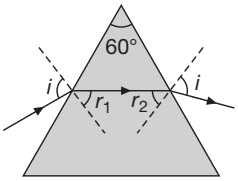
(to give $+3\ D$ net)

Hence, the correct answer is (A).

295. The slab does not affect the minimum deviation by prism. Thus for minimum deviation, we use

$r_1 = r_2 = 30°$ as shown in figure

CHAPTER 1



$\Rightarrow \quad \sin i = \sqrt{2}\sin(30°)$

$\Rightarrow \quad i = 45°$

Thus minimum deviation is

$2i - A = 90° - 60° = 30°$

$\Rightarrow \quad \sin i = \sqrt{2}\sin(30°)$

$\Rightarrow \quad i = 45°$

Hence, the correct answer is (A).

297. In the first case. Let x be the distance of object from the mirror. Then

$u = -x$, $v = +2x$ and $f = -f$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{2x}-\frac{1}{x}=-\frac{1}{f}$$

$$\Rightarrow \quad x=\frac{f}{2}$$

In the second case, let y be the distance of object from the mirror. Then

$u = -y$, $v = -2y$ and $f = -f$

$$\Rightarrow \quad \frac{1}{-2y}-\frac{1}{y}=-\frac{1}{f}$$

$$\Rightarrow \quad y=\frac{3}{2}f$$

So, object will have to be moved by a distance of $y - x = f$.

Hence, the correct answer is (C).

298. $\frac{f_0}{f_e} = -5$

Separation $= f_0 + f_e$

$36 = -4f_e$

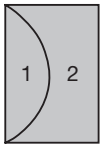
$\Rightarrow \quad f_e = -9$ cm

$\Rightarrow \quad f_0 = 45$ cm

Hence, the correct answer is (A).

299. For lenses placed in contact, we have

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$



$$\Rightarrow \quad \frac{1}{F}=(\mu_1-1)\left(\frac{1}{\infty}+\frac{1}{R}\right)+(\mu_2-1)\left(\frac{1}{-R}-\frac{1}{\infty}\right)$$

$$\Rightarrow \quad \frac{1}{F}=\frac{\mu_1-\mu_2}{R}$$

$$\Rightarrow \quad F=\frac{R}{\mu_1-\mu_2}$$

Hence, the correct answer is (D).

300. $v_e = -D = -25$ cm, $f_e = 6.25$ cm

Since

$$\frac{1}{f_e}=\frac{1}{v_e}+\frac{1}{u_e}$$

$$\Rightarrow \quad \frac{1}{u_e}=\frac{1}{f_e}-\frac{1}{v_e}$$

$$\Rightarrow \quad \frac{1}{u_e}=\frac{1}{6.25}-\frac{1}{(-25)}$$

$\Rightarrow \quad u_e = 5$ cm

$\Rightarrow \quad v_0 = 15 - 5 = 10$ cm

Also, $f_0 = 2$ cm and

$$\frac{1}{f_0}=\frac{1}{v_0}-\frac{1}{u_0}$$

$$\Rightarrow \quad \frac{1}{2}=\frac{1}{10}-\frac{1}{u_0}$$

$\Rightarrow \quad u_0 = -2.5$ cm

Hence, the correct answer is (B).

301. $m = m_0 \times m_e = \frac{v_0}{u_0}\times\frac{v_e}{u_e}=\frac{10}{2.5}\times\frac{25}{5}$

$\Rightarrow \quad m = 20$

Hence, the correct answer is (C).

302. $(\text{Shift})_1 = t\left(1-\frac{1}{\mu_g}\right)=9\left(1-\frac{1}{3/2}\right)=3$ cm

$$(\text{Shift})_2 = t\left(1-\frac{\mu_w}{\mu_g}\right)=9\left(1-\frac{8}{9}\right)=1 \text{ cm}$$

So, distance between two images is given by

$(\text{Shift})_1 - (\text{Shift})_2 = 2$ cm

Hence, the correct answer is (B).

303. Both blocks loose contact immediately after the release because the springs are different and net forces on the two blocks are also not equal. The time period of oscillations of the two blocks is given by

$$T_P = 2\pi\sqrt{\frac{m}{4K}} \text{ and } T_Q = 2\pi\sqrt{\frac{m}{K}}$$

$$\Rightarrow \quad T_Q = 2T_P$$

Now, Q comes at lowest position at time $\frac{T_Q}{2}$ after travelling a distance $\frac{2mg}{K}$ downwards. In this time $\frac{T_Q}{2}$, which is also the time period of block P, the block P will come back to original position.
So, the distance between Q and its image will be

$$x = \frac{2mg}{K} \times 2 = \frac{4mg}{K}$$

Hence, the correct answer is (B).

304. $m = m_O \times m_e = m_O\left(1 + \frac{D}{f_e}\right)$

$$\Rightarrow \quad m_O = \frac{m}{\left(1 + \frac{D}{f_O}\right)}$$

$$\Rightarrow \quad m_O = \frac{30}{1+5} = 5$$

Hence, the correct answer is (A).

305. $m = 1 + \frac{D}{f}$

$$\Rightarrow \quad m = 1 + \frac{25}{5} = 6$$

Hence, the correct answer is (C).

306. $m = \frac{f_o}{f_e} = 10$

$$\Rightarrow \quad f_o = 50 \text{ cm}$$

Separation $= f_o + f_e = 55$ cm

Hence, the correct answer is (C).

307. $m = \frac{f_o}{f_e} = 5$

Hence, the correct answer is (A).

308. Separation $= f_0 + f_e$

Separation $= 0.3 + 0.05 = 0.35$ m

Hence, the correct answer is (A).

310. For an equilateral prism, we have

$$A = 60°$$

Since, the ray inside the prism is parallel to its base, so we have the condition of minimum deviation. So, $i = e = 60°$ and $\delta_m = i + e - A = (60° + 60°) - 60° = 60°$

Since, $\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$$\Rightarrow \quad \mu = \frac{\sin(60°)}{\sin(30°)} = \sqrt{3}$$

Hence, the correct answer is (D).

311. $\frac{f_0}{f_e}$ must be maximum

Hence, the correct answer is (A).

313. There is no effect on the spot. Rotating the glass slab will shift a ray parallel to axis. The direction of a ray before and after the glass rotates is unaffected. All parallel rays are focussed at the focal point of the lens according to ray optics, and there is no effect on the focussed spot.
Hence, the correct answer is (A).

315. Linear magnification $m = \frac{-2 \text{ cm}}{1 \text{ cm}} = -2$

So, the image is real and inverted

$$|v| = 2|u|$$

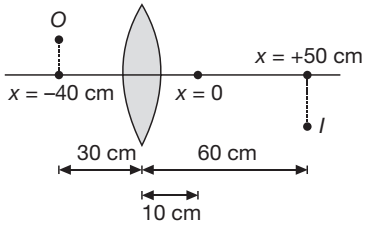
Let $|u| = x$ then $|v| = 2x$

Now $|u| + |v| = 50 - (-40) = 90$

$$\Rightarrow \quad x + 2x = 90$$

$$\Rightarrow \quad x = 30 \text{ cm}$$

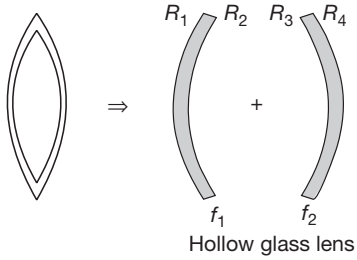
So, the distance of object from the lens is 30 cm and of object is 60 cm, i.e., the lens must be located at $x = -10$ cm as shown in figure.



Hence, the correct answer is (B).

318. Hollow convex lens is as shown in figure. Applying Lens Maker's Formula, we get

CHAPTER 1



Hollow glass lens

$$\frac{1}{f_1}=(\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=0 \qquad \{\because R_1=R_2\}$$

$\Rightarrow \quad f_1\to\infty$

Similarly, $f_2\to\infty$

So, a hollow, convex lens of any material will behave like a glass plate.

Hence, the correct answer is (D).

320. If distance of image is v from lens and $u\to-\infty$, so we use

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{f}+\frac{1}{f}+\left(\frac{2}{R_1}+\frac{2}{R_2}\right)$$

Because the ray of light passes through lens thrice and reflected twice from the two spherical surfaces of lens.

$$\Rightarrow \quad \frac{1}{v}=\frac{3}{f}+2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

By Lens Maker's formula, we have

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

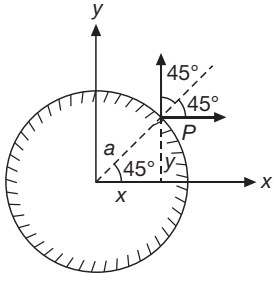
$$\Rightarrow \quad \frac{1}{v}=\left(\frac{1}{R_1}+\frac{1}{R_2}\right)(3\mu-3+2)$$

Since $\mu=1.5$

$$\Rightarrow \quad v=\frac{f(\mu-1)}{3\mu-1}=\frac{f}{7}$$

Hence, the correct answer is (A).

322. The ray diagram is as shown below



Since, $x=\frac{a}{\sqrt{2}}$

and $y=\frac{a}{\sqrt{2}}$

$$\Rightarrow \quad P\equiv\left(\frac{a}{\sqrt{2}},\frac{a}{\sqrt{2}}\right)$$

Hence, the correct answer is (D).

324. $$-\frac{f}{f-16}=\frac{f}{f-6}$$

$$\Rightarrow \quad -f+16=f-6$$

$$\Rightarrow \quad 2f=22$$

$$\Rightarrow \quad f=11\text{ cm}$$

Hence, the correct answer is (C).

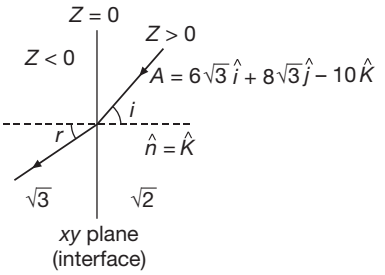
Multiple Correct Choice Type Questions

1. The normal to the interface is along $\hat{k}$.

$$\therefore \quad \cos i=\frac{\vec{A}\cdot(-\hat{k})}{|\vec{A}||\hat{k}|}=\frac{+10}{20}=+\frac{1}{2}$$

$$\Rightarrow \quad i=60^\circ$$

{i is the angle which incident ray makes with –Z-axis}



But according to Snell's Law

$$\sqrt{2}\sin 60=\sqrt{3}\sin r$$

$$\Rightarrow \quad \sin r=\frac{\sqrt{2}}{\sqrt{3}}\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \sin r=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad r=45^\circ$$

Hence, (B) and (C) are correct.

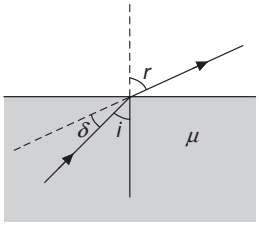
2. **The correct answer is (A).**

3. **The correct answer is (A, D).**

Combined Solution to 2 & 3

For $i<C$, no TIR will take place, so we have deviation (δ) given by

$\delta = r - i$



Now, according to Snell's Law, we have

$\mu \sin i = \sin r$

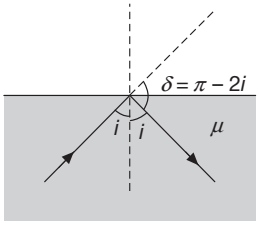
$\Rightarrow \quad r = \sin^{-1}(\mu \sin i)$

$\Rightarrow \quad \delta = \sin^{-1}(\mu \sin i) - i \qquad \ldots(1)$

This is a non-linear variation of δ with i. Also, we have δ to be maximum when $i = C$ and hence $r = \frac{\pi}{2}$. So

$$\delta_{max} = \delta_1 = \frac{\pi}{2} - C \qquad \ldots(2)$$

Further when $i > C$, then TIR takes place and the incident ray is reflected back in the denser medium as shown in the figure.



So, $\delta = \pi - 2i$

i.e., δ decreases linearly with i. So,

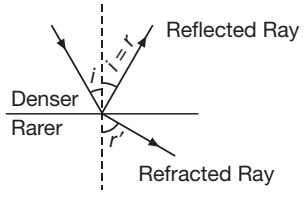
$$\delta_{max} = \delta_2 = \pi - 2C \qquad \ldots(3)$$

From (2) and (3), we get

$\delta_2 = 2\delta_1$

4. $\sin C = \frac{\mu_{rarer}}{\mu_{denser}} = \frac{\mu_r}{\mu_d}$

$i + r' = 90°$



According to Snell's Law

$\mu_d \sin i = \mu_r \sin(r')$

$\Rightarrow \quad \frac{\sin i}{\sin(r')} = \frac{\mu_r}{\mu_d} = \sin C$

$\Rightarrow \quad \frac{\sin i}{\sin(90 - i)} = \sin C$

$\Rightarrow \quad \sin C = \tan i$

$\Rightarrow \quad C = \sin^{-1}(\tan i)$

$\Rightarrow \quad C = \sin^{-1}(\tan r) \qquad \{\text{Because } i = r\}$

Hence, (A) and (B) are correct.

6. According to Snell's Law, we have

$\mu_1 \lambda_1 = \mu_2 \lambda_2$

$\Rightarrow \quad (1)(6000) = (1.5)\lambda_2$

$\Rightarrow \quad \lambda_2 = 4000 \text{ Å}$

Since frequency does not change when light goes from one medium to another, so

$v = \frac{c}{\lambda_1}$

$\Rightarrow \quad v = \frac{3 \times 10^8}{6000 \times 10^{-10}} = 5 \times 10^{14} \text{ Hz}$

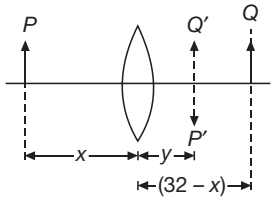
Hence, (A) and (C) are correct.

9. See Displacement Method.

Hence, (B), (C) and (D) are correct.

12. Since images are formed at the same place, So one image must be real and other must be virtual.

$|x| + |y| = 32$



For P

$\frac{1}{y} - \frac{1}{-x} = \frac{1}{15}$

$\Rightarrow \quad \frac{1}{y} + \frac{1}{x} = \frac{1}{15} \qquad \ldots(1)$

For Q

$\frac{1}{-y} - \frac{1}{-(32 - x)} = \frac{1}{15}$

CHAPTER 1

$$\Rightarrow \quad \frac{1}{-y}+\frac{1}{32-x}=\frac{1}{15} \qquad \ldots(2)$$

Adding (1) and (2), we get

$$\frac{1}{x}+\frac{1}{32-x}=\frac{2}{15}$$

$$\frac{32}{x(32-x)}=\frac{2}{15}$$

$$\Rightarrow \quad 32x-x^2=240$$

$$\Rightarrow \quad x^2-32x+240=0$$

$$\Rightarrow \quad x^2-20x-12x+240=0$$

$$\Rightarrow \quad x(x-20)-12(x-20)=0$$

$$\Rightarrow \quad (x-12)(x-20)=0$$

$$\Rightarrow \quad x=12 \text{ cm}, 20 \text{ cm}$$

Hence, (A) and (D) are correct.

13. $m=\dfrac{f-v}{f}$

$$\Rightarrow \quad m=1-\frac{v}{f}$$

$$\Rightarrow \quad \text{Slope}=-\frac{1}{f}=\frac{b}{c}$$

$$\Rightarrow \quad f=\frac{c}{b}$$

Also at $m=0$, $v=a$

$$\Rightarrow \quad 0=f-a$$

$$\Rightarrow \quad f=a$$

Hence, (B) and (C) are correct.

14. All are the consequences of the "DISPLACEMENT METHOD TO FIND FOCAL LENGTH OF A CONVEX LENS"

Hence, (A), (B), (C) and (D) are correct.

15. $\dfrac{1}{f}=\dfrac{1}{-15}+\dfrac{1}{30}$

$$\Rightarrow \quad f=-30 \text{ cm} \quad \text{(Diverging in nature)} \quad \{\text{OPTION (D)}\}$$

Since red deviates the least and violet deviates the maximum. So a coloured pattern with red on the outerside is observed.

Hence, (B) and (D) are correct.

16. Optical path length in passing from 1st medium is n_1s_1.

Optical path length in passing from 2nd medium is n_2s_2 and so on.

So total optical path length $=\sum_{i=1}^{m} n_is_i$ {OPTION (C)}

$$\Rightarrow \quad \text{Total time of flight } t=\frac{1}{c}\sum_{i=1}^{m} n_is_i \qquad \{\text{OPTION (A)}\}$$

For inhomogeneous media optical path length is $O.P.L.=\int_A^B n(s)ds$ and the ray must travel along a path in which time taken to go from A to B is minimum. Such paths are called stationary pathways and this is the statement of Fermat's Least Action Principle or Fermat's Principle of Least Time.

Hence, (A), (C) and (D) are correct.

17. The final image is formed at infinity when the combined focal length of the two lenses (in contact) is 30 cm i.e.,

$$\frac{1}{30}=\frac{1}{20}+\frac{1}{f}$$

$$f=-60 \text{ cm}$$

So, when another concave lens of focal length 60 cm is kept in contact with the first lens.

Similarly, if μ be the refractive index of a liquid in which focal length of the given convex lens becomes 30 cm. Then from Lens Maker's Formula, we have

$$\frac{1}{20}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \ldots(1)$$

$$\frac{1}{30}=\left(\frac{3/2}{\mu}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\mu=\frac{9}{8}$$

Hence, (A) and (D) are correct.

18. $m=+2$, means image is virtual, erect and magnified. A virtual and magnified image can be formed only by a concave mirror and that too when object lies between pole and focus.

Hence, (A) and (C) are correct.

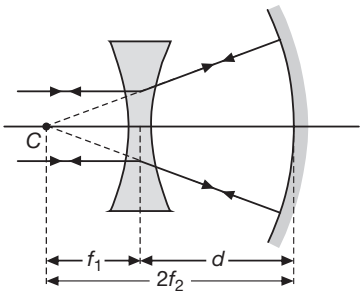
19. When passing from vacuum to a medium, frequency remains unchanged while speed and wavelength decreases μ times.

Hence, (B), (C) and (D) are correct.

20. For convex mirror (having positive focal length) the image is always smaller in size. For concave mirror (having negative focal length) the image is smaller when the object lies beyond $2f$.

Hence, (A), (B) and (C) are correct.

21.



Hence, (A) and (B) are correct.

22. The tube length of an astronomical telescope, in normal adjustment, is $(f_o + f_e)$ and that of Galilean telescope, in normal adjustment is $(f_o - f_e)$ where f_o and f_e are focal lengths of objective and eye piece respectively. In this case, $f_e = f$, so difference in tube lengths is

$$\ell_1 - \ell_2 = (f_o + f_e) - (f_o - f_e) = 2f_e = 2f$$

Hence, (B) and (C) are correct.

25. A concave mirror and a convex lens give virtual magnified image for a particular object position (i.e. when object lies between F and P (or C))

Hence, (B) and (C) are correct.

26. A concave mirror can give both real and virtual magnified images. Since nothing is specified, so

$$m = \pm 3$$

$$\Rightarrow \quad \pm 3 = \frac{-15}{-15 - u}$$

$$+u = -10 \text{ cm}$$

$$-u = -20 \text{ cm}$$

Hence, (B) and (D) are correct.

27. When the image is virtual, then

$$u = -x \text{ , so } v = +2x$$

From the mirror formula, we get

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{2x} - \frac{1}{x} = \frac{1}{-f}$$

$$\Rightarrow \quad x = \frac{f}{2}$$

When the image is real, then

$$u = -y \text{ , so } v = -2y$$

Again applying the mirror formula, we get

$$\frac{1}{-2y} - \frac{1}{y} = -\frac{1}{f}$$

$$\Rightarrow \quad \frac{3}{2y} = \frac{1}{f}$$

$$\Rightarrow \quad y = \frac{3}{2}f$$

Hence, (C) and (D) are correct.

28. When the object moves from infinity to the pole of the mirror, the virtual image moves from focus to the pole.
Hence, (A) and (C) are correct.

29. Since, for refraction at a plane surface, we have

$$\frac{\mu_1}{u} = \frac{\mu_2}{v} \qquad \ldots(1)$$

If x be the height of the bird above the water surface, then for the light travelling from the bird to the fish, we have

$$\mu_1 = 1, \ \mu_2 = \mu \text{ and } u = -x$$

So, from (1), we get

$$\frac{1}{(-x)} = \frac{\mu}{v}$$

$$\Rightarrow \quad v = -\mu x$$

$$\Rightarrow \quad |v| = \mu x$$

Now speed of the bird is $\frac{dx}{dt}$

So, apparent speed of the bird is

$$\left|\frac{dv}{dt}\right| = \mu \frac{dx}{dt}$$

$$\Rightarrow \quad \left|\frac{dv}{dt}\right| > \frac{dx}{dt} \qquad \{\because \mu > 1\}$$

Hence, (B) and (D) are correct.

30. TIR takes place when ray of light travels from denser to rarer medium.

Further, $\sin C_{12} = \frac{\mu_2}{\mu_1}$ and $\sin C_{13} = \frac{\mu_3}{\mu_1}$

Since, $\frac{\mu_2}{\mu_1} > \frac{\mu_3}{\mu_1}$

$$C_{12} > C_{13}$$

CHAPTER 1

Smaller the value of critical angle, more are the chances of TIR.

Hence, (A), (C) and (D) are correct.

31. Since, $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$

For $\mu = \sqrt{2}$ and $A = 60°$, we get

$$\delta_m = 30°$$

Further, at minimum deviation, we have

$$r_1 = r_2 = \frac{A}{2} = 30°$$

Applying Snell's Law, we get

$$\sin i_1 = \mu \sin r_1$$

$$\Rightarrow \quad \sin i_1 = (\sqrt{2})\sin(30°) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad i_1 = 45°$$

Hence, (A) and (C) are correct.

32. Using, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$, we get

$$\frac{1.5}{v} - \frac{1}{\infty} = \frac{1.5-1}{20}$$

$$\Rightarrow \quad v = +60 \text{ cm}$$

Since v is positive, the rays actually meet.

Hence, (B) and (C) are correct.

33. In air, $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$ …(1)

When immersed in a liquid of refractive index $\dfrac{\mu}{2}$,

$$\frac{1}{f_1} = \left(\frac{\mu}{\mu/2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f_1} = \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots(2)$$

From (1) and (2), we get

$$\frac{1}{f} = \frac{\mu - 1}{f_1}$$

$$\Rightarrow \quad f_1 = \frac{\mu - 1}{f}$$

When immersed in a liquid of refractive index 2μ,

$$\frac{1}{f_2} = \left(\frac{\mu}{2\mu} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f_2} = -\frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots(3)$$

From (1) and (3), we get

$$\frac{1}{f} = (\mu - 1)\left(-\frac{2}{f_2}\right)$$

$$\Rightarrow \quad f_2 = -\frac{2(\mu - 1)}{f}$$

Hence, the correct answer is (B).

34. When in contact, we have

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \quad 10 = \frac{1}{f_1} + \frac{1}{f_2} \qquad \left\{\because P = \frac{1}{F} = 10\right\}$$

$$\Rightarrow \quad f_1 + f_2 = 10 f_1 f_2 \qquad \ldots(1)$$

When at a separation of 0.25 m, we have

$$\frac{1}{F'} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

$$\Rightarrow \quad 6 = \frac{1}{f_1} + \frac{1}{f_2} - \frac{0.25}{f_1 f_2}$$

$$\Rightarrow \quad 6 = 10 - \frac{0.25}{f_1 f_2}$$

$$\Rightarrow \quad f_1 f_2 = \frac{1}{16} \qquad \ldots(2)$$

From (1), we get

$$f_1 + f_2 = \frac{10}{16} = \frac{5}{8}$$

Now, from (2) we get

$$f_1\left(\frac{5}{8} - f_1\right) = \frac{1}{16}$$

$$\Rightarrow \quad 16 f_1^2 - 10 f_1 + 1 = 0$$

$$\Rightarrow \quad 16 f_1^2 - 8 f_1 - 2 f_1 + 1 = 0$$

$$\Rightarrow \quad 8 f_1 (2 f_1 - 1) - 1(2 f_1 - 1) = 0$$

$$\Rightarrow \quad (8 f_1 - 1)(2 f_1 - 1) = 0$$

$$\Rightarrow \quad f_1 = \frac{1}{8} \text{ or } f_1 = \frac{1}{2}$$

$$\Rightarrow \quad P_1 = 8D \text{ or } P_1 = 2D$$

Hence, the correct answer is (B).

35. Since, $\frac{\mu_1}{-u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$

where $\mu_1=\mu$, $\mu_2=1$, $u=-R$, $R=-R$

$$\Rightarrow \quad \frac{1}{v}-\frac{\mu}{(-R)}=\frac{1-\mu}{-R}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{R}$$

$$\Rightarrow \quad v=R$$

Hence, (A) and (D) are correct.

36. Real image is smaller in size if object lies beyond $2f$ and it is larger if object lies between f and $2f$.
Hence, (A) and (D) are correct.

37. A ray can pass undeviated when $\mu_1=\mu_2$ or the ray is incident normally i.e., angle of incidence is $0°$.
Hence, (A) and (C) are correct.

38. Applying Lens Maker's Formula, we get

$$\frac{1}{f_{\text{air}}}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_{\text{water}}}=\left(\frac{3/2}{4/3}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

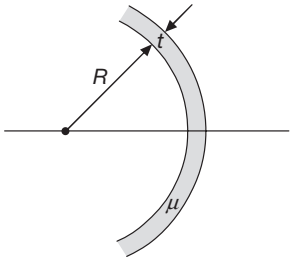
From these two equations, we get

$$f_{\text{water}}=4f_{\text{air}}=4f$$

In air, the image was inverted, real and magnified i.e., the object must be lying between f and $2f$. Now the focal length has changed to $4f$. Therefore, the object now lies between pole and focus and so the new image formed will be virtual and magnified.

Hence, (A) and (C) are correct.

40. Let t be the thickness of the watch glass, R be the radius of curvature of inner surface, so $(R+t)$ is radius of curvature of outer surface. If μ be the refractive index of this thin lens (watch glass), from Lens Maker's Formula, we get



$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

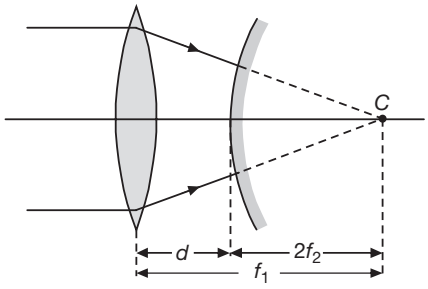
$$\Rightarrow \quad \frac{1}{v}=\frac{1}{f}=(\mu-1)\left[\frac{1}{-R}-\frac{1}{-(R+t)}\right]$$

$$\Rightarrow \quad \frac{1}{f}=(\mu-1)\left(\frac{1}{R+t}-\frac{1}{R}\right)=-\frac{(\mu-1)t}{R(R+t)}<0$$

So, the watch glass will have a diverging nature.

Hence, the correct answer is (C).

41.



Hence, (A) and (B) are correct.

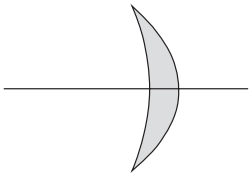
42. A concave mirror can give a virtual image (object lies between F and P) but the image is always magnified and a convex mirror can never give a real image.

Hence, (B) and (C) are correct.

43. Since, the lights used are of different colours, so they have different frequencies and hence (B) is also correct.

Hence, (A), (B) and (C) are correct.

44. $\frac{1}{f}=(\mu-1)\left(\frac{1}{-2R}-\frac{1}{-R}\right)$



$$\Rightarrow \quad \frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{2R}\right)$$

$$\Rightarrow \quad \frac{1}{f}=\frac{\mu-1}{2R}$$

$$\Rightarrow \quad f=\frac{2R}{\mu-1}$$

The focal length of lens has nothing to do with the direction from which the light is incident on it.

Hence, (B) and (C) are correct.

49. For a concave mirror, when object is placed at $2F$ (or C) real image of the same size as that object is formed at $2F$ (or C).

CHAPTER 1

$$\Rightarrow \quad m_{\text{concave}} = \frac{-f}{-f-(-2f)} = -1$$

For convex mirror

$$\Rightarrow \quad m_{\text{convex}} = \frac{f}{f-(-2f)} = \frac{1}{3}$$

Hence, (B) and (C) are correct.

Reasoning Based Questions

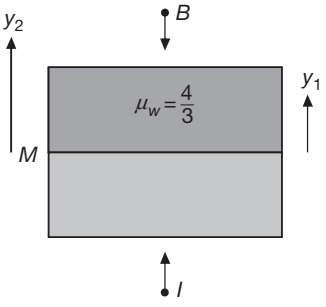
1. Lateral displacement, $\Delta x = t\sin i\left(1-\frac{1}{\mu}\right) < t$.

Hence, the correct answer is (D).

2. Both Statements are true but Statement-2 is not the correct explanation to Statement-1.

Hence, the correct answer is (B).

6.



$$\frac{y_{I/M}}{\frac{4}{3}} = \frac{y_2 - y_1}{1} + \frac{y_1}{\frac{4}{3}}$$

$$\Rightarrow \quad y_{I/M} = y_1 + \frac{4}{3}(y_2 - y_1) \neq y_2$$

But $y_{I/B} = 2\left[\frac{3y_1}{4} + (y_2 - y_1)\right]$

$$\Rightarrow \quad \frac{dy_{I/B}}{dt} = 2\frac{dy_2}{dt} = 2v_0$$

Hence, the correct answer is (C).

7. When the object is virtual, a real image can be formed by a plane or convex mirror.

Hence, the correct answer is (D).

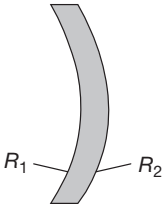
8. Both Statements are correct but Statement-2 is not correct explanation of Statement-1.

Hence, the correct answer is (B).

10. For a mirror, $m = \frac{f}{f-u}$.

Hence, the correct answer is (D).

11.



According to Lens Maker's Formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Since, $R_1 = R_2$

$$\Rightarrow \quad \frac{1}{f} = 0$$

So, power $= \frac{1}{f} = 0$

Hence, the correct answer is (A).

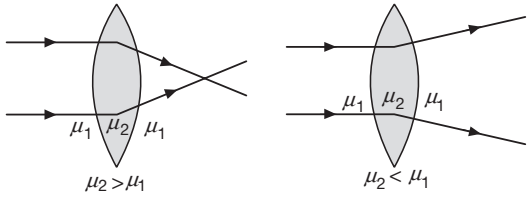
16. When light goes from one medium to another, its frequency remains unchanged.

Hence, the correct answer is (B).

19. Both the Statements are true, and Statement-2 is the correct explanation to Statement-1.

Hence, the correct answer is (A).

20. Statement-1 is true & Statement-2 is false.



Hence, the correct answer is (C).

21. $A = 60°$

$\mu = \sqrt{2}$

$$i = \frac{A + D_m}{2} = \frac{60 + 30}{2} = 45°$$

Since, $r_2 = \frac{A}{2} = 30°$

So, According to Snell's Law, we have

$$\frac{\sin i}{\sin r} = \frac{\sin(45°)}{\sin(30°)} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \sqrt{2}$$

Both Statement-1 and Statement-2 are true but Statement-2 is not the correct explanation of Statement-1.

Hence, the correct answer is (B).

22. Both Statements are true and Statement-2 is correct explanation to Statement-1.

Hence, the correct answer is (A).

23. Both Statements are true and Statement-2 is correct explanation of Statement-1.

Hence, the correct answer is (B).

24. Statement-1 false and Statement-2 is true.

Hence, the correct answer is (D).

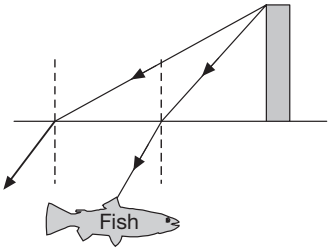
25. When the object is virtual, the convex mirror will give a real image.

Hence, the correct answer is (D).

26. After refraction at two parallel faces of a glass slab, a ray of light emerges in a direction parallel to the direction of incidence of white light on the slab. As rays of all colours emerge in the same direction (of incidence of white light), hence there is no dispersion, but only lateral displacement.

Hence, the correct answer is (B).

28.



Hence, the correct answer is (C).

29. Angle of incidence at any location should be greater than critical angle.

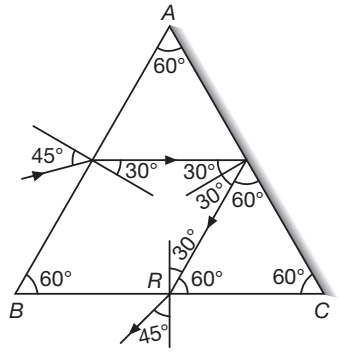
Hence, the correct answer is (C).

30. In search light, we need intense parallel beam of light. When source of light is placed at focus of concave mirror, only paraxial rays are rendered parallel due to large aperture of mirror. Marginal rays give a divergent beam but in case of parabolic mirror, when source is at focus, beam of light produced over the entire cross-section of mirror is a parallel beam.

Hence, the correct answer is (C).

Linked Comprehension Type Questions

1. Different angles by geometry and the given conditions are shown in figure.



$$\mu = \frac{\sin 45^\circ}{\sin 30^\circ} = \sqrt{2}$$

Hence, the correct answer is (B).

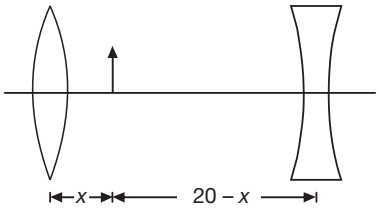
2. $S_{\text{Total}} = S_P + S_Q + S_R$

$$S_{\text{Total}} = (45^\circ - 30^\circ) + (180^\circ - 2\times 30^\circ) + (45^\circ - 30^\circ)$$

$$\Rightarrow \quad S_{\text{Total}} = 150^\circ$$

Hence, the correct answer is (C).

3. Image formed by concave lens is virtual for all positions of object i.e., image by concave lens lies between the two lenses. For both the images to coincide, image by convex lens should also lie in between the two lenses the two lens or image by convex lens should also be virtual.



For convex lens:

$$\frac{1}{-v} - \frac{1}{-x} = \frac{1}{20}$$

$$\Rightarrow \quad \frac{1}{x} - \frac{1}{v} = \frac{1}{20} \qquad \ldots(1)$$

For concave lens:

$$\frac{1}{-(20-v)} - \frac{1}{-(20-x)} = \frac{1}{-10}$$

$$\Rightarrow \quad \frac{1}{20-x} - \frac{1}{20-v} = \frac{1}{-10}$$

CHAPTER 1

Solving this equation, we get

$$x = \frac{20(\sqrt{3}-1)}{\sqrt{3}} \text{ cm} \qquad \ldots(2)$$

$$\Rightarrow \quad v = 20(\sqrt{3}-1) \text{ cm}$$

Hence, the correct answer is (B).

4. Magnification by convex lens is given by

$$m_1 = \frac{v_1}{u_1} = \frac{-v}{-x} = \frac{v}{x} = \frac{20(\sqrt{3}-1)}{20(\sqrt{3}-1)/\sqrt{3}} = \sqrt{3}$$

Magnification by concave lens is given by

$$m_2 = \frac{v_2}{u_2} = \frac{-(20-v)}{-(20-x)} = \frac{20-v}{20-x}$$

$$m_2 = \frac{40-20\sqrt{3}}{20-\dfrac{20(\sqrt{3}-1)}{\sqrt{3}}} = (2\sqrt{3}-3)$$

Hence, the correct answer is (D).

8. Since refractive index decreases with increase of λ and velocity decreases with increase of refractive index, so we have

$$v_{red} > v_{orange} > v_{yellow}$$

Hence, the correct answer is (A).

9. Colour and frequency remain unchanged as it is property of source.
Hence, the correct answer is (D).

10. Dispersion depends on wavelength
Hence, the correct answer is (B).

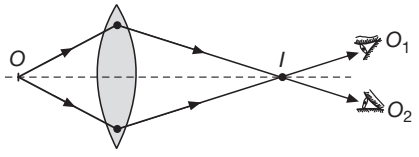
11. Since light takes shortest path, so

$$t = \frac{nx_0}{c}$$

Hence, the correct answer is (D).

12. Speed of light changes with refractive index.
Hence, the correct answer is (C).

13. Since, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$



$$\Rightarrow \quad \frac{1}{v} + \frac{1}{20} = \frac{1}{15}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{15} - \frac{1}{20}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{60}$$

$$\Rightarrow \quad v = 60 \text{ cm}$$

Hence, the correct answer is (D).

14. Observer O_1 cannot see the image because light will be absorbed by the blackened portion of the lens.
Hence, the correct answer is (A).

16. According to Snell's Law, we have

$$\frac{\sin i}{\sin r} = \frac{\mu_1}{\mu_2} = K \; i = 60°$$

$$\Rightarrow \quad \sin(60°) = K_1 \; r = 90°$$

$$\Rightarrow \quad K_1 = \frac{\sqrt{3}}{2}$$

Hence, the correct answer is (D).

17. $\theta_1 = \dfrac{\pi}{6}$

Hence, the correct answer is (C).

18. Since, $\theta = 0°$

$$\Rightarrow \quad i = r$$

$$\Rightarrow \quad K_2 = 1$$

Hence, the correct answer is (A).

19. Since, $f = R - \frac{R}{2}\sec\theta$

So, for $\theta > 0$, $f < \frac{R}{2}$

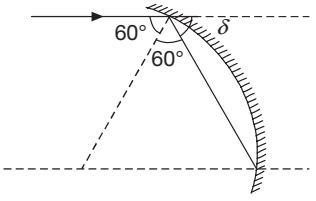
$$\Rightarrow \quad f_p < f_m$$

Hence, the correct answer is (C).

20. $f_m = R - \frac{R}{2}\sec(60°) = 0$

Hence, the correct answer is (D).

21.



$$\delta = 180 - 120 = 60°$$

Laws of reflection is universally true
Hence, the correct answer is (A).

22. $A = \sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + (-10)^2}$

$A = 20$ units
Hence, the correct answer is (D).

23. $\cos i = \dfrac{\vec{A}\cdot(-\hat{k})}{|\vec{A}|} = \dfrac{\left(6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k}\right)\cdot(-\hat{k})}{20}$

$\cos i = \dfrac{10}{20} = \dfrac{1}{2};\ i = 60°$

Hence, the correct answer is (C).

24. $\dfrac{\sin i}{\sin r} = \dfrac{\sqrt{3}}{\sqrt{2}},\ \sin r = \dfrac{\sqrt{2}}{\sqrt{3}}\sin i$

$\dfrac{\sin i}{\sin r} = \dfrac{\sqrt{2}}{\sqrt{3}}\sin 60° = \dfrac{\sqrt{2}}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad r = 45°$

Hence, the correct answer is (B).

25 Let the vector representing the refracted ray be

$\vec{A}' = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} + C\hat{k}$

Since, $\cos r = \dfrac{\vec{A}'\cdot(-\hat{k})}{|\vec{A}'|}$

$\Rightarrow \quad \cos r = \dfrac{\left(6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} + C\hat{k}\right)\cdot(-\hat{k})}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + C^2}}$

$\Rightarrow \quad \cos r = \dfrac{-C}{\sqrt{108 + 192 + C^2}}$

$\Rightarrow \quad \cos 45° = \dfrac{-C}{\sqrt{300 + C^2}};\ \dfrac{1}{\sqrt{2}} = \dfrac{-C}{\sqrt{300 + C^2}}$

$\Rightarrow \quad C = \pm 10\sqrt{3}$

Since the refracted ray travels downwards,

$\Rightarrow \quad C = -10\sqrt{3}$

$\Rightarrow \quad \vec{A}' = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\sqrt{3}\hat{k}$

Hence, the correct answer is (D).

26. $|A'| = \sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + (-10\sqrt{3})^2} = 10\sqrt{6}$

Hence, the correct answer is (C).

27. $\hat{n} = \dfrac{\vec{A}'}{|\vec{A}'|} = \dfrac{3}{5\sqrt{2}}\hat{i} + \dfrac{4}{5\sqrt{2}}\hat{j} - \dfrac{1}{\sqrt{2}}\hat{k}$

Hence, the correct answer is (A).

28. Optical path length $(OPL) = \int ndx$

$\Rightarrow \quad OPL = \int_0^1 (1 + x^2)\,dx$

$\Rightarrow \quad OPL = \left(x + \dfrac{x^3}{3}\right)\Big|_0^1 = \dfrac{4}{3}$ m

Hence, the correct answer is (A).

31. $P_{eq} = 2P_{lens} + P_{mirror}$

where $P_{lens} = \dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

and $P_{mirror} = -\dfrac{1}{f} = -\dfrac{2}{R}$

$\dfrac{1}{f_1} = 2(\mu - 1)\left(\dfrac{1}{R} - \dfrac{1}{(-R)}\right) - \dfrac{2}{(-R)}$

$\Rightarrow \quad \dfrac{1}{f_1} = \dfrac{4(\mu - 1)}{R} + \dfrac{2}{R}$

$\Rightarrow \quad \dfrac{1}{f_1} = \dfrac{4\mu}{R} - \dfrac{4}{R} + \dfrac{2}{R}$

$\Rightarrow \quad \dfrac{1}{f_1} = \dfrac{4\mu}{R} - \dfrac{2}{R}$

$\Rightarrow \quad \dfrac{1}{f_1} = \dfrac{2}{R}(2\mu - 1)$

$\Rightarrow \quad f_1 = \dfrac{R}{2(2\mu - 1)}$

Hence, the correct answer is (C).

32. For a plano-convex lens, we have

$\dfrac{1}{f_2} = (\mu - 1)\left(\dfrac{1}{R} - \dfrac{1}{\infty}\right)$

$\Rightarrow \quad \dfrac{1}{f_2} = \dfrac{\mu - 1}{R}$

$\Rightarrow \quad f_2 = \dfrac{R}{\mu - 1}$

Hence, the correct answer is (C).

33. When plane surface is silvered, then

$\dfrac{1}{f_3} = \dfrac{2}{f_\ell} - \dfrac{1}{\infty} \qquad \{\because f_m \to \infty\}$

$\dfrac{1}{f_3} = \dfrac{2(\mu - 1)}{R}$

$\Rightarrow \quad f_3 = \dfrac{R}{2(\mu - 1)}$

Hence, the correct answer is (B).

CHAPTER 1

34. When curved surface is silvered, then

$$\frac{1}{f_4}=\frac{2}{f_\ell}-\frac{1}{f_m}$$

$$\Rightarrow \quad \frac{1}{f_4}=2(\mu-1)\left(\frac{1}{\infty}-\frac{1}{-R}\right)-\frac{2}{(-R)}$$

$$\Rightarrow \quad \frac{1}{f_4}=\frac{2(\mu-1)}{R}+\frac{2}{R}$$

$$\Rightarrow \quad \frac{1}{f_4}=\frac{2\mu}{R}$$

$$\Rightarrow \quad f_4=\frac{R}{2\mu}$$

Hence, the correct answer is (D).

35. Focal length of lens L_1 and L_2 is given by using the Lens Maker's Formula, so

$$\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)$$

$$\Rightarrow \quad f_1=50 \text{ cm}$$

Similarly, $\frac{1}{f_2}=(\mu_2-1)\left(\frac{1}{\infty}-\frac{1}{(-R)}\right)$

$$\Rightarrow \quad f=40 \text{ cm}$$

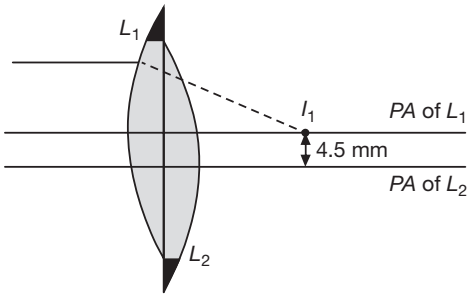
So, equivalent focal length is given by

$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\Rightarrow \quad f_{eq}=\frac{200}{9} \text{ cm}$$

Hence, the correct answer is (D).

36. Image formed by L_1 lies 50 cm behind it and on principal axis of L_1. This will act as an object for L_2. So, for L_2,



we have

$$u=+50 \text{ cm}, \quad f_2=+40 \text{ cm}$$

Applying Len's Formula, we get

$$\frac{1}{v}-\frac{1}{50}=\frac{1}{40}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{50}+\frac{1}{40}$$

$$\Rightarrow \quad v=\frac{200}{9} \text{ cm}$$

Magnification produced by L_2 is $m_2=\frac{v}{u}=\frac{4}{9}$

However, for L_2, the image I_1 is at a distance of 4.5 mm above its principal axis i.e., (PA of L_2). So, distance of image I_2 from PA of L_2 is

$$y=m_2(4.5 \text{ mm})$$

$$\Rightarrow \quad y=\frac{4}{9}(4.5 \text{ mm})=2 \text{ mm}$$

Hence I_2 is at a distance of $(4.5-2)$ mm $=2.5$ mm from PA of L_1.

Hence, the correct answer is (C).

37. Applying refraction at curved surface formula, i.e.,

$\frac{\mu_1}{-u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$, we get

$$\frac{1}{x}+\frac{2}{v_1}=\frac{1}{R}$$

$$\Rightarrow \quad v_1=\frac{2xR}{x-R} \qquad \{\text{from pole of curved surface}\}$$

This image formed will be at a distance $v'=(v_1-R)$ from the plane surface. Now again applying the above formula, but at the plane surface, we get

$$\frac{1}{v}-\frac{2}{v'}=0 \qquad \{\because \text{ for plane surface } R\to\infty\}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{2}{v_1-R}$$

Substituting $v_1=\frac{2xR}{x-R}$, we get

$$\frac{1}{v}=\frac{2(x-R)}{R(x+R)}$$

For the image to be virtual, we have

$$\frac{1}{v}<0$$

$$\Rightarrow \quad \frac{2(x-R)}{R(x+R)}<0$$

$\Rightarrow \quad x < R$

This condition is only satisfied by $x = \dfrac{R}{3}$.

Hence, the correct answer is (C).

38. Since, $v_1 = \dfrac{(2x)R}{x - R}$

So, for $x = 2R$, we get

$v_1 = 4R$

Also, $u = -2R$

Since, $m_1 = \left(\dfrac{\mu_1}{\mu_2}\right)\left(\dfrac{v}{u}\right)$

$\Rightarrow \quad m_1 = \left(\dfrac{1}{2}\right)\left(\dfrac{4R}{-2R}\right)$

$\Rightarrow \quad m_1 = -1$

However, for plane mirror, we have

$m_2 = 1$

$m_{net} = m_1 m_2 = -1$

Since, $m_{net} = \ominus$, so the final image formed is real, inverted and of same size.

Hence, the correct answer is (D).

39. $\dfrac{\sin(90°)}{\sin r} = 2$

$\Rightarrow \quad \sin r = \dfrac{1}{2}$

$\Rightarrow \quad r = 30°$

In triangle OAC, we have

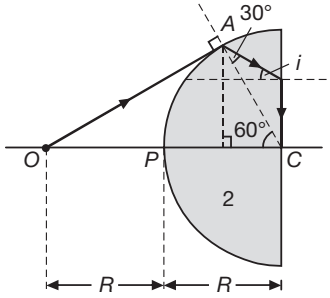
$OC = 2R$, $AC = R$, so, by Pythagora's Theorem, we get

$OA = \sqrt{3}R$

$\Rightarrow \quad \angle ACP = 60°$

Using geometry, we get

$i = 30°$



Since, refractive index of this is 2, so the critical angle C is given by

$C = \sin^{-1}\left(\dfrac{1}{2}\right) = 30°$

Since, $i = C$, so the ray grazes the plane surface.

Hence, the correct answer is (C).

40. Since yellow is the mean colour, so

$\mu_y = \dfrac{\mu_b + \mu_r}{2} = \dfrac{1.51 + 1.49}{2} = 1.50$

Hence, the correct answer is (C).

41. Again yellow is the mean colour, so

$\mu'_y = \dfrac{\mu'_b + \mu'_r}{2} = \dfrac{1.77 + 1.73}{2} = 1.75$

Hence, the correct answer is (D).

42. For no deviation to take place, deviation (given by $\delta = (\mu - 1)A$ by one must be cancelling the deviation due to the other. For this

(a) the prisms must be arranged upside down

(b) $\dfrac{A'}{A} = -\left[\dfrac{\mu_y - 1}{\mu'_y - 1}\right]$

$\Rightarrow \quad A' = -\left(\dfrac{1.50 - 1}{1.75 - 1}\right)6° = -4°$

Hence, the correct answer is (D).

43. Net dispersion $= (\delta_b - \delta_r) + (\delta'_b - \delta'_r)$

$= (\mu_b - \mu_r)A + (\mu'_b - \mu'_r)A'$

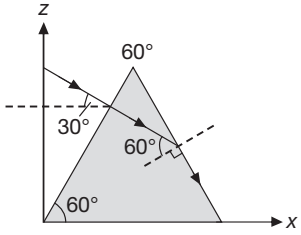
$= (1.51 - 1.49)6° - (1.77 - 1.73)4°$

$= 0.02 \times 6° - 0.04 \times 4°$

$= 0.12° - 0.16° = -0.04°$

Hence, the correct answer is (D).

44.



Since, $\sqrt{3}\,z + x = 10$

$\Rightarrow \quad z = \dfrac{-x}{\sqrt{3}} + \dfrac{10}{\sqrt{3}}$

$\Rightarrow \quad \text{Slope} = \dfrac{-1}{\sqrt{3}}$

$\Rightarrow \quad \tan\theta = \dfrac{-1}{\sqrt{3}}$

CHAPTER 1

$\Rightarrow \quad \theta = -30°$

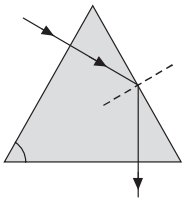
So, the ray is incident normally on the face AB. Thus, angle of incidence on face AC is $60°$.
For grazing the face AC, we have

$$\mu \sin(60°) = 1 \times \sin(90°)$$

$$\Rightarrow \quad \mu = \frac{2}{\sqrt{3}}$$

Hence, the correct answer is (C).

45. For $\mu = \frac{3}{2}$ the ray will be internally reflected, so, the ray is normal at face BC. Hence finally, the refracted ray is parallel to z-axis.



Hence, the correct answer is (B).

46. If BC is silvered, the ray will retrace its path. Hence equation of ray coming out of the prism is $\sqrt{3}z + x = 10$.
Hence, the correct answer is (B).

47. The objective lens must form a real image for eyepiece to magnify it.
Hence, the correct answer is (C).

48. The image formed by the objective must lie within the focus of the eyepiece.
Hence, the correct answer is (A).

49. To obtain best magnification, the object must be placed just beyond the focus of the objective lens. In this case, the first image distance from the objective is very large.
Hence, the correct answer is (A).

50. $i + \theta = 90°$, $\theta = 90° - i$,

Slope of tangent $= \tan\theta = \tan(90° - i) = \cot i$

Hence, the correct answer is (C).

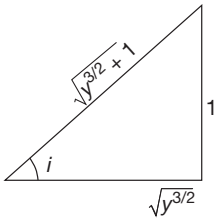
51. But $\tan\theta = \frac{dy}{dx}$

$$\therefore \quad \frac{dy}{dx} = \cot i \qquad \ldots(1)$$

Applying Snell's Law at A and B

$$n_A \sin i_A = n_B \sin i_B$$

$n_A = 1$ because $y = 0$



$\sin i_A = 1$ because $i_A = 90°$ (Grazing incidence)

$$n_B = \sqrt{Ky^{3/2} + 1} = \sqrt{y^{3/2} + 1}$$

because $K = 1.0(m)^{-3/2}$

$$\therefore \quad (1)(1) = \sqrt{\left(y^{3/2} + 1\right)} \sin i$$

$$\Rightarrow \quad \sin i = \frac{1}{\sqrt{y^{3/2} + 1}}$$

$$\therefore \quad \cot i = \sqrt{y^{3/2}} \text{ or } y^{3/4} \qquad \ldots(2)$$

Equating equations (1) and (2), we get

$$\frac{dy}{dx} = y^{3/4}$$

$$\Rightarrow \quad y^{-3/4} dy = dx$$

$$\Rightarrow \quad \int_0^y y^{-3/4} dy = \int_0^x dx$$

$$\Rightarrow \quad 4y^{1/4} = x \qquad \ldots(3)$$

The required equation of trajectory is $4y^{1/4} = x$

$$\Rightarrow \quad y = \frac{x^4}{4^4} = \frac{x^4}{256}$$

Hence, the correct answer is (D).

52. At the point of intersection on the upper surface,

$$y = 1 \text{ m}$$

$$\Rightarrow \quad x = (256)^{1/4} = 4 \text{ m}$$

So the co-ordinates are $(4 \text{ m}, 1 \text{ m})$

Hence, the correct answer is (D).

53. As $n_A \sin i_A = n_P \sin i_P$ and as $n_A = n_P = 1$

Therefore, $i_P = i_A = 90^o$ i.e., the ray will emerge parallel to the boundary at P i.e., at grazing emergence.
Hence, the correct answer is (A).

54. $-\frac{1}{F} = \frac{2}{f_\ell} - \frac{1}{f_m}$

$$\Rightarrow \quad \frac{1}{-F} = 2(1.5 - 1)\left(\frac{1}{20} - \frac{1}{60}\right) - \frac{2}{(-20)}$$

$$\Rightarrow \quad \frac{1}{-F}=\frac{1}{20}-\frac{1}{60}+\frac{2}{20}=\frac{8}{60}$$

$$\Rightarrow \quad F=-\frac{60}{8}=-7.5 \text{ cm}$$

Hence, the correct answer is (C).

55. Since $F=\ominus$, so combination behaves like a concave mirror.

Hence, the correct answer is (B).

56. $$m=\frac{f}{f-u}=\frac{-7.5}{-7.5+30}=\frac{-7.5}{22.5}=-\frac{1}{3}$$

Hence, the correct answer is (A).

57. **The correct answer is (C).**

58. **The correct answer is (B).**

59. **The correct answer is (B).**

Combined solution of 57, 58, & 59

Each part will work as a separate lens and will form its own image. For any part, we have $u=-0.3$ m, $f=+0.2$ m.

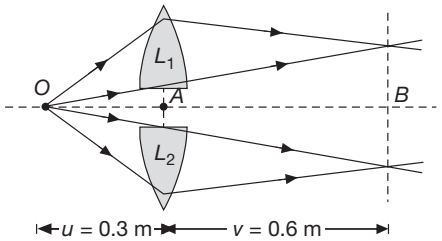
Therefore, from lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{0.3}=\frac{1}{0.2}$$

$$\Rightarrow \quad v=0.6 \text{ m}$$

So, each part forms a real image of the point object O at 0.6 m from the lens, as shown in figure.



Since the triangles OL_1L_2 and OI_1I_2 are similar. So, we have

$$\frac{I_1I_2}{L_1L_2}=\frac{OB}{OA}=\frac{u+v}{u}$$

$$\Rightarrow \quad \frac{I_1I_2}{L_1L_2}=\frac{0.3+0.6}{0.3}=\frac{0.9}{0.3}=3$$

$$\Rightarrow \quad I_1I_2=3(L_1L_2)=3(2\times 0.0005)=0.003 \text{ m}$$

60. $$\frac{1}{f_1}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_1}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R}-\frac{1}{-R}\right)=\frac{1}{2}\times\frac{2}{R}=\frac{1}{R}$$

$$\Rightarrow \quad f_1=R$$

Hence, the correct answer is (A).

61. $$\frac{1}{f_2}=\left(\frac{4}{3}-1\right)\left(\frac{1}{-R}-\frac{1}{\infty}\right)=-\frac{1}{3R}$$

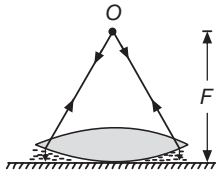
$$\Rightarrow \quad f_2=-3R$$

Hence, the correct answer is (C).

62. Now, $$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{R}-\frac{1}{3R}=\frac{2}{3R}$$

$$\Rightarrow \quad F=\frac{3R}{2}$$

Since, image coincides with the object so, clearly rays of light must have retraced their path after reflection. This is possible only when rays of light must have fallen normally on the plane mirror. For this, the object is at the focus of the lens system.



$$\Rightarrow \quad F=\text{distance of object}=15 \text{ cm}$$

$$\Rightarrow \quad \frac{3R}{2}=15 \text{ cm}$$

$$\Rightarrow \quad R=10 \text{ cm}$$

Hence, the correct answer is (C).

63 If f is the focal length of the liquid concave lens, then

$$\frac{1}{25}=\frac{1}{10}+\frac{1}{f} \quad \text{or} \quad \frac{1}{f}=\frac{-3}{50}$$

$$\Rightarrow \quad f=-\frac{50}{3} \text{ cm}$$

Hence, the correct answer is (C).

64. Using Lens-Maker's Formula for the liquid concave lens,

$$\frac{1}{-\frac{50}{3}}=(\mu-1)\left(\frac{1}{-R}-\frac{1}{\infty}\right)=-\frac{\mu-1}{R}=-\frac{\mu-1}{10}$$

CHAPTER 1

$$\Rightarrow \quad \frac{3}{50}=\frac{\mu-1}{10}$$

$$\Rightarrow \quad \mu=1+0.6=1.6$$

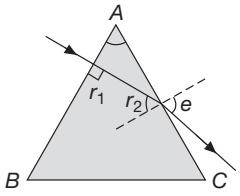
Hence, the correct answer is (D).

65. Since the ray is incident normally, so

$$r_1=0$$

Further, $r_1+r_2=A=60°$

$$\Rightarrow \quad r_2=A=60°$$



Applying Snell's Law at AC, we get

$$\mu\sin r_2=\mu_1\sin e$$

$$\Rightarrow \quad \mu_1\sin e=\frac{4}{\sqrt{3}}\times\sin 60°$$

$$\Rightarrow \quad \sin e=\frac{4}{\sqrt{3}\mu_1}\times\frac{\sqrt{3}}{2}=\frac{2\sqrt{3}}{\sqrt{3}}\times\frac{1}{\mu_1}=\frac{2}{\mu_1} \qquad \ldots(1)$$

Further, deviation is given by

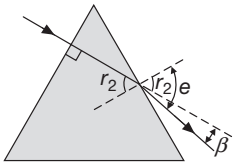
$$\beta=e-r_2$$

From the graph, we observe that when $\mu_1=k_2$, $\beta=0$

So, $\beta=e-r_2=0$

$$\Rightarrow \quad e=r_2=60°$$

So, equation (1) becomes



$$\sin(60°)=\frac{2}{\mu_1}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2}=\frac{2}{\mu_1}$$

$$\Rightarrow \quad \mu_1=\frac{4}{\sqrt{3}}$$

$$\Rightarrow \quad k_2=\frac{4}{\sqrt{3}}$$

Hence, the correct answer is (B).

66. When $\mu_1<k_1$ light will not emerge, it can be seen from graph, so we have

$$r_2=C \text{ when } \mu_1=k_1$$

$$\Rightarrow \quad \sin r_2=\sin C=\frac{\mu}{\mu_1}$$

$$\Rightarrow \quad \sin(60°)=\frac{\mu}{\mu_1} \qquad \{\because\ r_2=60°\}$$

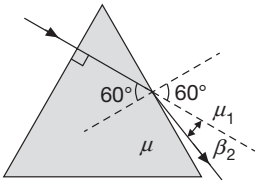
$$\Rightarrow \quad \frac{\sqrt{3}}{2}=\frac{4}{\sqrt{3}}\times\frac{1}{\mu_1}$$

$$\Rightarrow \quad \mu_1=\frac{8}{3}$$

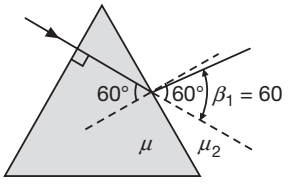
$$\Rightarrow \quad k_1=\frac{8}{3}$$

Hence, the correct answer is (C).

67. From the graph, we observe that β_1 is the maximum deviation and β_2 is the minimum deviation



So, $\beta_2=\frac{\pi}{2}-60°=30°$ and $\beta_1=60°$



$$\Rightarrow \quad \beta_1-\beta_2=60°-30°=30°$$

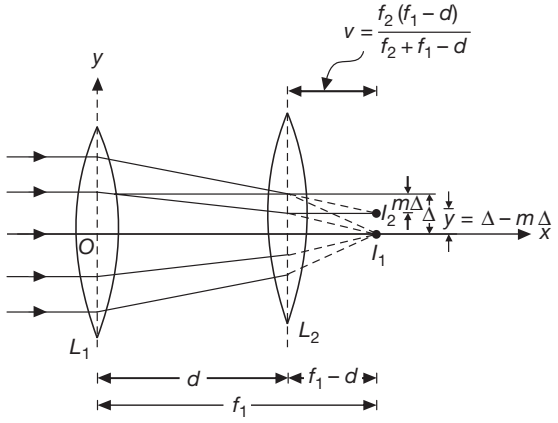
Hence, the correct answer is (B).

68. From the first lens parallel beam of light is focussed at its focus i.e., at a distance f_1 from it. This image I_1 acts as virtual object for second lens L_2. Therefore, for L_2

$$u=+(f_1-d),\ f=+f_2$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{f_2}+\frac{1}{f_1-d}$$

$$\Rightarrow \quad v=\frac{f_2(f_1-d)}{f_2+f_1-d}$$



Therefore, x-coordinate of its focal point will be

$$x = d + v = d + \frac{f_2(f_1-d)}{f_2+f_1-d}$$

$$\Rightarrow \quad x = \frac{f_1f_2 + d(f_1-d)}{f_1+f_2-d}$$

Hence, the correct answer is (A).

69. Linear magnification for L_2, is given by

$$m = \frac{v}{u} = \left(\frac{f_2(f_1-d)}{f_2+f_1-d}\right)\left(\frac{1}{f_1-d}\right) = \frac{f_2}{f_2+f_1-d}$$

Therefore, second image will be formed at a distance of $m\Delta$ or $\left(\frac{f_2}{f_2+f_1-d}\right)(\Delta)$ below its optic axis.

Therefore, y-coordinate of the focus of system is given by

$$y = \Delta - \left(\frac{f_2\Delta}{f_2+f_1-d}\right)$$

$$\Rightarrow \quad y = \frac{(f_1-d)\Delta}{f_2+f_1-d}$$

Hence, the correct answer is (A).

72. For both the halves, position of object and image is same, however the only difference is of magnification. Magnification for one of the halves is given as $2(>1)$. This can be for the first one, because for this, $|v|>|u|$. Therefore, magnification, $|m| = \left|\frac{v}{u}\right| > 1$. So, for the first half, we have

$$\left|\frac{v}{u}\right| = 2$$

$$\Rightarrow \quad |v| = 2|u|$$

Let $u = -x$, then $v = +2x$

and $|u|+|v| = 1.8$ m

$$\Rightarrow \quad 3x = 1.8 \text{ m}$$

$$\Rightarrow \quad x = 0.6 \text{ m}$$

Hence, $u = -0.6$ m and $v = +1.2$ m

Using $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{1.2} - \frac{1}{-0.6} = \frac{1}{0.4}$

$$\Rightarrow \quad f = 0.4 \text{ m}$$

For the second half, we have

$$\frac{1}{f} = \frac{1}{1.2-d} + \frac{1}{-(0.6+d)}$$

$$\Rightarrow \quad \frac{1}{0.4} = \frac{1}{1.2-d} + \frac{1}{(0.6+d)}$$

Solving this, we get $d = 0.6$ m

Magnification for the second half will be

$$m_2 = \frac{v}{u} = \frac{0.6}{-(1.2)} = -\frac{1}{2}$$

and magnification for the first half is

$$m_1 = \frac{v}{u} = \frac{1.2}{-(0.6)} = -2$$

Hence, the correct answer is (D).

73. For direct observation, the eye E appears to be farther that it actually is. So,

$$x = \mu H + \frac{H}{2} = H\left(\mu + \frac{1}{2}\right)$$

Hence, the correct answer is (A).

74. The distance of eye from mirror is

$$\mu H + H$$

So, distance of eye as seen by the fish in the mirror is

$$x = (\mu H + H) + \frac{H}{2}$$

$$\Rightarrow \quad x = \frac{3H}{2} + \mu H$$

$$\Rightarrow \quad x = H\left(\mu + \frac{3}{2}\right)$$

Hence, the correct answer is (B).

75. For direct observation by the eye E, the fish appears to be closer than it actually is. So, we have

$$x = H + \frac{H}{2\mu}$$

$$\Rightarrow \quad x = H\left(1 + \frac{1}{2\mu}\right)$$

Hence, the correct answer is (C).

CHAPTER 1

76. The distance of the mirror as observed by the eye is $H+\frac{H}{\mu}$. So, the distance of the fish as seen by the eye in the mirror is

$$x = H+\frac{H}{\mu}+\frac{H}{2\mu}$$

$$\Rightarrow \quad x = H\left(1+\frac{3}{2\mu}\right)$$

Hence, the correct answer is (D).

Matrix Match/Column Match Type Questions

1. A → (r, s)
B → (p, q, r, s, t)
C → (q, r, s, t)
D → (p, r, s)

(A) $m<0$, means real image, possible for concave mirror and convex lens. So, (A) → (r, s).

(B) $m>0$, means virtual image, possible for all i.e., plane mirror, convex mirror (always), concave mirror (when object lies between focus and pole), concave lens (always) and convex lens (when object lies between optical centre and focus). So, (B) → (p, q, r, s, t).

(C) $|m|<1$, means diminished image, possible for concave mirror, convex lens, convex mirror and concave lens. So, (C) → (q, r, s, t).

(D) $|m|\geq 1$, means magnified (>1) and same sized $(=1)$ image, possible for concave mirror/convex lens (both $|m|>1$) and plane mirror $(|m|=1)$. So, (D) → (p, r, s).

2. A → (s, t)
B → (p, t)
C → (s, t)
D → (q, t)

$$\vec{v}_A = \hat{i}+\vec{a}t = \hat{i}+\left(2\hat{i}+\hat{j}\right)(2) = 5\hat{i}+2\hat{j}$$

$$\Rightarrow \quad \vec{v}_{A'} = -5\hat{i}+2\hat{j}$$

$$\Rightarrow \quad \vec{v}_{A'A} = \vec{v}_{A'}-\vec{v}_A = -10\hat{i}$$

Similarly, $\vec{v}_B = \left(-\hat{i}+3\hat{j}\right)$

$$\Rightarrow \quad \vec{v}_{B'} = \left(\hat{i}+3\hat{j}\right)$$

$$\Rightarrow \quad \vec{v}_{B'B} = \vec{v}_{B'}-\vec{v}_B = 2\hat{i}$$

4. A → (r)
B → (q)
C → (p)
D → (p)

(A) Velocity of fish in air $= 8\times\frac{3}{4} = 6\uparrow$

Velocity of fish w.r.t. bird $= 6+6 = 12\uparrow$

(B) Velocity of image of fish after reflection from mirror in air $= 8\times\frac{3}{4} = 6\downarrow$

w.r.t. bird $= -6+6 = 0$

(C) Velocity of bird as seen from water $= 6\times\frac{4}{3} = 8\downarrow$

Velocity of bird w.r.t. fish $= 8+8 = 16\downarrow$

(D) Velocity of bird in water after reflection from mirror $= 8\downarrow$

w.r.t. fish $= 8-8 = 0$

11. A → (p, s)
B → (p, q, r, s)
C → (p, q, r, s)
D → (q, s)

(A) For convex mirror:

$$\frac{1}{v}+\frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f}-\frac{1}{u}$$

$$\Rightarrow \quad v = \left(\frac{uf}{u-f}\right)$$

Now, v may be positive and negative, depending on values of u and f.

Since, $|m| = \left|\frac{v}{u}\right| = \left(\frac{f}{u-f}\right) = \left(\frac{1}{\frac{u}{f}-1}\right)$

Again $|m|$ can be greater than or less than 1.

(B) For concave mirror:

$$\frac{1}{v}+\frac{1}{+u} = \frac{1}{-f}$$

$$\Rightarrow \quad v = -\left(\frac{fu}{f+u}\right)$$

So, v is always negative i.e., image is always real.

Further, $|m| = \left|\frac{v}{u}\right| = \frac{1}{\frac{u}{f}+1}$

i.e., m is always less than 1 or image is always diminished.

12. A → (s)
B → (p)
C → (q)
D → (r)

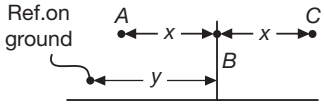
$A \rightarrow$ when object lies between pole and focus image is virtual, magnified and erect.

$B \rightarrow$ when object lies between focus and centre of curvature, image is real, inverted and magnified.

$C \rightarrow$ when object lies at centre of curvature, image is real, inverted and of equal size.

$D \rightarrow$ when object lies beyond centre of curvature, image is real, inverted and smaller in size.

13. A → (s)
B → (q)
C → (p)
D → (q)



(A) $\frac{d}{dt}(x+y)=\frac{dx}{dt}+\frac{dy}{dt}=5+2=7 \text{ ms}^{-1}$

(B) $\frac{dx}{dt}=3+2=5 \text{ ms}^{-1}$

(C) $\frac{d(2x)}{dt}=2\frac{dx}{dt}=10 \text{ ms}^{-1}$

(D) $\frac{dx}{dt}=5 \text{ ms}^{-1}$

Integer/Numerical Answer Type Questions

1. Forward shift of point of incidence due to a single reflection $x=0.2\tan(30°)=\frac{0.2}{\sqrt{3}}$.

Hence required number of reflections required is

$$N=\frac{\text{Mirror Length}}{\text{Forward Shift}}=\frac{2\sqrt{3}}{\left(\frac{0.2}{\sqrt{3}}\right)}=30$$

2. Since, for a concave mirror, we can have both virtual and real image, so
for real image, $m=-5$ and
for virtual image, $m=+5$

Since we know that $m=\frac{f}{f-u}$

where $f=-30$ cm and $m=\pm5$ because the image can be real or virtual.
For real image, $m=-5$

$\Rightarrow \quad -5=\frac{-30}{-30-u}$

$\Rightarrow \quad u=-36$ cm

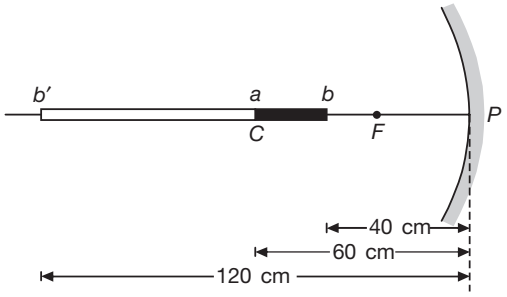
For virtual image, $m=+5$

$\Rightarrow \quad 5=\frac{-30}{-30-u}$

$\Rightarrow \quad u=-24$ cm

Hence, the object must be placed at 24 cm or 36 cm in front of the concave mirror

3. The image of the end a of the rod which lies at the centre of curvature C is formed at C.



The position of the image of end b can be obtained using mirror formula, according to which we have

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

where $u=-40$ cm, $f=-30$ cm

$\Rightarrow \quad \frac{1}{v}+\frac{1}{-40}=\frac{1}{-30}$

$\Rightarrow \quad v=-120$ cm

Length of the image (rod) is given by

$\ell_i=120-60=60$ cm

So, magnification $m=-\frac{dv}{du}=-\left(\frac{v_2-v_1}{u_2-u_1}\right)$

$\Rightarrow \quad m=-\left(\frac{120-60}{60-40}\right)$

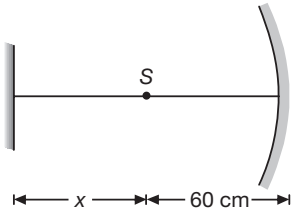
$\Rightarrow \quad m=-3$

$\Rightarrow \quad |m|=3$

4. Applying mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ for concave mirror, we get

$$\frac{1}{v}-\frac{1}{60}=\frac{1}{(-40)}$$

$\Rightarrow \quad v=-120$ cm



CHAPTER 1

Now, for the rays to again converge at S, (after reflection from the plane mirror)

$$\begin{pmatrix}\text{Distance of } S\\ \text{from Plane Mirror}\end{pmatrix}=\begin{pmatrix}\text{Distance of Image}\\ \text{formed by}\\ \text{Concave Mirror}\end{pmatrix}$$

$\Rightarrow \quad x=120-(x+60)$

$\Rightarrow \quad x=30$ cm

So, the desired distance is 90 cm

From reversibility principle, it hardly matters whether the ray of light is first reflected from the concave mirror or plane mirror.

5. Since the image formed is real, so for the first case, we have

$$m=-3$$

$$\Rightarrow \quad -\frac{v_1}{u_1}=-3$$

$$\Rightarrow \quad v_1=3u_1$$

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{(-3u_1)}+\frac{1}{(-u_1)}=\frac{1}{-f}$$

$$\Rightarrow \quad f=\frac{3u_1}{4} \qquad \ldots(1)$$

Similarly, for the second case, we have

$$m=-2$$

$$\Rightarrow \quad -\frac{v_2}{u_2}=-2$$

$$\Rightarrow \quad v_2=2u_2$$

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{-(2u_2)}+\frac{1}{(-u_2)}=\frac{1}{-f}$$

$$\Rightarrow \quad f=\frac{2u_2}{3} \qquad \ldots(2)$$

From (1) and (2), we get

$$\frac{3u_1}{4}=\frac{2u_2}{3}$$

$$\Rightarrow \quad u_2=\frac{9u_1}{8} \qquad \ldots(3)$$

But according to the problem, we are given that the shift of the object is 6 cm, so we get

$$u_2-u_1=6 \text{ cm}$$

$$\Rightarrow \quad \frac{9}{8}u_1-u_1=6$$

$$\Rightarrow \quad u_1=48 \text{ cm}$$

Since, $v_1=3u_1$

$$\Rightarrow \quad v_1=144 \text{ cm}$$

So, $f=\frac{3u_1}{4}=\frac{3}{4}(48)=36$ cm

Now $u_2=\frac{9u_1}{8}=\frac{9}{8}(48)=54$ cm

and $v_2=2u_1=2(54)=108$ cm

So, shift of the screen is (v_1-v_2)

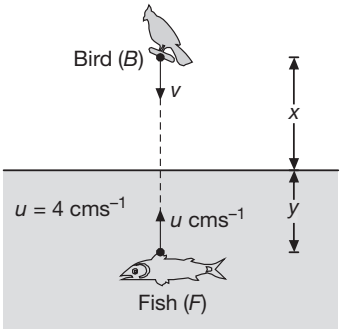
$$\Rightarrow \quad (v_1-v_2)=144-108=36 \text{ cm}$$

6. Let at some instant bird is at a height of x from the water surface and it is diving downwards with v cms^{-1}

At this instant fish is at a depth y below water surface. Then the distance between fish and image of bird at this instance will be,

$$s=\mu x+y$$

$$\Rightarrow \quad s=\frac{4}{3}x+y$$



Differentiating w.r.t. time, we get

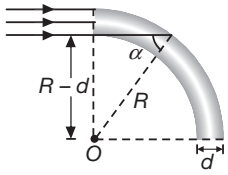
$$\left(-\frac{ds}{dt}\right)=\frac{4}{3}\left(-\frac{dx}{dt}\right)+\left(-\frac{dy}{dt}\right)$$

$\Rightarrow \quad 16=\frac{4}{3}(v)+u$, where $u=4$ cms^{-1}

$$\Rightarrow \quad v=9 \text{ cms}^{-1}$$

7. The necessary and sufficient condition for all the rays to pass around the arc is that the ray with least angle of incidence should get internally reflected i.e., should suffer TIR.

From the figure, it becomes obvious that the ray with least angle of incidence is the one which is incident almost grazingly with the inner wall.



For this ray, let α be the angle of incidence, then we observe that

$$\sin\alpha = \frac{R-d}{R} \text{ where } d \text{ is the diameter of tube}$$

For TIR, $\alpha \geq C$

$\Rightarrow \sin\alpha \geq \sin C$

$\Rightarrow \frac{R-d}{R} \geq \frac{1}{\mu}$

$\Rightarrow R \geq \frac{\mu d}{\mu - 1}$

Since, $\mu = 1.5$ and $d = 4$ cm, so we get

$R = 12$ cm

Hence, the least radius required is 12 cm.

8. Since, $\theta = 90° - i$

$\Rightarrow \tan\theta = \cot i$

$\Rightarrow \frac{dy}{dx} = \cot i$ ...(1)

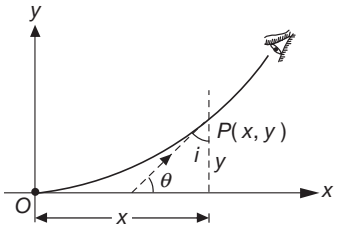
According to Snell's Law at O and P, we have

$\mu_0 \sin i_0 = \mu_P \sin i_P$

Since $\mu = \sqrt{1+ay}$

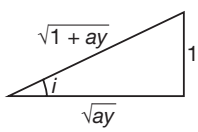
$\Rightarrow$ At $y = 0$, $\mu = 1$

$\Rightarrow \sin(90°) = \left(\sqrt{1+ay}\right)\sin i$



$\Rightarrow \sin i = \frac{1}{\sqrt{1+ay}}$

$\Rightarrow \cot i = \sqrt{ay} = \frac{dy}{dx}$



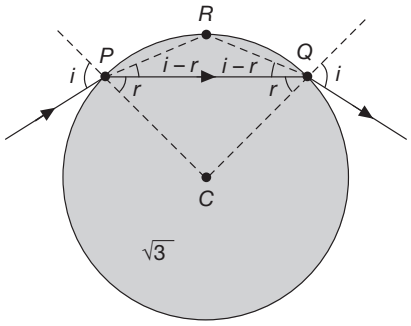
$\Rightarrow \int_0^y \frac{dy}{\sqrt{ay}} = \int_0^x dx$

$\Rightarrow x = 2\sqrt{\frac{y}{a}}$

Substituting $y = 2$ m and $a = 2\times10^{-6}$ m^{-1}, we get

$x_{max} = 2000$ m $= 2$ km

9. The ray diagram for the situation is shown in figure.



Since, $\angle PCQ = \pi - 2r$, $\angle PRQ = \pi - 2(i-r)$

From the property of a circle, we get

$2(\angle PRQ) + \angle PCQ = 2\pi$

$\Rightarrow 2\pi - 4(i-r) + \pi - 2r = 2\pi$

$\Rightarrow r = 2i - \frac{\pi}{2}$

According to Snell's Law, we have

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin\left(2i - \frac{\pi}{2}\right)}$$

$\Rightarrow \sqrt{3} = \frac{\sin i}{-\cos 2i} = \frac{\sin i}{2\sin^2 i - 1}$

$\Rightarrow 2\sqrt{3}\sin^2 i - \sin i - \sqrt{3} = 0$

$\Rightarrow \sin i = \frac{1 \pm \sqrt{1+24}}{4\sqrt{3}} = \frac{1\pm5}{4\sqrt{3}}$

Rejecting the negative value, we get

$\sin i = \frac{\sqrt{3}}{2}$

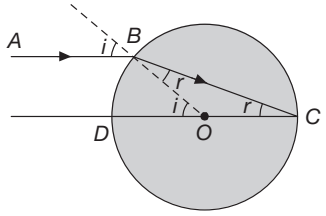
$\Rightarrow i = 60°$

10. From the figure, we observe that

$BO = OC$

So, if $\angle OBC = \angle BCO = r$ (say) and i be the angle of incidence, then

$i = r + r = 2r$

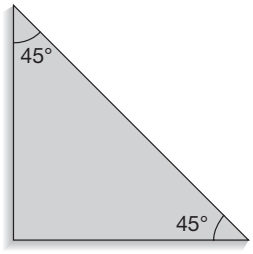


Since, we know that

$$\mu = \frac{\sin i}{\sin r}$$

$$\Rightarrow \quad \mu = \frac{\sin 2r}{\sin r} \approx \frac{2r}{r} = 2$$

11. After reflecting twice from two plane mirrors at right angles, a ray of light gets deviated by $180°$, irrespective the angle of incidence, so the emergent ray is anti-parallel to incident ray and hence angle of deviation is $180°$.



12. From Archimedes Principle, we know that

$$\frac{V_{\text{immersed}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{liquid}}}$$

$$\Rightarrow \quad \frac{V_{\text{immersed}}}{V} = \frac{\rho_{\text{sphere}}}{\rho_{\text{liquid}}} = \frac{\rho}{2\rho}$$

$$\Rightarrow \quad \frac{V_{\text{immersed}}}{V} = \frac{1}{2}$$

i.e., half the sphere is inside the liquid. For the image to coincide with the object, light should fall normally on the sphere. Using, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ twice, we have

$$\frac{\frac{3}{2}}{v_1} - \frac{1}{(-8)} = \frac{\left(\frac{3}{2}\right) - 1}{+2}$$

$$\Rightarrow \quad v_1 = 12 \text{ cm}$$

Further, $\frac{\frac{4}{3}}{h-10} - \frac{\frac{3}{2}}{8} = \frac{\frac{4}{3} - \frac{3}{2}}{-2}$

Solving this equation, we get

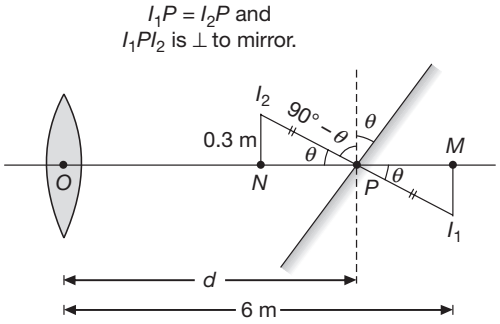
$$h = 15 \text{ cm}$$

13. For the lens, $u = -2$ m , $f = +1.5$ m

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{-2} = \frac{1}{1.5}$$

$$\Rightarrow \quad v = 6 \text{ m}$$

Since, $m = \frac{6}{(-2)} = -3$



Therefore, y co-ordinate of image formed by the lens is given by

$$y = m(0.1) = -0.3 \text{ m}$$

In triangle PNI_2, we have

$$\tan\theta = \frac{I_2N}{NP}$$

$$\Rightarrow \quad \tan\theta = \frac{0.3}{NP} = 0.3 \qquad \{\because \tan\theta = 0.3\}$$

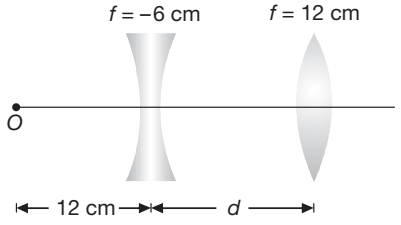
$$\Rightarrow \quad NP = MP = 1 \text{ m}$$

$$\Rightarrow \quad d = 6 - 1 = 5 \text{ m}$$

and x co-ordinate of final image I_2 is,

$$x = d - 1 = 4 \text{ m}$$

14. Applying lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ twice we get



$$\frac{1}{v_1} - \frac{1}{-12} = \frac{1}{-6} \qquad \ldots(1)$$

$$\frac{1}{\infty} - \frac{1}{v_1 - d} = \frac{1}{12} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$v_1 = -4$ cm

and $d = 8$ cm

15. Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v_1} + \frac{1}{40} = \frac{1}{20}$$

$\Rightarrow \quad v_1 = 40$ cm

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v_2} + \frac{1}{10} = \frac{1}{-10}$$

$\Rightarrow \quad v_2 = -5$ cm

so, the final image is formed at a distance of 5 cm from the mirror towards lens.

16. From Lens Maker's Formula, we get

$$\frac{1}{20} = (1.5-1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$\Rightarrow \quad R = 20$ cm

Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ twice with the condition that rays must fall normally on the concave mirror, we get

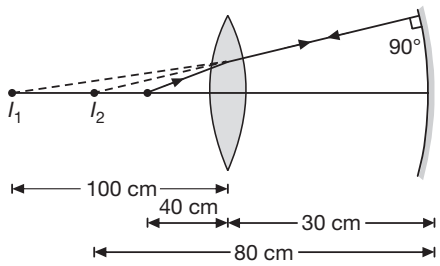
$$\frac{1.5}{v_1} - \frac{1.2}{-40} = \frac{1.5-1.2}{+20} \qquad \ldots(1)$$

$$\frac{2}{d-80} - \frac{1.5}{v_1} = \frac{2-1.5}{-20} \qquad \ldots(2)$$

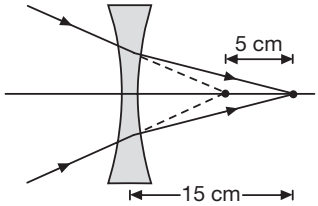
Solving equations (1) and (2), we get

$d = 30$ cm and $v_1 = -100$ cm

The ray diagram is as shown in figure.



17. Using the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get



$$\frac{1}{f} = \frac{1}{15} - \frac{1}{10} = -\frac{1}{30}$$

$\Rightarrow \quad f = -30$ cm

18. Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v_1} - \frac{1}{(-40)} = \frac{1}{30}$$

$\Rightarrow \quad v_1 = 120$ cm

Shift due to the slab $\Delta x = \left(1 - \frac{1}{\mu}\right)d = \left(1 - \frac{1}{1.8}\right)9 = 4$ cm

So, $u' = -(40 - \Delta x) = -36$ cm

$$\Rightarrow \quad \frac{1}{v_2} - \frac{1}{(-36)} = \frac{1}{30}$$

$\Rightarrow \quad v_2 = 180$ cm

Therefore, the screen has to be shifted away from the lens by a distance

$x = v_2 - v_1 = 60$ cm

19. Lateral magnification in first case is -3, so

if $u = -x$ then $v = +3x$

Since $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{+3x} + \frac{1}{x} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{4}{3x} = \frac{1}{f}$$

$$\Rightarrow \quad x = \frac{4f}{3} \qquad \ldots(1)$$

In the second case magnification is -2, so now we have

CHAPTER 1

$u = -(x+1.5)$, $v = 2(x+1.5)$

Since $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{2(x+1.5)} + \frac{1}{(x+1.5)} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{3}{2(x+1.5)} = \frac{1}{f}$$

$$\Rightarrow \quad f = \frac{2}{3}(x+1.5) \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$f = 9$ cm

20. Using Lens Maker's Formula for both the cases, we get

$$\frac{1}{f_{\text{air}}} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots(1)$$

and $$\frac{1}{f_{\text{water}}} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots(2)$$

Dividing equation (1) by (2), we get

$$\frac{f_{\text{water}}}{f_{\text{air}}} = \frac{(\mu_g - 1)}{\left(\frac{\mu_g}{\mu_w} - 1\right)}$$

Substituting the values, we get

$$f_{\text{water}} = \frac{\left(\frac{3}{2} - 1\right)}{\left(\frac{\frac{3}{2}}{\frac{4}{3}} - 1\right)} f_{\text{air}}$$

$$\Rightarrow \quad f_{\text{water}} = 4 f_{\text{air}} = 4 \times 10 = 40 \text{ cm}$$

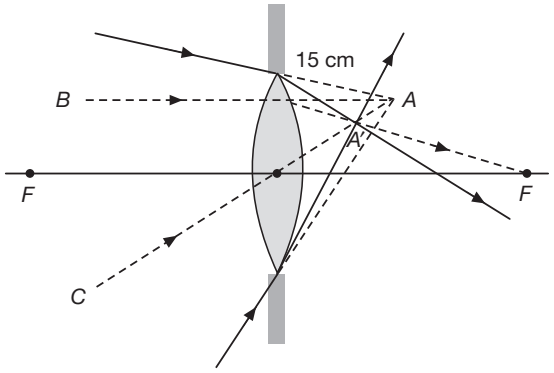
21. Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$u = +15$ cm, $f = +30$ cm

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{15} = \frac{1}{30}$$

$$\Rightarrow \quad v = +10 \text{ cm}$$

Therefore, the focus of the rays will move 5 cm closer to the screen. The ray diagram is as shown in figure.



22. Since we know that

$$f = \frac{D^2 - x^2}{4D}$$

Substituting $f = 16$ cm and $x = 60$ cm, we get

$$16 = \frac{D^2 - (60)^2}{4D}$$

$$\Rightarrow \quad D^2 - 3600 = 64D$$

$$\Rightarrow \quad D^2 - 64D - 3600 = 0$$

$$\Rightarrow \quad D^2 - 100D + 36D - 3600 = 0$$

$$\Rightarrow \quad D(D-100) + 36(D-100) = 0$$

$$\Rightarrow \quad (D-100)(D+36) = 0$$

But $D \neq 36$ cm, because it happens to be less than 60 cm. So,

$D = 100$ cm

23. For the first flare spot, the lens acts as if its right face is silvered. The equivalent focal length is given by

$$\frac{1}{F} = \frac{2\left(\frac{\mu_2}{\mu_1}\right)}{R_2} - \frac{2\left(\frac{\mu_2}{\mu_1} - 1\right)}{R_1}$$

$$\Rightarrow \quad \frac{1}{F} = \frac{2 \times 1.5}{-60} - \frac{2(1.5-1)}{+30}$$

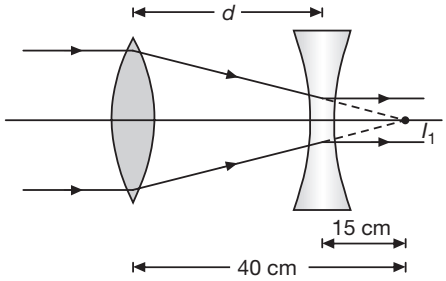
$$\Rightarrow \quad F = -12 \text{ cm}$$

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{F}$ with $u \to \infty$, we get

$$\frac{1}{v} + \frac{1}{\infty} = \frac{1}{-12}$$

$$\Rightarrow \quad v = -12 \text{ cm}$$

24. **Method 1:** Ray diagram is as shown



$d = 25$ cm

Method 2: Since this combination just behaves as a plane glass plate, so the power of the combination is zero. Since

$$P_{comb} = \frac{1}{f_{comb}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

$$\Rightarrow \quad O = \frac{1}{40} + \frac{1}{(-15)} - \frac{x}{(40)(-15)}$$

$$\Rightarrow \quad \frac{x}{(40)(15)} = \frac{1}{15} - \frac{1}{40}$$

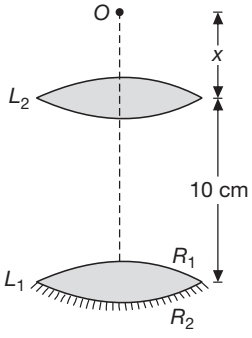
$$\Rightarrow \quad \frac{x}{(40)(15)} = \frac{40-15}{(40)(15)}$$

$$\Rightarrow \quad x = 25 \text{ cm}$$

25. Applying Lens Maker's Formula, we get

$$\frac{1}{40} = (1.5-1)\left(\frac{1}{120} + \frac{1}{R_1}\right)$$

$$\Rightarrow \quad R_1 = 24 \text{ cm}$$



Applying lens formula, for L_2, we get

$$\frac{1}{v_1} + \frac{1}{x} = \frac{1}{20} \qquad \ldots(1)$$

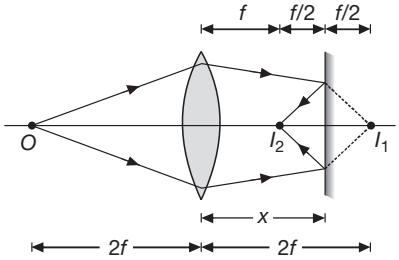
Using, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ for unsilvered side of L_1, we get

$$\frac{1.5}{(-120)} - \frac{1}{(v_1 - 10)} = \frac{1.5-1}{24} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$x = 10$ cm

26. The ray diagram is as shown in figure for first two steps. If the rays reflected from the mirror are parallel after passing through the lens for the second time, then I_2 must lie at first focus of lens. So, the desired distance is given by



$$x = (2f + 2f) - \frac{f}{2}$$

$$\Rightarrow \quad x = \frac{3f}{2} = \frac{3(30)}{2} = 45 \text{ cm}$$

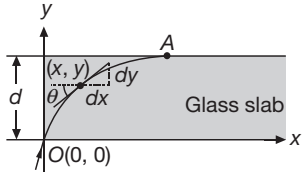
27. $\mu \sin\theta = 1 \times \sin 90°$

$$\Rightarrow \quad \sin\theta = \frac{1}{\mu}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{\mu^2 - 1}} = \frac{1}{\sqrt{1 + e^{\frac{x}{d}} - 1}} = e^{-\frac{x}{2d}}$$

$$\Rightarrow \quad \frac{dy}{dx} = e^{-\frac{x}{2d}}$$

$$\Rightarrow \quad y = 2d\left(1 - e^{-\frac{x}{2d}}\right) \qquad \ldots(1)$$



At A, $y = d$, so from equation (1), we get

$$\frac{1}{2} = 1 - e^{\frac{x}{2d}}$$

$$\Rightarrow \quad e^{\frac{x}{2d}} = 2$$

CHAPTER 1

$\Rightarrow \quad x = 2d\ln 2 = d\ln(4)$

$\Rightarrow \quad \alpha = 4$

$\Rightarrow \quad \mu = \sqrt{1+e^{\frac{2d\ln 2}{d}}} = \sqrt{1+e^{\ln 4}} = \sqrt{1+4} = \sqrt{5}$

$\Rightarrow \quad \beta = 5$

28. For refraction formula, we use

$u \to \infty$, $R = +10$ cm, $v = 2R = +20$ cm

$\mu_1 = 1$ and $\mu_2 = \mu$

Since, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \quad \frac{\mu}{20} = \frac{\mu - 1}{10}$

$\Rightarrow \quad \mu = 2\mu - 2$

$\Rightarrow \quad \mu = 2$

29. Let AB be the object placed at a distance of 15 cm from the lens as shown in figure. We shall calculate the position of image formed by this lens in absence of plane mirror.

For lens formula we use $\mu_1 = -15$ cm, and $f = +10$ cm

$\Rightarrow \quad \frac{1}{v_1} = \frac{1}{10} - \frac{1}{15} = \frac{3-2}{30} = \frac{1}{30}$

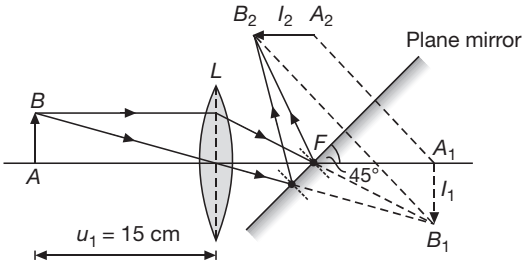
$\Rightarrow \quad v_1 = +30$ cm

So the image would be formed at 30 cm from the lens to the right of it

Magnification by lens is $m_1 = \frac{v_1}{u_1} = \frac{30}{15} = 2$

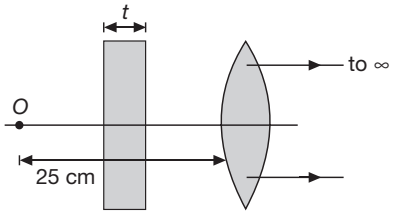
$\Rightarrow$ Size of the image $= 2 \times 4 = 8$ cm

The image produced by lens is I_1 which is shown by A_1B_1 in figure.



The image I_1 acts as the object for the plane mirror and after reflection of light rays from the plane mirror final image produced is A_2B_2. In a plane mirror, the image formed is at same distance at which object is kept from it and size remain same. So the final image is produced at a distance $30 - 10 = 20$ cm as shown in figure at an angle $90°$ to the principal axis as mirror rotates the reflected rays by twice the angle at which mirror is rotated.

30. For final image at infinity object must at focal point of lens i.e., 20 cm.



As actual object is kept at 25 cm so 5 cm is the shift due to glass slab, gives as

$\text{Shift} = t\left(1 - \frac{1}{\mu}\right)$

$\Rightarrow \quad 5 = t\left(1 - \frac{1}{1.5}\right)$

$\Rightarrow \quad t = 15$ cm

31. Using lens formula for objective, we have

$\frac{1}{v_0} - \frac{1}{-5} = \frac{1}{+0.25}$

$\Rightarrow \quad v_0 = -0.2632$ m

Now using lens formula for eyepiece, we have

$\frac{1}{(-0.25)} - \frac{1}{-u_e} = \frac{1}{+0.025}$

$\Rightarrow \quad \frac{1}{u_e} = 4 + 40 = 44$

$\Rightarrow \quad u_e = 0.0227$ m

Tube length of the telescope is

$L = 0.2632 + 0.0227 = 0.2859$ m $= 28.59$ cm

$\Rightarrow \quad L \approx 29$ cm

Magnifying power $M = \frac{v_o}{u_e} = \frac{0.2632}{0.0227} = 11.6$

$\Rightarrow \quad 10M = 116$

ARCHIVE: JEE MAIN

1. $1\times\sin(40°)=1.31\sin\theta$

$\Rightarrow \sin\theta=\dfrac{0.64}{1.31}$

$\Rightarrow \theta\approx 30°$

$\ell=(20\ \mu m)\cot\theta$

$\Rightarrow N=\dfrac{2}{20\times10^{-6}\times\cot\theta}$

$\Rightarrow N=\dfrac{2\times10^{6}}{20\times\sqrt{3}}=57735$

$\Rightarrow N\approx 57000$

Hence, the correct answer is (D).

2. $\dfrac{1}{v_1}-\dfrac{1}{u_1}=\dfrac{1}{f_1}$

$\Rightarrow v_1=\dfrac{40\times20}{(40-20)}=40$ cm

Now $u_2=60-40=20$ cm

Since $\dfrac{1}{v_2}+\dfrac{1}{u_2}=\dfrac{1}{f_2}$

$\Rightarrow v_2=\dfrac{20\times10}{(20-10)}=20$ cm

So, image traces back to object itself because image formed by the lens is at the centre of curvature of mirror.

Hence, the correct answer is (C).

3. $\theta=\dfrac{1.22\lambda}{D}$

$\Rightarrow \theta=\dfrac{1.22\times500\times10^{-9}}{200\times10^{-2}}=\dfrac{1.22\times500\times10^{-9}}{2}$

$\Rightarrow \theta=305\times10^{-9}$ radian

Hence, the correct answer is (B).

4.

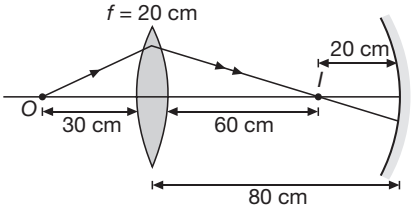


Image formed by lens

$\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$

$\Rightarrow \dfrac{1}{v}+\dfrac{1}{30}=\dfrac{1}{20}$

$\Rightarrow v=+60$ cm

If image position does not change even when mirror is removed, it means that image formed by lens is formed at centre of curvature of spherical mirror. Radius of curvature of mirror is

$R=80-60=20$ cm

So, focal length of mirror is $f=10$ cm

For virtual image, object is to be kept between focus and pole and hence maximum distance of object from spherical mirror for which virtual image is formed, is 10 cm

Hence, the correct answer is (D).

5. $\dfrac{1}{v}+\dfrac{1}{u}=-\dfrac{1}{40}$

Given, $\dfrac{v}{u}=-5$

$\Rightarrow -\dfrac{1}{5u}+\dfrac{1}{u}=-\dfrac{1}{40}$

$\Rightarrow u=-32$ cm

Hence, the correct answer is (A)

6. Since, $\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$

Given that $v=\pm2u$

$\Rightarrow \dfrac{1}{2u}+\dfrac{1}{u}=\dfrac{1}{20}$

$\Rightarrow \dfrac{3}{2u}=\dfrac{1}{20}$

$\Rightarrow u_1=x_1=30$ cm

And $\dfrac{1}{u}-\dfrac{1}{2u}=\dfrac{1}{20}$

$\Rightarrow \dfrac{1}{2u}=\dfrac{1}{20}$

$\Rightarrow u_2=x_2=10$

$\Rightarrow \dfrac{x_1}{x_2}=\dfrac{30}{10}=3$

Hence, the correct answer is (A).

CHAPTER 1

7. $\theta = \dfrac{1.22\lambda}{D}$

$\Rightarrow \quad \theta = \dfrac{1.22 \times 600 \times 10^{-9}}{250} \times 100 = 2.92 \times 10^{-7}$

$\Rightarrow \quad \theta = 3 \times 10^{-7}$ rad

Hence, the correct answer is (B).

8. $f_L = 18$ cm

$\Rightarrow \quad \dfrac{1}{18} = 0.5 \times \dfrac{2}{R}$

$\Rightarrow \quad R = 18$ cm

Since, $\dfrac{1}{f_2} = (\mu_\ell - 1)\left(-\dfrac{1}{18}\right)$ and $OA' = 27$ cm

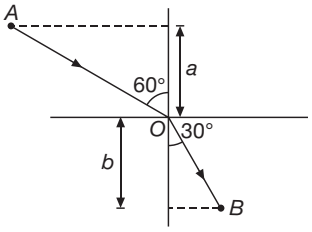
$\Rightarrow \quad \dfrac{1}{27} = \dfrac{1}{18} - \dfrac{(\mu_\ell - 1)}{18} = \dfrac{1 - \mu_\ell + 1}{18}$

$\Rightarrow \quad 2 = 3(2 - \mu_\ell) = 6 - 3\mu_\ell$

$\Rightarrow \quad \mu_\ell = \dfrac{4}{3}$

Hence, the correct answer is (B).

9. From the given figure



$\dfrac{a}{AO} = \cos(60°)$

$\Rightarrow \quad AO = 2a$

$\dfrac{b}{BO} = \cos 30°$

$\Rightarrow \quad BO = \dfrac{2b}{\sqrt{3}}$

So, length of optical path is

$L = AO + BO \times \sqrt{3}$

$\Rightarrow \quad L = 2a + 2b$

Hence, the correct answer is (D).

10. Focal length of plano-convex lens is

$$f_1 = \frac{R}{(\mu_1 - 1)}$$

Focal length of plano concave lens is

$$f_2 = \frac{-R}{(\mu_2 - 1)}$$

For the combination of two lenses, we have

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{\mu_1 - 1}{R} - \frac{\mu_2 - 1}{R} = \frac{\mu_1 - \mu_2}{R}$$

$\Rightarrow \quad f_{eq} = \dfrac{R}{\mu_1 - \mu_2}$

Hence, the correct answer is (C).

11. As the graph between magnification (m) and image distance (v) varies linearly, then

$m = k_1 v + k_2$

$\Rightarrow \quad \dfrac{v}{u} = k_1 v + k_2$

$\Rightarrow \quad \dfrac{1}{u} = k_1 + \dfrac{k_2}{v}$

$\Rightarrow \quad \dfrac{k_2}{v} - \dfrac{1}{u} = k_1$

Clearly, from above, we have

$k_1 = \dfrac{1}{f}$ and $k_2 = 1$

$\Rightarrow \quad f = \dfrac{1}{\text{slope of m-v graph}} = \dfrac{b}{c}$

Hence, the correct answer is (D).

12. Numerical aperature (NA) of the microscope is

$$NA = \frac{0.61\lambda}{d}$$

where d is the minimum separation between two points to be seen as distinct.

$\Rightarrow \quad d = \dfrac{0.61\lambda}{NA} = \dfrac{(0.61) \times (5000 \times 10\ \text{m}^{-10})}{1.25}$

$\Rightarrow \quad d = 2.4 \times 10^{-7}$ m

$\Rightarrow \quad d = 0.24\ \mu m$

Hence, the correct answer is (A).

13. $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$\Rightarrow \quad \dfrac{1}{v} + \dfrac{1}{(-5)} = \dfrac{1}{(-20)}$

$\Rightarrow \quad \dfrac{1}{v} = -\dfrac{1}{20} + \dfrac{1}{5}$

$$\Rightarrow \quad \frac{1}{v}=\frac{3}{20}$$

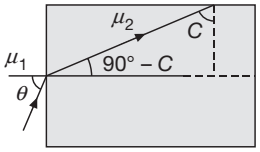
$$\Rightarrow \quad d=\frac{\left(\frac{20}{3}+5\right)}{\mu}=\frac{\frac{35}{3}}{\frac{4}{3}}$$

$$\Rightarrow \quad d=\frac{35}{4}$$

$$\Rightarrow \quad d=8.8 \text{ cm}$$

Hence, the correct answer is (D).

14.



$$\sin C=\frac{\mu_1}{\mu_2} \qquad \ldots(1)$$

$$\mu_1 \sin\theta=\mu_2 \sin(90°-C)=\mu_2 \cos C$$

From (1), we get $\cos C=\sqrt{1-\sin^2 C}=\sqrt{1-\frac{\mu_1^2}{\mu_2^2}}$

$$\Rightarrow \quad \sin\theta=\frac{\mu_2\sqrt{1-\frac{\mu_1^2}{\mu_2^2}}}{\mu_1}$$

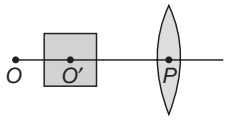
$$\theta=\sin^{-1}\left(\sqrt{\frac{\mu_2^2-\mu_1^2}{\mu_1^2}}\right)=\sin^{-1}\left(\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}\right)$$

For TIR,

$$\theta<\sin^{-1}\left(\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}\right)$$

Hence, the correct answer is (B).

15. Since $2f=10$ cm



$$\Rightarrow \quad f=5 \text{ cm}$$

$$\text{Shift } =OO'=1.5\left(1-\frac{2}{3}\right)=0.5 \text{ cm}$$

$$\Rightarrow \quad O'P=(10-0.5) \text{ cm}=9.5 \text{ cm}$$

Since, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{(-9.5)}=\frac{1}{5}$$

$$\Rightarrow \quad \frac{1}{v}+\frac{1}{9.5}=\frac{1}{5}$$

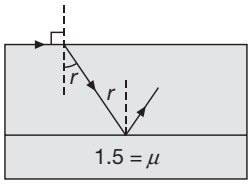
$$\Rightarrow \quad v=10.55 \text{ cm}$$

Hence shift is $(10.55-10)$ cm $=0.55$ cm away from lens.

Hence, the correct answer is (D).

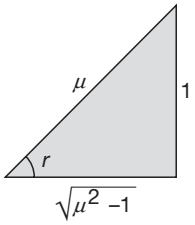
16. For air-medium interface maximum angle of incidence can be 90°

$$\Rightarrow \quad \frac{\sin(90°)}{\sin r}=\mu$$



$$\Rightarrow \quad \sin r=\frac{1}{\mu}$$

For light reflected from liquid glass interface to be never completely polarised, we must have angle of incidence (r) at the liquid glass interface to be less than the polarising angle p at the interface



$$\Rightarrow \quad r<p$$

$$\Rightarrow \quad \tan r<\tan p \text{ , where}$$

$$\tan p=\frac{3}{2\mu} \qquad \{\text{from Brewster's law}\}$$

$$\Rightarrow \quad \frac{1}{\sqrt{\mu^2-1}}<\frac{3}{2\mu}$$

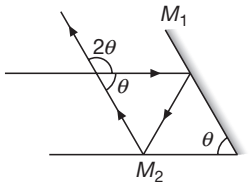
$$\Rightarrow \quad \mu \geq \frac{3}{\sqrt{5}}$$

$$\Rightarrow \quad \mu_{\min}=\frac{3}{\sqrt{5}}$$

Hence, the correct answer is (C).

CHAPTER 1

17. The ray diagram for the situation asked in the problem is shown in figure.



From above, we see that

$3\theta = 180°$

$\Rightarrow \quad \theta = 60°$

Hence, the correct answer is (D).

18. $\dfrac{1}{f_1} = (\mu_1 - 1)\dfrac{1}{R}$

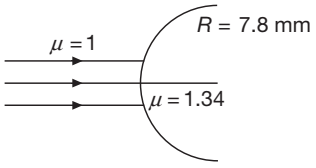
$\dfrac{1}{f_2} = (\mu_2 - 1)\dfrac{1}{R}$

$\Rightarrow \quad 2(\mu_1 - 1)\dfrac{1}{R} = (\mu_2 - 1)\dfrac{1}{R}$

$\Rightarrow \quad 2\mu_1 - \mu_2 = 1$

Hence, the correct answer is (A).

19. $\dfrac{1.34}{v} - \dfrac{1}{u} = \dfrac{1.34 - 1}{(7.8)}$



Since $u \to -\infty$

$\Rightarrow \quad v = f$

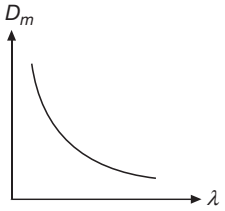
$\Rightarrow \quad \dfrac{1.34}{f} = \dfrac{0.34}{7.8 \text{ mm}}$

$\Rightarrow \quad f = \left(\dfrac{1.34 \times 7.8}{0.34}\right)$ mm

$\Rightarrow \quad f = 3.07 \text{ cm} \approx 3.1 \text{ cm}$

Hence, the correct answer is (C).

20. Since for a thin prism, we have $D_m = (\mu - 1)A$



Hence, the correct answer is (A).

21. $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

Since, $u = -20$ m, $f = 0.3$

$\Rightarrow \quad \dfrac{1}{v} = \dfrac{1}{0.3} - \dfrac{1}{20}$

$\Rightarrow \quad \dfrac{1}{v} = \dfrac{10}{3} - \dfrac{1}{20}$

$\Rightarrow \quad v = \dfrac{60}{197}$ m

$\Rightarrow \quad m = \dfrac{v}{u} = \dfrac{3}{197}$

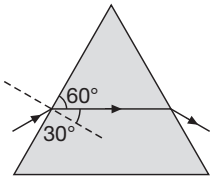
Since $(\text{velocity})_{\text{image}} = m^2 (\text{velocity})_{\text{object}}$

$\Rightarrow \quad v_{\text{image}} = \left(\dfrac{3}{197}\right)^2 \times 5$

$\Rightarrow \quad v_{\text{image}} = 1.16 \times 10^{-3}$ ms^{-1} toward the lens.

Hence, the correct answer is (C).

22. For minimum deviation the ray passes symmetrically



$\Rightarrow \quad r = 30°$

$\sin i = \sqrt{3} \sin 30° = \dfrac{\sqrt{3}}{2}$

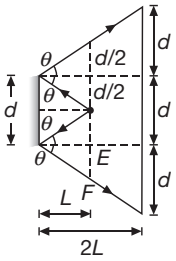
$\Rightarrow \quad i = 60°$

Hence, the correct answer is (D).

23. $\dfrac{d}{d/2} = \dfrac{y}{2L}$

$\Rightarrow \quad y = d$

Hence, the distance over which the image can be seen is $d + d + d = 3d$.



Hence, the correct answer is (B).

24. For lens A

$\dfrac{1}{v} - \dfrac{1}{(-20)} = \dfrac{1}{5}$

$\Rightarrow \quad v = \dfrac{20}{3}$ cm

For lens B

$u = \dfrac{20}{3} - 2$

$$u = \frac{14}{3} \text{ cm}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{\frac{14}{3}} = -\frac{1}{5}$$

$$\Rightarrow \quad v = 70 \text{ cm}$$

Image is real and right of B.

Hence, the correct answer is (B).

25. Using Lens Maker's Formula, we get

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\frac{2}{R}$$

$$\Rightarrow \quad f = R$$

In water $\left(\mu = \frac{4}{3}\right)$, we get the new focal length f' as

$$\frac{1}{f'} = \left(\frac{3/2}{4/3} - 1\right)\frac{2}{R}$$

$$\Rightarrow \quad f' = 4f \qquad \{\because R = f\}$$

Now since the object in placed is between focus and lens, so no real image on the screen is obtained.

Hence, the correct answer is (C).

26. For plano convex lens, we have

$$\frac{1}{f_2} = \frac{\mu_2 - 1}{R} \qquad \ldots(1)$$

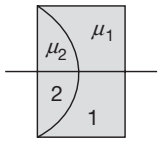
For plano concave lens, we have

$$\frac{1}{f_1} = -\left(\frac{\mu_1 - 1}{R}\right) \qquad \ldots(2)$$

Now for combination of both as shown, we have

For 2, $\frac{\mu_2}{v_1} - \frac{1}{\infty} = \frac{\mu_2 - 1}{\infty}$

For 1, $\frac{\mu_1}{v_1'} - \frac{\mu_2}{v_1} = \frac{\mu_2 - \mu_1}{R}$



For combination, $\frac{1}{f} - \frac{\mu_1}{v_1'} = \frac{1 - \mu_1}{\infty}$

$$\Rightarrow \quad \frac{1}{f} = \frac{(\mu_2 - \mu_1)}{R} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \quad f = \frac{R}{\mu_2 - \mu_1}$$

Hence, the correct answer is (B).

27. When object is at 8 cm, image distance

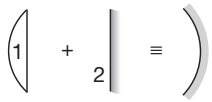
$$v_1 = \frac{f \times u}{u - f} = \frac{(-5)(-8)}{-8 + 5} = -\frac{40}{3} \text{ cm}$$

When object is at 12 cm, image distance $v_2 = -\frac{60}{7}$ cm

$$\Rightarrow \quad \text{Separation} = |v_1 - v_2| = \frac{100}{21} \text{ cm}$$

Hence, the correct answer is (C).

28. For figure (A),

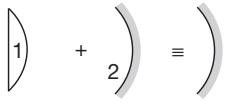


$$\frac{1}{f_\ell} = \left(\frac{\mu - 1}{R}\right) \text{ and } f_{eq} = -28 \text{ cm}$$

$$P = 2P_\ell + P_m = 2P_\ell + 0$$

$$\Rightarrow \quad -\frac{1}{28} = -2\left(\frac{\mu - 1}{R}\right) \qquad \ldots(1)$$

For figure (B),



$$\frac{1}{f_\ell} = \left(\frac{\mu - 1}{R}\right), \; f_2 = -\frac{R}{2} \text{ and } f_{eq} = -10 \text{ cm}$$

Since, $P = 2P_\ell + P_m$

$$\Rightarrow \quad -\frac{1}{10} = -2\left(\frac{\mu - 1}{R}\right) - \frac{2}{R}$$

$$\Rightarrow \quad \frac{1}{10} = \frac{1}{28} + \frac{2}{R}$$

$$\Rightarrow \quad \frac{2}{R} = \frac{1}{10} - \frac{1}{28} = \frac{18}{280}$$

$$\Rightarrow \quad R = \frac{280}{9} \text{ cm}$$

Substituting R in equation (1), we get

$$\frac{1}{28} = 2\left(\frac{\mu - 1}{280}\right)9$$

$$\Rightarrow \quad \mu - 1 = \frac{5}{9}$$

$$\Rightarrow \quad \mu = 1 + \frac{5}{9} = \frac{14}{9} = 1.55$$

Hence, the correct answer is (C).

29. Since, $\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$

$$\Rightarrow \quad \frac{1}{10} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{2}{f_1 f_2}$$

CHAPTER 1

Substituting the values of f_1 and f_2 option wise, we see that

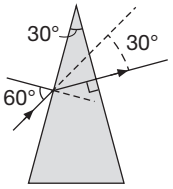
$$f_{eq} = 10 \text{ cm for } f_1 = 18 \text{ cm and } f_2 = 20 \text{ cm}.$$

Hence, the correct answer is (A).

30. As $\delta = i + e - A$

$\Rightarrow \quad 30° = 60° + e - 30°$

$\Rightarrow \quad e = 0$



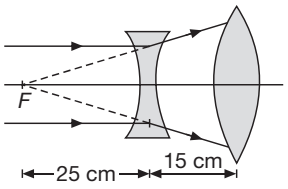
So, required angle with face of prism is 90°

Hence, the correct answer is (C).

31. Given that focal length of concave lens is $f = -25$ cm

Focal length of convex lens is $f' = 20$ cm

The formation of image is shown here.



The image for diverging lens will form at F. i.e. at focal length of concave lens.

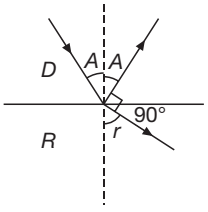
Now, this image will serve as the object for convex lens. It is at twice the focal length of the convex lens (i.e. $2f$). Hence, the final image will also form at $2f$, which is at a distance of 40 cm from the convergent lens. Also, the image formed is real.

Hence, the correct answer is (A).

32. Refractive index of denser medium with respect to rarer medium, $n_{12} = \dfrac{n_D}{n_R} = \dfrac{1}{\sin\theta_C}$

$$\Rightarrow \quad \frac{n_R}{n_D} = \sin\theta_C \qquad \ldots(1)$$

Using Snell's law at the interface of two media,



$$n_D \sin A = n_R \sin r$$

$$\Rightarrow \quad \frac{n_R}{n_D} = \frac{\sin A}{\sin(90 - A)} = \frac{\sin A}{\cos A} = \tan A$$

$$\Rightarrow \quad \tan A = \sin\theta_C$$

$$\Rightarrow \quad A = \tan^{-1}(\sin\theta_C)$$

Hence, the correct answer is (C).

33. Telescope resolves and brings the objects closer which is far away from the telescope. Hence for telescope with magnifying power 20, the tree appears 20 times nearer.

Hence, the correct answer is (D).

34. For convex lens, $u_1 = -60$ cm, $f_1 = 30$ cm

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{30} - \frac{1}{60} = \frac{1}{60}$$

$$\Rightarrow \quad v_1 = 60 \text{ cm}$$

For concave lens

$$u_2 = 60 - 20 = 40 \text{ cm}, \quad f_2 = -120 \text{ cm}$$

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{120} + \frac{1}{40} = \frac{2}{120}$$

$$\Rightarrow \quad v_2 = 60 \text{ cm}$$

For plane mirror, virtual object is 10 cm behind the mirror.

Hence, real image is formed 10 cm in front of the mirror.

Now, again for concave lens, $u_2 = 40$ cm i.e., light rays from the object retrace their path after striking the plane mirror, Hence the final image is formed at the object itself.

Hence, the correct answer is (A).

35. The refractive index is given by

$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\Rightarrow \quad \mu = \frac{\text{Reading 3} - \text{Reading 1}}{\text{Reading 3} - \text{Reading 2}}$$

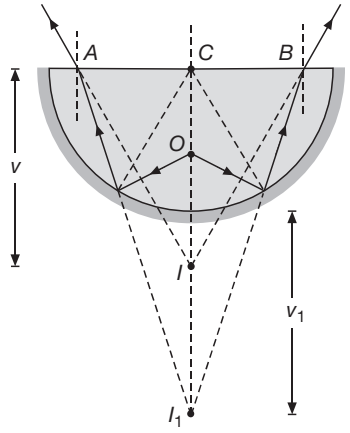
Therefore, the minimum of three readings are required.

Hence, the correct answer is (C).

36. Since, $R = -10$ cm

So, object distance from mirror is

$$u = -(10 - 6) = -4 \text{ cm}$$



Focal length of mirror is

$$f = -\frac{R}{2} = -\frac{10}{2} = -5 \text{ cm}$$

Image distance $= v = ?$

Using mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$\Rightarrow \quad \frac{1}{v_1} + \frac{1}{-4} = \frac{1}{-5}$$

$$\Rightarrow \quad \frac{1}{v_1} = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$$

$$\Rightarrow \quad v_1 = 20 \text{ cm}$$

Now, I_1 acts as object for plane glass surface AB

$$\Rightarrow \quad \text{Apparent depth } v = \frac{R + v_1}{\mu} = \frac{30}{1.5} = 20 \text{ cm}$$

Hence, the position of the image of the air bubble made by the mirror is seen 20 cm below the flat surface.

Hence, the correct answer is (B).

37. Here, $\lambda = 600 \text{ nm} = 6 \times 10^{-7}$ m

$D = 30 \text{ cm} = 30 \times 10^{-2}$ m $= 0.3$ m

$R = 10 \text{ ly} = 10 \times 9.46 \times 10^{15}$ m

$\ell = ?$

The limit of resolution of a telescope $\Delta\theta = \frac{1.22\lambda}{D} = \frac{\ell}{R}$

$$\ell = \frac{1.22\lambda R}{D} = \frac{1.22 \times 6 \times 10^{-7} \times 10 \times 9.46 \times 10^{15}}{0.3}$$

$$\Rightarrow \quad \ell = 2.31 \times 10^{8} \text{ km}$$

Hence, the correct answer is (A).

38. Given, wavelength of light, $\lambda = 500 \text{ nm} = 500 \times 10^{-9}$ m

Least distance of distinct vision, $D = 25 \text{ cm} = 25 \times 10^{-2}$ m

Radius of pupil, $r = 0.25$ cm

$\Rightarrow$ Diameter of pupil, $d = 2r = 0.50 \text{ cm} = 0.50 \times 10^{-2}$ m

Resolving power of eye, $\Delta\theta = \frac{1.22\lambda}{d}$

$$\Rightarrow \quad \Delta\theta = \frac{1.22 \times 500 \times 10^{-9}}{0.50 \times 10^{-2}} = 1.22 \times 10^{-4} \text{ rad}$$

So, minimum separation that eye can resolve is

$$x = D\Delta\theta = 1.22 \times 10^{-4} \times 25 \times 10^{-2}$$

$$\Rightarrow \quad x = 30.5 \times 10^{-6} \text{ m} = 30 \ \mu\text{m}$$

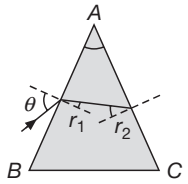
Hence, the correct answer is (D).

39. According to Snell's law

$$\sin\theta = \mu \sin r_1$$

$$\Rightarrow \quad \sin r_1 = \frac{\sin\theta}{\mu}$$

$$\Rightarrow \quad r_1 = \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$$



Now, $A = r_1 + r_2$

$$\Rightarrow \quad r_2 = A - r_1 = A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) \qquad \ldots(1)$$

For the ray to get transmitted through the face AC, r_2 must be less than critical angle, i.e., $r_2 < \sin^{-1}\left(\frac{1}{\mu}\right)$

$$\Rightarrow \quad A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) < \sin^{-1}\left(\frac{1}{\mu}\right) \qquad \text{(using (1))}$$

$$\Rightarrow \quad \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) > A - \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\Rightarrow \quad \frac{\sin\theta}{\mu} > \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)$$

$$\Rightarrow \quad \theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$$

Hence, the correct answer is (C).

CHAPTER 1

40. Here, $u = -10$ cm , $v = +15$ cm

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{15}-\frac{1}{10}=\frac{1}{f}=\frac{2}{R}$$

$$\Rightarrow \quad -\frac{5}{150}=\frac{2}{R}$$

$$R=-\frac{300}{5}=-60 \text{ cm}$$

Hence, the correct answer is (C).

41. Angular magnification $m=\frac{f_0}{f_e}=\frac{150}{5}=30$

$$\Rightarrow \quad \frac{\tan\beta}{\tan\alpha}=30$$

$$\Rightarrow \quad \tan\beta=\tan\alpha\times 30=\left(\frac{50}{1000}\right)\times 30=\frac{15}{10}=\frac{3}{2}$$

$$\Rightarrow \quad \beta=\tan^{-1}\left(\frac{3}{2}\right)$$

$$\Rightarrow \quad \theta=\beta\approx 60^\circ$$

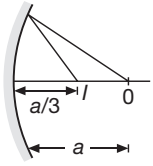
Hence, the correct answer is (D).

42. This combination will behave like a mirror of power,

$$P_{\text{eq}}=2P_L+P_M$$

$$\Rightarrow \quad P_{eq}=2\frac{1}{f}+0$$

$$\Rightarrow \quad f_{eq}=\frac{f}{2}$$



So, the behaviour of the system, will be like a mirror of focal length $-\frac{f}{2}$

Using mirror equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f_{eq}}$, we get

Since, $u=-a$, $v=-\frac{a}{3}$, $f_{eq}=-\frac{f}{2}$

$$\Rightarrow \quad \frac{1}{-\frac{a}{3}}+\frac{1}{-a}=\frac{-1}{\frac{f}{2}}$$

$$\Rightarrow \quad \frac{4}{a}=\frac{2}{f}$$

$$\Rightarrow \quad a=2f$$

Hence, the correct answer is (B).

43. Given $\mu=\frac{3}{2}$ (crown glass) and focal length $=f$

Let focal length be f_1 when lens is placed in liquid of refractive index $\mu_1=\frac{4}{3}$

Let focal length be f_2 when lens is placed in liquid of refractive index $\mu_2=\frac{5}{3}$

According to Lens Makers Formula, we have

$$\frac{1}{f}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Using Lens maker's formula $\frac{1}{f_1}=\left(\frac{\mu}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$, we get

$$\frac{1}{f_1}=\left(\frac{\frac{3}{2}}{\frac{4}{3}}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{8}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Similarly, $\frac{1}{f_2}=\left(\frac{\mu}{\mu_2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\Rightarrow \quad \frac{1}{f_2}=\left(\frac{\frac{3}{2}}{\frac{5}{3}}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{-1}{10}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

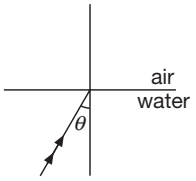
$$\Rightarrow \quad f_1=4f \text{ and } f_2=-5f$$

Hence, the correct answer is (C).

44. Since, $\sin C=\sin\theta=\frac{1}{\mu}$ and $\mu\lambda=$ constant

$$\Rightarrow \quad \sin\theta\propto\lambda$$

Also, refractive index (μ) of the medium depends on the wavelength of the light. μ is less for the greater wavelength (i.e. lesser frequency).



So, θ will be more for lesser frequency of light. Hence, the spectrum of visible light whose frequency is less than that of green light will come out to the air medium.
Hence, the correct answer is (C).

45. According to Lens Maker's Formula, we have

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

Since the lens is plano-convex, so

$$R_1 = R\ ,\ R_2 \to \infty$$

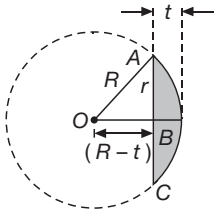
$$\Rightarrow \quad \frac{1}{f} = \frac{(\mu - 1)}{R}$$

$$\Rightarrow \quad f = \frac{R}{(\mu - 1)} \qquad \ldots(1)$$

The speed of light in the medium of lens is $2 \times 10^8\ \text{ms}^{-1}$

$$\Rightarrow \quad \mu = \frac{c}{v} = \frac{3 \times 10^8\ \text{ms}^{-1}}{2 \times 10^8\ \text{ms}^{-1}} = \frac{3}{2} \qquad \ldots(2)$$

If r is the radius and t is the thickness of lens (at the centre), the radius of curvature R of its curved surface in accordance with figure will be given by using Pythagoras Theorem.



$$R^2 = r^2 + (R - t)^2$$

$$\Rightarrow \quad R^2 = r^2 + R^2 + t^2 - 2Rt$$

Since $r \gg t$

$$\Rightarrow \quad 2Rt = r^2 + t^2$$

$$\Rightarrow \quad R = \frac{r^2}{2t} \qquad \ldots(3)$$

Since, $r = 3$ cm , $t = 3$ mm $= 0.3$ cm

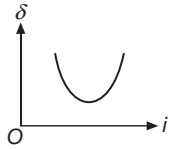
$$\Rightarrow \quad R = \frac{(3)^2}{2 \times 0.3} = 15 \text{ cm}$$

On substituting the values of μ and R from Equations (2) and (3) in (1), we get

$$f = \frac{15 \text{ cm}}{(1.5 - 1)} = 30 \text{ cm}$$

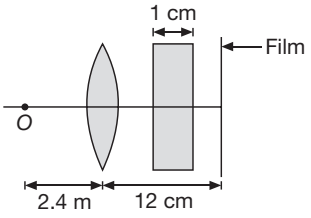
Hence, the correct answer is (D).

46. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is as shown in the adjacent figure.



Hence, the correct answer is (D).

47.



According to thin lens formula, we have

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Since, $u = -2.4$ m $= -240$ cm , $v = 12$ cm

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{12} - \frac{1}{(-240)} = \frac{1}{12} + \frac{1}{240}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{21}{240}$$

$$\Rightarrow \quad f = \frac{240}{21} \text{ cm}$$

When a glass plate is interposed between lens and film, so shift produced by the glass plate will be

$$\text{Shift} = t\left(1 - \frac{1}{\mu}\right) = 1\left(1 - \frac{1}{1.5}\right) = 1\left(1 - \frac{2}{3}\right) = \frac{1}{3} \text{ cm}$$

To get image at film, lens should form an image at a distance

$$v' = 12 - \frac{1}{3} = \frac{35}{3} \text{ cm}$$

Again, using lens formula, we get

$$\frac{21}{240} = \frac{3}{35} - \frac{1}{u'}$$

$$\Rightarrow \quad \frac{1}{u'} = \frac{3}{35} - \frac{21}{240} = \frac{1}{5}\left[\frac{3}{7} - \frac{21}{48}\right]$$

$$\Rightarrow \quad \frac{1}{u'} = \frac{1}{5}\left[\frac{144 - 147}{336}\right]$$

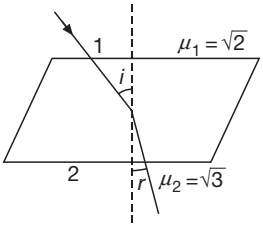
$$\Rightarrow \quad \frac{1}{u'} = -\frac{3}{1680}$$

$$\Rightarrow \quad u' = -560 \text{ cm} = -5.6 \text{ m}$$

$$\Rightarrow \quad |u'| = 5.6 \text{ m}$$

Hence, the correct answer is (C).

48.



Since, $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$

$$\Rightarrow \quad \cos i = \frac{10}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + (-10)^2}} = \frac{10}{20}$$

$$\Rightarrow \quad \cos i = \frac{1}{2}$$

$$\Rightarrow \quad i = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

Using Snell's Law, $\mu_1 \sin i = \mu_2 \sin r$, we get

$$\sqrt{2}\sin(60°) = \sqrt{3}\sin r$$

$$\Rightarrow \quad r = 45°$$

Hence, the correct answer is (B).

49. Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

$$\Rightarrow \quad \frac{dv}{dt} = -\frac{v^2}{u^2}\left(\frac{du}{dv}\right)$$

where $\frac{dv}{dt}$ is velocity of image and $\frac{du}{dt}$ is velocity of object.

Given that, $f = 20$ cm

$$\Rightarrow \quad \frac{1}{u} + \frac{1}{(-280)} = \frac{1}{20}$$

$$\Rightarrow \quad v = \frac{280}{15} \text{ cm}$$

So, velocity of image is given by

$$\frac{dv}{dt} = -\left(\frac{280}{15 \times 280}\right)^2 \times 15$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{1}{15} \text{ ms}^{-1}$$

Hence, the correct answer is (B).

50. As the beam is initially parallel, the shape of wavefront is planar.

Hence, the correct answer is (A).

51. Given $\mu = \mu_0 + \mu_2 \text{I}$

As $\mu = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$

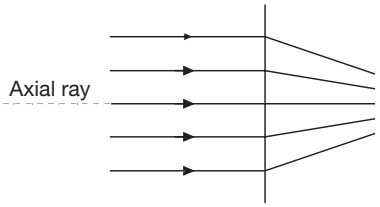
$$\mu = \frac{c}{v}$$

$$\Rightarrow \quad v = \frac{c}{\mu} = \frac{c}{\mu_0 + \mu_2 \text{I}}$$

As the intensity is maximum on the axis of the beam, therefore v is minimum on the axis of the beam.

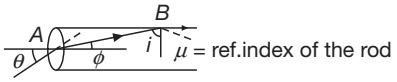
Hence, the correct answer is (B).

52. When the beam enters the medium, the axial ray will travel slowest and hence it will lag behind. To compensate for the path, the rays will bend towards the axis.



Hence, the correct answer is (C).

53. For light ray to graze along the wall of rod, $i = C$



where, C is the critical angle given by

$$C = \sin^{-1}\left(\frac{1}{\mu}\right)$$

Since $C = 90° - \phi$

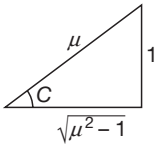
$$\Rightarrow \quad \phi = 90° - C$$

Applying Snell's Law at A, we get

$$\frac{\sin\theta}{\sin\phi} = \mu = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{\sin\theta}{\cos C} = \mu \qquad \{\because \phi = 90 - C\}$$

Since, $\cos C = \frac{\sqrt{\mu^2 - 1}}{\mu}$



$$\Rightarrow \quad \sin\theta = \mu\frac{\sqrt{\mu^2 - 1}}{\mu} = \sqrt{\mu^2 - 1}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\sqrt{\frac{4}{3} - 1}\right) = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

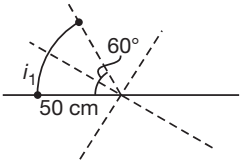
Hence, the correct answer is (D).

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. Image formed by the lens is at $(75, 0)$

This acts as a virtual image for the mirror



Assuming that the mirror is not tilted, then for mirror

$u = +25,\ v = ?,\ f = +50$

$\Rightarrow \quad \frac{1}{v} + \frac{1}{25} = \frac{1}{50}$

$\Rightarrow \quad v = -50 \text{ cm}$ ...(1)

This image is formed on x-axis

Now, when the mirror is rotated clockwise by $30°$, image rotates by $60°$ clockwise. Since the ray strikes the pole of the mirror, so this ray rotates by $60°$ and image lies on this ray. Hence image rotates by $60°$.

New co-ordinates of image will be

$x = 50 - 50\cos(60°) = 25 \text{ cm}$

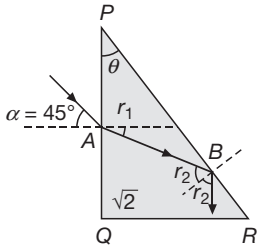
$y = 50\sin(60°) = \frac{50\sqrt{3}}{2} \text{ cm} = 25\sqrt{3} \text{ cm}$

Hence, the correct answer is (A).

2. For Refraction at A, we have

$\frac{\sin(45°)}{\sin r_1} = \sqrt{2}$

$\Rightarrow \quad \sin r_1 = \frac{1}{2}$



$\Rightarrow \quad r_1 = 30°$

For TIR at B, we have

$r_2 \geq C$

Since, $\sin C = \frac{1}{\mu} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad C = 45°$

$\Rightarrow \quad r_2 \geq 45°$

In ΔPAB, $(90° + r_1) + \theta + (90° - r_2) = 180°$

$\Rightarrow \quad r_2 = r_1 + \theta$

$\Rightarrow \quad 45° = 30° + \theta$

$\Rightarrow \quad \theta = 15°$

Hence, the correct answer is (A).

3. $R = 10 \text{ cm}$

Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\frac{1}{v} - \frac{1.5}{-50} = \frac{1 - 1.5}{-10}$

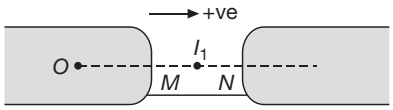
$\Rightarrow \quad \frac{1}{v} + \frac{1.5}{50} = \frac{0.5}{10}$

$\Rightarrow \quad \frac{1}{v} = \frac{0.5}{10} - \frac{1.5}{50} = \frac{2.5 - 1.5}{50}$

$\Rightarrow \quad v = 50$

Now, $MN = d$, $MI_1 = 50 \text{ cm}$,

$NI_1 = (d - 50) \text{ cm}$



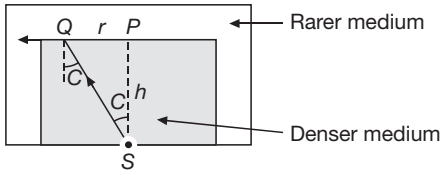
Again, $\frac{1.5}{\infty} - \frac{1}{-(d-50)} = \frac{1.5 - 1}{10}$

$\frac{1}{d - 50} = \frac{1}{20}$

$d = 70$

Hence, the correct answer is (B).

4. At point Q angle of incidence is critical angle C, where



$\sin C = \frac{\mu_\ell}{\mu_{\text{block}}}$

In ΔPQS, $\sin C = \frac{r}{\sqrt{r^2 + h^2}}$

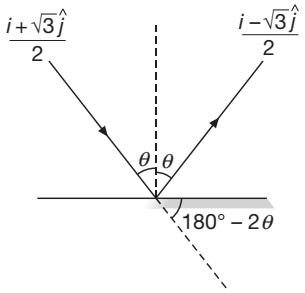
$\Rightarrow \quad \frac{\mu_\ell}{\mu_{\text{block}}} = \frac{r}{\sqrt{r^2 + h^2}}$

$\Rightarrow \quad \mu_\ell = \frac{r}{\sqrt{r^2 + h^2}} \times 2.72 = \frac{5.77}{11.54} \times 2.72 = 1.36$

Hence, the correct answer is (C).

CHAPTER 1

5. $\cos(180^\circ - 2\theta) = \dfrac{\left(\dfrac{i+\sqrt{3}\hat{j}}{2}\right)\left(\dfrac{i-\sqrt{3}\hat{j}}{2}\right)}{\left|\dfrac{i+\sqrt{3}\hat{j}}{2}\right|\left|\dfrac{i-\sqrt{3}\hat{j}}{2}\right|}$



$$\Rightarrow \quad -\cos 2\theta = \frac{(1-3)}{\frac{4}{1}}$$

$$\Rightarrow \quad -\cos 2\theta = \frac{-1}{2}$$

$$\Rightarrow \quad \cos 2\theta = \frac{1}{2}$$

$$\Rightarrow \quad 2\theta = 60^\circ$$

$$\Rightarrow \quad \theta = 30^\circ$$

Hence, the correct answer is (A).

6. $\mu = \dfrac{c}{v_{med}} = \dfrac{f\lambda_{air}}{f\lambda_{med}} = \dfrac{3}{2}$

Now, $v = +8$ m, $m = -\dfrac{1}{3} = \dfrac{v}{u}$

$$\Rightarrow \quad u = -24 \text{ m}$$

Since, $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{8} + \frac{1}{24} = \frac{4}{24}$$

$$\Rightarrow \quad f = 6 \text{ m}$$

For a plano-convex lens, we have

$$f = \frac{R}{\mu - 1}$$

$$\Rightarrow \quad 6 = \frac{R}{0.5}$$

$$\Rightarrow \quad R = 3 \text{ m}$$

Hence, the correct answer is (C).

7. $\dfrac{1}{f} = (1.5-1)\left(\dfrac{1}{14} - \dfrac{1}{\infty}\right) + (1.2-1)\left(\dfrac{-1}{\infty} - \dfrac{1}{-14}\right)$

$$\Rightarrow \quad f = 20 \text{ cm}$$

Since, object distance $= 40$ cm $= 2f$

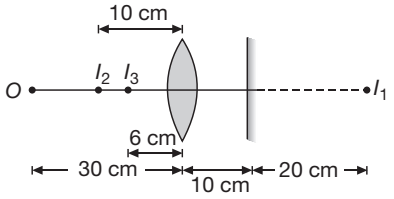
$\Rightarrow$ Image distance $= 40$ cm

Hence, the correct answer is (B).

8. When $\theta < C$ partial transmission and reflection will occur. When $\theta > C$, only reflection takes place.

Hence, the correct answer is (C).

9. Since object is placed at distance $2f$ from the lens, so first image I_1 will be formed at distance $2f$ on other side. This image I_1 will behave like a virtual object for mirror. The second image I_2 will be formed at distance 20 cm in front of the mirror, or at distance 10 cm to the left hand side of the lens.



Now, applying lens formula, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

where, $v = ?$, $u = +10$ cm, $f = +15$ cm

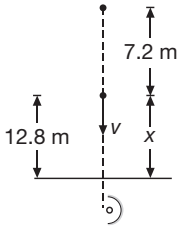
$$\Rightarrow \quad \frac{1}{v} - \frac{1}{+10} = \frac{1}{+15}$$

$$\Rightarrow \quad v = 6 \text{ cm}$$

So, the final image is at distance 16 cm from the mirror. But, this image will be real, because the ray of light is travelling from right to left.

Hence, the correct answer is (B).

10. $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 7} = 12 \text{ ms}^{-1}$



Position of ball with respect to fish is

$$x_{app} = \mu x$$

$$\Rightarrow \quad \frac{dx_{app}}{dt} = \mu\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \quad v_{app} = \mu v = \frac{4}{3} \times 12 = 16 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

11. At minimum deviation $(\delta = \delta_m)$

$$r_1 = r_2 = \frac{A}{2} = \frac{60°}{2} = 30° \qquad \text{(For both colours)}$$

Hence, the correct answer is (A).

12. Critical angle from Region III to Region IV

$$\sin C = \frac{\frac{n_0}{8}}{\frac{n_0}{6}} = \frac{3}{4}$$

Now, applying Snell's law in Region I and Region III, we get

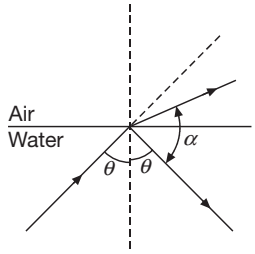
$$n_0 \sin\theta = \frac{n_0}{6}\sin C$$

$$\Rightarrow \quad \sin\theta = \frac{1}{6}\sin C = \frac{1}{6}\left(\frac{3}{4}\right) = \frac{1}{8}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{1}{8}\right)$$

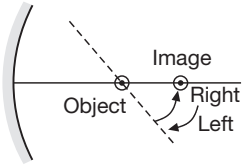
Hence, the correct answer is (B).

13. Since $\theta < C$, both reflection and refraction will take place. From the figure we can see that angle between reflected and refracted rays α is less than $180° - 2\theta$.



Hence, the correct answer is (C).

14. Since object and image move in opposite directions, the positioning should be as shown in the figure.



Object lies between focus and centre of curvature $f < x < 2f$.

Hence, the correct answer is (B).

15. For refraction from lens, we have $\frac{1}{v_1} - \frac{1}{(-20)} = \frac{1}{15}$

$$\Rightarrow \quad v_1 = 60 \text{ cm}$$

i.e., first image is formed at 60 cm to the right of lens system.

Reflection from mirror

After reflection from the mirror, the second image will be formed at a distance of 60 cm to the left of lens system.

Refraction from lens

$$\frac{1}{v_3} - \frac{1}{60} = \frac{1}{15}$$

$$\Rightarrow \quad v_3 = 12 \text{ cm}$$

Therefore, the final image is formed at 12 cm to the left of the lens system.

Hence, the correct answer is (C).

16. From the lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, we have

$$\frac{1}{f} = \frac{1}{10} - \frac{1}{-10}$$

$$\Rightarrow \quad f = +5 \text{ cm}$$

Further from the graph, $\Delta u = 0.1$ cm and $\Delta v = 0.1$ cm

Now, differentiating the lens formula we get

$$\frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}$$

$$\Rightarrow \quad \Delta f = \left(\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right) f^2$$

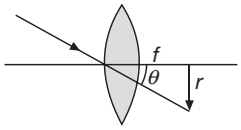
Substituting the values, we get

$$\Delta f = \left(\frac{0.1}{10^2} + \frac{0.1}{10^2}\right)(5)^2 = 0.05 \text{ cm}$$

$$\Rightarrow \quad f \pm \Delta f = (5 \pm 0.05) \text{ cm}$$

Hence, the correct answer is (B).

17.



$$r = f\tan\theta$$

$$\Rightarrow \quad r \propto f$$

$$\Rightarrow \quad \pi r^2 \propto f^2$$

Hence, the correct answer is (B).

18. Let focal length of convex lens is $+f$, then length of concave lens would be $-\frac{3}{2}f$.

Since $\frac{1}{f_{comb}} = \frac{1}{f_1} + \frac{1}{f_2}$

CHAPTER 1

$$\Rightarrow \quad \frac{1}{30}=\frac{1}{f}-\frac{2}{3f}=\frac{1}{3f}$$

$$\Rightarrow \quad f=10 \text{ cm}$$

Therefore, focal length of convex lens is $+10$ cm and that of concave lens is -15 cm

Hence, the correct answer is (D).

19. Distance of object from mirror is $15+\frac{33.25}{1.33}=40$ cm

Distance of image from mirror is $15+\frac{25}{1.33}=33.8$ cm

Applying mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get

$$\Rightarrow \quad \frac{1}{-33.8}+\frac{1}{-40}=\frac{1}{f}$$

$$\Rightarrow \quad f=-18.3 \text{ cm}$$

Hence, the correct answer is (C).

20. Critical angle $C=\sin^{-1}\left(\frac{1}{\mu}\right)$

Wavelength increases in the sequence of VIBGYOR. According to Cauchy's formula refractive index (μ) decreases as the wavelength increases. Hence the refractive index will increase in the sequence of ROYGBIV. The critical angle C will thus increase in the same order VIBGYOR.

For green light the incidence angle is just equal to the critical angle. For yellow, orange and red the critical angle will be greater than the incidence angle. So these colours will emerge from the glass air interface.

Hence, the correct answer is (A).

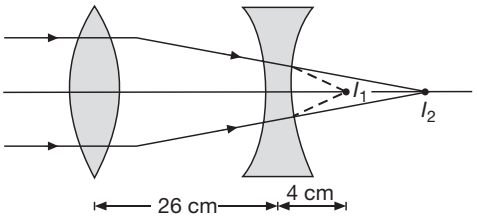
21. For minimum deviation the ray inside the prism is parallel to the base of the prism in case of an equilateral prism.

Hence, the correct answer is (B).

22. When the object is placed at the centre of the glass sphere, the rays from the object fall normally at the surface of the sphere and emerge undeviated.

Hence, the correct answer is (C).

23. Image formed by convex lens at I_1 will act as a virtual object for concave lens.



For concave lens applying $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{v}-\frac{1}{4}=\frac{1}{-20}$$

$$\Rightarrow \quad v=5 \text{ cm}$$

Magnification for concave lens

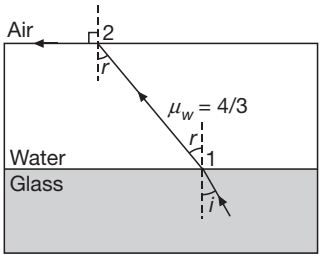
$$m=\frac{v}{u}=\frac{5}{4}=1.25$$

Since, size of the image at I_1 is 2 cm, therefore, size of image at I_2 will be $2\times1.25=2.5$ cm

Hence, the correct answer is (B).

24. Applying Snell's Law $(\mu\sin i=\text{constant})$ at 1 and 2, we get

$$\mu_1\sin i_1=\mu_2\sin i_2$$



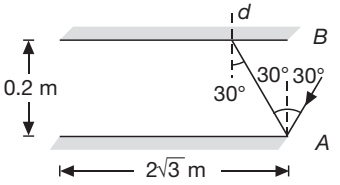
$$\Rightarrow \quad \mu_g\sin i=\mu_w\sin r=1\sin(90°)$$

$$\Rightarrow \quad \mu_g\sin i=(1)(\sin 90°)$$

$$\Rightarrow \quad \mu_g=\frac{1}{\sin i}$$

Hence, the correct answer is (B).

25. $\frac{d}{0.2}=\tan(30°)$



$$\Rightarrow \quad d=\frac{0.2}{\sqrt{3}}=\frac{2}{10\sqrt{3}}$$

Total number of reflections $= n=\frac{\ell}{d}=\frac{2\sqrt{3}}{\left(\frac{2}{10\sqrt{3}}\right)}$

$$\Rightarrow \quad n=30$$

Hence, the correct answer is (B).

26. $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

For no dispersion, we have $d\left(\frac{1}{f}\right)=0$

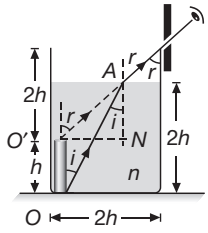
$\Rightarrow \quad d\mu\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=0$

$\Rightarrow \quad R_1=R_2$

Hence, the correct answer is (C).

27. $\frac{\sin i}{\sin r}=\frac{1}{n}$

Since



$\tan r=\frac{2h}{2h}=1$

$\Rightarrow \quad r=45°$

$\Rightarrow \quad \sin i=\frac{h}{h\sqrt{5}}$

$\Rightarrow \quad \sin i=\frac{1}{\sqrt{5}}$

$\Rightarrow \quad \frac{1}{n}=\frac{\frac{1}{\sqrt{5}}}{\frac{1}{\sqrt{2}}}$

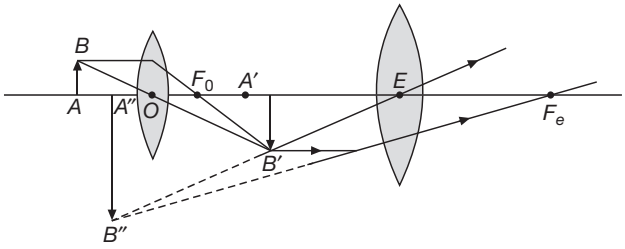
$\Rightarrow \quad n=\sqrt{\frac{5}{2}}$

Hence, the correct answer is (B).

28. CD is parallel to AB, so both media must have equal refractive indices.
Hence, the correct answer is (D).

29. Since all the prisms P, Q and R are made of same material, so the deviation suffered by the combination remains the same as the combination forms a part of the bigger sphere.
Hence, the correct answer is (C).

30. The ray diagram is shown in Figure.



From the figure it is clear that image formed by objective (or the intermediate image) is real, inverted and magnified.
Hence, the correct answer is (C).

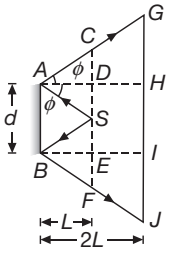
31. The lens makers' formula is

$$\frac{1}{f}=\left(\frac{n_L}{n_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

where $n_L=$ Refractive index of lens and $n_m=$ Refractive index of medium.
In case of double concave lens, R_1 is negative and R_2 is positive. Therefore $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ will be negative.
For the lens to be diverging in nature, focal length f should be negative or $\left(\frac{n_L}{n_m}-1\right)$ should be positive or $n_L>n_m$ but since $n_2>n_1$ (given), therefore the lens should be filled with L_2 and immersed in L_1.
Hence, the correct answer is (D).

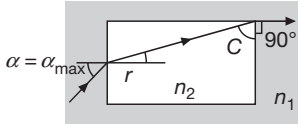
32. Since rays after passing through the glass slab just suffer lateral displacement hence we have angle between the emergent rays as α.
Hence, the correct answer is (B).

33.



Hence, the correct answer is (D).

34.



CHAPTER 1

$$\frac{\sin\alpha}{\sin r}=\frac{n_1}{n_2}$$

$$\Rightarrow \quad \sin\alpha=\frac{n_1}{n_2}\sin r \qquad \ldots(1)$$

For TIR at other end

$$\Rightarrow \quad \sin C=\frac{n_2}{n_1}$$

Also $C=90-r$

$$\Rightarrow \quad \sin\alpha=\frac{n_1}{n_2}\sin(90-C) \qquad \{\text{Put } r=90-C \text{ in (1)}\}$$

$$\Rightarrow \quad \sin\alpha=\frac{n_1}{n_2}\cos C$$

$$\Rightarrow \quad \sin\alpha=\frac{n_1}{n_2}\left[\cos\left[\sin^{-1}\left(\frac{n_2}{n_1}\right)\right]\right]$$

Hence, the correct answer is (A).

35. $$\frac{1}{f'}=\left(\frac{1.5}{1.75}-1\right)\left(-\frac{2}{R}\right)$$

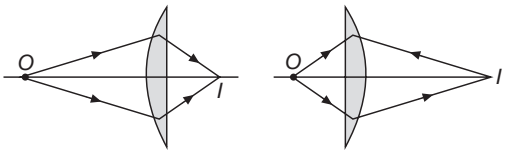
$$\Rightarrow \quad \frac{1}{f'}=\frac{0.25}{1.75}\times\frac{2}{R}$$

$$\Rightarrow \quad f'=\frac{R}{2}\left(\frac{175}{25}\right)$$

$$\Rightarrow \quad f'=3.5R$$

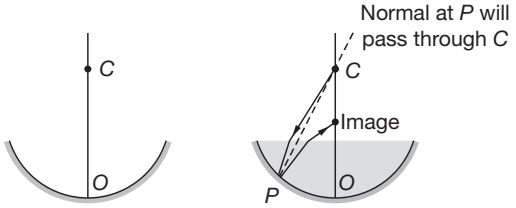
Hence, the correct answer is (A).

36. In general spherical aberration is minimum when the total deviation produced by the system is equally divided on all refracting surfaces. A planoconvex lens is used for this purpose. In order that the total deviation be equally divided on two surfaces, it is essential that more parallel beam (or the incident and refracted) be incident on the convex side. Thus, when the object is far away from the lens, incident rays will be more parallel than the refracted rays, therefore, the object should face the convex side, but if the object is near the lens, the object should face the plane side. This has been shown in figure.



Hence, the correct answer is (B).

37. The ray diagram is shown in figure. Therefore, the image will be real and between C and O.



Hence, the correct answer is (D).

38. $$\frac{1}{-(-x)}+\frac{1.5}{x}=\frac{1.5-1}{R}$$

$$\Rightarrow \quad \frac{2.5}{x}=\frac{0.5}{R}$$

$$\Rightarrow \quad x=5R$$

Hence, the correct answer is (A).

39. The focal length of combination is given by

$$\frac{1}{f_{\text{comb}}}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\Rightarrow \quad \frac{1}{f_{\text{comb}}}=\frac{1}{40}-\frac{1}{25}$$

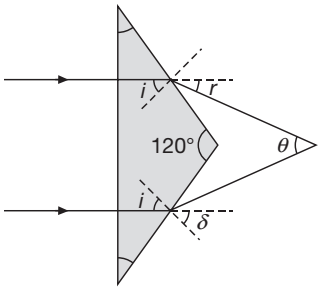
$$\Rightarrow \quad f_{\text{comb}}=-\frac{200}{3}\text{ cm}=-\frac{2}{3}\text{ m}$$

Power of the combination (in dioptre) is $P=\frac{1}{f}$

$$\Rightarrow \quad P=-\frac{3}{2}=-1.5 \qquad \left\{P=\frac{1}{f(m)}\right\}$$

Hence, the correct answer is (B).

40. The diagramatic representation of the given problem is shown in figure.



From figure it follows that $\angle i=\angle A=30°$

From Snell's Law, we get

$$n_1\sin i=n_2\sin r$$

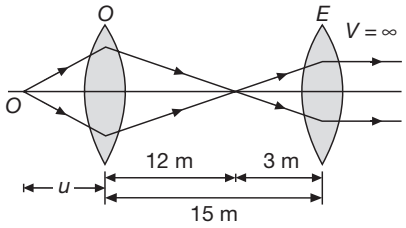
$$\Rightarrow \quad \sin r=\frac{1.44\sin(30°)}{1}=0.72$$

Now, $\angle\delta = \angle r - \angle i = \sin^{-1}(0.72) - 30°$

$\Rightarrow \quad \theta = 2(\angle\delta) = 2\left(\sin^{-1}(0.72) - 30°\right)$

Hence, the correct answer is (C).

41. Since, the final image is formed at infinity, the image formed by the objective will be at the focal point of the eye-piece, which is 3 cm. The image formed by the objective will be at a distance of $12\text{ cm}\left(=15\text{ cm}-3\text{ cm}\right)$ from the objective.



If u is the distance of the object from the objective, then by applying lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we get

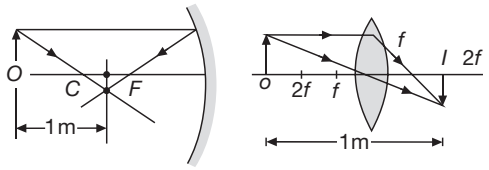
$$\frac{1}{12}-\frac{1}{u}=\frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{u}=\frac{1}{12}-\frac{1}{2}=\frac{2-12}{24}=-\frac{10}{24}$$

$$\Rightarrow \quad u=-2.4\text{ cm}$$

Hence, the correct answer is (A).

42. Image can be formed on the screen if it is real. Real image of reduced size can be formed by a concave mirror or a convex lens as shown in figure.



A diminished real image is formed by a convex lens when the object is placed beyond $2f$ and the image of such object is formed beyond $2f$ on other side.

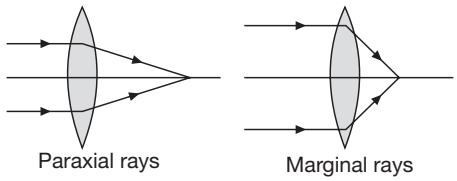
Thus, $d > (2f + 2f)$

$\Rightarrow \quad 4f < 0.1\text{ m}$

$\Rightarrow \quad f < 0.25\text{ m}$

Hence, the correct answer is (C).

43. Spherical aberration is caused due to spherical nature of lens. Paraxial and marginal rays are focussed at different places on the axis of the lens. Therefore, image so formed is blurred. This aberration can be reduced by either stopping paraxial rays or marginal rays, which can be done by using a circular annular mask over the lens.



Hence, the correct answer is (C).

44. For the refraction through the first lens, we have $u \to \infty$, so

$$v_1 = f_1$$

Since, $d < f_2$, the first image (formed by L_1) lies to the right of second lens L_2, so

$$u_2 = +(f_1 - d)$$

Applying Lens Formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{v_2}-\frac{1}{(f_1-d)}=\frac{1}{f_2}$$

$$\Rightarrow \quad v_2=\frac{f_2(f_1-d)}{f_1+f_2-d}$$

$$\Rightarrow \quad x = v_2 + d = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$$

Magnification for second lens is given by

$$m=\frac{v_2}{u_2}=\frac{f_2}{f_1+f_2-d}$$

The image due to the second lens is formed below its principal axis and is of the size mb. So, the y coordinate of the focal point system is given by

$$y = b - mb$$

$$\Rightarrow \quad y = b - \frac{f_2 b}{f_1 + f_2 - d}$$

$$\Rightarrow \quad y = \frac{(f_1 - d)b}{f_1 + f_2 - d}$$

So, the coordinates of the focal point of this system are

$$(x,\ y)=\left[\frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d},\ \frac{(f_1 - d)b}{f_1 + f_2 - d}\right]$$

Hence, the correct answer is (C).

CHAPTER 1

45. $A_2 = \left(\frac{n_1 - 1}{n_2 - 1}\right) A_1$

$\Rightarrow \quad A_2 = \left(\frac{1.54 - 1}{1.72 - 1}\right) 4$

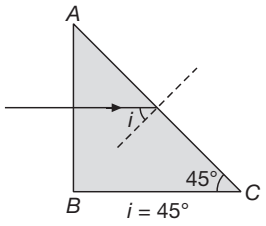
$\Rightarrow \quad A_2 = 3°$

Hence, the correct answer is (C).

46. The colours for which $i > C$, will get total internal reflection :

$i > C$

$\Rightarrow \quad \sin i > \sin C$



$\Rightarrow \quad \sin 45° > \frac{1}{\mu}$

$\Rightarrow \quad \frac{1}{\sqrt{2}} > \frac{1}{\mu}$

or for which $\mu > \sqrt{2}$ or $\mu > 1.414$.

Hence, the rays for which $\mu > 1.414$ will get TIR.

For green and blue $\mu > 1.414$, so they will suffer TIR on face AC only red comes out from this face.

Hence, the correct answer is (A).

47. $f_o + f_e = 36 \text{ cm} \qquad \ldots(1)$

$\frac{f_o}{f_e} = 5 \qquad \ldots(2)$

$\Rightarrow \quad f_o = 30 \text{ cm}$ and $f_e = 6 \text{ cm}$.

Hence, the correct answer is (D).

48. From the mirror formula

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad (f = \text{constant}) \qquad \ldots(1)$

$-v^{-2} dv - u^{-2} du = 0$

$\Rightarrow \quad |dv| = \left|\frac{v^2}{u^2}\right| |du| \qquad \ldots(2)$

Here, $|dv|$ = size of image.

$|du|$ = size of object (short) lying along the axis $= b$

Further, from equation (1), we can find

$\frac{v^2}{u^2} = \left(\frac{f}{u - f}\right)^2$

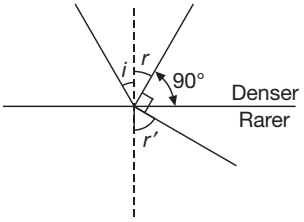
Substituting these values in equation (2), we get

size of image $= b\left(\frac{f}{u - f}\right)^2$

Hence, the correct answer is (D).

49. $r + r' + 90° = 180°$

$\Rightarrow \quad r' = 90 - r$



Further, $i = r$

Applying Snell's Law,

$\mu_D \sin i = \mu_R \sin r'$

$\Rightarrow \quad \mu_D \sin r = \mu_R \sin(90 - r) = \mu_R \cos r$

$\Rightarrow \quad \frac{\mu_R}{\mu_D} = \tan r$

$\theta_C = \sin^{-1}\left(\frac{\mu_R}{\mu_D}\right) = \sin^{-1}(\tan r)$

Hence, the correct answer is (A).

50. $\frac{1}{f} = \frac{1}{40} + \frac{1}{-25}$

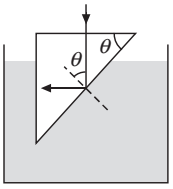
$\Rightarrow \quad f = \frac{-40 \times 25}{40 - 25}$

$\Rightarrow \quad f = -66.67 \text{ cm}$

$\Rightarrow \quad P = \frac{10}{f(\text{in cm})} = -1.5 \text{ D}$

Hence, the correct answer is (A).

51. $\sin C = \frac{\mu_r}{\mu_d} = \frac{4/3}{3/2} = \frac{8}{9}$



$$\Rightarrow \quad C = \sin^{-1}\left(\frac{8}{9}\right)$$

For TIR

$$i > C$$

$$\Rightarrow \quad \theta > C$$

$$\Rightarrow \quad \sin\theta > \sin C$$

$$\Rightarrow \quad \sin\theta > \frac{8}{9}$$

Hence, the correct answer is (A).

52. According to Snell's Law

$$\mu\lambda = \text{constant}$$

$$\Rightarrow \quad \mu_a\lambda_a = \mu_g\lambda_g$$

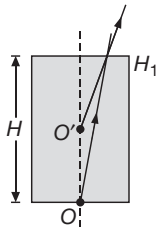
$$\Rightarrow \quad \lambda_g = \frac{\mu_a\lambda_a}{\mu_g}$$

$$\Rightarrow \quad \lambda_g < \lambda_a$$

Hence, the correct answer is (A).

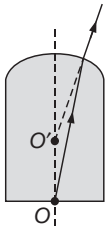
Multiple Correct Choice Type Problems

1. **CASE-1:**



$$H_1 = \frac{H}{\mu} = \frac{30\times 2}{3} = 20 \text{ cm} \quad \text{(below)}$$

CASE-2:



$$\frac{1}{V} + \frac{3}{2H} = \frac{1-1.5}{-R}$$

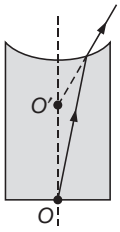
$$\Rightarrow \quad \frac{1}{V} + \frac{1}{20} = \frac{1}{2\times 300}$$

$$\Rightarrow \quad \frac{1}{V} = \frac{1}{600} - \frac{1}{20}$$

$$\Rightarrow \quad \frac{1}{V} = \frac{-29}{600}$$

$$\Rightarrow \quad H_2 = \frac{600}{29} = 20.68 \text{ cm} \quad \text{(below)}$$

CASE-3:



$$\frac{1}{V} + \frac{3}{2H} = \frac{-1}{2\times 300}$$

$$\Rightarrow \quad \frac{1}{V} = \frac{-1}{600} - \frac{1}{20}$$

$$\Rightarrow \quad \frac{1}{V} = \frac{-31}{600}$$

$$\Rightarrow \quad H_3 = \frac{600}{31} = 19.35 \text{ cm}$$

$$H_2 - H_1 \simeq 20.68 - 20 \simeq 0.68$$

Since $H_2 > H_1$ and $H_2 > H_3$

$$\Rightarrow \quad H_1 > H_3$$

Hence, (B) and (D) are correct.

2. For the combination of lens, we have

$$\frac{1}{f} = (n_1 - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) + (n_2 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$\Rightarrow \quad \frac{1}{f} = \frac{(n_1-1)}{R} + \frac{(n_2-1)}{R} = \frac{(n_1+n_2-2)}{R} \qquad \ldots(1)$$

For $n_1 = n_2 = n$, we get

$$\frac{1}{f} = (n-1)\frac{2}{R} \qquad \ldots(2)$$

$$\Rightarrow \quad -\frac{\Delta f}{f^2} = \frac{\Delta n}{R} \qquad \ldots(3)$$

Since, from (1), we get

$$\frac{f}{R} = \frac{1}{n_1+n_2-2}, \text{ so, equation (3) becomes}$$

$$-\frac{\Delta f}{f} = \frac{\Delta n}{(n_1+n_2-2)}$$

CHAPTER 1

Since $n_1 = n$ and $n_2 = n + \Delta n$, so $\dfrac{\Delta f}{f} = \dfrac{\Delta n}{(2n + \Delta n - 2)}$

When $n_1 = n_2 = 1.5$, $\Delta n = 10^{-3}$, $f = 20$ cm we get

$R = 20$ cm

$$|\Delta f| = \frac{10^{-3} \times 20}{(2 \times 1.5 + 10^{-3} - 2)} \approx 0.02 \text{ cm}$$

When $\dfrac{\Delta n}{n} < 0$ then, $\dfrac{\Delta f}{f} > 0$

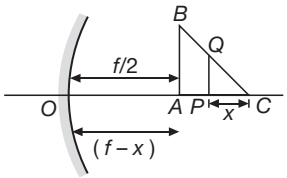
When the surfaces are replaced by concave surfaces of same radius of curvature, then but has same magnitude such that

$$-\frac{\Delta f}{f} \approx \frac{\Delta n}{(2n-2)} = \frac{\Delta n}{2(n-1)}$$

i.e., both $\dfrac{\Delta f}{f}$ and $\dfrac{\Delta n}{n}$ remain unchanged.

Hence, (B), (C) and (D) are correct.

3. For image of point A, we have $u = -\dfrac{f}{2}$, $v = ?$, $f = -f$



Since $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$$\Rightarrow \quad \frac{1}{v} + \frac{1}{-\left(\frac{f}{2}\right)} = \frac{1}{-f}$$

$$\Rightarrow \quad v = f$$

$$\Rightarrow \quad \frac{I_{AB}}{AB} = -\frac{f}{-\frac{f}{2}}$$

$$\Rightarrow \quad I_{AB} = 2AB$$

For height of PQ, we have $u = -(f - x)$, $v = ?$, $f = -f$

Since $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$$\Rightarrow \quad \frac{1}{v} + \frac{1}{-(f-x)} = \frac{1}{-f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{(f-x)} - \frac{1}{f}$$

$$\Rightarrow \quad v = \frac{f(f-x)}{x}$$

$$\Rightarrow \quad \frac{I_{PQ}}{PQ} = -\frac{f(f-x)}{x[-(f-x)]} = \left(\frac{f}{x}\right)$$

$$\Rightarrow \quad I_{PQ} = \frac{f}{x} PQ$$ Since from triangle ABC and PQC, we have

$$\frac{AB}{\left(\frac{f}{2}\right)} = \frac{PQ}{x}$$

$$\Rightarrow \quad PQ = \frac{2(AB)x}{f}$$

$$\Rightarrow \quad I_{PQ} = \frac{f}{x}\left[\frac{2(AB)x}{f}\right] = 2AB$$

$$\Rightarrow \quad I_{PQ} = 2AB$$

So, Size of image is independent of x and hence the final image will be of same height terminating at infinity.

Hence, the correct answer is (D).

4. The minimum deviation produced by a prism

$$\delta_m = 2i - A = A$$

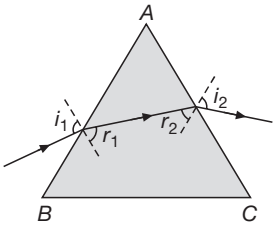
$$\Rightarrow \quad i_1 = i_2 = A \text{ and } r_1 = r_2 = \frac{A}{2}$$

$$\Rightarrow \quad r_1 = \frac{i_1}{2}$$

Now, using Snell's law

$$\sin A = \mu \sin\left(\frac{A}{2}\right)$$

$$\Rightarrow \quad \mu = 2\cos\left(\frac{A}{2}\right)$$



For this prism when the emergent ray at the second surface is tangential to the surface

$$i_2 = \frac{\pi}{2}$$

$$\Rightarrow \quad r_2 = \theta_c$$

$$\Rightarrow \quad r_1 = A - \theta_c$$

$\Rightarrow \quad \sin i_1 = \mu \sin(A - \theta_c)$

$\Rightarrow \quad i_i = \sin^{-1}\left[\sin A\sqrt{4\cos^2\left(\frac{A}{2}\right) - 1} - \cos A\right]$

For minimum deviation through isosceles prism, the ray inside the prism is parallel to the base of the prism if $\angle B = \angle C$.

But it is not necessarily parallel to the base if,

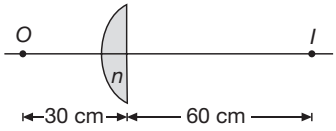
$\angle A = \angle B$

$\Rightarrow \quad \angle A = \angle C$

Hence, (A), (B) and (C) are correct.

5. Since $m = \frac{v}{u} = 2$

$\Rightarrow \quad v = 60$ cm



Using lens formula, we get

$$\frac{1}{60} - \frac{1}{(-30)} = \frac{1}{f_1}$$

$\Rightarrow \quad \frac{1}{f_1} = \frac{1}{60} + \frac{2}{60}$

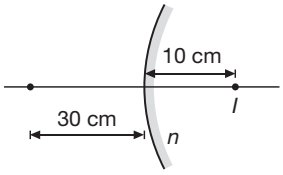
$\Rightarrow \quad f_1 = 20$ cm

Further, $\frac{1}{f_1} = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$

$\Rightarrow \quad f_1 = \frac{R}{n-1}$

Since $f_1 = 20$ cm

$\Rightarrow \quad \frac{R}{n-1} = +20$ cm



Now using mirror formula, we get

$$\frac{1}{10} + \frac{1}{-30} = \frac{1}{f_2}$$

$\Rightarrow \quad \frac{3}{30} - \frac{1}{30} = \frac{1}{f_2} = \frac{2}{30}$

$\Rightarrow \quad f_2 = 15 = \frac{R}{2}$

$\Rightarrow \quad R = 30$ cm

$\Rightarrow \quad \frac{R}{n-1} = 20 \text{ cm} = \frac{30}{n-1}$

$\Rightarrow \quad \frac{30}{n-1} = 20$

$\Rightarrow \quad 2n - 2 = 3$

$\Rightarrow \quad n = 2.5$

$\Rightarrow \quad f_1 = +20$ cm

So, refractive index of lens is 2.5, radius of curvature of convex surface is 30 cm and the faint image is erect and virtual focal length of lens is 20 cm.

Hence, (A) and (D) are correct.

6. For, film, $\frac{1}{f_{\text{film}}} = (n_1 - 1)\left(\frac{1}{R} - \frac{1}{R}\right)$

$\Rightarrow \quad f_{\text{film}} \to \infty$

$\Rightarrow \quad P = 0$

$\Rightarrow$ There is no effect of presence of film.

From Air to Glass

Using the equation $\frac{n_2}{v} - \frac{1}{u} = \frac{n_2 - 1}{R}$, we get

$$\frac{1.5}{v} - \frac{1}{\infty} = \frac{1.5 - 1}{R}$$

$\Rightarrow \quad v = 3R$

Since parallel rays will meet at focus, so

$f_1 = 3R$

From Glass to Air

Again using the same equation, $\frac{1}{v} - \frac{n_2}{u} = \frac{1 - n_2}{-R}$, we get

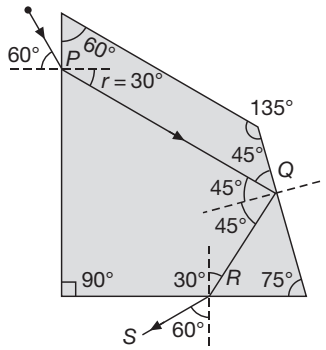
$$\frac{1}{v} - \frac{1.5}{\infty} = \frac{1 - 1.5}{-R}$$

$\Rightarrow \quad v = 2R$

$\Rightarrow \quad f_2 = 2R$

Hence, (A) and (C) are correct.

7. $\sqrt{3} = \frac{\sin(60°)}{\sin r}$



CHAPTER 1

$\Rightarrow \quad r = 30°$

Since, $\theta_C = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) = 0.577$

At point Q, angle of incidence inside the prism is $i = 45°$.

Since $\sin i = \frac{1}{\sqrt{2}}$ is greater than $\sin\theta_C = \frac{1}{\sqrt{3}}$, ray gets totally internally reflected at face CD. Path of ray of light after point Q is shown in figure.

From the figure, we can see that angle between incident ray OP and emergent ray RS is $90°$.

Hence, (A), (B) and (C) are correct.

8. Values of options (C) and (D) don't match with the mirror formula.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Hence, (C) and (D) are correct.

9. When upper half of the lens is covered, image is formed by the rays coming from lower half of the lens i.e., the image will be formed by lesser number of rays. Therefore, intensity of image will decrease. However, complete image will be formed.

Hence, (B) and (D) are correct.

10. Objective and eye piece are separated by a distance $(f_o + f_e) = 16.02$ m OPTION **(A)**}

Angular Magnification $= -\frac{f_o}{f_e} = -\frac{16}{0.02} = -800$

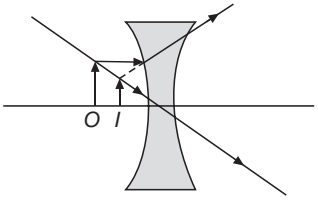
{OPTION **(B)**}

A telescope produces an image which is always inverted. {OPTION **(C)**}

In a telescope an objective is larger than the eye piece. OPTION **(D)**}

Hence, (A), (B), (C) and (D) are correct.

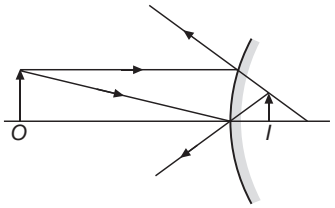
11. For a lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$



i.e., $\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$

For a concave lens, f and u are negative, i.e., v will always be negative and image will always be virtual.

For a mirror



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

I.e., $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

Here, f is positive and u is negative for a convex mirror. Therefore, v is always positive and image is always virtual.

Hence, (C) and (D) are correct.

12. For TIR, we have

$i > C$

$\Rightarrow \quad \sin i > \sin C$

$\Rightarrow \quad \sin(45°) > \sin C$

Since, $\sin C = \frac{1}{n}$

$\Rightarrow \quad \frac{1}{\sqrt{2}} > \frac{1}{n}$

$\Rightarrow \quad n > \sqrt{2}$

$\Rightarrow \quad n > 1.414$

Hence, (C) and (D) are correct.

Comprehension Type Questions

1. For S_1 immersed in water, we have

$$\frac{4}{3}\sin i = \frac{\sqrt{45}}{4}\sin(90 - \theta_c) = \frac{\sqrt{45}}{4}\cos\theta_c$$

Since, $\sin\theta_c = \frac{n_2}{n_1}$

$\Rightarrow \quad \cos\theta_c = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$

$\Rightarrow \quad \frac{4}{3}\sin i = \frac{\sqrt{45}}{4}\frac{3}{\sqrt{45}}$

$\Rightarrow \quad \sin i = \frac{9}{16}$

In second case, i.e. for S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$, we have

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{7}{8}$$

$$\Rightarrow \quad \cos\theta_c = \frac{\sqrt{15}}{8}$$

Since $\frac{16}{3\sqrt{15}}\sin i = \frac{8}{5}\sin(90-\theta_c)$

Simplifying we get,

$$\sin i = \frac{9}{16}$$

Same approach can be adopted for other options. Correct answers are (A) and (C).

Hence, the correct answer is (A).

2. Since, $1\sin i_m = n_1 \sin(90-\theta_c)$

$$\Rightarrow \quad \sin i_m = n_1 \cos\theta_c$$

$$\Rightarrow \quad NA = n_1\sqrt{1-\sin^2\theta_c}$$

$$\Rightarrow \quad NA = n_1\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$\Rightarrow \quad NA = \sqrt{n_1^2 - n_2^2}$$

Substituting the values we get,

$$NA_1 = \frac{3}{4} \text{ and } NA_2 = \frac{\sqrt{15}}{5} = \sqrt{\frac{3}{4}}$$

$$\Rightarrow \quad NA_2 < NA_1$$

Therefore, the numerical aperture of combined structure is equal to the lesser of the two numerical aperture, which is NA_2.

Hence, the correct answer is (D).

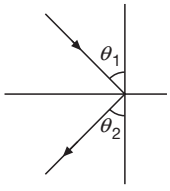
3. Since value of n in meta-material is negative.

$$\therefore \quad v = \frac{c}{|n|}$$

Hence, the correct answer is (B).

4. $1\sin\theta_1 = -n\sin\theta_2$

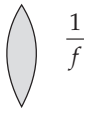
$$\sin\theta_2 = -\frac{1}{n}\sin\theta_1$$



Hence, the correct answer is (C).

Matrix Match/Column Match Type Questions

1. A → (p)
B → (s)
C → (r)
D → (p)

(P)  $\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{1}{r}+\frac{1}{r}\right) = \frac{1}{r}$

$\Rightarrow \quad f = r$

$$\Rightarrow \quad \frac{1}{f_{eq}} = \frac{1}{f}+\frac{1}{f} = \frac{2}{r}$$

$$\Rightarrow \quad f_{eq} = \frac{r}{2}$$

(Q) $\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{1}{r}\right)$

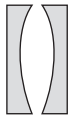
$$\Rightarrow \quad f = 2r$$

$\Rightarrow \quad \frac{1}{\phi}+\frac{1}{\phi} = \frac{2}{\phi} = \frac{1}{\rho}$

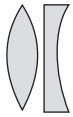
$$\Rightarrow \quad f_{eq} = r$$

(R) $\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(-\frac{1}{r}\right) = -\frac{1}{2r}$

$$\Rightarrow \quad f = -2r$$

 $\Rightarrow \quad \frac{1}{f_{eq}} = \frac{1}{f}+\frac{1}{f} = -\frac{2}{2r}$

$$\Rightarrow \quad f_{eq} = -r$$

(S)  $\Rightarrow \quad \frac{1}{f_{eq}} = \frac{1}{r}+\frac{1}{-2r} = \frac{1}{2r}$

$$\Rightarrow \quad f_{eq} = 2r$$

2. A → (q)
B → (r)
C → (s)
D → (p)

CHAPTER 1

For $e \to i$

$\Rightarrow \quad \sin 45° > \theta_c$

$\Rightarrow \quad \sin 45° > \sin\theta_c$

$\Rightarrow \quad \frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1}$

$\Rightarrow \quad \mu_1 > \sqrt{2}\mu_2$

For $e \to f$

angle of refraction is lesser than angle of incidence, so $\mu_2 > \mu_1$ and then $\mu_2 > \mu_3$

For $e \to g$, $\mu_1 = \mu_2$

for $e \to h$, $\mu_2 < \mu_1 < \sqrt{2}\mu_2$ and $\mu_2 > \mu_3$

3. A → (p, r)
B → (q, s, t)
C → (p, r, t)
D → (q, s)

(A)→ since $\mu_1 < \mu_2$, the ray of light will bend towards normal after first refraction.

(B) → $\mu_1 > \mu_2$, the ray of light will bend away from the normal after first refraction.

(C) → since $\mu_2 = \mu_3$ means in second refraction there will be no change in the path of ray of light.

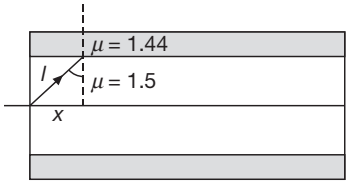
(D) → Since $\mu_2 > \mu_3$, ray of light will bend away from the normal after second refraction.

Therefore, the correct options are as under.
(A) → p, r; (B) → q, s, t; (C) → p, r, t; (D) → q, s

Integer/Numerical Answer Type Questions

1. $1.5\sin\theta_C = 1.44\sin(90°)$

$\Rightarrow \quad \sin\theta_C = \frac{24}{25}$



Since $\ell = \frac{x}{\sin\theta_C} = \frac{25}{4}x$

So, total length for light to travel is given by

$$\ell' = \frac{25}{4} \times 9.6 = 10 \text{ m}$$

Hence total time is given by

$$T = \frac{\ell'}{c/1.5} = 5 \times 10^{-8} \text{ s}$$

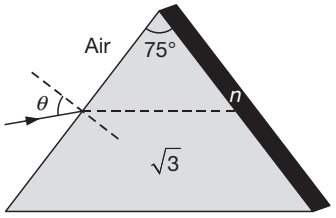
$\Rightarrow \quad T = 50 \times 10^{-9} \text{ s} = t \times 10^{-9} \text{ s}$

$\Rightarrow \quad t = 50.00$

2. For TIR at other face, we have

$$\sin C = \frac{n}{n_0}$$

$\Rightarrow \quad \sin C = \frac{n}{\sqrt{3}}$ …(1)



Applying Snell's Law at first surface, we get

$$\sin\theta = \sqrt{3}\sin(75 - C)$$

For $\theta = 60°$, we have

$$\frac{\sqrt{3}}{2} = \sqrt{3}\sin(75 - C)$$

$\Rightarrow \quad \sin(75 - C) = \frac{1}{2}$

$\Rightarrow \quad C = 45°$

From (1), we get

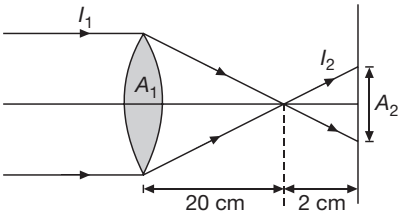
$$\frac{1}{\sqrt{2}} = \frac{n}{\sqrt{3}}$$

$\Rightarrow \quad n = \sqrt{\frac{3}{2}}$

$\Rightarrow \quad n^2 = \frac{3}{2} = 1.5$

3. $I_1 = \frac{P}{A_1}$

$\Rightarrow \quad P = I_1 A_1$ …(1)



Also, $I_2 = \frac{P}{A_2}$

$\Rightarrow \quad P = I_2 A_2$ …(2)

From (1) and (2), we get

$$I_2 = \frac{I_1 A_1}{A_2} = (1.3)\left(\frac{20}{2}\right)^2 = 130 \text{ kWm}^{-2}$$

4. $1.6\sin\theta = (n - m\Delta n)\sin(90°)$

$\Rightarrow \quad 1.6\sin\theta = n - m\Delta n$

$\Rightarrow \quad 1.6 \times \frac{1}{2} = 1.6 - m(0.1)$

$\Rightarrow \quad 0.8 = 1.6 - m(0.1)$

$\Rightarrow \quad m \times 0.1 = 0.8$

$\Rightarrow \quad m = 8$

Theoretically $m = 8$ satisfies $m > 1$ but such a large value of refractive index may not be physically possible to obtain.

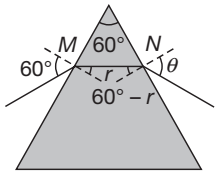
5. Applying Snell's law at M and N,

$\sin(60°) = n\sin r$ …(1)

$\sin\theta = n\sin(60 - r)$ …(2)

Differentiating we get

$$\cos\theta\frac{d\theta}{dn} = -n\cos(60 - r)\frac{dr}{dn} + \sin(60 - r)$$



Differentiating Equation (1), we get

$$n\cos r\frac{dr}{dn} + \sin r = 0$$

$\Rightarrow \quad \frac{dr}{dn} = -\frac{\sin r}{n\cos r} = \frac{-\tan r}{n}$

$\Rightarrow \quad \cos\theta\frac{d\theta}{dn} = -n\cos(60 - r)\left(\frac{-\tan r}{n}\right) + \sin(60 - r)$

$$\frac{d\theta}{dn} = \frac{1}{\cos\theta}[\cos(60 - r)\tan r + \sin(60 - r)]$$

From Equation (1), for $n = \sqrt{3}$, we get $r = 30°$

$\Rightarrow \quad \frac{d\theta}{dn} = \frac{1}{\cos(60°)}[\cos(30°) \times \tan(30°) + \sin(30°)]$

$\Rightarrow \quad \frac{d\theta}{dn} = 2\left(\frac{1}{2} + \frac{1}{2}\right) = 2$

$\Rightarrow \quad m = 2$

6. **CASE-1: Arrangement kept in air**

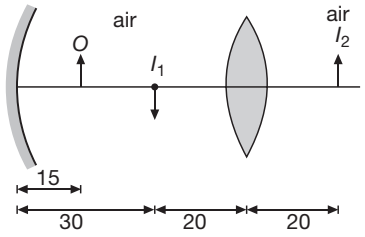
Applying mirror formula

$\frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f_1}$, we get

$$\frac{1}{v_1} + \frac{1}{(-15)} = \frac{1}{(-10)}$$

$\Rightarrow \quad v_1 = -30$

$\Rightarrow \quad \left|\frac{v_1}{u_1}\right| = \frac{30}{15} = 2$



For lens

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$\Rightarrow \quad \frac{1}{10} = \frac{1}{v_2} - \frac{1}{-20}$

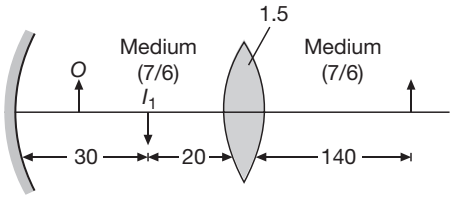
$\Rightarrow \quad v_2 = 20$

$\Rightarrow \quad \left|\frac{v_2}{u_2}\right| = \frac{20}{20} = 1$

$\Rightarrow \quad |M_1| = \left|\frac{v_1}{u_1}\right|\left|\frac{v_2}{u_2}\right| = \left(\frac{30}{15}\right)\left(\frac{20}{20}\right) = 2 \times 1 = 2$ (in air)

CASE-2: Arrangement kept in a medium with refractive index $\frac{7}{6}$.

For mirror, there is no change.



For lens, $\frac{1}{f_{air}} = \frac{1}{10} = \left(\frac{\frac{3}{2}}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, because $f_{air} = 10$ cm

$\Rightarrow \quad \frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{5}$

CHAPTER 1

Since $\frac{1}{f_{\text{medium}}} = \left(\frac{\frac{3}{2}}{\frac{7}{6}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\Rightarrow \quad \frac{1}{f_{\text{medium}}} = \frac{2}{35}\ \text{cm}^{-1}$

Applying lens formula, we get

$$\frac{1}{v_2} - \frac{1}{(-20)} = \frac{2}{35}$$

$\Rightarrow \quad \frac{1}{v_2} + \frac{7}{140} = \frac{8}{140}$

$\Rightarrow \quad \frac{1}{v_2} = \frac{8}{140} - \frac{7}{140} = \frac{1}{140}$

$\Rightarrow \quad v_2 = 140\ \text{cm}$

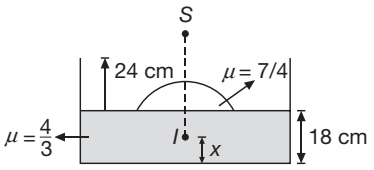
$\Rightarrow \quad \left|\frac{v_2}{u_2}\right| = \frac{140}{20} = 7$

$\Rightarrow \quad |M_2| = \left|\frac{v_1}{u_1}\right|\left|\frac{v_2}{u_2}\right| = \left(\frac{30}{15}\right)\left(\frac{140}{20}\right),$

$\Rightarrow \quad M_2 = 2 \times 7 = 14$

$\Rightarrow \quad \left|\frac{M_2}{M_1}\right| = \frac{14}{2} = 7$

7.



Two refractions will take place, first, from spherical surface and the other from the plane surface.
So, applying

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

two times with proper sign convention
Ray of light is travelling downwards. Therefore, downward direction is taken as positive direction.

$$\frac{\frac{7}{4}}{v} - \frac{1.0}{(-24)} = \frac{\frac{7}{4} - 1.0}{+6} \qquad \ldots(1)$$

$$\frac{\frac{4}{3}}{(18 - x)} - \frac{\frac{7}{4}}{v} = \frac{\frac{4}{3} - \frac{7}{4}}{\infty} \qquad \ldots(2)$$

Solving these equations, we get, $x = 2$ cm

8. Applying mirror formula when image is at $\frac{25}{3}$ m, we get

$$\frac{1}{+\frac{25}{3}} + \frac{1}{-u_1} = \frac{1}{+10}$$

$\Rightarrow \quad \frac{1}{u_1} = \frac{3}{25} - \frac{1}{10}$

$\Rightarrow \quad u_1 = 50\ \text{m}$

Again applying mirror formula when image is at $\frac{50}{7}$ m, we get

$$\frac{1}{\left(+\frac{50}{7}\right)} + \frac{1}{-u_2} = \frac{1}{+10}$$

$\Rightarrow \quad \frac{1}{u_2} = \frac{7}{50} - \frac{1}{10}$

$\Rightarrow \quad u_2 = 25\ \text{m}$

Speed of object $= \frac{u_1 - u_2}{\text{time}} = \frac{25}{30}\text{ms}^{-1} = 3\ \text{kmh}^{-1}$

9. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \quad \frac{u}{v} - 1 = \frac{u}{f}$

$\Rightarrow \quad \frac{u}{v} = \left(\frac{u + f}{f}\right)$

$\Rightarrow \quad m = \frac{v}{u} = \left(\frac{f}{u + f}\right)$

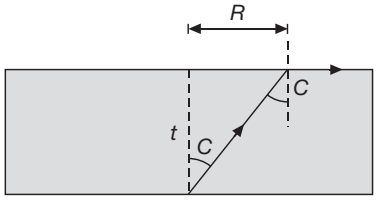
$\Rightarrow \quad \frac{m_{25}}{m_{50}} = \frac{\left(\frac{20}{-25 + 20}\right)}{\left(\frac{20}{-50 + 20}\right)} = 6$

10. Since $\frac{R}{t} = \tan C$

$\Rightarrow \quad R = t(\tan C)$

But, $\sin C = \frac{1}{\mu} = \frac{3}{5}$

$\Rightarrow \quad \tan C = \frac{3}{4}$



$\Rightarrow \quad R = \frac{3}{4}t = \frac{3}{4}(8\ \text{cm}) = 6\ \text{cm}$

CHAPTER 2: WAVE OPTICS

Test Your Concepts-I (Based on Interference)

1. (a) $\left(1-\frac{1}{\mu}\right)t=\frac{3\lambda}{\mu}$

$\Rightarrow \quad t=\frac{3\lambda}{(\mu-1)}=\frac{3\times 0.78}{1.3-1}=7.8\ \mu\text{m}$

(b) Upwards

$$\frac{yd}{D}-\left(1-\frac{1}{\mu}\right)t=\frac{4\lambda}{\mu}$$

$\Rightarrow \quad y=4.2$ mm

Downwards

$$t\left(1-\frac{1}{\mu}\right)+\frac{yd}{D}=\frac{4\lambda}{\mu}$$

$\Rightarrow \quad y=0.6$ mm

2. Since we are given that
Shift = 5 (fringe width)

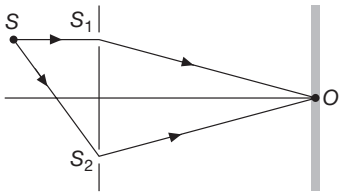
$$\Rightarrow \quad \frac{(\mu-1)tD}{d}=\frac{5\lambda D}{d}$$

$$\Rightarrow \quad t=\frac{5\lambda}{\mu-1}=\frac{25000}{1.5-1}=50{,}000\ \text{Å}=5\ \mu\text{m}$$

3. $\Delta x=SS_2O-SS_1O=\sqrt{d^2+\ell^2}-\ell$

$$\Rightarrow \quad \phi=\frac{2\pi}{\lambda}\cdot\Delta x$$

$$\Rightarrow \quad \phi=\frac{2\pi}{\lambda}\left(\sqrt{d^2+\ell^2}-\ell\right)$$



(a) Geometric path difference between S_2P and S_1P is

$$y=\frac{\left(\frac{d}{2}\right)d}{D}=\frac{d^2}{2D}$$

Their optical path difference $=\frac{\mu d^2}{2D}$

Therefore, net path difference

$$\Delta x=\left(\sqrt{d^2+\ell^2}-\ell\right)+\frac{\mu d^2}{2D}$$

So, phase difference is given by

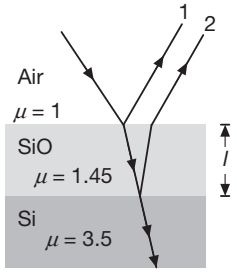
$$\phi=\frac{2\pi}{\lambda}\left[\left(\sqrt{d^2+\ell^2}-d\right)+\frac{\mu d^2}{2D}\right]$$

(b) When liquid is filled between slits and source S, then

$$\Delta x=\mu\left(\left(\sqrt{d^2+\ell^2}-\ell\right)+\frac{d^2}{2D}\right)$$

$$\Rightarrow \quad \phi=\frac{2\pi}{\lambda}\left(\mu\left(\sqrt{d^2+\ell^2}-\ell\right)+\frac{d^2}{2D}\right)$$

4. The reflected light will be minimum when rays 1 and 2 (shown in figure) meet the condition of destructive interference.



Rays 1 and 2 both suffer an additional phase change of π (or 180°) after being reflected at surface of a denser medium. The net change in phase due to reflection is therefore zero and the condition for a reflection minimum requires a path difference of $\frac{\lambda}{2}$. Hence,

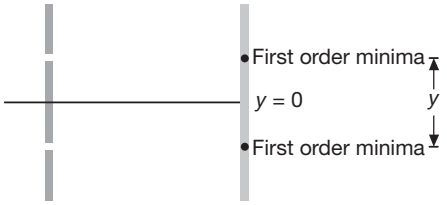
$$2\mu t=\frac{\lambda}{2}$$

$$\Rightarrow \quad t=\frac{\lambda}{4\mu}=\frac{550}{4(1.45)}=94.8\ \text{nm}$$

5. The desired distance is

$$y=\text{fringe width}=\frac{\lambda D}{d}=\frac{546\times 10^{-9}\times 0.4}{0.4\times 10^{-3}}$$

$$\Rightarrow \quad y=0.546\times 10^{-3}\ \text{m}=0.546\ \text{mm}$$



6. For constructive interference in case of soap film,

$$2\mu t = \left(n - \frac{1}{2}\right)\lambda \qquad n = 1, 2, 3, ...$$

For minimum thickness t, $n = 1$

$$\Rightarrow \quad 2\mu t = \frac{\lambda}{2}$$

$$\Rightarrow \quad t = \frac{\lambda}{4\mu} = \frac{600}{4 \times 1.33} = 112.78 \text{ nm}$$

7. (a) $\beta = \dfrac{\lambda D}{d} = \dfrac{(5000 \times 10^{-10})(1)}{1 \times 10^{-3}} = 0.5$ mm

(b) $y = (2n-1)\dfrac{\lambda D}{d}$

For $n = 5$

$y = 2.25$ mm

(c) At $y = \dfrac{1}{3}$ mm, $y \ll D$

$$\Rightarrow \quad \Delta x = \frac{yd}{D}$$

$$\Rightarrow \quad \Delta\phi = \frac{2\pi}{\lambda}\Delta x = 2\pi\frac{yd}{\lambda D} = \frac{4\pi}{3}$$

Now resultant intensity, is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$$

$$\Rightarrow \quad I = 4I_0 + I_0 + 2\sqrt{4I_0^2}\cos\Delta\phi$$

$$\Rightarrow \quad I = 5I_0 + 4I_0\cos\left(\frac{4\pi}{3}\right) = 3I_0$$

(d) $\dfrac{d}{\lambda} = \dfrac{10^{-3}}{0.5 \times 10^{-6}} = 2000$

$n = 1000$ is not very much less than 2000

Hence now $\Delta x = d\sin\theta$ must be used, so we get

$$d\sin\theta = n\lambda = 1000\lambda$$

$$\Rightarrow \quad \sin\theta = 1000\frac{\lambda}{d} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30°$$

Since, $y = D\tan\theta$

$$\Rightarrow \quad y = \frac{1}{\sqrt{3}} \text{ meter}$$

(e) Highest order maxima

$$n_{\max} = \left[\frac{d}{\lambda}\right] = 2000$$

Hence, $n = 5000$ is not possible.

8. (a) $\Delta x = 0.5\ \mu\text{m} = 5 \times 10^{-7}$ m

So, phase difference is given by

$$\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x) = \left(\frac{2\pi}{400 \times 10^{-9}}\right)(5 \times 10^{-7})$$

$$\Rightarrow \quad \phi = \frac{5\pi}{2}$$

(b) $\Delta x' = (\mu - 1)t = (1.5 - 1)(1.5) = 0.75\ \mu\text{m}$

$$\Rightarrow \quad (\Delta x)_{\text{net}} = \Delta x + \Delta x' = 1.25\ \mu\text{m} = 1.25 \times 10^{-6} \text{ m}$$

$$\Rightarrow \quad \phi' = \left(\frac{2\pi}{400 \times 10^{-9}}\right)(1.25 \times 10^{-6})$$

$$\Rightarrow \quad \phi' = \frac{25\pi}{4}$$

9. Since, shift $= 2(\text{Fringe Width})$

$$\Rightarrow \quad \frac{(\mu - 1)tD}{d} = \frac{\lambda(2D)}{d}$$

$$\Rightarrow \quad \lambda = \frac{(\mu - 1)t}{2} = \frac{(1.6 - 1)(1.964 \times 10^{-6})}{2}$$

$$\Rightarrow \quad \lambda = 0.589 \times 10^{-6} \text{ m}$$

$$\Rightarrow \quad \lambda = 5890\ \text{Å}$$

10. Shift $= 5$ (fringe width)

$$\Rightarrow \quad (\mu - 1)\frac{tD}{d} = \frac{\lambda D}{d}$$

$$\Rightarrow \quad t = \frac{\lambda}{\mu - 1} = \frac{6 \times 10^{-7}}{(1.5 - 1)} = 1.2 \times 10^{-6} \text{ m}$$

$$\Rightarrow \quad t = 1.2\ \mu\text{m}$$

11. $\beta = \dfrac{(14.75 - 12.50)}{10 - 1}$ mm $= 0.25$ mm

$$\beta' = \frac{\lambda'}{\lambda}\beta = \frac{5500}{6000} \times 0.25 \text{ mm} = 0.23 \text{ mm}$$

Zero order maxima will remain unchanged. Earlier it was at 12.5 mm – 12.25 mm.

Tenth order will now be at $12.25 \text{ mm} + 10\beta' = 14.55$ mm.

12. Given, $d = 0.1$ mm $= 10^{-4}$ m, $D = 0.5$ m and $\lambda = 5000$ Å $= 5 \times 10^{-7}$ m

$$\Delta y = (y_{11})_{\text{dark}} - (y_7)_{\text{bright}} = \frac{(2 \times 11 - 1)\lambda D}{2d} - \frac{7\lambda D}{d}$$

$$\Rightarrow \quad \Delta y = \frac{7\lambda D}{2d} = \frac{7\times 5\times 10^{-7}\times 0.5}{2\times 10^{-4}}$$

$$\Rightarrow \quad \Delta y = 8.75\times 10^{-3}\ \text{m} = 8.75\ \text{mm}$$

13. (a) Angular separation $\left(=\frac{\lambda}{d}\right)$ of the fringes remains constant. But the linear separation or fringe width increases in proportion to the distance (D) from the screen.

(b) As λ decreases, fringe width $(\beta \propto \lambda)$ decreases.

(c) As d increases, fringe width $\left(\beta \propto \frac{1}{d}\right)$ decreases.

(d) The interference pattern due to different component colours of white light overlap (in-coherently). The central bright fringes of different colours are at the same position. Therefore, the central fringe is white. Since blue colour has the lower λ, the fringe closest on either side of the central white fringe is blue; the farthest is red.

(e) Since in a medium the wavelength of light is $\lambda' = \frac{\lambda}{\mu}$, therefore the fringe width is given by

$$\beta = \frac{\lambda' D}{d} = \frac{\lambda D}{\mu d}.$$

Thus, fringe width decreases by μ.

14. For bright fringes to coincide, we have

$$\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{900}{750} = \frac{6}{5}$$

i.e., 5^{th} order maxima of larger wavelength λ_2 will overlap with 6^{th} order maxima of smaller wavelength λ_1.

So, $y_{\min} = \frac{6\lambda D}{d} = \frac{6\times 750\times 10^{-9}\times 2}{2\times 10^{-3}}$

$$\Rightarrow \quad y_{\min} = 4.5\times 10^{-3}\ \text{m}$$

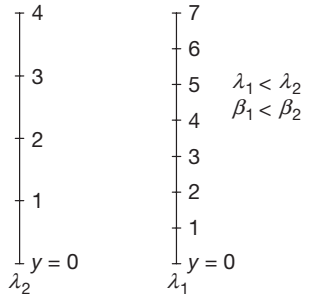
$$\Rightarrow \quad y_{\min} = 4.5\ \text{mm}$$

15. Let n_1 bright fringe of λ_1 overlaps with n_2 bright fringe of λ_2. Then,

$$\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$$

$$\Rightarrow \quad n_1\lambda_1 = n_2\lambda_2$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{700}{400} = \frac{7}{4}$$

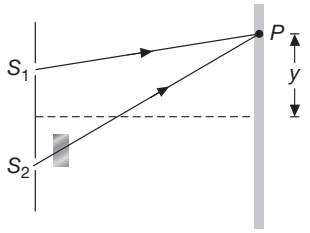


The ratio $\frac{n_1}{n_2} = \frac{7}{4}$ implies that 7^{th} bright fringe of λ_1 will overlap with 4^{th} bright fringe of λ_2. Similarly, 14^{th} of λ_1 will overlap with 8^{th} of λ_2 and so on.
So, the minimum order of λ_1 which overlaps with λ_2 is 7.

16. Without inserting the slab, path difference at P is given by

$$\Delta x = \frac{yd}{D} = \frac{0.15\times 10^{-3}\times 2\times 10^{-3}}{2}$$

$$\Rightarrow \quad \Delta x = 1.5\times 10^{-7}\ \text{m}$$



Corresponding phase difference at P is

$$\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x)$$

$$\Rightarrow \quad \phi = \left(\frac{2\pi}{6000\times 10^{-10}}\right)(1.5\times 10^{-7}) = \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{4}$$

Since, intensity at P is given by $I = 4I_0\cos^2\left(\frac{\phi}{2}\right)$

$$\Rightarrow \quad I = 4I_0\cos^2\left(\frac{\pi}{4}\right) = 2I_0$$

Phase difference after placing the glass sheet is

$$\phi' = \phi + \frac{2\pi}{\lambda}(\mu - 1)t$$

$$\Rightarrow \quad \phi' = \frac{\pi}{2} + \frac{2\pi}{6000\times 10^{-10}}(1.5-1)(8000+10^{-10})$$

$$\Rightarrow \quad \phi' = \frac{11\pi}{6}$$

CHAPTER 2

The intensity at P is now given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$$\Rightarrow \quad I' = I_0 + \eta I_0 + 2\sqrt{\eta I_0^2}\cos\left(\frac{11\pi}{6}\right) = 2I_0 \quad \{\because I' = 2I_0\}$$

Solving this equation, we get,

$$\eta = 0.21$$

17. (a) Since, $\Delta x = \frac{yd}{D} = \frac{(0.3\times10^{-3})(10\times10^{-3})}{1} = 3\times10^{-6}$ m

$$\Rightarrow \quad \phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x) = \left(\frac{2\pi}{546\times10^{-9}}\right)(3\times10^{-6})$$

$$\Rightarrow \quad \phi = 34.52 \text{ radian} = 1978^\circ$$

$$\Rightarrow \quad \frac{\phi}{2} = 989^\circ$$

Since, $I_P = I_0\cos^2\left(\frac{\phi}{2}\right)$

$$\Rightarrow \quad I_P = 3\times10^{-4} I_0$$

(b) Fringe width

$$\beta = \frac{\lambda D}{d} = \frac{(546\times10^{-9})(1)}{(0.3\times10^{-3})} = 1.82\times10^{-3} \text{ m}$$

$$\Rightarrow \quad \beta = 1.82 \text{ mm}$$

Therefore, number of fringes between point P and the central fringe are given by

$$N = \frac{10}{1.82} = 5.49$$

18. Let n_1 bright fringe corresponding to wavelength $\lambda_1 = 500$ nm coincides with n_2 bright fringe corresponding to wavelength $\lambda_2 = 700$ nm.

$$\Rightarrow \quad n_1\frac{\lambda_1 D}{d} = n_2\frac{\lambda_2 D}{d}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{7}{5}$$

This implies that 7^{th} maxima of λ_1 coincides with 5^{th} maxima of λ_2. Similarly 14^{th} maxima of λ_1 will coincides with 10^{th} maxima of λ_2 and so on.
So, minimum distance is

$$y_{\min} = \frac{n_1\lambda_1 D}{d} = 7\times5\times10^{-7}\times10^{3}$$

$$\Rightarrow \quad y_{\min} = 3.5\times10^{-3} \text{ m} = 3.5 \text{ mm}$$

19. Here $\lambda = 6\times10^{-5}$ cm, $t = 7.2\times10^{-4}$ cm

Displacement of the central bright fringe, $\Delta x = 6\beta$

Since $\Delta x = \frac{\beta}{\lambda}(\mu - 1)t$

$$\Rightarrow \quad \frac{\beta}{\lambda}(\mu - 1)t = 6\beta$$

$$\Rightarrow \quad \mu - 1 = \frac{6\lambda}{t} = \frac{6\times6\times10^{-5}}{7.2\times10^{-4}} = 0.5$$

$$\Rightarrow \quad \mu = 0.5 + 1 = 1.5$$

Test Your Concepts-II (Based on Diffraction)

1. For first dark band, $\sin\theta = \frac{\lambda}{d}$

As the diffraction pattern is obtained in the focal plane of lens, therefore

$$\tan\theta = \frac{x}{f}$$

For small θ, $\tan\theta \simeq \sin\theta$

$$\Rightarrow \quad \frac{x}{f} = \frac{\lambda}{d}$$

$$\Rightarrow \quad x = \left(\frac{\lambda}{d}\right)f$$

Since, $\lambda = 5900$ Å $= 59\times10^{-8}$ m, $f = 50$ cm $= 0.50$ m

$d = 0.025$ m $= 2.5\times10^{-5}$ m

$$\Rightarrow \quad x = \frac{59\times10^{-8}\times0.50}{2.5\times10^{-5}} = 11.8\times10^{-3} \text{ m} = 11.8 \text{ mm}$$

2. Given, $\lambda_1 = 5890\times10^{-10}$ m, $\lambda_2 = 5896\times10^{-10}$ m,

$d = 2\ \mu\text{m} = 10^{-6}$ m, $D = 2$ m

Distance of first secondary maximum from the centre of the screen is

$$x = \frac{3}{2}\frac{D\lambda}{d}$$

For the two wavelengths, we have

$$x_1 = \frac{3}{2}\frac{D\lambda_1}{d} \text{ and } x_2 = \frac{3}{2}\frac{D\lambda_2}{d}$$

Spacing between the first two maximum of sodium lines is given by

$$\Delta x = x_2 - x_1 = \frac{3D}{2d}(\lambda_2 - \lambda_1)$$

$$\Rightarrow \quad \Delta x = \frac{3\times2\times(5896-5890)\times10^{-6}}{2\times2\times10^{-6}} = 9\times10^{-4} \text{ m}$$

3. The linear separation between n bright fringes in an interference pattern on the screen is given by

$$x_n = \frac{n\lambda D}{d}$$

Since, $x_n \ll D$, the angular separation between n bright fringes should be

$$\theta_n = \frac{x_n}{D} = \frac{n\lambda}{d}$$

For 10 bright fringes, we get, $\theta_{10} = \dfrac{10\lambda}{D}$

The angular width of the central maximum in the diffraction pattern due to slit of width a is

$$2\theta_1 = \frac{2\lambda}{a}$$

Since, 10 maxima of the double slit pattern within the central maximum of the single slit pattern

$$\Rightarrow \quad 10\frac{\lambda}{d} < 2\frac{\lambda}{a}$$

$$\Rightarrow \quad a \le \frac{d}{5}$$

$$\Rightarrow \quad a = \frac{1}{5} \text{ mm} = 0.2 \text{ mm}$$

4. Since, $D_F = \dfrac{d^2}{\lambda}$

where $d = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$,

$$\lambda = 400 \text{ nm} = 4 \times 10^{-7} \text{ m}$$

$$\Rightarrow \quad D_F = \frac{(4 \times 10^{-3})^2}{4 \times 10^{-7}} = 40 \text{ m}$$

Thus, ray optics is valid upto a distance of 40 m from the aperture.

5. Here size of Fresnel zone D_F at the middle hill must be less that 50 m.
Distance of either of the two hills from the middle hill is

$$D = \frac{40}{2} \text{ km} = 20{,}000 \text{ m}$$

Since size of Fresnel's zone, $D_F = \sqrt{\lambda D}$

$$\Rightarrow \quad \sqrt{\lambda D} \ll 50$$

$$\Rightarrow \quad \lambda D \ll 2500$$

$$\Rightarrow \quad \lambda \ll \frac{2500}{D} = \frac{2500}{20{,}000} = 0.125 \text{ m} = 12.5 \text{ cm}$$

Thus, wavelengths longer than 12.5 cm will undergo serious diffraction effects.

6. Given, $\lambda = 6500 \text{ Å} = 6500 \times 10^{-10} \text{ m}$, $\theta = 30°$

For first minimum, $d \sin\theta = \lambda$

$$\Rightarrow \quad d = \frac{\lambda}{\sin\theta} = \frac{6500 \times 10^{-10}}{\sin 30°} = \frac{6500 \times 10^{-10}}{0.5}$$

$$\Rightarrow \quad d = 1.3 \times 10^{-6} \text{ m} = 1.3 \ \mu\text{m}$$

7. Given, $D = 2 \text{ m}$, $x_1 = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$,

$$\lambda = 5000 \text{ Å} = 5 \times 10^{-7} \text{ m}$$

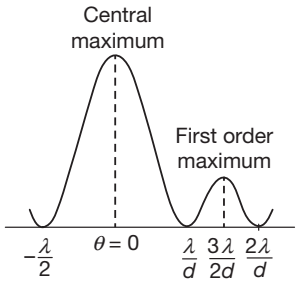
Width of central maximum is

$$\beta_0 = 2x_1 = 2 \times 5 \times 10^{-3} \text{ m} = 10^{-2} \text{ m}$$

So, slit width is

$$d = \frac{2D\lambda}{\beta_0} = \frac{2 \times 2 \times 5 \times 10^{-7}}{10^{-2}} = 2 \times 10^{-4} \text{ m} = 0.2 \text{ mm}$$

8. From figure, it is clear that the angular separation between central maximum and first order minimum is



$$\theta = \frac{3\lambda}{2d} - 0 = \frac{3\lambda}{2d}$$

$$\Rightarrow \quad \theta = \frac{3 \times 5890 \times 10^{-10}}{2 \times 0.25 \times 10^{-13}}$$

$$\Rightarrow \quad \theta = 3.534 \times 10^{-3} \text{ rad}$$

9. Angular spread of central maximum on either side of incident light is given by

$$\sin\theta = \frac{\lambda}{a}$$

So, slit width is $a = \dfrac{\lambda}{\sin\theta} = \dfrac{5000 \times 10^{-10}}{\sin 30°} = 10^{-6} \text{ m}$

For $\theta = 90°$, we get

$$a = \frac{\lambda}{\sin(90°)} = \frac{5000 \times 10^{-10}}{1} = 5 \times 10^{-7} \text{ m}$$

10. Given that $P = 20 \text{ mW} = 20 \times 10^{-3} \text{ W}$, $f = 0.05 \text{ m}$,

$$d = 1 \text{ mm} = 10^{-3} \text{ m}, \ \lambda = 7000 \text{ Å} = 7000 \times 10^{-10} \text{ m}$$

(a) Angular spread of the laser beam,

$$\theta = \frac{1.22\lambda}{d} = \frac{1.22 \times 7000 \times 10^{-10}}{1 \times 10^{-3}}$$

$$\Rightarrow \quad \theta = 8.54 \times 10^{-4} \text{ radian}$$

(b) Linear spread of the laser is

$$f \cdot \theta = 5 \times 10^{-2} \times 8.54 \times 10^{-4} \text{ m}$$

CHAPTER 2

So, areal spread of the laser, i.e., area of the target hit by it is

$$A = (5 \times 8.54 \times 10^{-6})^2 = 1.823 \times 10^{-15} \text{ m}^2$$

(c) Intensity of impact of the laser on the target is

$$I = \frac{\text{Power of laser}}{\text{Area hit}} = \frac{20 \times 10^{-3}}{1.823 \times 10^{-15}}$$

$$\Rightarrow \quad I = 10.97 \times 10^{12} \text{ Wm}^{-2}$$

Test Your Concepts-III (Based on Polarisation)

1. Since, $I = \frac{I_0}{2}$

Using Malus Law, $I = I_0 \cos^2\theta$

$$\Rightarrow \quad \frac{I_0}{2} = I_0 \cos^2\theta$$

$$\Rightarrow \quad \cos\theta = \pm\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \theta = \pm 45^\circ, \pm 135^\circ$$

The same effect occurs no matter which sheet is rotated or in which direction it is rotated.

2. According to Malus Law, $I = I_0 \cos^2\theta$

(a) Here $\theta = 30^\circ$

$$\Rightarrow \quad I = I_0 \cos^2(30^\circ) = I_0 \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} I_0$$

$$\Rightarrow \quad \frac{I}{I_0} = 0.75$$

(b) Here $\theta = 60^\circ$

$$\Rightarrow \quad I = I_0 \cos^2(60^\circ) = I_0 \left(\frac{1}{2}\right)^2 = \frac{I_0}{4}$$

$$\Rightarrow \quad \frac{I}{I_0} = 0.25.$$

3. Let I_0 be intensity of incident unpolarised light. Then the intensity of light emerging from the first polaroid will be

$$I_1 = \frac{I_0}{2}$$

If θ is the angle between the transmission directions of the two polaroids, then the intensity of light emerging from second polaroid is

$$I_2 = I_1 \cos^2\theta = \frac{I_0}{2}\cos^2\theta$$

Initially the two polaroids are crossed to each other i.e., $\theta = 90^\circ$.

When polaroid is rotated through 60°, the angle between their polarising directions will become

$$\theta = 90^\circ - 60^\circ = 30^\circ$$

$$\Rightarrow \quad I_2 = \frac{I_0}{2}\cos^2(30^\circ) = \frac{I_0}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8} I_0$$

$$\Rightarrow \quad \frac{I_2}{I_0} = 0.375$$

So, transmitted percentage is

$$\frac{I_2}{I_0} \times 100 = 0.375 \times 100 = 37.5\%$$

4. Obviously, the axis of P_3 is inclined at 45° to the axes of P_1 and P_2

Let amplitude of linearly polarised light emerging from P_1 be E_0

The amplitude of light emerging from P_3 is

$$E_0 \cos 45^\circ = \frac{E_0}{\sqrt{2}}$$

Next the amplitude of light emerging from P_2 is

$$\frac{E_0}{\sqrt{2}} \cos 45^\circ = \frac{E_0}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{E_0}{2}$$

Let intensity transmitted by $P_1 = I_0 \propto E_0^2$

So, intensity after passing through P_2 and P_3 is

$$I \propto \left(\frac{E_0}{2}\right)^2$$

$$\Rightarrow \quad I = \frac{I_0}{4}$$

Thus, the intensity becomes one-fourth of the maximum transmitted intensity.

5. The planes of polarisation of light beams A and B are mutually at right angles. Initially, the beam B shows zero intensity. Therefore, $\theta = 90^\circ$ for beam B and $\theta = 0^\circ$ for beam A. When the polaroid is rotated through 30°, we have

$\theta = 60^\circ$ for beam B and

$\theta = 30^\circ$ for beam A

In this position, we have

$$\left(\begin{matrix}\text{Intensity of emerging} \\ \text{beam } A\end{matrix}\right) = \left(\begin{matrix}\text{Intensity of emerging} \\ \text{beam } B\end{matrix}\right)$$

$$\Rightarrow \quad I_A \cos^2(30^\circ) = I_B \cos^2(60^\circ)$$

$$\Rightarrow \quad \frac{I_A}{I_B} = \frac{\cos^2(60^\circ)}{\cos^2(30^\circ)} = \frac{\left(\frac{1}{2}\right)^2}{\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{3} = 1:3$$

6. From Snell's Law, $\frac{\sin i_p}{\sin r_p} = \mu$

From Brewster Law, $\tan i_p = \frac{\sin i_p}{\cos i_p} = \mu$

$\Rightarrow \quad \frac{\sin i_p}{\sin r_p} = \frac{\sin i_p}{\cos i_p}$

$\Rightarrow \quad \sin r_p = \cos i_p$

$\Rightarrow \quad \sin r_p = \sin\left(90° - i_p\right)$

$\Rightarrow \quad r_p = 90° - i_p$

$\Rightarrow \quad i_p + r_p = 90°$

Hence the reflected and transmitted rays are perpendicular to each other.

7. Let P_1, P_2, P_3 be the three polarisers and θ be the angle between the transmission axes of P_1 and P_2. As P_1 and P_3 are crossed, the angle between P_2 and P_3 is $(90° - \theta)$.

Let I_0 be the intensity of the unpolarised light falling on P_1. Then the intensity of light emerging from P_1 will be

$$I_1 = \frac{I_0}{2}$$

By Malus Law, the intensity of light emerging from P_2 is given by

$$I_2 = I_1 \cos^2\theta = \frac{I_0}{2}\cos^2\theta$$

The intensity of light emerging from P_3 is given by

$$I_3 = I_2 \cos^2(90° - \theta) = \frac{1}{2} I_0 \cos^2\theta \sin^2\theta = \frac{1}{8} I_0 \sin^2(2\theta)$$

$\Rightarrow \quad \sin^2(2\theta) = \frac{8I_3}{I_0} = \frac{8\times 3}{32} = \frac{3}{4}$

$\Rightarrow \quad \sin(2\theta) = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad 2\theta = 60°$

$\Rightarrow \quad \theta = 30°$

So, $I_3 = \frac{I_0}{8}\sin^2(2\theta)$

I_3 will be maximum when $\sin^2(2\theta) = 1$ (maximum), i.e., $\sin(2\theta) = 1 = \sin(90°)$

$\Rightarrow \quad \theta = 45°$

8. Here $\mu = 1.6640$

By Brewster law, $\tan i_p = \mu$

$\Rightarrow \quad \tan i_p = 1.6640$

$\Rightarrow \quad i_p = \tan^{-1}(1.6640) = 59°$

If r is the angle of refraction, then $i_p + r = 90°$

$\Rightarrow \quad r = 90° - i_p = 90° - 59° = 31°$

9. (a) Thickness of a quarter wave plate is

$$t_{1/4} = \frac{\lambda}{4(\mu_0 - \mu_e)} = \frac{5000\times 10^{-10}}{4(1.544 - 1.533)}$$

$\Rightarrow \quad t_{1/4} = 2.136\times 10^{-5}$ m

(b) Thickness of a half-wave plate is

$$t_{1/2} = \frac{\lambda}{2(\mu_0 - \mu_e)} = \frac{5000\times 10^{-10}}{2(1.544 - 1.533)}$$

$\Rightarrow \quad t_{1/2} = 2.762\times 10^{-5}$ m

10. Specific rotation, $\alpha = \frac{\theta}{\ell c} = \frac{13.2}{2\times 0.1}$

where $\theta = 13.2°$, $c = 10\% = 0.1$ g cm^{-1}

$\ell = 20$ cm $= 2$ decimetre

$\Rightarrow \quad \alpha = 60°$ cm^{3}g^{-1}(dm)$^{-1}$

11. Since 5% solution in a decimetre tube i.e. $\ell_1 = 10$ cm causes a rotation of $20°$. Also, $\alpha = \frac{\theta}{\ell c}$

$\Rightarrow \quad 20 = \alpha \times \ell_1 \times \frac{5}{100}$ …(1)

Let ℓ_2 length of the 10% solution of the same substance cause a rotation of $30°$. Then

$$30 = \alpha \times \ell_2 \times \frac{10}{100} \quad \ldots(2)$$

Dividing (1) by (2), we get

$$\frac{20}{30} = \frac{\alpha \times \ell_1 \times 0.05}{\alpha \times \ell_2 \times 0.10}$$

$\Rightarrow \quad \ell_2 = 0.75\ell_1 = (0.75)(10) = 7.5$ cm

Single Correct Choice Type Questions

1. $I_{max} = I = (a + a)^2$

$\Rightarrow \quad I = 4a^2 = 4I_0$

When either of the two slits is covered then

$$I' = (a + 0)^2 = a^2 = \frac{I}{4}$$

Hence, the correct answer is (B).

2. $I' = \frac{3}{4}(4I) = 3I$

$\Rightarrow \quad 3I = I + I + 2I\cos\phi$

$\Rightarrow \quad \cos\phi = \frac{1}{2}$

CHAPTER 2

$\Rightarrow \quad \phi = \dfrac{\pi}{3}$

For second minima, we have $\phi = 3\pi$ and for third maxima, we have $\phi = 6\pi$. So, this value should lie between 3π and 6π.

Hence, $\dfrac{\pi}{3}$ cannot be a possible value.

Hence, the correct answer is (A).

3. Specific rotation (α) is given by

$$\alpha = \frac{\theta}{\ell c}$$

$$\Rightarrow \quad c = \frac{\theta}{\alpha \ell} = \frac{0.4}{0.01 \times 0.25} = 160 \text{ kgm}^{-3}$$

So, percentage purity of sugar solution is

$$\% \text{ purity } = \frac{160}{200} \times 100 = 80\%$$

Hence, the correct answer is (A).

4. Since, angular width $= \dfrac{\lambda}{d} = 10^{-3}$ (Given)

So, number, of fringes within $0.12°$ will be

$$n = \frac{0.12 \times 2\pi}{360 \times 10^{-3}} \cong (2.09)$$

The number of bright spots will be three.

Hence, the correct answer is (B).

8. $\Delta\lambda = \dfrac{v_s}{c}\lambda$

$$\Rightarrow \quad v_s = \frac{\Delta\lambda \cdot c}{\lambda} = \frac{47 \times 3 \times 10^8}{4700}$$

$\Rightarrow \quad v_s = 3 \times 10^6 \text{ ms}^{-1}$ away from earth

Hence, the correct answer is (D).

9. The relation among angle of diffraction θ, order n and number of lines per cm of the grating N is $\sin\theta = Nn\lambda$. The maximum value of $\sin\theta = 1$. Hence maximum value for $n = \dfrac{1}{N\lambda}$. $\lambda = 6000 \times 10^{-8}$ cm, $N = 5000$ lines/cm, we get $n = 3.33$. The order of spectrum is an integer. Thus we cannot see the fourth order, but can see the third order.

Hence, the correct answer is (B).

10. Since $\mu\lambda = \text{constant}$

$\Rightarrow \quad \mu_1\lambda_1 = \mu_2\lambda_2$

$\Rightarrow \quad (1)\lambda_1 = n\lambda_2$

$\Rightarrow \quad \dfrac{\lambda_1}{\lambda_2} = n$

Hence, the correct answer is (A).

11. $n_1\lambda_1 = n_2\lambda_2$

$\Rightarrow \quad 62 \times 5893 = n_2 \times 4358$

$\Rightarrow \quad n_2 = 84$

Hence, the correct answer is (D).

12. $\beta = \dfrac{\lambda D}{d}$

$$\Rightarrow \quad \beta = \frac{(5000 \times 10^{-10})(2)}{(10^{-3})}$$

$\Rightarrow \quad \beta = 10^{-3}$ m $= 1$ mm

Hence, the correct answer is (D).

13. Let the required thickness be t Å. So, number of wavelengths in vacuum is $\dfrac{t}{6000}$.

$\Rightarrow$ Number of wavelengths in air is $\dfrac{1.003t}{6000}$

According to the problem

$$\frac{t}{6000} = \frac{1.003t}{6000} + 1$$

$\Rightarrow \quad t = 2$ mm

Hence, the correct answer is (A).

14. $I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 9I$

$I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = I$

Hence, the correct answer is (C).

15. Theoretically infinite colours are possible, hence infinite wavelengths.

Hence, the correct answer is (D).

17. $\Delta x = (\mu_2 - \mu_1)t$

$$\Rightarrow \quad \delta = \frac{2\pi}{\lambda}(\mu_2 - \mu_1)t$$

Hence, the correct answer is (C).

18. Because sound waves are longitudinal in nature.

Hence, the correct answer is (D).

19. The blue filter will allow only blue light to pass through. So, light from the object passes through filter. Similarly, the white background will also look blue through the filter. Thus, we have a blue object under a blue background, which makes it indistinguishable.

Hence, the correct answer is (D).

20. Intensity of illumination at a point is decided by three factors; (i) power of the source P (ii) distance from the source r and (iii) angle of incidence of rays $\cos\theta$

$$\Rightarrow \quad I = P\cos\frac{\theta}{r^2}$$

At noon the sun's rays are normally incident making $\theta = 0$. Hence I is maximum at noon.
Hence, the correct answer is (B).

21. Distance of third maxima from central maxima is

$$x = \frac{3\lambda D}{d} = \frac{3\times 5000\times 10^{-10}\times(200\times 10^{-2})}{0.2\times 10^{-3}} = 1.5 \text{ cm}$$

Hence, the correct answer is (B).

23. For observation of colours, the thickness of the film should be of the order of the wavelength of visible light.
Hence, the correct answer is (C).

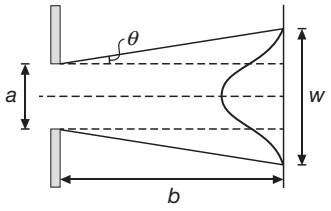
24. The red filter will pass only red light. Hence a blue cross will appear as black. The white background will appear as red when seen through a red filter. Thus, the observer will see a black cross under a red background.
Hence, the correct answer is (D).

25. The direction in which the first minima occurs is θ (say). Then

$$a\sin\theta = \lambda$$

$$\Rightarrow \quad a\theta = \lambda \qquad \{\because \theta \approx \sin\theta \text{ when } \theta \text{ small}\}$$

$$\Rightarrow \quad \theta = \frac{\lambda}{a}$$



Width of the central maximum is

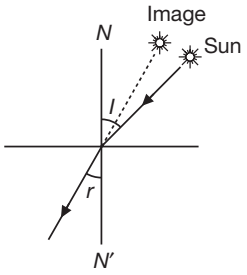
$$w = 2b\theta + a = 2b\left(\frac{\lambda}{a}\right) + a$$

Hence, the correct answer is (C).

26. Newton's concept of light is that it is made of corpuscles or particles. All particles are deflected by earth's gravitational field. Hence, they are also deflected by gravitational field. In the general theory of relativity. Einstein predicts deflection of light by the gravitational field. The angle of deflection predicted by both are however not the same. Einstein's prediction agrees with experimental results.
Hence, the correct answer is (B).

27. In photoelectric effect, the photon is treated as a particle having a quantum of energy $h\nu$. It emits one electron. Here the particle and wave aspect of light appear in two sides of equation. Einstein's photoelectric equation appears to be interlinking of the two aspects.
Hence, the correct answer is (D).

28. This is the effect of refraction. Rays from the sun are passing from vacuum to air when they enter into the earth's atmosphere. They bend towards the normal making the sun to appear at a higher altitude as shown in figure.



Hence, the correct answer is (A).

30. In the presence of thin glass plate, the fringe pattern shifts, but no change in fringe width.
Hence, the correct answer is (D).

31. If an unpolarised light is converted into plane polarised light by passing through a polaroid, it's intensity becomes half.
Hence, the correct answer is (C).

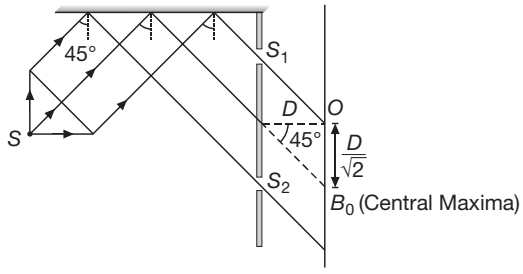
32. P to Q convergence increasing and Q to R direction changing.
Hence, the correct answer is (D).

33. $\beta = \frac{\lambda D}{d}$ and

$$\beta' = \frac{\lambda(D/2)}{2d} = \frac{\beta}{4}$$

Hence, the correct answer is (A).

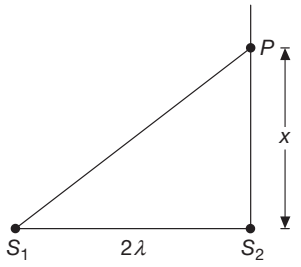
34. From the figure shown below it is clear that the distance of central maxima from O is $\frac{D}{\sqrt{2}}$.



Hence, the correct answer is (A).

CHAPTER 2

35. Path difference at s_2 is 2λ. Therefore, for minimum intensity at P



Let x be the minimum distance from s_2. Then

$$s_1P - s_2P = \frac{3\lambda}{2} \neq \frac{\lambda}{2} \quad \ldots(1)$$

$$\Rightarrow \quad \sqrt{4\lambda^2 + x^2} - x = \frac{3\lambda}{2}$$

Solving this equation, we get

$$x = \frac{7\lambda}{12}$$

NOTE: If we substitute $s_1P - s_2P = \frac{\lambda}{2}$ in equation (1) we get $x = \frac{15\lambda}{4}$ which is greater than $\frac{7\lambda}{12}$.

Hence, the correct answer is (A).

36. Shift $= \frac{(\mu - 1)tD}{d}$

$$x = \frac{(1.5 - 1)tD}{d} \quad \ldots(1)$$

and $$\frac{3}{2}x = \frac{(\mu - 1)tD}{d} \quad \ldots(2)$$

Dividing equation (1) by (2)

$$\frac{2}{3} = \frac{0.5}{\mu - 1}$$

$$2\mu - 2 = 1.5$$

$$2\mu = 3.5$$

$$\mu = 1.75$$

Hence, the correct answer is (A).

37. Phase difference corresponding to $y_1 = -\frac{\pi}{2}$ and that for $y_2 = +\frac{\pi}{2}$

$\Rightarrow$ Average intensity between y_1 and y_2

$$\Rightarrow \quad I_{av} = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} I_{\max}\cos^2\left(\frac{\phi}{2}\right)d\phi = I_{\max}\frac{(\pi + 2)}{2\pi}$$

So, the required ratio is $\frac{1}{2}\left(1 + \frac{2}{\pi}\right)$

Hence, the correct answer is (A).

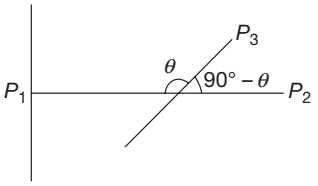
38. Intensity at the centre will be zero if path difference $= \frac{\lambda}{2}$

$$\Rightarrow \quad (\mu - 1)t = \frac{\lambda}{2}$$

$$\Rightarrow \quad t = \frac{\lambda}{2(\mu - 1)}$$

Hence, the correct answer is (C).

39. No light is emitted from the second polaroid, so P_1 and P_2 are perpendicular to each other



Let the initial intensity of light is I_0. So, Intensity of light after transmission from first polaroid $= \frac{I_0}{2}$.
Intensity of light emitted from P_3

$$I_1 = \frac{I_0}{2}\cos^2\theta$$

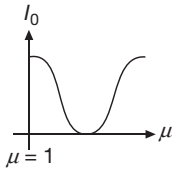
Intensity of light transmitted from last polaroid i.e., from

$$P_2 = I_1\cos^2(90^o - \theta) = \frac{I_0}{2}\cos^2\theta\sin^2\theta$$

$$P_2 = \frac{I_0}{8}(2\sin\theta\cos\theta)^2 = \frac{I_0}{8}\sin^2 2\theta$$

Hence, the correct answer is (A).

40. In absence of film, intensity is maximum at the mid-point of screen. As the value of μ is increased, intensity decreases and then increases alternately. Hence the correct variation is



Hence, the correct answer is (C).

41. For secondary maxima, we have

$$a\sin\theta = \frac{5\lambda}{2}$$

$$\Rightarrow \quad a\theta = a\left(\frac{y}{D}\right) = \frac{5\lambda}{2}$$

Since $D \approx f$

$$\Rightarrow \quad 2y = \frac{5\lambda f}{a} = \frac{5\times 0.8\times 10^{-7}}{4\times 10^{-4}} = 6\times 10^{-3}\ \text{m} = 6\ \text{mm}$$

Hence, the correct answer is (A).

43. Path difference at P is

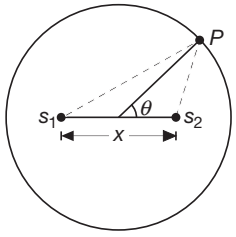
$$\Delta x = 2\left(\frac{x}{2}\cos\theta\right) = x\cos\theta$$

For intensity to be maximum

$$\Delta x = n\lambda \qquad \{n = 0, 1, 2, 3, ...\}$$

$$\Rightarrow \quad x\cos\theta = n\lambda$$

$$\Rightarrow \quad \cos\theta = \frac{n\lambda}{x}$$



Since, $\cos\theta \not> 1$

$$\Rightarrow \quad \frac{n\lambda}{x} \not> 1$$

$$\Rightarrow \quad n \not> \frac{x}{\lambda}$$

Substituting $x = 5\lambda$, we get

$$n \not> 5$$

$$\Rightarrow \quad n = 1, 2, 3, 4, 5, ...$$

Therefore, in all four quadrants there can be 20 maximas. There are more maximas at $\theta = 0°$ and $\theta = 180°$. But $n = 5$ corresponds to $\theta = 90°$ and $\theta = 270°$ which are coming only twice while we have multiplied it four times. Therefore, total number of maximas are still 20 i.e., $n = 1$ to 4 in four quadrants (total 16) plus four more at $\theta = 0°$, $90°$, $180°$ and $270°$.

Hence, the correct answer is (A).

44. Laser is intense, coherent (all photons in phase) and monochromatic (only single wavelength).

Hence, the correct answer is (A).

45. The focal length increases with increases of wavelength. $\frac{1}{f} \propto (n-1)$ nearly $\propto \frac{1}{\lambda^2}$ where λ is wavelength. In the given set of radiations. X-rays have the minimum wavelength. Hence focal length is minimum for X-rays.

Hence, the correct answer is (D).

46. As the star accelerates towards the earth, its velocity will increase. By Doppler effect in light, a source approaching an observer shows violet shift. So, the colour of the star will shift towards the shorter wavelength region, that is blue.

Hence, the correct answer is (D).

48. The direction of light is given by the normal vector $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$. So, angle made by the $\vec{n}$ with y-axis is given by $\cos\beta = \frac{2}{\sqrt{1^2+2^2+3^2}} = \frac{2}{\sqrt{14}}$

Hence, the correct answer is (C).

50. $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

$$\Rightarrow \quad \frac{0.05}{100} = \frac{v}{3\times 10^8}$$

$$\Rightarrow \quad v = 1.5\times 10^5$$

(Since wavelength is decreasing, so star coming closer)

Hence, the correct answer is (B).

51. Momentum of the electron will increase. So, the wavelength $\left(\lambda = \frac{h}{p}\right)$ of electrons will decrease and fringe width decreases as $\beta \propto \lambda$.

Hence, the correct answer is (B).

52. $I = 4I_0\cos^2\left(\frac{\phi}{2}\right)$

At central position $I_1 = 4I_0$ ...(1)

Since the phase difference between two successive fringes is $2x$, the phase difference between two points separated by a distance equal to one quarter of the distance between the two, successive fringes are equal to

$$\delta = (2\pi)\left(\frac{1}{4}\right) = \frac{\pi}{2}\ \text{radian}$$

$$\Rightarrow \quad I_2 = 4I_0\cos^2\left(\frac{\frac{\pi}{2}}{2}\right) = 2I_0 \qquad ...(2)$$

Using (1) and (2), $\frac{I_1}{I_2} = \frac{4I_0}{2I_0} = 2$

Hence, the correct answer is (A).

53. $I = I_0\left[\frac{\sin\alpha}{\alpha}\right]^2$, where $\alpha = \frac{\phi}{2}$

For n^{th} secondary maxima $d\sin\theta = \left(\frac{2n+1}{2}\right)\lambda$

$$\Rightarrow \quad \alpha = \frac{\phi}{2} = \frac{\pi}{\lambda}[d\sin\theta] = \left(\frac{2n+1}{2}\right)\pi$$

$$\therefore \quad I = I_0\left(\frac{\sin\left(\frac{2n+1}{2}\right)\pi}{\left(\frac{2n+1}{n}\right)\pi}\right)^2 = \frac{I_0}{\left\{\frac{(2n+1)}{2}\pi\right\}^2}$$

So $I_0 : I_1 : I_2 = I_0 : \frac{4}{9\pi^2}I_0 : \frac{4}{25\pi^2}I_0 = 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2}$

Hence, the correct answer is (C).

CHAPTER 2

54. Let t be the thickness so corresponding

$$\Delta x = \mu t = \frac{3}{2}t$$

Also $I_{max} = 4I$

$$\Rightarrow \quad I_R = I = \frac{I_{max}}{2} = 2I$$

Since, $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$$\Rightarrow \quad 2I_0 = I_0 + I_0 + 2\sqrt{I_0^2}\cos\left(\frac{3\pi t}{\lambda}\right)$$

$$\Rightarrow \quad \cos\left(\frac{3\pi t}{\lambda}\right) = 0$$

$$\Rightarrow \quad \frac{3\pi t}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots$$

$$\Rightarrow \quad t = \frac{\lambda}{6}, \frac{3\lambda}{6}, \frac{5\lambda}{6}, \ldots\ldots..$$

$$\Rightarrow \quad t = \frac{\lambda}{2}$$

Hence, the correct answer is (C).

55. If I is the final intensity and I_0 is the initial intensity then

$$I = \frac{I_0}{2}\left(\cos^2 30°\right)^5$$

$$\Rightarrow \quad \frac{I}{I_0} = \frac{1}{2}\times\left(\frac{\sqrt{3}}{2}\right)^{10} = 0.12$$

Hence, the correct answer is (D).

56. For maxima $\Delta = d\sin\theta = n\lambda$

$$\Rightarrow \quad 2\lambda\sin\theta = n\lambda$$

$$\Rightarrow \quad \sin\theta = \frac{n}{2}$$

since value of $\sin\theta$ cannot be greater 1.

$$\therefore \quad n = 0, 1, 2$$

Therefore, only five maximas can be obtained on both side of the screen.

Hence, the correct answer is (B).

57. In this case, we can assume as if both the source and the observer are moving towards each other with speed v. Hence

$$v' = \frac{c - u_o}{c - u_s}v = \frac{c - (-v)}{c - v}v = \frac{c + v}{c - v}v$$

$$v' = \frac{(c + v)(c - v)}{(c - v)^2}v = \frac{c^2 - v^2}{c^2 + v^2 - 2vc}v$$

Since $v \ll c$, therefore $v' = \frac{c^2}{c^2 - 2vc} = \frac{c}{c - 2v}v$

Hence, the correct answer is (B).

58. Fringe width $\beta \propto \lambda$. Therefore, λ and hence β decreases 1.5 times when immersed in liquid. The distance between central maxima and 10^{th} maxima is 3 cm in vacuum. When immersed in liquid it will reduce to 2 cm. Position of central maxima will not change while 10^{th} maxima will be obtained at $y = 4$ cm.

Hence, the correct answer is (C).

59. For maximum intensity on the screen

$$d\sin\theta = n\lambda$$

$$\Rightarrow \quad \sin\theta = \frac{n\lambda}{d} = \frac{n(2000)}{7000} = \frac{n}{3.5}$$

Since maximum value of $\sin\theta$ is 1

So $n = 0, 1, 2, 3$ only. Thus, only seven maximas can be obtained on both sides of the screen.

Hence, the correct answer is (B).

60. Optical path difference $(O.P.D.)$ between P and Q is

$$(\text{O.P.D.}) = (2.25\lambda_0)(1) + (3.5\lambda_0)(2) + (3\lambda_0)(3)$$

$$\Rightarrow \quad (\text{O. P. D.}) = 18.25\lambda_0$$

Phase difference $\Delta\phi = \left(\frac{2\pi}{\lambda_0}\right)\Delta x = \frac{\pi}{2}$

Hence, the correct answer is (C).

61. If $d\sin\theta = (\mu - 1)t$, central fringe is obtained at O

If $d\sin\theta > (\mu - 1)t$, central fringe is obtained above O and

If $d\sin\theta < (\mu - 1)t$, central fringe is obtained below O

Hence, the correct answer is (D).

62. Resultant intensity $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

At central position with coherent source and $I_1 = I_2 = I_0$

$$I_{con} = 4I_0 \qquad \ldots(1)$$

In case of incoherent at a given point, ϕ varies randomly with time so $(\cos\phi)_{av} = 0$

$$\therefore \quad I_{Incoh} = I_1 + I_2 = 2I_0 \qquad \ldots(2)$$

Hence $\frac{I_{coh}}{I_{Incoh}} = \frac{2}{1}$

Hence, the correct answer is (B).

63. According to given condition

$$(\mu - 1)t = n\lambda \text{ for minimum } t,\ n = 1$$

So, $(\mu - 1)t_{min} = \lambda$

$$t_{min} = \frac{\lambda}{\mu - 1} = \frac{\lambda}{1.5 - 1} = 2\lambda$$

Hence, the correct answer is (A).

64. $\left(\frac{I_1+I_2}{I_1-I_2}\right)^2 \to \infty$

$\Rightarrow \quad I_1 - I_2 \to 0$

$\Rightarrow \quad I_1 \simeq I_2$

Hence, the correct answer is (B).

65. Path difference between the waves reaching O will be

$$\Delta x = 2d\sin\alpha$$

For dark fringe at O, we have

$$2d\sin\alpha = \frac{\lambda}{2}$$

$$\Rightarrow \quad \sin\alpha = \frac{\lambda}{4d}$$

$$\Rightarrow \quad \alpha = \sin^{-1}\left(\frac{\lambda}{4d}\right)$$

Hence, the correct answer is (B).

66. When unpolarised light is made incident at polarising angle, the reflected light is plane polarised in a direction perpendicular to the plane of incidence.

Therefore $\vec{E}$ in reflected light will vibrate in vertical plane with respect to plane of incidence.

Hence, the correct answer is (A).

67. If I is the intensity of the incident unpolarised light, the intensity transmitted by the first is $\frac{I}{2}$. This is the intensity of incident light on the second polaroid. Intensity transmitted by the second polaroid is $\left(\frac{I}{2}\right)\cos^2\theta$, where θ is the angle between the axes. Here $\sin\theta = \frac{3}{5}$, $\cos\theta$ is therefore $\frac{4}{5}$.

$$\frac{I}{2}\cos^2\theta = \frac{I}{2}\times\left(\frac{4}{5}\right)^2 = \frac{8}{25}I$$

$\frac{8}{25}$ is the required ratio.

Hence, the correct answer is (D).

69. If you divide the original slit into N strips and represents the light from each strip, when it reaches the screen, by a phasor, then at the central maximum in the diffraction pattern you add N phasors, all in the same direction and each with the same amplitude. The intensity is therefore N^2. If you double the slit width, you need $2N$ phasors, if they are each to have the amplitude of the each to have the amplitude of the phasors you used for the narrow slit. The intensity at the central maximum is proportional to $(2N)^2$ and is, therefore, four times the intensity for the narrow slit.

Hence, the correct answer is (D).

70. $\sin C = \frac{3}{5}$. Hence $n = \frac{5}{3}$. If i is the polarising angle $\tan i = n$. $i = \tan^{-1}(n) = \tan^{-1}\left(\frac{5}{3}\right)$.

Hence, the correct answer is (B).

71. Since $\theta \propto \ell$

Since the volume ratio is given to be $1:2$, so in a tube of length 30 cm means 10 cm length of first solution and 20 cm length of second solution.

Rotation produced by 10 cm length of first solution is

$$\theta_1 = \frac{38^\circ}{20}\times 10 = 19^\circ$$

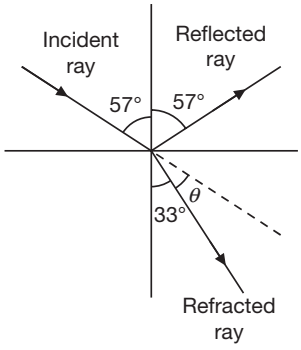
Rotation produced by 20 cm length of second solution is

$$\theta_2 = \frac{24^\circ}{30}\times 20 = 16^\circ$$

So, net rotation produced is $\theta = 19^\circ - 16^\circ = 3^\circ$

Hence, the correct answer is (D).

72.



For glass of refractive index 1.5, polarising angle is 57°, $\tan i = 1.5$. At this stage the angle between the refracted ray and the reflected ray is 90°. The required angle from figure is $\theta = 57 - 33 = 24^\circ$.

Hence, the correct answer is (C).

73. The property of double refraction is shown by quartz, calcite and ice also.

Hence, the correct answer is (D).

74. For first minimum

$$\lambda = a\sin\theta$$

$$\Rightarrow \quad 6.5\times10^{-7} = a\sin 30$$

$$\Rightarrow \quad a = 13\times10^{-7}\ \text{m}$$

$$\Rightarrow \quad a = 1.3\ \mu\text{m}$$

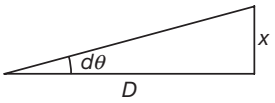
Hence, the correct answer is (C).

CHAPTER 2

75. The resolving power of the human eye is $\frac{1}{d\theta}$ is the smallest angle subtended by two objects just distinguishable, $d\theta = 1' = \frac{1}{60^\circ}$. Converting this into radian and taking 1 radian as nearly 60° (Actually $57^\circ 18'$) we get the resolving power as 3600.

Hence, the correct answer is (B).

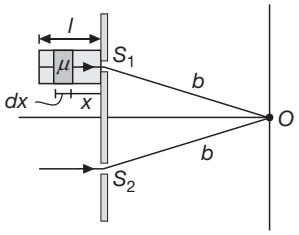
77.



If x is the distance between poles and D is the distance of the eye from the poles as shown in figure, we have $d\theta = \tan d\theta = \frac{x}{D}$. Here $D = 3.6$ km $= 3600$ m and $d\theta = \frac{1}{3600}$ radian. Hence $x = 1$ m.

Hence, the correct answer is (C).

78. When light passes through a medium of refractive index μ, the optical path it travels is μt.



Therefore, before reaching O, light through S_1 travels a distance $(\mu\ell + b)$ while that through S_2 travels a distance $(\ell + b)$

So, path difference $\Delta x = (\mu\ell + b) - (\ell + b) = (\mu - 1)\ell$

$$\Delta x = (\mu - 1)dx = (1 + ax - 1)dx$$

$$\Rightarrow \quad \Delta x = axdx$$

For the whole length, we have

$$\Delta x_{\text{total}} = \int_0^\ell axdx = \frac{a\ell^2}{2}$$

For a minima to be formed at O, we have

$$\Delta x_{\text{total}} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \quad \frac{a\ell^2}{2} = (2n+1)\frac{\lambda}{2}$$

For minimum a, n should be minimum, so $n = 0$

$$\Rightarrow \quad \frac{a\ell^2}{2} = \frac{\lambda}{2}$$

$$\Rightarrow \quad a = \frac{\lambda}{\ell^2}$$

Hence, the correct answer is (B).

79. $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

$$\Rightarrow \quad 1 = \frac{v}{c}$$

$$\Rightarrow \quad v = c$$

Hence, the correct answer is (A).

80. $\lambda' = \lambda\left(1 - \frac{v}{c}\right) = 5890\left(1 - \frac{4.5\times10^6}{3\times10^8}\right) \approx 5802$ Å

Hence, the correct answer is (C).

81. $\frac{\lambda_1}{\lambda_2} = \frac{4500}{6000} = \frac{3}{4}$

$$\Rightarrow \quad 4\lambda_1 = 3\lambda_2$$

i.e. the fourth bright fringe of λ_1 coincides with the third bright fringe of λ_2.

Hence, the correct answer is (C).

82. From here, we must take a note that central maxima $(n = 0)$ is flanked by first minima $(n = 1)$ and then first maxima $(n = 1)$.

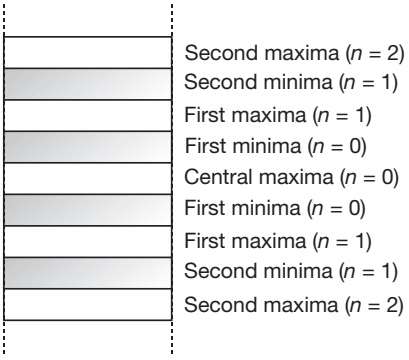
Position of the $(n+1)$th dark fringe is

$$x_{n+1} = (2n+1)\frac{\lambda D}{2d}$$

So, position of second dark fringe is

$$x_2 = [2(1)+1]\frac{\lambda D}{2d}$$

$$\Rightarrow \quad x_2 = \frac{3\lambda D}{2d}$$


Second maxima (n = 2)
Second minima (n = 1)
First maxima (n = 1)
First minima (n = 0)
Central maxima (n = 0)
First minima (n = 0)
First maxima (n = 1)
Second minima (n = 1)
Second maxima (n = 2)

Similarly, position of nth bright fringe is

$$x_n = n\left(\frac{\lambda D}{d}\right)$$

$$\Rightarrow \quad x_4 = \frac{4\lambda D}{d}$$

So, separation is

$$\Delta x = x_4 - x_2 = \frac{5}{2}\left(\frac{\lambda D}{d}\right)$$

$\Rightarrow \quad \Delta x = 1.5$ mm

Hence, the correct answer is (C).

83. Shift, $\Delta x = \left[(\mu - 1)t\right]\frac{D}{d}$

$\Rightarrow \quad 20\beta = (\mu - 1)\frac{tD}{d}$

$\Rightarrow \quad 20\left(\frac{\lambda D}{d}\right) = (\mu - 1)\frac{tD}{d}$

$\Rightarrow \quad 20\lambda = (\mu - 1)t$

$\Rightarrow \quad \mu = \frac{20\lambda}{t} + 1$

$\Rightarrow \quad \mu = \frac{20 \times 5000 \times 10^{-10}}{2.5 \times 10^{-5}} + 1$

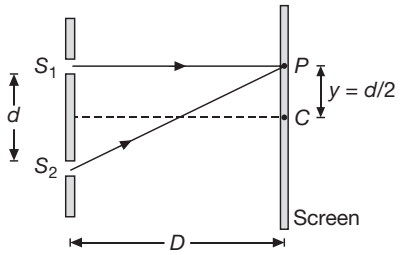
$\Rightarrow \quad \mu = \frac{4}{10} + 1$

$\Rightarrow \quad \mu = 1.4$

Hence, the correct answer is (C).

84. Suppose P is a point in front of one slit at which intensity is I_0. From figure, it is clear that $y = \frac{d}{2}$. Path difference between the waves reaching at P is

$$\Delta x = \frac{yd}{D} = \frac{\left(\frac{d}{2}\right)d}{10d} = \frac{d}{20} = \frac{5\lambda}{20} = \frac{\lambda}{4}$$



Hence corresponding phase difference is given by

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

Resultant intensity at P is

$$I = I_{\max}\cos^2\left(\frac{\phi}{2}\right) = I_o\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$$

Hence, the correct answer is (A).

85. For dark fringe at P

$$S_1P - S_2P = \Delta = \frac{(2n-1)\lambda}{2}$$

Here $n = 3$ and $\lambda = 6000$

So, $\Delta = \frac{5\lambda}{2} = 5 \times \frac{6000}{2} = 15000$ Å $= 1.5$ micron

Hence, the correct answer is (B).

86. Fringe width $\omega = \frac{\lambda D}{d}$. When the apparatus is immersed in a liquid, λ and hence w is reduced μ (refractive index) times.

$10\omega' = (5.5)\omega$

$\Rightarrow \quad 10\lambda'\left(\frac{D}{d}\right) = (5.5)\frac{\lambda D}{d}$

$\Rightarrow \quad \frac{\lambda}{\lambda'} = \frac{10}{5.5} = \mu$

$\Rightarrow \quad \mu = 1.8$

Hence, the correct answer is (A).

87. The intensity of light received is inversely proportional to the square of distance. Hence the amplitude of the light wave received is inversely proportional to the distance.

Hence, the correct answer is (B).

88. Fringe width $\beta \propto \lambda$. Therefore, λ and hence w will decrease 1.5 times when immersed in liquid. The distance between central maxima and 10^{th} maxima is 3 cm in vacuum. When immersed in liquid it will reduce to 2 cm. Position of central maxima will not change while 10^{th} maxima will be obtained at $y = 4$ cm.

Hence, the correct answer is (C).

89. $I = 4I_0\cos^2\left(\frac{\phi}{2}\right)$

$I_0 = 4I_0\cos^2\left(\frac{\phi}{2}\right)$

$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \frac{1}{2}$

$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{3}$

$\Rightarrow \quad \phi = \frac{2\pi}{3} = \left(\frac{2\pi}{\lambda}\right)\Delta x$

$\Rightarrow \quad \frac{1}{3} = \left(\frac{1}{\lambda}\right)y\frac{d}{D} \qquad \left\{\because \Delta x = \frac{yd}{D}\right\}$

$\Rightarrow \quad y = \frac{\lambda}{3 \times \frac{d}{D}} = \frac{6 \times 10^{-7}}{3 \times 10^{-4}}$

$\Rightarrow \quad y = 2 \times 10^{-3}$ m $= 2$ mm

Hence, the correct answer is (A).

CHAPTER 2

90. For third maximum

$$x = n\left(\frac{\lambda D}{d}\right)$$

$$\Rightarrow \quad x = (3)\frac{5000\times10^{-10}\times2}{0.2\times10^{-3}}$$

$$\Rightarrow \quad x = 1.5 \text{ cm}$$

Hence, the correct answer is (B).

91. Rotation produced $\theta = \alpha\ell c$

Net rotation produced $\theta_r = \theta_1 - \theta_2 = \ell(\alpha_1 c_1 - \alpha_2 c_2)$

$$\Rightarrow \quad \theta_r = 0.29\times(0.01\times60 - 0.02\times30) = 0$$

Hence, the correct answer is (B).

92. Blue of the sky is explained by scattering. Scattered light obeys Rayleigh's law. $I \propto \frac{1}{\lambda^4}$. Hence shorter wavelengths are scattered with more intensity. In visible light, violet and blue have the minimum wavelength. Of the two, blue is more intense.
Hence, the correct answer is (D).

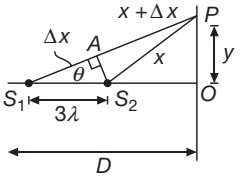
93. The interference fringes for two slits are hyperbolic.
Hence, the correct answer is (D).

94. Total phase difference
= Initial phase difference + Phase difference due to path

$$\Rightarrow \quad \phi = 66° + \frac{360°}{\lambda}\times\Delta x = 66° + \frac{360°}{\lambda}\times\frac{\lambda}{4} = 66° + 90 = 156°$$

Hence, the correct answer is (A).

95.



For maxima, $\Delta x = n\lambda$

In this case path difference is $d\cos\theta$

$$\Rightarrow \quad n\lambda = \lambda = 3\lambda\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{1}{3}$$

$$\Rightarrow \quad \frac{D}{\sqrt{D^2 + y^2}} = \frac{1}{3}$$

$$\Rightarrow \quad 3D = \sqrt{D^2 + y^2}$$

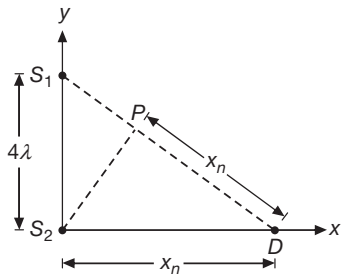
$$\Rightarrow \quad y = \sqrt{8}D = 2\sqrt{2}D$$

Hence, the correct answer is (B).

96. From $\Delta S_1 S_2 D$,

$$(S_1D)^2 = (S_1S_2)^2 + (S_2D)^2$$

$$(S_1P + PD)^2 = (S_1S_2)^2 + (S_2D)^2$$



Here S_1P is the path difference $= n\lambda$ for maximum intensity.

$$\therefore \quad (n\lambda + x_n)^2 = (4\lambda)^2 + (x_n)^2$$

$$\Rightarrow \quad x_n = \frac{16\lambda^2 - n^2\lambda^2}{2n\lambda}$$

Then $x_1 = \frac{16\lambda^2 - \lambda^2}{2\lambda} = 7.5\lambda$

$$x_2 = \frac{16\lambda^2 - 4\lambda^2}{4\lambda} = 3\lambda$$

$$x_3 = \frac{16\lambda^2 - 9\lambda^2}{6\lambda} = \frac{7}{6}\lambda$$

$$x_4 = 0$$

So, Number of points for maxima becomes 3.
Hence, the correct answer is (B).

97. If a is the width of the slit, the minimum of Fraunhofer diffraction pattern appears for $a\sin\theta = n\lambda$, where n is an integer. For the first minimum $n = 1$. Then

$$\sin\theta = \frac{\lambda}{a} = 10^{-3} \text{ rad ; } 1 \text{ rad} = 57°.$$

Hence $\theta = 0.057° = 0.057\times60' = 3.42'$.

Hence, the correct answer is (B).

98. Angular limit of resolution of human eye is

$$\frac{1.22\lambda}{a} = R(\text{say})$$

$$\Rightarrow \quad R = \frac{1.22\times5\times10^{-7}}{2\times10^{-3}} \text{rad}$$

$$\Rightarrow \quad R = \frac{1.22\times5\times10^{-7}}{2\times10^{-3}}\times\frac{180}{\pi} \text{ degree}$$

$$\Rightarrow \quad R = 0.0175 \text{ degree}$$

$$\Rightarrow \quad R = 0.0175\times60 \text{ minute}$$

$$\Rightarrow \quad R \simeq 1 \text{ minute}$$

Hence, the correct answer is (B).

99. $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1/I_2}+1}{\sqrt{I_1/I_2}-1}\right)^2 = \frac{9}{1}$

$\Rightarrow \quad \frac{x+1}{x-1} = 3 \qquad \{\because x = \sqrt{I_1/I_2}\}$

$\Rightarrow \quad x = 2$

$\Rightarrow \quad \frac{I_1}{I_2} = 4$

$\Rightarrow \quad I_1 = 4I_2$

i.e., if $I_2 = I_0$ then $I_1 = 4I_0$

$I_0 = 4I_0\cos^2\left(\frac{\phi}{2}\right)$

$\Rightarrow \quad \phi = \frac{2\pi}{3}$

$\Rightarrow \quad \left(\frac{2\pi}{\lambda}\right)\left(\frac{yd}{D}\right) = \frac{2\pi}{3}$

$\Rightarrow \quad y = \frac{\lambda D}{3d}$

Hence, the correct answer is (C).

100. $(n+1)5200 = n(7800)$

$\Rightarrow \quad n = 2$

Hence, the correct answer is (A).

101. Angular width $\beta = \frac{2\lambda}{a}$

$\Rightarrow \quad \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}$

where $\beta_1 = \beta$ and $\beta_2 = \beta - \left(\frac{30}{100}\right)\beta = \left(\frac{70}{100}\right)\beta$

$\Rightarrow \quad \frac{\beta}{\left(\frac{70}{100}\right)\beta} = \frac{6000}{\lambda_2} = 4200$ Å

Hence, the correct answer is (B).

103. Air has a refractive index slightly greater than one. So, when the chamber is evacuated, the refractive index decreases and hence wavelength increases as a result of which fringe width increases slightly.
Hence, the correct answer is (B).

104. Ultraviolet radiations, when fall on a fluorescent material, are converted into visible radiations. Hence, we should use ultraviolet radiation.
Hence, the correct answer is (D).

105. Using the same argument as in the SOLUTION 104.

$\frac{I}{2}\cos^2 30 = \frac{I}{2}\times\frac{3}{4} = \frac{3I}{8} = 0.375I$

Hence, the correct answer is (C).

106. $\lambda_{Red} > \lambda_{Violet}$

and all other colours have wavelength value lying in this region. Further, we have $X \propto \lambda$, so

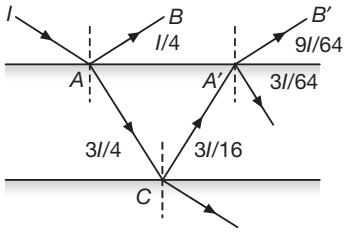
$X(\text{Blue}) < X(\text{Green})$

Hence, the correct answer is (B).

107. In double refraction, light rays always split into two rays (O-ray and E-ray). O-ray has same velocity in all directions but E-ray has different velocity in different directions. For calcite $\mu_E < \mu_O$, so $v_E > v_O$ and for quartz, we have $\mu_E > \mu_O$, so $v_O > v_E$.
Hence, the correct answer is (C).

108. From figure $I_1 = \frac{I}{4}$ and $I_2 = \frac{9I}{64}$

$\Rightarrow \quad \frac{I_2}{I_1} = \frac{9}{16}$



By using $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{\frac{I_2}{I_1}}+1}{\sqrt{\frac{I_2}{I_1}}-1}\right) = \left(\frac{\sqrt{\frac{9}{16}}+1}{\sqrt{\frac{9}{16}}-1}\right) = \frac{49}{1}$

Hence, the correct answer is (D).

109. Since, according to Malus Law

$I = I_0\cos^2\phi$

Hence, the correct answer is (D).

110. Since P is ahead of Q by 90° and path difference between P and Q is $\frac{\lambda}{4}$. Therefore at A, phase difference is zero, so intensity is $4I$. At C it is zero and at B, the phase difference is 90°, so intensity is $2I$.
Hence, the correct answer is (D).

111. P is the position of 11^{th} bright fringe from Q. From central position O, P will be the position of 10^{th} bright fringe.
Path difference between the waves reaching at

$P = S_1B = 10\lambda = 10\times6000\times10^{-10} = 6\times10^{-6}$ m

Hence, the correct answer is (A).

112. Since, $I = I_0 + I_0 + 2I_0\cos\phi$

$\Rightarrow \quad I_{max} = \left(\sqrt{I_0}+\sqrt{I_0}\right)^2 = 4I_0$

CHAPTER 2

Now, $I = 0.75I_{max}$

$\Rightarrow \quad I = 3I_0$

$\Rightarrow \quad 3I_0 = I_0 + I_0 + 2I_0\cos\phi$

$\Rightarrow \quad \cos\phi = \frac{1}{2}$

$\Rightarrow \quad \phi = \frac{\pi}{3}, \left(2\pi - \frac{\pi}{3}\right), \left(2\pi + \frac{\pi}{3}\right), \left(4\pi - \frac{\pi}{3}\right)$

$\Rightarrow \quad \phi = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \ldots\ldots$

$\Rightarrow \quad \left(\frac{2\pi}{\lambda}\right)\Delta x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \ldots\ldots$ ...(1)

where $\Delta x = (\mu - 1)t = \frac{t}{2}$ ...(2)

From Equation (1) and (2), we get

$$\left(\frac{2\pi}{\lambda}\right)\left(\frac{t}{2}\right) = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \ldots\ldots$$

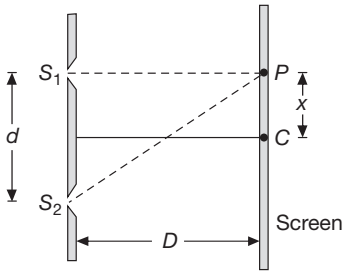
$\Rightarrow \quad t = \frac{\lambda}{3}, \frac{5\lambda}{3}, \frac{7\lambda}{3}, \frac{11\lambda}{3}, \ldots\ldots$

$\Rightarrow \quad t = 0.2\ \mu\text{m}, 1.0\ \mu\text{m}, 1.4\ \mu\text{m}, 2.2\ \mu\text{m}, \ldots$

Hence, the correct answer is (D).

113. Suppose P is a point infront of one slit at which intensity is to be calculated from figure it is clear that $x = \frac{d}{2}$. Path difference between the waves reaching at P

$$\Delta = \frac{xd}{D} = \frac{\left(\frac{d}{2}\right)d}{10d} = \frac{d}{20} = \frac{5\lambda}{20} = \frac{\lambda}{4}$$



Hence corresponding phase difference

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

Resultant intensity at P

$$I = I_{max}\cos^2\frac{\phi}{2} = \phi_0\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$$

Hence, the correct answer is (A).

114. If i is the polarising angle, $n = \tan i$ (Brewester's law). $n = \tan 67°$ is greater than 2. We also know $\frac{1}{\sin C} = n$. $\sin C = \frac{1}{n}$, where C is the critical angle, $\sin C$ is less than 0.5. Hence C is less than $30°$. The only possible answer is $22°$.

Hence, the correct answer is (A).

115. Only transverse waves can be polarised, while the other properties are common for all wave motion.

Hence, the correct answer is (D).

116. In the arrangement shown, the unpolarised light is incident at polarising angle of $90° - 33° = 57°$. The reflected light is thus plane polarised light. When plane polarised light is passed through Nicol prism (a polariser or analyser), the intensity gradually reduces to zero and finally increases.

Hence, the correct answer is (D).

117. If shift is equivalent to n fringes then

$$n = \frac{(\mu - 1)t}{\lambda}$$

$\Rightarrow \quad n \propto t$

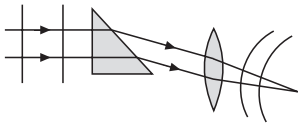
$\Rightarrow \quad \frac{t_2}{t_1} = \frac{n_2}{n_1}$

$\Rightarrow \quad t_2 = \frac{n_2}{n_1} \times t$

$$t_2 = \frac{20}{30} \times 4.8 = 3.2\ \text{mm}$$

Hence, the correct answer is (D).

118.

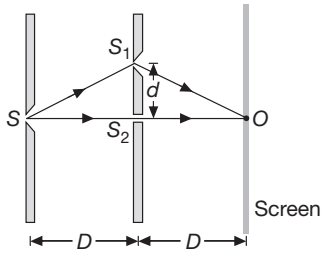


Hence, the correct answer is (B).

119. Path difference between the waves reaching at P,

$$\Delta = \Delta_1 + \Delta_2$$

where Δ_1 = Initial path difference



Δ_2 = Path difference between the waves after emerging from slits.

$$\Delta_1 = SS_1 - SS_2 = \sqrt{D^2 + d^2} - D$$

and $\Delta_2 = S_1O - S_2O = \sqrt{D^2 + d^2} - D$

$$\Rightarrow \quad \Delta = 2\left\{(D^2 + d^2)^{\frac{1}{2}} - D\right\} = 2\left\{\left(D^2 + \frac{d^2}{2D}\right) - D\right\}$$

$$\Rightarrow \quad \Delta = \frac{d^2}{D} \qquad \{\text{From Binomial expansion}\}$$

For obtaining dark at O, Δ must be equals to $(2n-1)\frac{\lambda}{2}$ i.e.,

$$\frac{d^2}{D} = (2n-1)\frac{\lambda}{2}$$

$$\Rightarrow \quad d = \sqrt{\frac{(2n-1)\lambda D}{2}}$$

For minimum distance $n = 1$ so $d = \sqrt{\frac{\lambda D}{2}}$

Hence, the correct answer is (C).

120. For destructive interference, path difference must be an odd multiple of $\frac{\lambda}{2}$.

Hence, the correct answer is (D).

121. These concentric bright and dark fringes are called Newton's Rings.

Hence, the correct answer is (C).

122. $I = I_0 + I_0 + 2I_0\cos(2\pi)$

$$\Rightarrow \quad I = 4I_0$$

For $x = \frac{\lambda}{4}$, $\phi = \frac{2\pi}{\lambda}x = \frac{\pi}{2}$

$$\Rightarrow \quad I' = I_0 + I_0 + 0$$

$$\Rightarrow \quad I' = 2I_0 = \frac{I}{2}$$

Hence, the correct answer is (B).

123. $\beta = \frac{\lambda D}{d}$

$$\Rightarrow \quad \beta \propto D$$

$$\Rightarrow \quad \frac{\beta_1}{\beta_2} = \frac{D_1}{D_2}$$

$$\Rightarrow \quad \frac{\beta_1 - \beta_2}{\beta_2} = \frac{D_1 - D_2}{D_2}$$

$$\Rightarrow \quad \frac{\Delta\beta}{\Delta D} = \frac{\beta_2}{D_2} = \frac{\lambda_2}{d_2}$$

$$= \lambda_2 = \frac{3\times10^{-5}}{5\times10^{-2}} \times 10^{-3} = 6\times10^{-7}\,\text{m} = 6000\ \text{Å}$$

Hence, the correct answer is (A).

124. Since, we have been given that, $\theta = a + \frac{\beta}{\lambda^2}$

$$\Rightarrow \quad 30 = a + \frac{b}{(5000)^2} \text{ and } 50 = a + \frac{b}{(4000)^2}$$

Solving for a, we get $a = \frac{50^\circ}{9}\ (\text{mm})^{-1}$

Hence, the correct answer is (B).

125. $[(n-1)t]\frac{D}{d} = \frac{\lambda(2D)}{d}$

$$\Rightarrow \quad \lambda = \frac{(n-1)t}{2} = 5892\ \text{Å}$$

Hence, the correct answer is (C).

127. $x = \frac{\lambda L}{d}$

$$\Rightarrow \quad \lambda = \frac{xd}{L}$$

Hence, the correct answer is (A).

128. Interference is a physical effect of superposition of wave motion. This can happen for any kind of wave motion, i.e., light waves, sound waves, matter waves, etc.

Hence, the correct answer is (C).

129. The extra path difference produced by the glass plate of thickness t and refractive index μ is $(\mu-1)t$. Due to this if n is the number of fringes which shift, $(\mu-1)t = n\lambda$. $t = 0.01\times10^{-3}$ m, $\lambda = 6000\times10^{-10}$ m, $\mu = 1.5$. This gives a value for $n = 8$.

Hence, the correct answer is (B).

131. $(\mu-1)t = n\lambda$

$$\Rightarrow \quad n = \frac{(\mu-1)t}{\lambda} = \frac{\left(\frac{5}{3}-1\right)t}{\lambda} = \frac{2t}{3\lambda}$$

Displacement of fringe system is $\Delta y = n\left(\frac{D\lambda}{2d}\right)$

$$\Rightarrow \quad \Delta y = \left(\frac{2t}{3\lambda}\right)\left(\frac{D\lambda}{2d}\right) = \frac{Dt}{3d}$$

Hence, the correct answer is (A).

133. $I = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$

Put $a_1^2 + a_2^2 = A$ and $a_1a_2 = B$

$$\Rightarrow \quad I = A + B\cos\phi$$

Hence, the correct answer is (D).

CHAPTER 2

134. $\Delta\lambda = \lambda\frac{v}{c}$ and $v = r\omega$

$$v = 7\times10^{8}\times\frac{2\pi}{25\times24\times3600},\ c = 3\times10^{8}\ \text{ms}^{-1}$$

$$\Rightarrow\quad \Delta\lambda = 0.04\ \text{Å}$$

Hence, the correct answer is (A).

135. Shift $= \frac{\beta}{\lambda}(\mu-1)t$

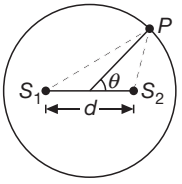
$$\Rightarrow\quad 7\beta = \frac{\beta}{\lambda}(\mu-1)t$$

$$\Rightarrow\quad t = \frac{7\lambda}{(\mu-1)} = \frac{7\times600}{(1.5-1)} = 8400\ \text{nm}$$

Hence, the correct answer is (C).

136. Here path difference at a point P on the circle is given by

$$\Delta x = d\cos\theta \qquad \ldots(1)$$



For maxima at P

$$\Delta x = n\lambda \qquad \ldots(2)$$

From equation (1) and (2)

$$n\lambda = d\cos\theta$$

$$\Rightarrow\quad \theta\cos^{-1}\left(\frac{n\lambda}{d}\right) = \cos^{-1}\left(\frac{4\lambda}{d}\right)$$

Hence, the correct answer is (B).

137. $S_1 = \frac{\Delta_1 D}{d} = 11\times10^{-3}$ and $S_2 = \frac{\Delta_2 D}{d} = 12\times10^{-3}$

$$\Rightarrow\quad \frac{\Delta_1}{\Delta_2} = \frac{11}{12}$$

$$\Rightarrow\quad 12\Delta_1 = 11\Delta_2$$

Hence, the correct answer is (B).

138. The film appears bright when the path difference $(2\mu t\cos r)$ is equal to odd multiple of $\frac{\lambda}{2}$

i.e., $2\mu t\cos r = \frac{(2n-1)\lambda}{2}$ where $n = 1, 2, 3$ .....

$$\Rightarrow\quad \lambda = \frac{4\mu t\cos r}{(2n-1)}$$

$$\Rightarrow\quad \lambda = \frac{4\times1.4\times10,000\times10^{-10}\times\cos0}{(2n-1)} = \frac{56000}{(2n-1)}\ \text{Å}$$

$$\Rightarrow\quad \lambda = 56000\ \text{Å},\ 18666\ \text{Å},\ 8000\ \text{Å},\ 6222\ \text{Å},\ 5091\ \text{Å},\ 4308\ \text{Å},\ 3733\ \text{Å}.$$

The wavelength which are not within specified range are to be refracted.

Hence, the correct answer is (A).

139. $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

$$\Rightarrow\quad \frac{(401.8-393.3)}{393.3} = \frac{v}{3\times10^{8}}$$

$$\Rightarrow\quad v = 6.48\times10^{6}\ \text{ms}^{-1} = 6480\ \text{kms}^{-1}$$

Hence, the correct answer is (A).

140. Intensity of each source is $I_0 = \frac{I_{\max}}{4} = \frac{100}{4} = 25$ unit

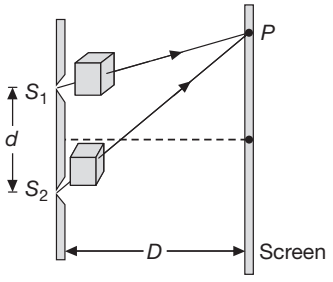
If the intensity of one of the sources is reduced by 36%, then $I_1 = 25$ unit and $I_2 = 25 - 25\times\frac{36}{100} = 16(\text{unit})$

Hence resultant intensity at the same point will now be

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} = 25 + 16 + 2\sqrt{25\times16} = 81\ \text{unit}$$

Hence, the correct answer is (D).

141. Shift $\Delta x = \frac{\beta}{\lambda}(\mu-1)t$



Shift due to one plate $\Delta x_1 = \frac{\beta}{\lambda}(\mu_1-1)$

Shift due to another path $\Delta x_2 = \frac{\beta}{\lambda}(\mu_2-1)t$

Net shift $\Delta x = \Delta x_2 - \Delta x_1 = \frac{\beta}{\lambda}(\mu_2-\mu_1)t$ $\ldots(1)$

Also it is given that $\Delta x = 5\beta$ $\ldots(2)$

Hence $5\beta = \frac{\beta}{\lambda}(\mu_1-\mu_2)t$

$$\Rightarrow\quad t = \frac{5\lambda}{(\mu_2-\mu_1)} = \frac{5\times4800\times10^{-10}}{(1.7-1.4)} = 8\times10^{-6}\ \text{m} = 8\ \mu\text{m}$$

Hence, the correct answer is (A).

143. At the polarising angle, the reflected ray is fully polarised while the transmitted ray is partially polarised. In fact a method to produce plane polarised light is by reflection at the polarising angle.
Hence, the correct answer is (B).

145. $a = \dfrac{\lambda}{\theta}$

$\Rightarrow \quad a = \dfrac{6500 \times 10^{-10}}{\left(\dfrac{\pi}{6}\right)}$

$\Rightarrow \quad a = 1.24 \times 10^{-6}$ m

$\Rightarrow \quad a = 1.24\ \mu$m

Hence, the correct answer is (B).

146. At the central maxima $\Delta x = 0$

$\Rightarrow \quad (2\mu - 1)t - (\mu - 1)(2t) - \dfrac{yd}{D} = 0$

$\Rightarrow \quad t - \dfrac{yd}{D} = 0$

$\Rightarrow \quad y = \dfrac{tD}{d}$

Hence, the correct answer is (B).

148. Using Malus Law, $I = I_0 \cos^2\theta$
As here polariser is rotating i.e., all the values of θ are possible.

$$I_{av} = \frac{1}{2\pi}\int_0^{2\pi} I\,d\theta = \frac{1}{2\pi} = \int_0^{2\pi} I_0 \cos^2\theta\,d\theta$$

On integration we get $I_{av} = \dfrac{I_0}{2}$

where $I_0 = \dfrac{\text{Energy}}{\text{Area} \times \text{Time}} = \dfrac{p}{A} = \dfrac{10^{-3}}{3 \times 10^{-4}} = \dfrac{10}{3}\dfrac{\text{Watt}}{\text{m}^2}$

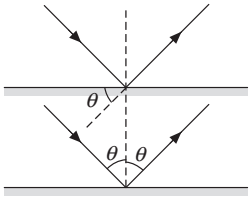
$\Rightarrow \quad I_{av} = \dfrac{1}{2} \times \dfrac{10}{3} = \dfrac{5}{3}$ Watt

and Time period $T = \dfrac{2\pi}{\omega} = \dfrac{2 \times 3.14}{31.4} = \dfrac{1}{5}$ sec

So, energy of light passing through the polariser per revolution $= I_{av} \times \text{Area} \times T = \dfrac{5}{3} \times 3 \times 10^{-4} \times \dfrac{1}{5} = 10^{-4}$ J

Hence, the correct answer is (A).

149.



Path difference $= 2d\sin\theta$
For constructive interference

$2d\sin\theta = n\lambda$

$\Rightarrow \quad \theta = \sin^{-1}\left(\dfrac{n\lambda}{2d}\right)$

Hence, the correct answer is (C).

151. During sunset and sunrise we see the unscattered part of the sunlight. Sun's rays have to travel long distance and hence are incident obliquely. All shorter wavelengths are scattered by Rayleigh's law for scattered intensity $I \propto \dfrac{1}{\lambda^4}$. Only the red-orange rays reach us. During noon the sun's rays are normally incident on the earth. Hence a portion of both the shorter wavelengths and longer wavelengths reach us. The sun looks white.
Hence, the correct answer is (D).

152. $y_3 - y_1 = \dfrac{3\lambda x}{a} - \dfrac{\lambda x}{a}$

$\Rightarrow \quad 3 \times 10^{-3} = \dfrac{2 \times 600 \times 10^{-9} \times 0.5}{a}$

$\Rightarrow \quad a = 0.2$ mm

Hence, the correct answer is (D).

153. $\beta = \dfrac{\lambda D}{d}$

So, angular width, 2θ is

$2\theta = \dfrac{2\beta}{D} = \dfrac{2\lambda}{d}$

$\theta = \dfrac{6328 \times 10^{-10}}{0.2 \times 10^{-3}}$

$\Rightarrow \quad \theta = 3164 \times 10^{-6}$ radian

$\Rightarrow \quad \theta = 3164 \times 10^{-6} \times \dfrac{180}{\pi}$ degree

$\Rightarrow \quad \theta = 0.18$ degree

$\Rightarrow \quad 2\theta = 0.36$ degree

Hence, the correct answer is (D).

154. For maxima $2\pi n = \dfrac{2\pi}{\lambda}(XO) - 2\pi\ell$

$\Rightarrow \quad \dfrac{2\pi}{\lambda}(XO) = 2\pi(n + \ell)$

$\Rightarrow \quad (XO) = \lambda(n + \ell)$

Hence, the correct answer is (A).

CHAPTER 2

155. From the given data, note that the fringe width (β_1) for $\lambda_1 = 900$ nm is greater than fringe width (β_2) for $\lambda_2 = 750$ nm. This means that at though the central maxima of the two coincide, but first maximum for $\lambda_1 = 900$ nm will be further away from the first maxima for $\lambda_2 = 750$ nm and so on. A stage may come when this mismatch equals β_2, then again maxima of $\lambda_1 = 900$ nm, will coincide with a maxima of $\lambda_2 = 750$ nm, let this correspond to n^{th} order fringe for λ_1. Then it will correspond to $(n+1)^{\text{th}}$ order fringe for λ_2.

Therefore $\frac{n\lambda_1 D}{d} = \frac{(n+1)\lambda_2 D}{d}$

$\Rightarrow \quad n \times 900 \times 10^{-9} = (n+1)750 \times 10^{-9}$

$\Rightarrow \quad n = 5$

Minimum distance y from central maxima is

$$y = \frac{n\lambda_1 D}{d} = \frac{5 \times 900 \times 10^{-9} \times 2}{2 \times 10^{-3}}$$

$\Rightarrow \quad y = 45 \times 10^{-4}\text{m} = 4.5\text{ mm}$

Hence, the correct answer is (C).

156. One of the essential condition to observe diffraction is that the dimension of the object should be of the order of wavelength of the wave. The velocity of sound in air as nearly $350\ \text{ms}^{-1}$. Hence the wavelength of sound waves produced by a tuning fork of frequency 384 Hz is $\lambda = \frac{V}{f} = 1$ m nearly. Hence the diameter of the sphere should be of the same order.
Hence, the correct answer is (C).

157. Distance between the first dark fringes is

$$\Delta x = \frac{2\lambda D}{d} = 2.4\text{ mm}$$

Hence, the correct answer is (D).

158. The action of a nicol prism is based on double refraction and dichroism. When a ray of light enters into a calcite crystal, it is split into two rays O and E. Both are plane polarised in mutually perpendicular planes. One of them suffers total reflection and absorption while the other comes out as plane polarised. Selective absorption is called dichroism.
Hence, the correct answer is (D).

159. The line joining sources is parallel to screen. Hence, shape of fringes on screen will be circular.
Hence, the correct answer is (B).

160. Shift, $\Delta x = \left[(\mu - 1)t\right]\frac{D}{d}$

Here $\Delta x = 5\beta$

$\Rightarrow \quad 5\left(\frac{\lambda D}{d}\right) = \left[(\mu-1)t\right]\frac{D}{d}$

$\Rightarrow \quad 5\lambda = (\mu - 1)t$

$\Rightarrow \quad \lambda = (\mu - 1)\frac{t}{5}$

$\Rightarrow \quad \lambda = \frac{(0.5)(6 \times 10^{-6})}{5}$

$\Rightarrow \quad \lambda = 6 \times 10^{-7}$ m

$\Rightarrow \quad \lambda = 6000$ Å

Hence, the correct answer is (B).

161. Only transverse waves can be polarised. (A) and (C) and (D) are electromagnetic radiations, which are transverse waves while (B), that is, β rays are electrons.
Hence, the correct answer is (B).

166. $\beta = \frac{\lambda D}{d}$

$\Rightarrow \quad \frac{6}{1000} = \frac{\lambda(1.2)}{\left(\frac{0.1}{1000}\right)}$

$\Rightarrow \quad \lambda = 5000$ Å

Hence, the correct answer is (B).

170. Using the equation $\beta = \frac{D\lambda}{d}$, $\beta \propto \lambda$. Of the given colours, yellow has the maximum wavelength and hence the maximum fringe width.
Hence, the correct answer is (C).

172. The tubelight has coating of fluorescent material which converts ultraviolet light into visible light. Hence the light is more intense.
Hence, the correct answer is (B).

173. We take one wavelength in the visible region and calculate the energy just to know its order. Let us take 6000 Å (red) for convenience of a rough calculation. Energy $E = \frac{hc}{\lambda}$. Taking $h = 6.6 \times 10^{-34}$, $c = 3 \times 10^8$, $\lambda = 6000 \times 10^{-10}$ m we get $\frac{hc}{\lambda}$ in joules. Dividing this by 1.6×10^{-19} to convert to eV, we get $E = 2\ eV$. For energy 1 eV, wavelength will be 12000 Å. Energy 5 eV means wavelength $\frac{2}{5}$ of 6000, that is 2400 Å. Both these are invisible, recalling that the visible range is 4000 to 7000 Å.
Hence, the correct answer is (B).

174. $\Delta x = (SA + AP) - SP$

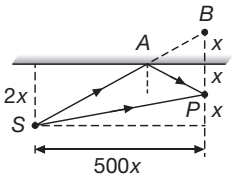
$\Delta x = (SA + AB) - SP = SB - SP$

$\Rightarrow \quad \Delta x = \sqrt{(500x)^2 + (3x)^2} - \sqrt{(500x)^2 + x^2}$

$\Rightarrow \quad \Delta x = 500x\left(\sqrt{1 + \frac{9}{500^2}} - \sqrt{1 + \frac{1}{500^2}}\right)$

$\Rightarrow \quad \Delta x = 500x\left(\left(1 + \frac{9}{2\times 500^2}\right) - \left(1 + \frac{1}{2\times 500^2}\right)\right)$

$\Rightarrow \quad \Delta x = \frac{(500)(8)x}{2(500)^2}$



Since reflection takes place at A i.e. denser medium, so for maxima, we have $\Delta x = (2n+1)\frac{\lambda}{2}$

$\Rightarrow \quad \Delta x = \frac{500x}{2(500)^2}(8) = \frac{\lambda}{2}$

$\Rightarrow \quad x = \frac{500}{8}\lambda = 62.5\lambda$

Hence, the correct answer is (C).

175. The torch produces light waves which are non-coherent in nature.
Hence, the correct answer is (C).

176. For unpolarised light

$I' = \frac{I_0}{2}\cos^2\phi$

$\Rightarrow \quad I' = \frac{I}{2}\cos^2\left(\frac{\pi}{4}\right)$

$\Rightarrow \quad I' = \frac{I}{4}$

Hence, the correct answer is (D).

178. The path difference of two waves producing destructive interference or darkness is $(2n+1)\left(\frac{\lambda}{2}\right)$ where n is an integer 0, 1, 2, 3, etc. A path difference of λ corresponds to a phase difference of 2π rad and a path difference of $\frac{\lambda}{2}$ corresponds to a phase difference of π. Hence the corresponding phase difference is $(2n+1)\pi$.
Hence, the correct answer is (C).

179. $\beta' = \frac{\beta}{\mu} < \beta$

Hence, the correct answer is (B).

180. $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{9}{4}$

$\Rightarrow \quad \frac{a_1}{a_2} = \frac{3}{2}$

Since, $\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{3+2}{3-2}\right)^2$

$\Rightarrow \quad \frac{I_{max}}{I_{min}} = \frac{25}{1}$

Hence, the correct answer is (A).

181. Position of first minima is same as the position of third maxima.

$\frac{1\times\lambda_1 D}{a} = \frac{(2\times 3+1)}{2}\frac{\lambda_2 D}{a}$

$\Rightarrow \quad \lambda_1 = 3.5\lambda_2$

Hence, the correct answer is (C).

182. Using the equation $\beta = \frac{D\lambda}{d}$; $\beta \propto \lambda$

Hence, the correct answer is (A).

185. In a biprism, the distance between sources is due to deviation produced by two small angled prisms. The deviation produced by each prism is $(\mu - 1)\alpha$. The total deviation is $2(\mu - 1)\alpha$. Separation of sources $2(\mu - 1)\alpha a$.

Hence, the correct answer is (B).

187. Shift $= \frac{\beta}{\lambda}(\mu - 1)t = \frac{\beta}{(5000\times 10^{-10})}(1.5)\times 2\times 10^{-6} = 2\beta$

i.e., 2 fringes upwards.
Hence, the correct answer is (A).

188. Separation n^{th} bright fringe and central maxima is $x_n = \frac{n\lambda D}{d}$

So, $x_3 = \frac{3\times 6000\times 10^{-10}\times 1}{0.5\times 10^{-3}} = 3.5$ mm.

Hence, the correct answer is (B).

189. $\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = 9$

Hence, the correct answer is (C).

190. For second dark fringe, $n = 1$

Since, $x = (2n+1)\frac{\lambda D}{2d}$

CHAPTER 2

$$\Rightarrow \quad 3 = \frac{3\lambda D}{2d} \qquad \ldots(1)$$

For fourth bright fringe, $n = 4$. Since

$$x = n\left(\frac{\lambda D}{d}\right)$$

$$\Rightarrow \quad x = 4\left(\frac{\lambda D}{d}\right) \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{x}{3} = \frac{4}{(3/2)}$$

$$\Rightarrow \quad x = 8 \text{ mm}$$

Hence, the correct answer is (B).

191. For same field of view

$$n_1\beta_1 = n_2\beta_2$$

$$\Rightarrow \quad n_1\left(\frac{\lambda_1 D}{d}\right) = n_2\left(\frac{\lambda_2 D}{d}\right)$$

$$\Rightarrow \quad n_1\lambda_1 = n_2\lambda_2$$

$$\Rightarrow \quad 60(4000) = n_2(6000)$$

$$\Rightarrow \quad n_2 = 40$$

Hence, the correct answer is (A).

192. $\beta' = \frac{\beta}{\mu} = 1$ mm

Hence, the correct answer is (B).

193. $\frac{a_1}{a_2} = \frac{5}{1}$

$$\Rightarrow \quad \frac{I_{\max}}{I_{\min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

$$\Rightarrow \quad \frac{I_{\max}}{I_{\min}} = \frac{36}{16} = \frac{9}{4}$$

Hence, the correct answer is (C).

194. $I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$$I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

For contrast to be good $I_1 = I_2$ and hence

$$I_{\max} = 4I_1$$

$$I_{\min} = 0$$

Hence, the correct answer is (D).

Multiple Correct Choice Type Questions

1. Shift due to the first plate is $x_1 = \frac{\beta}{\lambda}(\mu_1 - 1)t$

Shift due to the first plate is $x_1 = \frac{\beta}{\lambda}(\mu_1 - 1)t$

Net shift is towards the plate with higher μ, so

$$\Delta x = 3\beta = \frac{\beta}{\lambda}(\mu_2 - \mu_1)t$$

$$\Rightarrow \quad t = \frac{3\lambda}{\mu_2 - \mu_1} = \frac{3(4000 \times 10^{-10})}{1.7 - 1.4} = 4\ \mu\text{m}$$

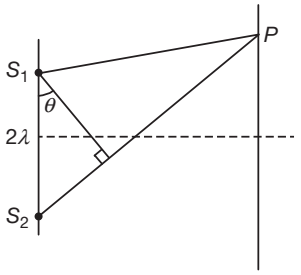
Hence, (A) and (C) are correct.

2. $AC = vt$, $BD = ct$, $\mu = \frac{c}{v} = \frac{ct}{vt} = \frac{BD}{AC}$

$$\mu = \frac{\sin\phi}{\sin\phi'} = \frac{\cos\theta}{\cos\theta'}$$

Hence, (A), (C) and (D) are correct.

3. The phase difference corresponding to $OS_1 - OS_2 = \frac{\lambda}{4}$ is $\frac{\pi}{2}$.



Hence, the net phase difference is given by

$$\phi = \frac{\pi}{2} - \frac{2\pi}{\lambda}(2\lambda\sin\theta)$$

$$\Rightarrow \quad \phi = \frac{\pi}{2} - 4\pi\sin\theta$$

For maxima, we have $\phi = n\pi$, where $n = 0, \pm 1, \pm 2 \ldots$

$$\Rightarrow \quad \frac{\pi}{2} - 4\pi\sin\theta = n\pi$$

$$\Rightarrow \quad \sin\theta = \frac{\frac{1}{2} - n}{4}$$

For, $n = 0$, $\sin\theta = \frac{1}{8}$

$n = \pm 1$, $\sin\theta = -\frac{1}{8}, +\frac{3}{8}$

$n = \pm 2$, $\sin\theta = -\frac{3}{8}, \frac{5}{8}$

$n = \pm 3$, $\sin\theta = -\frac{5}{8}, \frac{7}{8}$

$n = 4$, $\sin\theta = -\frac{7}{8}$

Hence, (A) and (D) are correct.

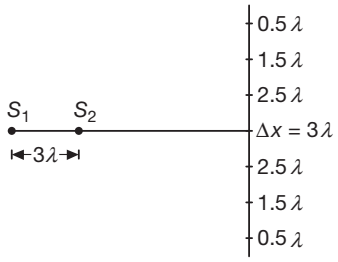
6. Path difference at O is d

if $d = \lambda, 2\lambda, 3\lambda, \ldots$,, then O is a maxima.

if, $d = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$ then O is a minima

The path difference on screen is > 0

For $d = 3\lambda$, we have



Hence, (A), (B) and (C) are correct.

10. $I_1 = I_2 = I$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(\sqrt{I} + \sqrt{\frac{I}{2}}\right)^2 < 4I$$

$$I_{min} = \left(\sqrt{I} - \sqrt{\frac{I}{2}}\right)^2 > 0$$

Hence, (A), (C) and (D) are correct.

11. Path difference $= \sqrt{D^2 + d^2} - D = 1$ cm

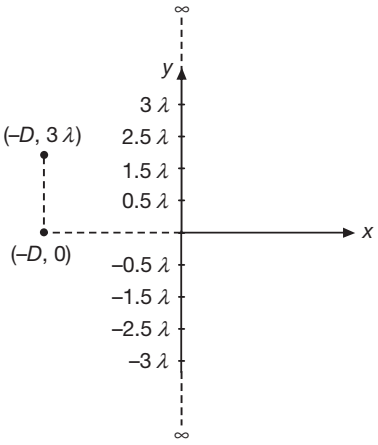
Also, $\left(\sqrt{D^2 + d^2} - D\right) = (2n-1)\frac{\lambda}{2}$

$$\Rightarrow \quad \lambda = \frac{2(1)}{2n-1}$$

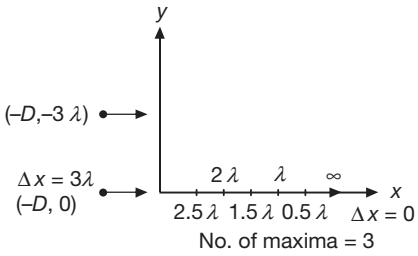
$$\Rightarrow \quad \lambda = 2 \text{ cm}, \frac{2}{3} \text{ cm}, \frac{2}{5} \text{ cm} \ldots\ldots\ldots$$

Hence, (A) and (C) are correct.

12.



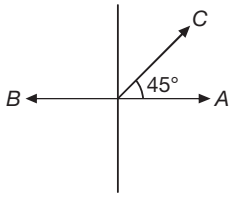
Δx at ∞ on y-axis is 3λ and -3λ there will be 6 minima and 5 maxima



Hence, (A), (B) and (C) are correct.

13. $\sqrt{D^2 + 4d^2} - D = \frac{\lambda}{2}$

gives, $\lambda = \frac{4d^2}{D}$



Resultant intensity at P is due to C only i.e. $I_R = I_0$

Hence, (B) and (D) are correct.

14. Direction of propagation of light is always perpendicular to the wave front, which is best satisfied by $x = c$

Hence, the correct answer is (A).

16. $\beta = \frac{\lambda D}{d}$

For increasing β, d must be decreased and λ must be increased (i.e. frequency must be decreased).

Hence, (A) and (B) are correct.

Reasoning Based Questions

2. Fringe width $\beta = \frac{\lambda D}{d}$ shall remain the same as the waves travel in air only, after passing through the thin transparent sheet. Due to introduction of thin sheet, only path difference of the wave is changed due to which there is shift of position of fringes only, which is given as $\Delta x = \frac{D(n-1)t}{d}$, where n is refractive index of thin sheet and t is its thickness.

Hence, the correct answer is (D).

3. If both the slits are illuminated by two bulbs of same power, no interference pattern will be observed on the screen. This is because waves reaching at any point on the screen do not have a constant phase difference.

CHAPTER 2

as phase difference from two non coherent sources changes randomly. Therefore, maxima and minima would also change their positions randomly and in quick succession. This will result in general illumination of the screen.
Hence, the correct answer is (D).

5. $\Delta x = d = n\lambda$, for $n = 1$, $d = \lambda$ and here we will have three maxima.
Hence, the correct answer is (D).

9. Statement-1 is false but Statement-2 is true.
Hence, the correct answer is (D).

14. Statement–1 is false, Statement–2 is true.
Hence, the correct answer is (D).

15. Statement-1 is true & Statement-2 is true and Statement-2 is correct explanation of the Statement-1.
Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. It will be shifted upward
Hence, the correct answer is (A).

2. Cannot be predicted without knowing the thickness of thin plate.
Hence, the correct answer is (D).

3. $(\mu_1 - \mu_2)t = \Delta x = 0.3t$
Hence, the correct answer is (B).

4. $(\mu_1 - \mu_2)t = \Delta x = \dfrac{\lambda}{2}$

$\Rightarrow \quad t = \dfrac{\lambda}{2(0.3)} = \dfrac{\lambda}{0.6}$

Hence, the correct answer is (A).

5. $\dfrac{3}{4}I_0 = I_0 \cos^2\dfrac{\phi}{2} = \dfrac{\sqrt{3}}{2}$

$\phi = \dfrac{\pi}{3}$

$\Rightarrow \quad \Delta x = \dfrac{\Delta\phi}{2\pi}\lambda = \dfrac{\lambda}{6}$

$(\mu_1 - \mu_2)t = \dfrac{\lambda}{6}t = \dfrac{\lambda}{(6)(0.3)} = \dfrac{5000\ \text{Å}}{1.8} = 2777.7\ \text{Å}$

Hence, the correct answer is (A).

6. Optical Path Difference $(OPD) = 2n_1 t$
Hence, the correct answer is (A).

7. For destructive interference, we have

$2n_1 t_{\min} = \dfrac{\lambda}{2} \qquad \{\because\ n_1 < n_2\}$

$\Rightarrow \quad t_{\min} = \dfrac{550}{4 \times 1.38} = 99.64\ \text{nm}$

Hence, the correct answer is (C).

8. $2n_1 t = \dfrac{3\lambda}{2}$

$\Rightarrow \quad t = 3t_{\min} \cong 298.9\ \text{nm}$

Hence, the correct answer is (A).

9. Power received at A,

$P_A = \dfrac{10}{\pi}\pi(0.001)^2 = 10^{-5}\ \text{W}$

Hence, the correct answer is (A).

10. Power received at B,

$P_B = \dfrac{10}{\pi}\pi(0.002)^2 = 4 \times 10^{-5}\ \text{W}$

Hence, the correct answer is (B).

11. Power transmitted through A,

$P'_A = \dfrac{10}{100} \times 10^{-5}\text{W} = 10^{-6}\ W$

Hence, the correct answer is (C).

12. Power transmitted through B,

$P'_B = \dfrac{10}{100} \times 4 \times 10^{-5}\ W = 4 \times 10^{-6}\ W$

Hence, the correct answer is (D).

13. Path difference,

$\Delta = (\mu - 1)t = (1.5 - 1) \times 2000 \times 10^{-10}\ \text{m}$

$\Delta = 10^{-7}\ \text{m}$

Hence, the correct answer is (C).

14. Phase difference, $\phi = \dfrac{2\pi}{\lambda}\Delta = \dfrac{2\pi \times 10^{-7}}{6000 \times 10^{-10}}$

$\Rightarrow \quad \phi = \dfrac{\pi}{3}$ radian

Hence, the correct answer is (C).

15. We know that, $P \propto a^2$ or $P = ka^2$, where k is any arbitrary positive constant.

Now, $a_A = \sqrt{\dfrac{10^{-6}}{k}}$, $a_B = \sqrt{\dfrac{4 \times 10^{-6}}{k}}$

Resultant amplitude,

$a_r = \sqrt{a_A^2 + a_B^2 + 2a_A a_B \cos\phi}$

Substituting values and simplifying,

$a_r^2 = \dfrac{7 \times 10^{-6}}{k}$

Resultant power, $P_r = ka_r^2 = 7\times10^{-6}$ W $= 7\ \mu$W

Hence, the correct answer is (D).

16. Anywhere on screen because there is no relation between θ, μ.

Hence, the correct answer is (D).

17. Total path difference is given by

$$\Delta x = (\mu-1)t - d\sin\theta$$

For central bright fringe to be obtained at O, we have

$$\Delta x = 0$$

$$\Rightarrow\quad (\mu-1)t = d\sin\theta$$

Hence, the correct answer is (A).

18. $\Delta\phi = (2n-1)\pi$, where $n = 5$

Hence, the correct answer is (B).

19. Fringe width $\beta = \dfrac{\lambda D}{d} = 4\times10^5$ nm

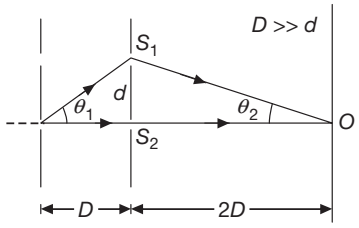
Hence, the correct answer is (B).

20. Since, $\Delta x = d\sin\theta - (\mu-1)t$, so if θ increases then $d\sin\theta$ will increase and hence Δx increases, so that the central maxima will shift upward.

Hence, the correct answer is (B).

21. Net path difference is given by

$$d\sin\theta_1 + d\sin\theta_2 = \frac{\lambda}{2} \qquad \{\text{for minimum } d\}$$



$$\Rightarrow\quad d\left(\frac{d}{D}\right) + d\left(\frac{d}{2D}\right) = \frac{\lambda}{2}\ (\sin\theta \approx \tan\theta)$$

$$\Rightarrow\quad \frac{3}{2}\frac{d^2}{D} = \frac{\lambda}{2}$$

$$\Rightarrow\quad d = \sqrt{\frac{\lambda D}{3}}$$

Hence, the correct answer is (B).

22. For central maxima, we have

$$d\sin\theta_1 + d\sin\theta_2 = 0$$

$$\Rightarrow\quad \frac{d}{D} + d\frac{y}{2D} = 0$$

$$\Rightarrow\quad y = -2d$$

So, the position of first bright fringe is given by

$$y = -2d + \frac{d}{2} = -\frac{3}{2}d$$

So, the first bright fringe is located $\dfrac{3}{2}d$ below O

Hence, the correct answer is (C).

23. $d\sin\theta_1 + d\sin\theta_2 = n\lambda$

$$\Rightarrow\quad \frac{d^2}{D} + \frac{nyd}{2D} = n\lambda \qquad \ldots(1)$$

Also, $\dfrac{d^2}{D} + \dfrac{nyd+1}{2D} = (n+1)\lambda \qquad \ldots(2)$

(2) – (1) we get $\dfrac{d}{2D}\beta = \lambda$

$$\Rightarrow\quad \beta = \frac{2D}{d}\lambda$$

Hence, the correct answer is (D).

24. In reflected light system, for constructive interference, we have

$$2\mu t = (2n-1)\frac{\lambda}{2}$$

For minimum thickness, we have

$$n = 1$$

$$\Rightarrow\quad 2\mu t = \frac{\lambda}{2}$$

$$\Rightarrow\quad t = \frac{900}{4\times1.5}$$

$$\Rightarrow\quad t = 150 \text{ nm}$$

Hence, the correct answer is (B).

25. In transmitted light system, for destructive interference, we have

$$2\mu t = (2n-1)\frac{\lambda}{2}$$

Again, for minimum thickness, we have

$$n = 1$$

$$\Rightarrow\quad t = \frac{900}{4\times1.5} = 150 \text{ nm}$$

Hence, the correct answer is (A).

26. Path difference between the waves reaching D is

$$\Delta x = \frac{3\pi R}{2} - \frac{\pi R}{2} = \frac{2\pi R}{2} = \pi R$$

Since, $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$$\Rightarrow\quad I_R = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \text{ when } \phi = 2n\pi,\ n = 0, 1, 2, 3,\ldots.$$

Since the wave generated is divided equally in two parts, so we have

CHAPTER 2

$$I_1 = I_2 = \frac{I_0}{2}$$

$$\Rightarrow \quad I_R = \left(\sqrt{\frac{I_0}{2}} + \sqrt{\frac{I_0}{2}}\right)^2$$

$$\Rightarrow \quad I_R = 2I_0$$

Hence, the correct answer is (C).

27. Since $\Delta x = \pi R$

For maxima, we have

$$\Delta\phi = (2n)\pi \text{ , where } n = 0, 1, 2, 3, \ldots.$$

$$\Rightarrow \quad \left(\frac{2\pi}{\lambda}\right)(\pi R) = (2n)\pi$$

$$\Rightarrow \quad \lambda_{\max} = \frac{\pi R}{n_{\min}} = \frac{\pi R}{1}$$

$$\Rightarrow \quad \lambda_{\max} = \pi R$$

Hence, the correct answer is (A).

28. Similarly, for minima, we have

$$\Delta\phi = (2n+1)\pi \text{ , where } n = 0, 1, 2, 3, \ldots.$$

$$\Rightarrow \quad \left(\frac{2\pi}{\lambda}\right)(\pi R) = (2n+1)\pi$$

$$\Rightarrow \quad \lambda_{\max} = \frac{2\pi R}{2n+1}$$

$$\Rightarrow \quad \lambda_{\max} = 2\pi R$$

Hence, the correct answer is (B).

29. Since, $d_{air} = 2a(\mu - 1)\alpha$

and $d_{\text{water}} = 2a\left(\frac{\mu}{\mu_w} - 1\right)\alpha$

$$\Rightarrow \quad \frac{d_{\text{water}}}{d_{\text{air}}} = \frac{\frac{\mu}{\mu_w} - 1}{\mu_w - 1}$$

$$\Rightarrow \quad \frac{d_{\text{water}}}{0.5} = \frac{\frac{3/2}{4/3} - 1}{\frac{3}{2} - 1} = \frac{1}{4}$$

$$\Rightarrow \quad d_{\text{water}} = \frac{1}{4} \times 0.5 = \frac{1}{8} \text{ mm}$$

Hence, the correct answer is (D).

30. Since, $\beta_{\text{air}} = \frac{D\lambda_{\text{air}}}{2a(\mu - 1)\alpha}$

and $\beta_{\text{water}} = \frac{D\lambda_{\text{air}}}{2a(\mu - \mu_w)\alpha}$

$$\Rightarrow \quad \frac{\beta_{\text{water}}}{\beta_{\text{air}}} = \frac{\mu - 1}{\mu - \mu_w} = \frac{\frac{3}{2} - 1}{\frac{3}{2} - \frac{4}{3}}$$

$$\Rightarrow \quad \beta_{\text{water}} = 3 \times 1 = 3 \text{ mm}$$

Hence, the correct answer is (C).

31. Since, $\beta = \frac{\lambda D}{d}$

$$\Rightarrow \quad 1 \times 10^{-3} = \frac{1 \times \lambda}{0.5 \times 10^{-3}}$$

$$\Rightarrow \quad \lambda = 5 \times 10^{-7} \text{ m} = 5000 \text{ Å}$$

Hence, the correct answer is (C).

32. At O, optical path difference is

$$\Delta x = (\mu_g - 1)t - \mu_w d \sin\theta$$

$$\Rightarrow \quad \Delta x = 0.5 \times 0.41 \times 10^{-3} - \frac{4}{3} \times 3 \times 10^{-4} \times \frac{1}{2}$$

$$\Rightarrow \quad \Delta x = 5 \times 10^{-6} \text{ m}$$

Since, $\lambda = 5000$ Å, so we observe that

$$\Delta x = 10\lambda$$

So, 10^{th} bright fringe, will be formed at 0.

Hence, the correct answer is (D).

33. For central maxima, we have

$$\Delta x = 0$$

where, $\Delta x = \frac{yd}{D} + (\mu_g - 1)t - \mu_w d \sin\theta$

$$\Rightarrow \quad \frac{yd}{D} + (\mu_g - 1)t - \mu_w d \sin\theta = 0$$

Substituting values, we get

$$\frac{yd}{D} = -10\lambda$$

$$\Rightarrow \quad y = -10\left(\frac{\lambda D}{d}\right) = -\frac{5}{3} \times 10^{-2} \text{ m}$$

Hence, the correct answer is (D).

34. At P, we have

$$\Delta x = \frac{yd}{D} + (\mu_g - 1)t - \mu_w d \sin\theta$$

$$\Rightarrow \quad \Delta x = \frac{10^{-2} \times 3 \times 10^{-4}}{8 \times 1} + 10\lambda = 10\lambda + \frac{3\lambda}{4}$$

$$\Delta\phi = \frac{2\pi}{\lambda}\left(10\lambda + \frac{3\lambda}{4}\right) = 20\pi + \frac{3\pi}{2}$$

So, $I_P = \sqrt{I^2 + I^2 + 2I^2 \cos\left(20\pi + \frac{3\pi}{2}\right)}$

Since $\cos\left(20\pi+\frac{3\pi}{2}\right)=\cos\left(\frac{3\pi}{2}\right)=0$

$\Rightarrow \quad I_P=\sqrt{2}I$

Further $I_{\max}=\frac{I}{2}+\frac{I}{2}+2\sqrt{\left(\frac{I}{2}\right)\left(\frac{I}{2}\right)}\cos(0°)$

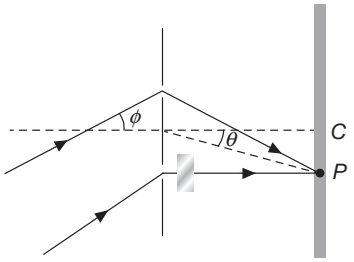
$\Rightarrow \quad I_{\max}=I+I=2I$

$\Rightarrow \quad \frac{I_P}{I_{\max}}=\frac{\sqrt{2}I}{2I}=\frac{1}{\sqrt{2}}$

Hence, the correct answer is (C).

35. Path difference is given by

$$\Delta x=d\sin\phi+d\sin\theta-(\mu-1)t$$



For central maxima, $\Delta x=0$

$\Rightarrow \quad \sin\theta=\frac{(\mu-1)t}{d}-\sin\phi$

$\Rightarrow \quad \sin\theta=\frac{\left(\frac{3}{2}-1\right)(0.1)}{50\times10^{-3}}-\sin(30°)=\frac{1}{2}$

$\Rightarrow \quad \theta=30°$

Hence, the correct answer is (C).

36. At C, $\theta=0°$, so we get

$$\Delta x=d\sin\phi-(\mu-1)t$$

$\Rightarrow \quad \Delta x=(50\times10^{-3})\left(\frac{1}{2}\right)-\left(\frac{3}{2}-1\right)(0.1)$

$\Rightarrow \quad \Delta x=0.025-0.05=-0.025$ mm

Substituting, $\Delta x=n\lambda$, we get

$$n=\frac{\Delta x}{\lambda}=\frac{-0.025}{500\times10^{-6}}=-50$$

Hence, at C there will be maxima. Therefore, the order of minima closest to the C are -49.

Hence, the correct answer is (B).

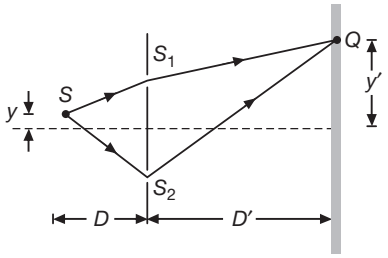
37. Number of fringes shifted upwards is

$$N=\frac{(\mu-1)t}{\lambda}=\frac{\left(\frac{3}{2}-1\right)(0.1)}{500\times10^{-6}}=100$$

Hence, the correct answer is (A).

38. Net path difference of the waves reacting at Q, is

$$\Delta x=\frac{yd}{D}+\frac{y'd}{D'}$$



For central maximum, $\Delta x=0$

$\Rightarrow \quad y'=-\frac{D'}{D}y'$

$\Rightarrow \quad y'=-\left(\frac{2}{1}\right)(0.5\sin(\pi t))$

$\Rightarrow \quad y'=-\sin(\pi t)$ mm

$\Rightarrow \quad y'=\sin(\pi+\pi t)=-\sin(\pi t)$

Hence, the correct answer is (D).

39. $y'=\frac{d}{2}$, at point P exactly in front of S_1, so we have

$$\Delta x=\left(\frac{yd}{D}\right)+\frac{\left(\frac{d^2}{2}\right)}{D'}$$

For maximum intensity, we have path difference to be an even multiple of $\frac{\lambda}{2}$, so

$$\Delta x=(2n)\frac{\lambda}{2}=n\lambda$$

Substituting the values, we get

$$0.5\sin(\pi t)+0.25=0.5n$$

$\Rightarrow \quad \sin(\pi t)=\frac{0.5n-0.25}{0.5}$

For minimum value of t, we have $n=1$

$\Rightarrow \quad \sin(\pi t)=0.5$

$\Rightarrow \quad \pi t=\frac{\pi}{6}$

$\Rightarrow \quad t=\frac{1}{6}$

Hence, the correct answer is (C).

40. For minimum intensity, path difference is an odd multiple of $\frac{\lambda}{2}$, so

$$\Delta x=(2n+1)\frac{\lambda}{2}$$

CHAPTER 2

$\Rightarrow \quad 0.5\sin(\pi t)+0.25=\dfrac{0.5}{2} \qquad \{\because n=1\}$

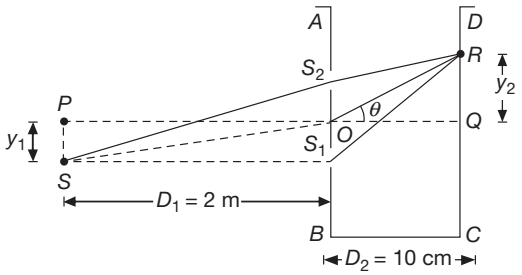
$\Rightarrow \quad \sin(\pi t)=0$

$\Rightarrow \quad \pi t=\pi$

$\Rightarrow \quad t=1\text{ s}$

Hence, the correct answer is (A).

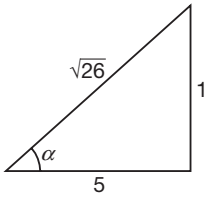
41. Given $y_1=40$ cm, $D_1=2$ m $=200$ cm, $D_2=10$ cm



$$\tan\alpha=\frac{y_1}{D_1}=\frac{40}{200}=\frac{1}{5}$$

$\Rightarrow \quad \alpha=\tan^{-1}\left(\dfrac{1}{5}\right)$

$\Rightarrow \quad \sin\alpha=\dfrac{1}{\sqrt{26}}\approx\dfrac{1}{5}=\tan\alpha$



Path difference between SS_1 and SS_2 is

$$\Delta x_1=SS_1-SS_2$$

$\Rightarrow \quad \Delta x_1=d\sin\alpha=(0.8\text{ mm})\left(\dfrac{1}{5}\right)$

$\Rightarrow \quad \Delta x_1=0.16\text{ mm} \qquad \ldots(1)$

Now, let at point R on the screen, central bright fringe is observed (i.e., net path difference $=0$).

Path difference between S_2R and S_1R is

$$\Delta x_2=S_2R-S_1R$$

$\Rightarrow \quad \Delta x_2=d\sin\theta \qquad \ldots(2)$

Central bright fringe will be observed when net path difference is zero.

$\Rightarrow \quad \Delta x_2-\Delta x_1=0$

$\Rightarrow \quad \Delta x_2=\Delta x_1$

$\Rightarrow \quad d\sin\theta=0.16$

$\Rightarrow \quad (0.8)\sin\theta=0.16$

$\Rightarrow \quad \sin\theta=\dfrac{0.16}{0.8}=\dfrac{1}{5}$

$\Rightarrow \quad \tan\theta=\dfrac{1}{\sqrt{24}}$

$\Rightarrow \quad \sin\theta\approx\dfrac{1}{5}$

$\Rightarrow \quad \tan\theta\approx\sin\theta=\dfrac{y_2}{D_2}=\dfrac{1}{5}$

$\Rightarrow \quad y_2=\dfrac{D_2}{5}=\dfrac{10}{5}=2\text{ cm}$

Therefore, central bright fringe is observed at 2 cm above point Q on side CD.

Hence, the correct answer is (B).

Alternate solution for (a)

Δx at R will be zero if $\Delta x_1=\Delta x_2$

$\Rightarrow \quad d\sin\alpha=d\sin\theta$

$\Rightarrow \quad \alpha=\theta$

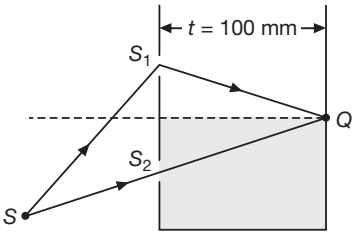
$\Rightarrow \quad \tan\alpha=\tan\theta$

$\Rightarrow \quad \dfrac{y_1}{D_1}=\dfrac{y_2}{D_2}$

$\Rightarrow \quad y_2=\dfrac{D_2}{D_1}\cdot y_1=\left(\dfrac{10}{200}\right)(40)\text{ cm}$

$\Rightarrow \quad y_2=2\text{ cm}$

42. The central bright fringe will be observed at point Q, if the path difference created by the liquid slab of thickness $t=10$ cm or 100 mm is equal to Δx_1, so that the net path difference at Q becomes zero.



$\Rightarrow \quad (\mu-1)t=\Delta x_1$

$\Rightarrow \quad (\mu-1)(100)=0.16$

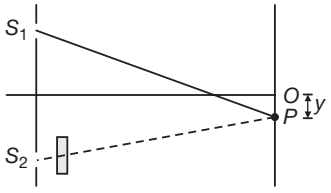
$\Rightarrow \quad \mu-1=0.0016$

$\Rightarrow \quad \mu=1.0016$

Hence, the correct answer is (C).

43. Given $\lambda=600\text{ nm}=6\times10^{-7}$ m,

$d=0.45\text{ mm}=0.45\times10^{-3}$ m and $D=1.5$ m



Thickness of glass sheet, $t = 10.4\ \mu\text{m} = 10.4 \times 10^{-6}$ m

Refractive index of the medium, $\mu_m = \dfrac{4}{3}$

And refractive index of glass sheet, $\mu_g = 1.5$

Let central maximum is obtained at a distance y below point O.

$$\Rightarrow \quad \Delta x_1 = S_1P - S_2P = \frac{yd}{D}$$

Path difference due to glass sheet is given by

$$\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$

Net path difference will be zero, when we have

$$\Delta x_1 = \Delta x_2$$

$$\Rightarrow \quad \frac{yd}{D} = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$

$$\Rightarrow \quad y = \left(\frac{\mu_g}{\mu_m} - 1\right)t\frac{D}{d}$$

Substituting the values, we get

$$y = \left(\frac{1.5}{4/3} - 1\right)\frac{10.4 \times 10^{-6}(1.5)}{0.45 \times 10^{-3}}$$

$$\Rightarrow \quad y = 4.33 \times 10^{-3}\ \text{m}$$

$$\Rightarrow \quad y = 4.33\ \text{mm} = \frac{13}{3}\ \text{mm}$$

Hence, the correct answer is (D).

44. At O, $\Delta x_1 = 0$ and $\Delta x_2 = \left(\dfrac{\mu_g}{\mu_m} - 1\right)t$

So, net path difference

$$\Delta x = \Delta x_2$$

Corresponding phase difference $\Delta\phi = \left(\dfrac{2\pi}{\lambda}\right)\Delta x$

Substituting the values, we get

$$\phi = \Delta\phi = \frac{2\pi}{6 \times 10^{-7}}\left(\frac{1.5}{4/3} - 1\right)\left(10.4 \times 10^{-6}\right)$$

$$\Rightarrow \quad \phi = \left(\frac{13}{3}\right)\pi$$

Now, $I(\phi) = I_{\max}\cos^2\left(\dfrac{\phi}{2}\right)$

$$\Rightarrow \quad I = I_{\max}\cos^2\left(\frac{13\pi}{6}\right)$$

$$\Rightarrow \quad I = \frac{3}{4}I_{\max}$$

Hence, the correct answer is (C).

45. At O, path difference is $\Delta x = \Delta x_2 = \left(\dfrac{\mu_g}{\mu_m} - 1\right)t$

For maximum intensity at O, we have

$$\Delta x = n\lambda, \text{ where } n = 1, 2, 3, \ldots\ldots$$

$$\Rightarrow \quad \lambda = \frac{\Delta x}{1}, \frac{\Delta x}{2}, \frac{\Delta x}{3}, \ldots\ldots \text{ and so on}$$

$$\Rightarrow \quad \Delta x = \left(\frac{1.5}{4/3} - 1\right)\left(10.4 \times 10^{-6}\ \text{m}\right)$$

$$\Rightarrow \quad \Delta x = \left(\frac{1.5}{4/3} - 1\right)\left(10.4 \times 10^{3}\ \text{nm}\right) = 1300\ \text{nm}$$

So, maximum intensity will be corresponding to

$$\lambda = 1300\ \text{nm}, \frac{1300}{2}\ \text{nm}, \frac{1300}{3}\ \text{nm}, \frac{1300}{4}\ \text{nm}, \ldots$$

$$\Rightarrow \quad \lambda = 1300\ \text{nm}, 650\ \text{nm}, 433.33\ \text{nm}, 325\ \text{nm}, \ldots.$$

The wavelength in the range 400 nm to 700 nm are 650 nm and $433.33\ \text{nm}\left(= \dfrac{1300}{3}\ \text{nm}\right)$

Hence, the correct answer is (C).

46. The optical path difference between the two waves arriving at P is

$$\Delta x = \frac{y_1 d}{D_1} + \frac{y_2 d}{D_2} = \frac{(1)(10)}{10^3} + \frac{(5)(10)}{2 \times 10^3}$$

$$\Rightarrow \quad \Delta x = 3.5 \times 10^{-2}\ \text{mm} = 0.035\ \text{mm}$$

To calculate the order of interference, we shall calculate

$$n = \frac{\Delta x}{\lambda}$$

$$\Rightarrow \quad n = \frac{0.035 \times 10^{-3}\ \text{m}}{5000 \times 10^{-10}\ \text{m}}$$

$$\Rightarrow \quad n = 70$$

$$\Rightarrow \quad \Delta x = 70\lambda$$

So, 70^{th} order maxima is obtained at P.

Hence, the correct answer is (C).

47. At O, $\Delta x = \dfrac{y_1 d}{D_1} = 10^{-2}\ \text{mm} = 0.01\ \text{mm}$

Now, we observe that $\Delta x = 20\lambda$

So, 20^{th} order maxima is obtained at O.

Hence, the correct answer is (D).

CHAPTER 2

48. $(\mu-1)t = 0.01$ mm

$\Rightarrow \quad t = \dfrac{0.01}{1.5-1} = 0.02$ mm $= 20\ \mu$m

Since the pattern has to be shifted upwards, therefore, the film must be placed infront of S_1.

Hence, the correct answer is (D).

49. Using the concept of Displacement Method, we get

$$O = \sqrt{I_1 I_2} = 3 \text{ mm}$$

Since, $m = \dfrac{f}{f+u}$

$\Rightarrow \quad \dfrac{100-x}{x} = 4$

$\Rightarrow \quad x = 20$ cm

So, we get $u = -20$, $v = +80$

Since $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$\Rightarrow \quad f = 16$ cm

50. $\beta = \dfrac{\lambda D}{d} = \dfrac{6000 \times 10^{-10} \times 1}{3 \times 10^{-3}} = 0.2$ mm

Hence, the correct answer is (B).

51. $\beta = \dfrac{\lambda D}{d}$

When D is increased, then β is increased.

Hence, the correct answer is (D).

52. At point P on the screen, y is the same. But $D' > D$.

Since, we know that $y = n\left(\dfrac{\lambda D}{d}\right) = n'\left(\dfrac{\lambda D'}{d}\right)$

$\Rightarrow \quad nD = n'D'$

Since, $D' > D$, so $n' < n$

Similarly, even if we would have been getting a minima at P, then too the order of the fringe will decrease at P.

Hence, the correct answer is (B).

53. At $t = 0$, at the point P, a fifth bright is formed. So, we get

$$y_P = y_{5B} = 5\left(\frac{\lambda D}{d}\right) \qquad \ldots(1)$$

At the point P, at time t, third minima is formed, so we have

$$y_P = y_{3B} = \left(3 - \frac{1}{2}\right)\frac{\lambda D'}{d}$$

$$\Rightarrow \quad y_P = y_{3B} = \frac{5}{2}\frac{\lambda(D+Vt)}{d} \qquad \ldots(2)$$

Equating (1) and (2), we get

$$D = Vt$$

$$\Rightarrow \quad t = \frac{D}{V}$$

Hence, the correct answer is (B).

54. Order of the fringe can be counted on either side of the central maximum, so third order bright is fringe number 5.

Hence, the correct answer is (D).

55. $\Delta\phi = 4\pi$

$\Rightarrow \quad \Delta\phi = 2(2\pi)$

So, this phase difference corresponds to second maxima, represented by fringe number 4.

Hence, the correct answer is (C).

56. $\Delta x_3 = \lambda$, $\Delta x_1 = \dfrac{\lambda}{2}$

$\Rightarrow \quad \Delta x_3 - \Delta x_1 = \dfrac{\lambda}{2} = 300$ nm

Hence, the correct answer is (B).

57. For reflected light, constructive interference takes place when

$$2\mu t = (2n-1)\frac{\lambda}{2}$$

For minimum thickness, we have

$$n = 1$$

$$\Rightarrow \quad 2\mu t = \frac{\lambda}{2}$$

$$\Rightarrow \quad t = \frac{900}{4 \times 1.5}$$

$$\Rightarrow \quad t = 150 \text{ nm}$$

Hence, the correct answer is (B).

58. For transmitted light, destructive interference takes place, when

$$2\mu t = (2n-1)\frac{\lambda}{2}$$

For minimum thickness $n = 1$

$$\Rightarrow \quad t = \frac{900}{4 \times 1.5} = 150 \text{ nm}$$

Hence, the correct answer is (A).

59. $d_{\text{air}} = 2a(\mu_p - 1)\alpha$,

where μ_p = Refractive Index of prism

$$d_{\text{water}} = 2a\left(\frac{\mu_p}{\mu_\ell} - 1\right)\alpha,$$

where μ_ℓ = Refractive Index of water/liquid.

$$\Rightarrow \quad \frac{d_{\text{water}}}{d_{\text{air}}} = \frac{\frac{\mu_p}{\mu_\ell} - 1}{\mu_p - 1}$$

$$\Rightarrow \quad \frac{d_{\text{water}}}{0.5} = \frac{\frac{3/2}{4/3} - 1}{\frac{3}{2} - 1} = \frac{1}{4}$$

$$\Rightarrow \quad d_{\text{water}} = \frac{1}{4} \times 0.5 = \frac{1}{8} \text{ mm}$$

Hence, the correct answer is (D).

60. $\beta_{\text{air}} = \dfrac{D\lambda_{\text{air}}}{2a(\mu_p - 1)\alpha}$

$$\Rightarrow \quad \beta_{\text{water}} = \frac{D\lambda_{\text{air}}}{2a(\mu_p - \mu_\ell)\alpha}$$

$$\Rightarrow \quad \frac{\beta_{\text{water}}}{\beta_{\text{air}}} = \frac{\mu_p - 1}{\mu_p - \mu_\ell} = \frac{3/2 - 1}{3/2 - 4/3}$$

$$\Rightarrow \quad \beta_{\text{water}} = 3 \times 1 = 3 \text{ mm}$$

Hence, the correct answer is (A).

61. $\beta = \dfrac{D\lambda}{d}$

$$\Rightarrow \quad 1 \times 10^{-3} = \frac{1 \times \lambda}{0.5 \times 10^{-3}}$$

$$\Rightarrow \quad \lambda = 5 \times 10^{-7} \text{ m} = 5000 \text{ Å}$$

Hence, the correct answer is (A).

62. $\Delta y = \dfrac{7\lambda' D}{d} - \dfrac{3\lambda' D}{d}$, where $\lambda' = \dfrac{\lambda}{\mu}$

Hence, the correct answer is (A).

63. Since, $\Delta y = \dfrac{D}{d}\left(\dfrac{\mu_g}{\mu_w} - 1\right)t$

Minimum separation between, maxima and minima, say n^{th} maxima and n^{th} minima is

$$\Delta y = \frac{n\lambda' D}{d} - \left(n - \frac{1}{2}\right)\frac{\lambda' D}{d} = \frac{\lambda' D}{2d}$$

where $\lambda' = \dfrac{\lambda}{\mu_w}$

$$\Rightarrow \quad \frac{\lambda' D}{2d} = \frac{D}{d}\left(\frac{\mu_g}{\mu_w} - 1\right)t$$

$$\Rightarrow \quad \left(\frac{\mu_g}{\mu_w} - 1\right)t = \frac{\lambda}{2\mu_w}$$

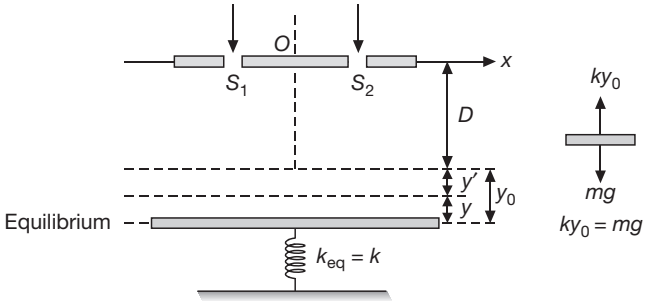
$$\Rightarrow \quad \left(\frac{1.53}{1.33} - 1\right)t = \frac{6300 \times 10^{-10}}{2(1.33)}$$

$$\Rightarrow \quad t = 15750 \times 10^{-10} \text{ m}$$

$$\Rightarrow \quad t = 1.575 \times 10^{-6} \text{ m} = 1.575 \ \mu\text{m}$$

Hence, the correct answer is (B).

64. Let us first replace two springs each of spring constant $\dfrac{k}{2}$ by a single spring of spring constant $k_{\text{eq}} = \dfrac{k}{2} + \dfrac{k}{2} = k$.



For equilibrium of plate, we have

$$mg = ky_0$$

$$\Rightarrow \quad y_0 = \frac{mg}{k}$$

The equation governing motion of plates is

$$y = y_0 \cos(\omega t)$$

Let y' be instantaneous position of plate from O, then

$$y' = y_0 - y$$

$$\Rightarrow \quad y' = y_0 - y_0 \cos(\omega t)$$

$$\Rightarrow \quad y' = y_0(1 - \cos(\omega t))$$

Instantaneous fringe width is given by

$$\beta = \frac{(D + y')\lambda}{d}$$

Rate of change of fringe width is

$$\frac{d\beta}{dt} = \left(\frac{\lambda}{d}\right)\frac{dy'}{dt} = \frac{\lambda}{d}\left[y_0 \omega \sin(\omega t)\right]$$

$$\Rightarrow \quad \frac{d\beta}{dt} = \left(\frac{\lambda}{d}\right)\left(\frac{mg}{k}\right)\sqrt{\frac{k}{m}}\sin(\omega t)$$

Now, acceleration of plates is zero at equilibrium i.e. at $t = 0$ i.e. when $\cos(\omega t) = 1$ i.e. when $\sin(\omega t) = 1$
So, rate at which fringe width will increase is

$$\frac{d\beta}{dt} = \frac{\lambda g}{d}\sqrt{\frac{m}{k}}$$

Hence, the correct answer is (A).

65. Plate is momentarily at rest, at the extreme positions i.e. when

$$y' = 0$$

CHAPTER 2

$\Rightarrow \quad \cos(\omega t) = 1$

So, fringe width is $\beta_1 = \dfrac{\lambda D}{d}$

Also, plate is momentary at rest when

$y' = 2y_0$

$\Rightarrow \quad \cos(\omega t) = -1$

So, fringe width is $\beta_2 = (D + 2y_0)\dfrac{\lambda}{d}$

$\Rightarrow \quad \Delta\beta = \beta_2 - \beta_1 = \dfrac{\lambda(D + 2y_0)}{d} - \dfrac{\lambda D}{d}$

$\Rightarrow \quad \Delta\beta = \dfrac{2y_0\lambda}{d} = \dfrac{2\lambda}{d}\left(\dfrac{mg}{k}\right) = \dfrac{2\lambda mg}{kd}$

Hence, the correct answer is (C).

66. Displacement of n^{th} bright fringe is given by

$$\Delta x = \frac{D}{d}(\mu - 1)t$$

Since first maxima shift to the position of central maxima, so

$$\Delta x = \beta = \frac{\lambda D}{d}$$

$\Rightarrow \quad \dfrac{\lambda D}{d} = \dfrac{D}{d}(\mu - 1)t \quad \Rightarrow \quad t = \dfrac{\lambda}{\mu - 1}$

Hence, the correct answer is (D).

Matrix Match/Column Match Type Questions

1. A → (p, s)
B → (q, r)
C → (p, q, r)
D → (q, r, s)
Concept based (see theory)

2. A → (p, q)
B → (s)
C → (p, q)
D → (r)
Concept based (see theory)

3. A → (q)
B → (p)
C → (s)
D → (r)
Concept based (see theory)

4. A → (r)
B → (s)
C → (q)
D → (p)
Concept based (see theory)

5. A → (p)
B → (s)
C → (r)
D → (q, r)
Concept based (see theory)

6. A → (r)
B → (t)
C → (r)
D → (p)
If glass slab is introduced across S_2 this effective path increases so central maxima will be shifted downward but fringe width remains same.

7. A → (p)
B → (s)
C → (r)
D → (q)
Concept based (see theory)

8. A → (p, q, r, s)
B → (p, q, r, s)
C → (p, q, r, s)
D → (p, q, r)
Concept based (see theory)

9. A → (q, r)
B → (q, s)
C → (p, s)
D → (p, q, r, s)
Concept based (see theory)

Integer/Numerical Answer Type Questions

1. Shift $\Delta y = \left|(\mu_2 - 1)t_2 - (\mu_1 - 1)t_1\right|\dfrac{D}{d}$

$\Rightarrow \quad \Delta y = \left|(1.2 - 1) \times 15 \times 10^{-6} - (1.6 - 1) \times 10 \times 10^{-6}\right|\left(\dfrac{1.5}{1.5 \times 10^{-3}}\right)$

$\Rightarrow \quad \Delta y = 3 \times 10^{-3}$ m

$\Rightarrow \quad \Delta y = 3$ mm

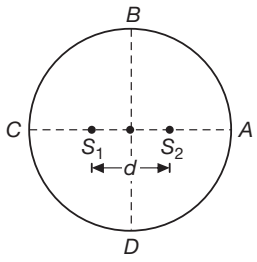
2. Since, $\lambda = 0.25$ m and $d = 2$ m $= 8\lambda$

At A and C, $\Delta x = d = 8\lambda$

Since this is an even multiple of λ, so maximum intensity is obtained.

At B and D, $\Delta x = 0$

Since this is an even multiple of λ, so again maximum intensity will be obtained.



Further, between A and B seven maximas corresponding to $\Delta x = 7\lambda$, 6λ, 5λ, 4λ, 3λ, 2λ and λ are obtained.

Similarly between B and C, C and D, and D and A.

So, total number of maximas is $N = 4 \times 7 + 4 = 32$

3. Ray 1 has a longer path than that of ray 2 by a distance $d\sin(45°)$, before reaching the slits. Afterwards ray 2 has a path longer than ray 1 by a distance $d\sin\theta$. The net path difference is therefore, $d\sin\theta - d\sin 45°$

(a) Central maximum is obtained where, net path difference is zero, so we have

$$d\sin\theta - d\sin(45°) = 0$$

$$\Rightarrow \quad \theta = 45°$$

(b) Third order maxima is obtained where net path difference is 3λ, so we have

$$d\sin\theta - d\sin(45°) = 3\lambda$$

$$\Rightarrow \quad \sin\theta = \sin(45°) + \frac{3\lambda}{d}$$

Substituting $d = 20\lambda$, we get

$$\sin\theta = \sin(45°) + \frac{3\lambda}{20\lambda}$$

$$\Rightarrow \quad \theta \approx 59°$$

4. (a) $$\frac{1}{10} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\Rightarrow \quad R = 10 \text{ cm}$$

Now applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ twice, we get

$$\frac{3/2}{v_1} - \frac{1}{-20} = \frac{3/2 - 1}{10} \text{ and} \qquad \ldots(1)$$

$$\frac{4/3}{v_2} - \frac{3/2}{v_1} = \frac{4/3 - 3/2}{-10} \qquad \ldots(2)$$

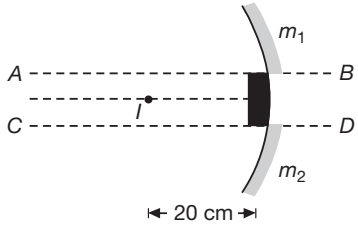
Adding equation (1) and (2), we get

$$v_2 = 80 \text{ cm}$$

(b) The image formed by (lens + water) system will act as an object for the mirror.

This is below the axis of m_1 and at the same distance as the centre is, therefore, its image will be formed vertically above at 1 mm from AB.

Similarly m_2 will form an image of I_1, 1 mm below CD.



$$\Rightarrow \quad I_1I_2 = 1 + 1 + 1 + 1 = 4 \text{ mm}$$

(c) $d = I_1I_2 = 4$ mm, $D = 80$ cm

Since, $\lambda = \frac{v}{f}$

$$\Rightarrow \quad \lambda = \frac{c}{\mu_w f} = \left(\frac{3 \times 10^8}{7.5 \times 10^{14}} \times \frac{3}{4}\right)$$

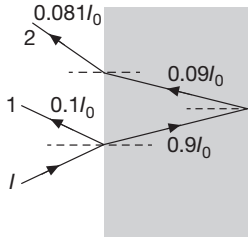
$$\Rightarrow \quad \beta = \frac{\lambda D}{d} = \frac{3}{4}\left(\frac{3 \times 10^8}{7.5 \times 10^{14}}\right)\left(\frac{80 \times 10^{-2}}{4 \times 10^{-3}}\right) \text{ m}$$

$$\Rightarrow \quad \beta = 6 \times 10^{-5} \text{ m}$$

$$\Rightarrow \quad \beta = 60\ \mu\text{m}$$

5. The intensities of the rays due to successive reflections and refractions are shown in figure. So,

$$I_1 = 0.1I_0,\ I_2 = 0.081I_0$$



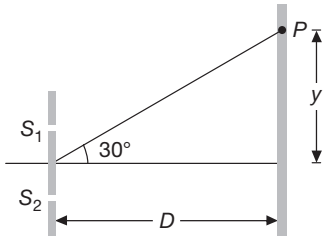
$$\Rightarrow \quad \sqrt{\frac{I_1}{I_2}} = \frac{10}{9}$$

$$\Rightarrow \quad \frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right)^2 = (19)^2 = 361$$

6. Fringe width $\beta = \dfrac{\lambda D}{d}$ and $y = \dfrac{D}{\sqrt{3}}$

Therefore, number of fringe widths in a distance y are given by

$$n = \frac{y}{\beta} = \frac{d}{\sqrt{3}\lambda} = \frac{0.32\times10^{-3}}{(\sqrt{3})(500\times10^{-9})} = 369.5$$



Therefore, total number of maxima obtained in the angular range $-30° < \theta < 30°$ (including the central one) is

$$N = 2\times369 + 1 = 739$$

7. Here, $d = 2\text{ mm} = 2\times10^{-3}\text{ m}$, $D = 10\text{ cm} = 0.10\text{ m}$, $t = 0.5\text{ mm} = 0.5\times10^{-3}\text{ m}$, $\Delta x = 5\text{ mm} = 5\times10^{-3}\text{ m}$, $\lambda = 6\times10^{-7}\text{ m}$

Since, $\Delta x = \dfrac{D}{d}(\mu - 1)t$

$$\Rightarrow \quad \mu - 1 = \frac{\Delta x d}{Dt} = \frac{5\times10^{-3}\times2\times10^{-3}}{0.10\times0.5\times10^{-3}} = 0.2$$

$$\mu = 1 + 0.2 = 1.2$$

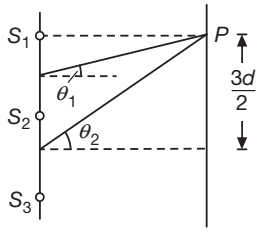
$$\Rightarrow \quad 5\mu = 6$$

9. $S_2P - S_1P = d\sin\theta_1 = d\left(\dfrac{\frac{d}{2}}{D}\right) = \dfrac{d^2}{2D} = \dfrac{\lambda}{3}$

$$\Rightarrow \quad \Delta\phi = \frac{2\pi}{3}$$

$$S_3P - S_2P = d\sin\theta_2 = d\left(\frac{\frac{3d}{2}}{D}\right) = \frac{3d^2}{2D} = \lambda$$

$$\Rightarrow \quad \Delta\phi = 2\pi$$



If I_R be the resultant intensity of the wave at the point P, then

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$$\Rightarrow \quad I_R = I_0 + 4I_0 + 2\sqrt{(I_0)(4I_0)}\cos(120°)$$

$$\Rightarrow \quad I_R = 3I_0 = 3(12\text{ Wm}^{-2}) = 36\text{ Wm}^{-2}$$

10. Angular fringe width $\theta = \dfrac{\beta}{D} = \dfrac{\lambda}{d}$

According to the given condition

$$\frac{\lambda}{d} \geq \frac{\pi}{180\times60}$$

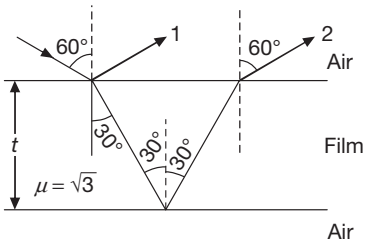
$$\Rightarrow \quad d < \frac{6\times10^{-7}\times180\times60}{\pi}$$

$$d_{\max} = 2.06\times10^{-3}\text{ m} = 2.06\text{ mm}$$

11. According to Snell's Law, we have

$$1\sin(30°) = \sqrt{3}\sin r$$

$$\Rightarrow \quad r = 30°$$



The Optical path difference is given by

$$\Delta x = 2\mu t\sec r - 2t\tan r\sin i$$

$$\Rightarrow \quad \Delta x = 2\sqrt{3}(t\sec30°) - 2t\tan(30°)\sin(60°)$$

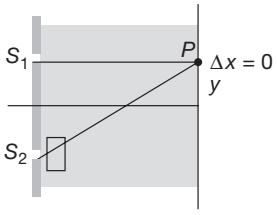
$$\Delta x = 4t - 2t\left(\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right) = 3t$$

Since the Ray 1 is reflected at the surface of the denser medium so it suffers an additional phase change of π or a path change of $\dfrac{\lambda}{2}$. So, for constructive interference, we have

$$3t = \frac{\lambda}{2}$$

$$\Rightarrow \quad t = \frac{\lambda}{6} = 1000\text{ Å} = 100\text{ nm}$$

12.



The path difference at O is

$$\Delta x = (S_2P - T)_{\text{liquid}} + T_{\text{glass}} - (S_1P)_{\text{liquid}}$$

$$\Delta x = \frac{yd}{D} + \left(\frac{\mu T}{\mu_\ell} - T\right)_{\text{liquid}}$$

At the central maxima, we have path difference

$$\Delta x = 0$$

$$\Rightarrow \quad y = -\frac{DT}{d}\left(\frac{\mu - \mu_\ell}{\mu_\ell}\right) = -\frac{TD}{d}\left(\frac{4-t}{10-t}\right)$$

Central maxima will be at O when $y = 0$

$$\Rightarrow \quad t = 4 \text{ s}$$

Now, the speed of the central maxima is given by

$$v = \left|\frac{dy}{dt}\right| = \frac{6DT}{(10-t)^2 d}$$

At $t = 4$ s, we have $v = \frac{6DT}{36d} = \frac{DT}{6d}$

$$\Rightarrow \quad v = \frac{(1)\left(36\times10^{-6}\right)}{(6)\left(2\times10^{-3}\right)} = 3\times10^{-3} \text{ ms}^{-1} = 3 \text{ mms}^{-1}$$

13. Fringe width, $\beta = \frac{\lambda D}{d} \propto \lambda$

Since the wavelength is decreasing from 600 nm to 400 nm, so fringe width also decreases by a factor of $\frac{4}{6} = \frac{2}{3}$. So the number of fringes will increase by a factor of $\frac{3}{2}$. Hence the new number of fringes formed is

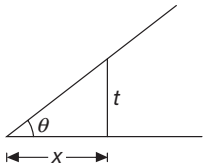
$$N = \frac{3}{2}(12) = 18$$

14. $\theta = 0.01$ radian

$n = 10$

$\lambda = 6000\times10^{-8}$ cm

$\Delta x = 2t = n\lambda$



Since, $\theta = \frac{t}{x}$

$$\Rightarrow \quad t = \theta x$$

$$\Rightarrow \quad 2\theta x = n\lambda$$

$$\Rightarrow \quad x = \frac{n\lambda}{2\theta} = 3\times10^{-1} \text{ m} = 3 \text{ mm}$$

15. When coherent, then Δx at centre is zero, so, we have the resultant intensity to be

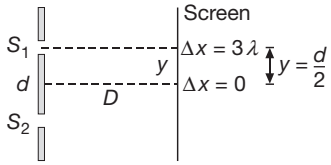
$$I_1 = 4I$$

When incoherent, then the sources will not interfere and it will be a general illumination at the point, so we have the resultant intensity to be,

$$I_2 = 2I$$

$$\Rightarrow \quad \frac{I_1}{I_2} = 2$$

16. Figure shows the $YDSE$ setup as described in the problem.



At a distance y from the centre of the screen, path different between the waves from S_1 and S_2 is given by

$$\Delta x = \frac{yd}{D}$$

Since, at $y = \frac{d}{2}$, path difference should be 3λ (because third bright fringe is located in front of one slit on the screen). So, we have at $y = \frac{d}{2}$ and $\Delta x = 3\lambda$. Since

$$\Delta x = \frac{yd}{D}$$

$$\Rightarrow \quad 3\lambda = \frac{\left(\frac{d}{2}\right)d}{D}$$

$$\Rightarrow \quad \frac{d^2}{2D} = 3\lambda$$

$$\Rightarrow \quad d = \sqrt{6\lambda D} = \sqrt{6\times6\times10^{-7}\times1}$$

$$\Rightarrow \quad d = \sqrt{36\times10^{-7}} \text{ m}$$

$$\Rightarrow \quad d = 0.6\sqrt{10} \text{ mm} = 1.9 \text{ mm}$$

$$\Rightarrow \quad d \approx 2 \text{ mm}$$

CHAPTER 2

ARCHIVE: JEE MAIN

1. $\frac{a_1}{a_2}=\frac{3}{1}$

$\Rightarrow \quad \frac{I_{max}}{I_{min}}=\left(\frac{a_2+a_1}{a_2-a_1}\right)^2=\left(\frac{3+1}{3-1}\right)^2=4$

Hence, the correct answer is (A).

2. Since $(\mu-1)t=n\lambda$

$\Rightarrow \quad t=\frac{n\lambda}{\mu-1}$

Hence, the correct answer is (B).

3. $I_1=4I_0,\ I_2=I_0$

$\Rightarrow \quad \frac{I_{max}}{I_{min}}=\frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}=\left(\frac{9}{1}\right)$

Hence, the correct answer is (D).

4. $(\mu-1)t=n\lambda$

$\Rightarrow \quad (\mu-1)t=\lambda$

$\Rightarrow \quad t=\frac{\lambda}{\mu-1}$

Hence, the correct answer is (D).

5. $I=\left(\frac{I_0}{2}\right)\cos^2(30°)\cos^2(60°)=\frac{I_0}{2}\times\frac{3}{4}\times\frac{1}{4}=\frac{3I_0}{32}$

$\Rightarrow \quad \frac{I_0}{I}=\frac{32}{3}=10.67$

Hence, the correct answer is (C).

6. $\frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}=16$

$\Rightarrow \quad \frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}=4$

$\Rightarrow \quad \frac{\sqrt{I_1}}{\sqrt{I_2}}=\frac{4+1}{4-1}=\frac{5}{3}$

$\Rightarrow \quad \frac{I_1}{I_2}=\frac{25}{9}$

Hence, the correct answer is (A).

7. $N=\frac{d\sin\theta}{\lambda}=\frac{0.320\times10^{-3}}{500\times10^{-9}}\times\frac{1}{2}$

$\Rightarrow \quad N=320$

Number of Bright fringes $=2(320)+1=641$

Hence, the correct answer is (D).

8. $\theta d=n_1\lambda_1$

$\theta d=n_2\lambda_2$

where $\theta d=\frac{0.1\times10^{-3}}{40}\times10^9$ nm

$\Rightarrow \quad \theta d=\frac{1}{40}\times10^5=2500$ nm

$\Rightarrow \quad n_1\lambda_1=n_2\lambda_2=2500$

$\Rightarrow \quad \theta d=2500$ is LCM of λ_1 and λ_2

$\Rightarrow \quad \lambda_1=625$ nm and $\lambda_2=500$ nm [By observation]

Hence, the correct answer is (B).

9. For 1st minima, we have $\Delta x=\frac{\lambda}{2}$

$\Rightarrow \quad \sqrt{d^2+(2d)^2}-2d=\frac{\lambda}{2}$

$\Rightarrow \quad \sqrt{5}d-2d=\frac{\lambda}{2}$

$\Rightarrow \quad d=\frac{\lambda}{2(\sqrt{5}-2)}$

Hence, the correct answer is (A).

10. $\Delta\phi=\frac{2\pi}{\lambda}\times\frac{\lambda}{8}=\frac{\pi}{4}$

Since, $I=I_{max}\cos^2\left(\frac{\phi}{2}\right)=4I_0\cos^2\left(\frac{\pi}{8}\right)$

$\Rightarrow \quad \frac{I}{4I_0}=\cos^2\left(\frac{\pi}{8}\right)=0.85$

Hence, the correct answer is (D).

11. Angular width between second and first diffraction minima $=\frac{\lambda}{a}$

Angular width of a fringe $=\frac{\lambda}{d}$

$\Rightarrow \quad n=\frac{d}{a}=\frac{19.44}{4.05}$

$\Rightarrow$ Number of bright fringes $=04$

Hence, the correct answer is (D).

12. Polarizers A and B are oriented with parallel pass axis. Suppose polarizer C is at angle θ with A then it also makes angle θ with B,
Using Malus' law, we get

$I_C=\frac{I}{2}\cos^2\theta$ and

$I_B=I_C\cos^2\theta=\frac{I}{2}\cos^4\theta$

$$\Rightarrow \quad \frac{I}{8} = \frac{I}{2}\cos^4\theta \qquad \left\{\because I_B = \frac{I}{8}\right\}$$

$$\cos^4\theta = \frac{1}{4} = \frac{1}{(\sqrt{2})^4}$$

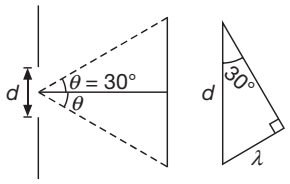
$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}} = \cos(45°)$$

$$\Rightarrow \quad \theta = 45°$$

Hence, the correct answer is (C).

13. In a single slit diffraction, $a\sin\theta = \lambda$.

$$\Rightarrow \quad a\sin(30°) = \lambda$$



$$\Rightarrow \quad \lambda = \frac{a}{2} = \frac{1\times10^{-6}\text{ m}}{2}$$

$$\Rightarrow \quad \lambda = 5000\text{ Å}$$

In Young's double slit experiment, fringe width is

$$\beta = \frac{\lambda D}{d} \quad (d \text{ is slit separation})$$

$$\Rightarrow \quad 10^{-2} = \frac{5000\times10^{-10}\times0.5}{d}$$

$$\Rightarrow \quad d = 25\times10^{-6}\text{ m} = 25\ \mu\text{m}$$

Hence, the correct answer is (A).

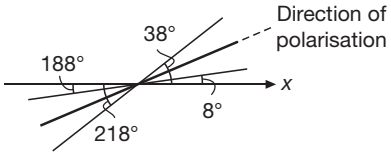
14. If the angular position of second minima from central maxima is θ then, $\sin\theta = \frac{2\lambda}{a} = \frac{2\times550\times10^{-9}}{22\times10^{-7}} = \frac{1}{2}$

$$\Rightarrow \quad \theta = \frac{\pi}{6}\text{ rad}$$

Hence, the correct answer is (C).

15. The direction of polarisation must be equally inclined to 8° and 38° OR 188° and 218° positions. So, the required angle is

$$\frac{8°+38°}{2} = 23° \quad \text{OR} \quad \frac{188°+218°}{2} = 203°$$



Hence, the correct answer is (A).

16. Polarizers A and B are oriented with parallel pass axis. Suppose polarizer C is at angle θ with A then it also makes angle θ with B.
Using Malus' law,

$$I_C = \left(\frac{I}{2}\cos^2\theta\right) \text{ and } I_B = I_C\cos^2\theta = \frac{I}{2}\cos^4\theta$$

$$\Rightarrow \quad \frac{I}{3} = \frac{I}{2}\cos^4\theta \qquad \left\{\because I_B = \frac{I}{3}\right\}$$

$$\Rightarrow \quad \cos^4\theta = \frac{2}{3}$$

$$\Rightarrow \quad \cos\theta = \left(\frac{2}{3}\right)^{\frac{1}{4}}$$

Hence, the correct answer is (B).

17. Frequency of the microwave measured by the observer will be given by Doppler's effect of light.

$$\frac{\nu'}{\nu} = \sqrt{\frac{1+\beta}{1-\beta}} \qquad \left\{\text{Here } \beta = \frac{v}{c}\right\}$$

$$\Rightarrow \quad \frac{\nu'}{\nu} = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} = \sqrt{\frac{c+v}{c-v}}$$

$$\nu' = \nu\sqrt{\frac{\left(c+\frac{c}{2}\right)}{c-\frac{c}{2}}} = 10\sqrt{3}\text{ GHz}$$

$$\Rightarrow \quad \nu' = 17.3\text{ GHz}$$

Hence, the correct answer is (C).

18. Let y be the distance from the central maximum to the point where the bright fringes due to both the wavelengths coincides.

Now, for λ_1, $y = \frac{n_1\lambda_1 D}{d}$

For λ_2, $y = \frac{n_2\lambda_2 D}{d}$

$$\Rightarrow \quad n_1\lambda_1 = n_2\lambda_2$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{520}{650} = \frac{4}{5}$$

i.e. with respect to central maximum 4^{th} bright fringe of λ_1 coincides with 5^{th} bright fringe of λ_2

Now, $y = \frac{4\times650\times10^{-9}\times1.5}{0.5\times10^{-3}}$ m

$$\Rightarrow \quad y = 7.8\times10^{-3}\text{ m}$$

$$\Rightarrow \quad y = 7.8\text{ mm}$$

Hence, the correct answer is (B).

CHAPTER 2

19. For single slit diffraction, $\sin\theta = \frac{n\lambda}{b}$

Position of n^{th} minima from central maxima $= \frac{n\lambda D}{b}$

When $n = 2$, then $x_2 = \frac{2\lambda D}{b} = 0.03$ …(1)

When $n = 4$, then $x_4 = \frac{4\lambda D}{b} = 0.06$ …(2)

Equation (2) − Equation (1) gives

$$x_4 - x_2 = \frac{4\lambda D}{b} - \frac{2\lambda D}{b} = 0.03$$

$$\Rightarrow \quad \frac{\lambda D}{b} = \frac{0.03}{2}$$

So, width of central maximum is

$$\frac{2\lambda D}{b} = 2 \times \frac{0.03}{2} = 0.03 \text{ m} = 3 \text{ cm}$$

Hence, the correct answer is (D).

20. Here, $d = 0.1$ mm, $\lambda = 6000$ Å, $D = 0.5$ m

For third dark band, $d\sin\theta = 3\lambda$

$$\Rightarrow \quad \sin\theta = \frac{3\lambda}{d} = \frac{y}{D}$$

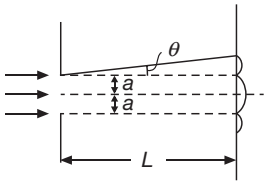
$$\Rightarrow \quad y = \frac{3D\lambda}{d} = \frac{3 \times 0.5 \times 6 \times 10^{-7}}{0.1 \times 10^{-3}} = 9 \times 10^{-3} \text{ m} = 9 \text{ mm}$$

Hence, the correct answer is (C).

21. Size of spot, b = Geometrical spread + diffraction spread

$$\Rightarrow \quad b = a + L\frac{\lambda}{a}$$

Now, value of b would be minimum if $\frac{db}{da} = 0$



$$\Rightarrow \quad 1 + L\left(\frac{-\lambda}{a^2}\right) = 0$$

$$\Rightarrow \quad a^2 = \lambda L$$

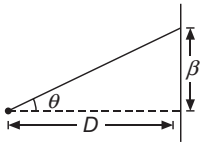
$$\Rightarrow \quad a = \sqrt{\lambda L}$$

$$\Rightarrow \quad b_{\min} = \sqrt{\lambda L} + \frac{\lambda L}{\sqrt{\lambda L}} = 2\sqrt{\lambda L} = \sqrt{4\lambda L}$$

Hence, the correct answer is (C).

22. For a particular distance d_0 between the slits, the eye is not able to resolve two consecutive bright fringes.

Now, $\theta = \frac{\beta}{D}$ but $\beta = \frac{\lambda D}{d_0}$

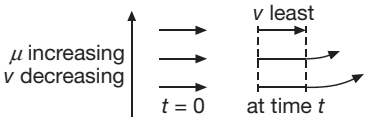


$$\Rightarrow \quad \theta = \frac{\lambda}{d_0}$$

$$\Rightarrow \quad d_0 = \frac{\lambda}{\theta} = \frac{600 \times 10^{-9} \text{ m}}{\frac{1}{60} \times \frac{\pi}{180} \text{ rad}} = 2.06 \times 10^{-3} \text{ m} \approx 2 \text{ mm}$$

Hence, the correct answer is (C).

23. Consider a light beam travelling horizontally. Since refractive index of air increases with height, so speed of wavefront decreases with height, due to which the light beam bends upwards.



Hence, the correct answer is (B).

24. Intensity at any point on the screen is given by

$$I = 4I_0\cos^2\left(\frac{\phi}{2}\right), \text{ where } I_{\max} = 4I_0$$

Since, $I = \frac{I_{\max}}{2} = 2I_0 = 4I_0\cos^2\left(\frac{\phi}{2}\right)$

$$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{4}$$

$$\Rightarrow \quad \phi = \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{2\pi}{\lambda}\Delta x = \frac{\pi}{2}$$

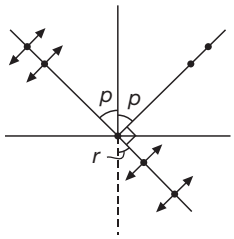
$$\Rightarrow \quad \Delta x = \frac{\lambda}{4}$$

$$\Rightarrow \quad y\frac{d}{D} = \frac{\lambda}{4}$$

$$\Rightarrow \quad y = \frac{\lambda D}{4d} = \frac{\beta}{4}$$

Hence, the correct answer is (B).

25. At Brewster's angle, $p = \tan^{-1}(\mu)$, the reflected light is completely polarized, whereas refracted light is partially polarized. Thus, the reflected ray will have lesser intensity compared to refracted ray.



$\Rightarrow \quad I_{\text{reflected}} < \dfrac{I_0}{2}$

Hence, the correct answer is (C).

26. When a polaroid rotated through $30°$ with respect to beam A, then beam B is at $60°$ with it.

So, $I_A \cos^2(30°) = I_B \cos^2(60°)$

$\Rightarrow \quad I_A\left(\dfrac{3}{4}\right) = I_B\left(\dfrac{1}{4}\right)$

$\Rightarrow \quad \dfrac{I_A}{I_B} = \dfrac{1}{3}$

Hence, the correct answer is (A).

27. When the screen is placed perpendicular to the line joining the sources, the fringes will be concentric circles.

Hence, the correct answer is (A).

28. Intensity of unpolarised light after passing polaroid A is

$$I_1 = \frac{I_0}{2}$$

Now this light will pass through the second polaroid B whose axis is inclined at an angle of $45°$ to the axis of polaroid A. So, in accordance with Malus law, the intensity of light emerging from polaroid B is

$$I_2 = I_1 \cos^2(45°) = \left(\frac{I_0}{2}\right)\left(\frac{I}{\sqrt{2}}\right)^2 = \frac{I_0}{4}$$

Hence, the correct answer is (D).

29. Here, $A_2 = 2A_1$

$\Rightarrow \quad \text{Intensity} \propto (\text{Amplitude})^2$

$\Rightarrow \quad \dfrac{I_2}{I_1} = \left(\dfrac{A_2}{A_1}\right)^2 = \left(\dfrac{2A_1}{A_1}\right)^2 = 4$

$\Rightarrow \quad I_2 = 4I_1$

Maximum intensity, $I_m = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$\Rightarrow \quad I_m = \left(\sqrt{I_1} + \sqrt{4I_1}\right)^2 = \left(3\sqrt{I_1}\right)^2 = 9I_1$

$\Rightarrow \quad I_1 = \dfrac{I_m}{9} \qquad \ldots(1)$

Resultant intensity, $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$\Rightarrow \quad I_R = I_1 + 4I_1 + 2\sqrt{I_1(4I_1)}\cos\phi$

$\Rightarrow \quad I_R = 5I_1 + 4I_1\cos\phi = I_1 + 4I_1 + 4I_1\cos\phi$

$\Rightarrow \quad I_R = I_1 + 4I_1(1 + \cos\phi)$

$\Rightarrow \quad I_R = I_1 + 8I_1\cos^2\dfrac{\phi}{2} \qquad \left\{\because\ 1 + \cos\phi = 2\cos^2\dfrac{\phi}{2}\right\}$

$\Rightarrow \quad I_R = I_1\left(1 + 8\cos^2\dfrac{\phi}{2}\right)$

Putting the value of I_1 from equation (1), we get

$$I_R = \frac{I_m}{9}\left(1 + 8\cos^2\frac{\phi}{2}\right)$$

Hence, the correct answer is (C).

30. The centre of interference pattern is dark thus showing that the phase difference between two interfering waves is π. However, this does not imply Statement-1.

Hence, the correct answer is (C).

31. For interference, by Young's double slits, the path difference is

$x = n\left(\dfrac{\lambda D}{d}\right)$ for bright fringes and

$x = \left(n - \dfrac{1}{2}\right)\dfrac{\lambda D}{d}$ for dark fringes

The central fringe, when $x = 0$, coincides for all wavelengths. The third fringe of $\lambda_1 = 590$ nm coincides with the fourth bright fringe of unknown wavelength λ_2. So, $3\lambda_1 = 4\lambda_2$

$\Rightarrow \quad 3 \times 590 \text{ nm} = 4\lambda_2$

$\Rightarrow \quad \lambda_2 = \dfrac{3 \times 590}{4} = 442.5 \text{ nm}$

Hence, the correct answer is (C).

CHAPTER 2

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. $I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$

$$\frac{1}{2} = \cos^2\left(\frac{\phi}{2}\right)$$

$$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \pm\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$$

$$\Rightarrow \quad \phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$$

$$\Rightarrow \quad \Delta x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$

$$\Rightarrow \quad \Delta x = (2n+1)\frac{\lambda}{4}$$

Hence, the correct answer is (B).

2. As $\lambda_R > \lambda_G > \lambda_B$

$$\Rightarrow \quad \beta_R > \beta_G > \beta_B \text{ as } \beta \propto \lambda$$

Hence, the correct answer is (D).

3. $I = 4I_0\cos^2\left(\frac{\phi}{2}\right)$

$$\Rightarrow \quad \phi = \frac{2\pi}{3}$$

$$\Rightarrow \quad \Delta x \times \left(\frac{2\pi}{\lambda}\right) = \frac{2\pi}{3} = \frac{\lambda}{3}$$

$$\Rightarrow \quad \sin\theta = \frac{\Delta x}{d}$$

$$\Rightarrow \quad \sin\theta = \frac{\lambda}{3d}$$

Hence, the correct answer is (C).

4. Let n^{th} minima of 400 nm coincides with mth minima of 560 nm , then

$$(2n-1)\left(\frac{400}{2}\right) = (2m-1)\left(\frac{560}{2}\right)$$

$$\Rightarrow \quad \frac{2n-1}{2m-1} = \frac{7}{5} = \frac{14}{10} = \ldots\ldots$$

i.e., 4^{th} minima of 400 nm coincides with 3^{rd} minima of 560 nm.

Location of this minima is,

$$Y_1 = \frac{(2\times 4-1)(1000)\left(400\times 10^{-6}\right)}{2\times 0.4} = 14 \text{ mm}$$

Next 11^{th} minima of 400 nm will coincide with 8^{th} minima of 560 nm

Location of this minima is,

$$Y_2 = \frac{(2\times 11-1)(1000)\left(400\times 10^{-6}\right)}{2\times 0.1} = 42 \text{ mm}$$

$\therefore$ Required distance $= Y_2 - Y_1 = 28$ mm

Hence, the correct answer is (D).

5. $PR = d$

$\therefore \quad PO = d\sec\theta$

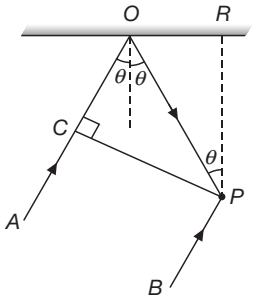
and $CO = PO\cot 2\theta = d\sec\theta\cos 2\theta$

Path difference between the two rays is

$$\Delta x = CO + PO = (d\sec\theta + d\sec\theta\cos 2\theta)$$

Phase difference between the two rays is

$\Delta\phi = \pi$ (one is reflected, while another is direct)



Therefore, condition for constructive interference should be

$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}.....$$

$$\Rightarrow \quad d\sec\theta(1+\cos 2\theta) = \frac{\lambda}{2}$$

$$\Rightarrow \quad \left(\frac{d}{\cos\theta}\right)(2\cos^2\theta) = \frac{\lambda}{2}$$

$$\Rightarrow \quad \cos\theta = \frac{\lambda}{4d}$$

Hence, the correct answer is (B).

6. Path difference due to slab should be integral multiple of λ or $\Delta x = n\lambda$

$\Rightarrow \quad (\mu-1)t = n\lambda \qquad n = 1, 2, 3, ...$

$\Rightarrow \quad t = \dfrac{n\lambda}{\mu-1}$

For minimum value of t, $n = 1$

$\Rightarrow \quad t = \dfrac{\lambda}{\mu-1} = \dfrac{\lambda}{1.5-1} = 2\lambda$

Hence, the correct answer is (A).

7. $n_1\lambda_1 = n_2\lambda_2$

$\Rightarrow \quad n_2 = n_1 \times \dfrac{\lambda_1}{\lambda_2} = 12 \times \dfrac{600}{400} = 18$

Hence, the correct answer is (B).

8. $I(\phi) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \qquad ...(1)$

Here $I_1 = I$ and $I_2 = 4I$

At point A, $\phi = \dfrac{\pi}{2}$

$\Rightarrow \quad I_A = I + 4I = 5I$

At point B, $\phi = \pi$

$\Rightarrow \quad I_B = I + 4I - 4I = I$

$\Rightarrow \quad I_A - I_B = 4I$

Hence, the correct answer is (B).

Equation (1) for resultant intensity can be applied only when the sources are coherent. In the question it is given that the rays interfere. Interference takes place only when the sources are coherent. That is why we applied equation number (1). When the sources are incoherent, the resultant intensity is given by $I = I_1 + I_2$.

9. In interference we know that

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \text{ and } I_{min} = \left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2$$

Under normal conditions (when the widths of both the slits are equal)

$I_1 \approx I_2 = I$ (say)

$\Rightarrow \quad I_{max} = 4I$ and $I_{min} = 0$

When the width of one of the slits is increased. Intensity due to that slit would increase, while that of the other will remain same. So let

$I_1 = I$ and $I_2 = \eta I \qquad \{\eta > I\}$

Then, $I_{max} = I\left(1 + \sqrt{\eta}\right)^2 > 4I$

and $I_{min} = I\left(\sqrt{\eta} - 1\right)^2 > 0$

$\Rightarrow$ Intensity of both maxima and minima is increased.

Hence, the correct answer is (A).

10. Diffraction is obtained when the slit width is of the order of wavelength of light (or any electromagnetic wave) used. Here, wavelength of x-rays $(1-100\ \text{Å}) \ll$ slit width $(0.6\ \text{mm})$. Therefore, no diffraction pattern will be observed.

Hence, the correct answer is (D).

11. Locus of equal path difference is the lines running parallel to the axis of the cylinder. Hence, straight fringes are obtained.

Hence, the correct answer is (A).

Circular rings (also called Newton's rings) are observed in interference pattern when a plano-convex lens of large focal length is placed with its convex surface in contact with a plane glass plate because locus of equal path difference in this case is a circle.

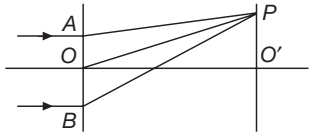
12. At first minima of the diffraction pattern, path difference is

$\Delta x = b\sin\theta = \lambda$

$\Rightarrow \quad \Delta\phi = \dfrac{2\pi}{\lambda}\Delta x = \left(\dfrac{2\pi}{\lambda}\right)\lambda = 2\pi$

Hence, the correct answer is (D).

13. The diagrammatic representation of the given problem is shown in figure.



For P to be a minimum, the phase difference of secondary waves from O and A (or B) is π (Out of phase) Hence, the phase difference between the wavelets from the opposite edges is 2π.

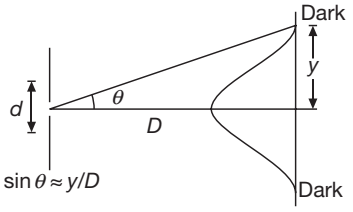
Hence, the correct answer is (C).

14. For first dark fringe on either side $d\sin\theta = \lambda$

$\Rightarrow \quad \dfrac{yd}{D} = \lambda$

$\Rightarrow \quad y = \dfrac{\lambda D}{d}$

CHAPTER 2



Therefore, distance between two dark fringes on either side is $2y = \dfrac{2\lambda D}{d}$

Substituting the values, we get

$$2y = \frac{2(600\times10^{-6}\text{ mm})(2\times10^{3}\text{ mm})}{(1.0\text{ mm})} = 2.4\text{ mm}$$

Hence, the correct answer is (D).

15. $I_{max} = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2 = \left(\sqrt{4I}+\sqrt{I}\right)^2 = 9I$

$I_{min} = \left(\sqrt{I_1}-\sqrt{I_2}\right)^2 = \left(\sqrt{4I}-\sqrt{I}\right)^2 = I$

Hence, the correct answer is (C).

16. $\beta = \dfrac{\lambda D}{d}$

d is halved and D is doubled

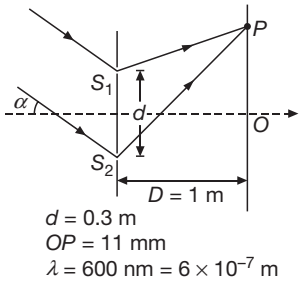
So, Fringe width β will become four times.

Hence, the correct answer is (D).

Multiple Correct Choice Type Problems

1. Total geometric path difference for point P is

$$\Delta x = d\sin\alpha + \frac{yd}{D}$$



$d = 0.3$ m
$OP = 11$ mm
$\lambda = 600$ nm $= 6\times10^{-7}$ m

For **OPTION (A),**

$$\alpha = \frac{0.36}{\pi}\text{ degree}$$

Since α is small, so

$$\Delta x \approx \alpha d + \frac{yd}{D}$$

$\Rightarrow \quad \Delta x = 3900$ nm

Since $\Delta x = 3900$ nm is an odd multiple of $\dfrac{\lambda}{2}$ i.e., 300 nm.

$\Rightarrow \quad (2n-1)\dfrac{\lambda}{2} = \Delta x$ (for destructive interference)

$\Rightarrow \quad n = 7$

For **OPTION (B),**

Fringe width in all the above case remain unchanged.

For **OPTION (C),**

$\Delta x_0 = 0$ and $\dfrac{yd}{D} = 3300$ nm

As $(\alpha = 0)$ (Destructive interference at P)

For **OPTION (D)**

If $\alpha = \dfrac{0.36}{\pi}$ degree

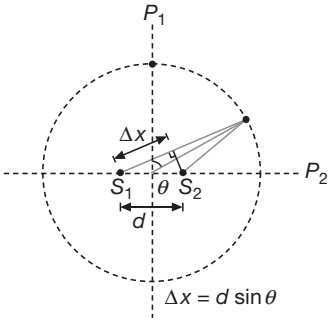
Then $\Delta x_0 = \alpha d = 600$ nm

$\Rightarrow \quad \Delta x_0 = n\lambda$ (for $n = 1$) (Constructive interference)

Hence, the correct answer is (A).

2. From the figure, we observe that the path difference x between the waves arriving at the circle from sources S_1 and S_2 is

$$x = d\sin\theta$$



At P_1, $\theta = 0°$

$\Rightarrow \quad x = 0$

At P_2, $\theta = 90°$

$\Rightarrow \quad x = d$

Since for maxima, we have $d\sin\theta = n\lambda$

$\Rightarrow \quad \sin\theta = \dfrac{n\lambda}{d}$

Since at P_2, $\theta = 90°$

$\Rightarrow \quad \dfrac{n\lambda}{d} = 1$

$\Rightarrow \quad n = \dfrac{d}{\lambda} = \dfrac{1.8\times10^{-3}}{600\times10^{-9}} = 3000$

So, total number of fringes produced between P_1 and P_2 in first quadrant is close to 3000.

Since, $x = d\sin\theta$

$\Rightarrow \quad \Delta x = \Delta(d\sin\theta)$

Since for maxima, we have $x = (2n)\dfrac{\lambda}{2} = n\lambda$

$\Rightarrow \quad \Delta(n\lambda) = d\Delta(\sin\theta)$

$\Rightarrow \quad \lambda\Delta n = (d\cos\theta)\Delta\theta$

Now, between any two consecutive maxima, we have

$|\Delta n| = 1$

$\Rightarrow \quad \lambda = (d\cos\theta)\Delta\theta$

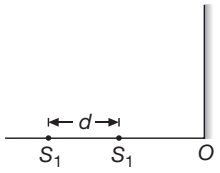
$\Rightarrow \quad \Delta\theta = \dfrac{\lambda}{d\cos\theta}$

Now, as we move from P_1 to P_2, θ increases, $\cos\theta$ decreases and hence $\Delta\theta$ increases.

Hence, (C) and (D) are correct.

3. Path difference at point O is $d = 0.6003$ mm $= 600300$ nm.

We observe this path difference to be equal to $\left(100\lambda + \dfrac{\lambda}{2}\right)$



So, minima is formed at point O.

Line S_1S_2 and screen are $\perp$ to each other, so fringe pattern is circular (semi-circular because only half of screen is available)

Hence, (A) and (B) are correct.

4. Fringe width $\beta = \dfrac{\lambda D}{d}$

$\Rightarrow \quad \beta \propto \lambda$

Since, $\lambda_2 > \lambda_1$

$\Rightarrow \quad \beta_2 > \beta_1$

Number of fringes in a given width

$m = \dfrac{y}{\beta}$

$\Rightarrow \quad m \propto \dfrac{1}{\beta}$

$\Rightarrow \quad m_2 < m_1$ as $\beta_2 > \beta_1$

Distance of 3rd maximum (third bright) of λ_2 from central maximum

$$y_{3B} = \frac{3\lambda_2 D}{d} = \frac{1800D}{d}$$

Distance of 5th minimum (fifth dark) of λ_1 from central maximum

$$y_{5D} = \frac{9\lambda_1 D}{2d} = \frac{1800D}{d}$$

So, 3rd maximum of λ_2 will overlap with 5th minimum of λ_1.

Angular separation (or angular fringe width) $= \dfrac{\lambda}{d} \propto \lambda$

$\Rightarrow$ Angular separation for λ_1 will be lesser.

Hence, (A) and (C) are correct.

5. $d = \dfrac{\lambda}{2\sin\theta}$

$\Rightarrow \quad \log_e d = \log_e\lambda - \log_e 2 - \log_e(\sin\theta)$

$\Rightarrow \quad \dfrac{\Delta(d)}{d} = 0 - 0 - \dfrac{1}{\sin\theta} \times \cos\theta(\Delta\theta)$

Fractional error $= \left|\dfrac{\Delta d}{d}\right| = (\cot\theta)\Delta\theta$

Absolute error $\Delta d = (d\cot\theta)\Delta\theta = \left(\dfrac{\lambda}{2\sin\theta}\right)\left(\dfrac{\cos\theta}{\sin\theta}\right)\Delta\theta$

Now, given that $\Delta\theta =$ constant

As θ increases, $\sin\theta$ increases, $\cos\theta$ and $\cot\theta$ decrease.

$\Rightarrow$ Both fractional and absolute errors decrease.

Hence, the correct answer is (D).

6. For $d = \lambda$, there will be only one, central maxima.

For $\lambda < d < 2\lambda$, there will be three maximas on the screen corresponding to path difference, $\Delta x = 0$ and $\Delta x = \pm\lambda$.

Hence, (A) and (B) are correct.

7. The intensity of light is $I(\theta) = I_0\cos^2\left(\dfrac{\delta}{2}\right)$

where $\delta = \dfrac{2\pi}{\lambda}(\Delta x) = \left(\dfrac{2\pi}{\lambda}\right)(d\sin\theta)$

(i) For $\theta = 30°$, $\lambda = \dfrac{c}{v} = \dfrac{3\times10^8}{10^6} = 300$ m and $d = 150$ m, we get

$$\delta = \left(\frac{2\pi}{300}\right)(150)\left(\frac{1}{2}\right) = \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{\delta}{2} = \frac{\pi}{4}$$

$$\Rightarrow \quad I(\theta) = I_0 \cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2} \qquad \{\text{option (A)}\}$$

(ii) For $\theta = 90°$

$$\delta = \left(\frac{2\pi}{300}\right)(150)(1) = \pi$$

$$\Rightarrow \quad \frac{\delta}{2} = \frac{\pi}{2} \text{ and } I(\theta) = 0$$

(iii) For $\theta = 0°$, $\delta = 0$ or $\frac{\delta}{2} = 0$

$$I(\theta) = I_0 \qquad \{\text{option (C)}\}$$

Hence, (A) and (C) are correct.

8. $x = \text{Path Difference} = \sqrt{b^2 + d^2} - d$

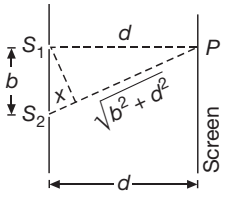
$$\Rightarrow \quad x = d\left(\sqrt{1 + \frac{b^2}{d^2}} - 1\right)$$

$$\Rightarrow \quad x \simeq d\left(1 + \frac{b^2}{2d^2} - 1\right)$$

$$\left\{\text{Since } \frac{b^2}{d^2} \ll 1, \text{ Hence } \sqrt{1 + \frac{b^2}{d^2}} \simeq 1 + \frac{b^2}{2d^2}\right\}$$

$$\Rightarrow \quad x \simeq \frac{b^2}{2d}$$

For wavelengths to be missing we must find positions of minima.



At minima path difference is an odd multiple of $\frac{\lambda}{2}$

$$\Rightarrow \quad x = \frac{b^2}{2d} = (2n-1)\frac{\lambda}{2} \quad n = 1, 2, 3, \ldots\ldots$$

$$\Rightarrow \quad \lambda = \frac{b^2}{(2n-1)d}; \; n = 1, 2, \ldots\ldots\ldots\ldots$$

$$\Rightarrow \quad \lambda = \frac{b^2}{d}, \frac{b^2}{3d}, \frac{b^2}{5d}, \ldots\ldots\ldots\ldots$$

Hence, (A) and (C) are correct.

9. $$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = 9$$

$$\Rightarrow \quad \sqrt{I_1} + \sqrt{I_2} = 3\left(\sqrt{I_1} - \sqrt{I_2}\right)$$

$$\Rightarrow \quad 2\sqrt{I_1} = 4\sqrt{I_2}$$

$$\Rightarrow \quad I_1 = 4I_2$$

Since, $\frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2$

$$\Rightarrow \quad \frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}} = 2$$

Hence, (B) and (D) are correct.

Comprehension Type Questions

1. Wavefronts are parallel in both media. Therefore, light which propagates perpendicular to wavefront travels as a parallel beam in each medium.

Hence, the correct answer is (A).

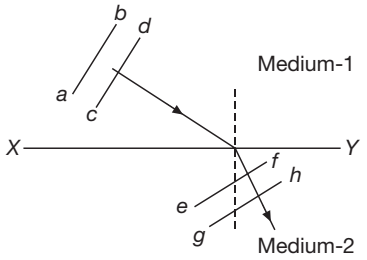
2. All points on a wavefront are at the same phase

$$\Rightarrow \quad \phi_d = \phi_c \text{ and } \phi_f = \phi_e$$

$$\Rightarrow \quad \phi_d - \phi_f = \phi_c - \phi_e$$

Hence, the correct answer is (C).

3. In medium-2 wavefront bends away from the normal after refraction. Therefore, ray of light which is perpendicular to wavefront bends towards the normal in medium-2 during refraction. So, medium-2 is denser or its speed in medium-1 is more.



Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A → (p, s)
B → (q)
C → (t)
D → (r, s, t)

(A) → (p, s) → Intensity at P_0 is maximum. It will continuously decrease from P_0 towards P_2

(B) → (q) → Path difference due to slap will be compensated by geometrical path difference. Hence, $\delta(P_1)=0$.

(C) → (t) → $\delta(P_0)=\frac{\lambda}{2}$, $\delta(P_1)=\frac{\lambda}{2}-\frac{\lambda}{4}=\frac{\lambda}{4}$ and $\delta(P_2)=\frac{\lambda}{2}-\frac{\lambda}{3}=\frac{\lambda}{6}$. When path difference increases from 0 to $\frac{\lambda}{2}$, intensity will decrease from maximum to zero. Hence, in this case,

$$I(P_2)>I(P_1)>I(P_0)$$

(D) → (r, s, t)

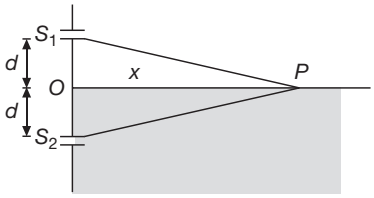
$$\delta(P_0)=\frac{3\lambda}{4},\ \delta(P_1)=\frac{3\lambda}{4}-\frac{\lambda}{4}=\frac{\lambda}{2}$$

and $\delta(P_2)=\frac{3\lambda}{4}-\frac{\lambda}{3}=\frac{5\lambda}{12}$.

In this case $I(P_1)=0$.

Integer/Numerical Answer Type Questions

1.



$$\mu(S_2P)-S_1P=m\lambda$$

$$\Rightarrow\quad \mu\sqrt{d^2+x^2}-\sqrt{d^2+x^2}=m\lambda$$

$$\Rightarrow\quad (\mu-1)\sqrt{d^2+x^2}=m\lambda$$

$$\Rightarrow\quad \left(\frac{4}{3}-1\right)\sqrt{d^2+x^2}=m\lambda$$

$$\Rightarrow\quad \sqrt{d^2+x^2}=3m\lambda$$

Squaring this equation, we get

$$x^2=9m^2\lambda^2-d^2$$

$$\Rightarrow\quad p^2=9$$

$$\Rightarrow\quad p=3$$

CHAPTER 2

This page is intentionally left blank